Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.2-c-x-^m-a+b-x^2-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1071]. This is test number [19].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1071)	% 0. (0)
Mathematica	% 100. (1071)	% 0. (0)
Maple	% 70.49 (755)	% 29.51 (316)
Maxima	% 36.88 (395)	% 63.12 (676)
Fricas	% 62.93 (674)	% 37.07 (397)
Sympy	% 90.57 (970)	% 9.43 (101)
Giac	% 59.66 (639)	% 40.34 (432)

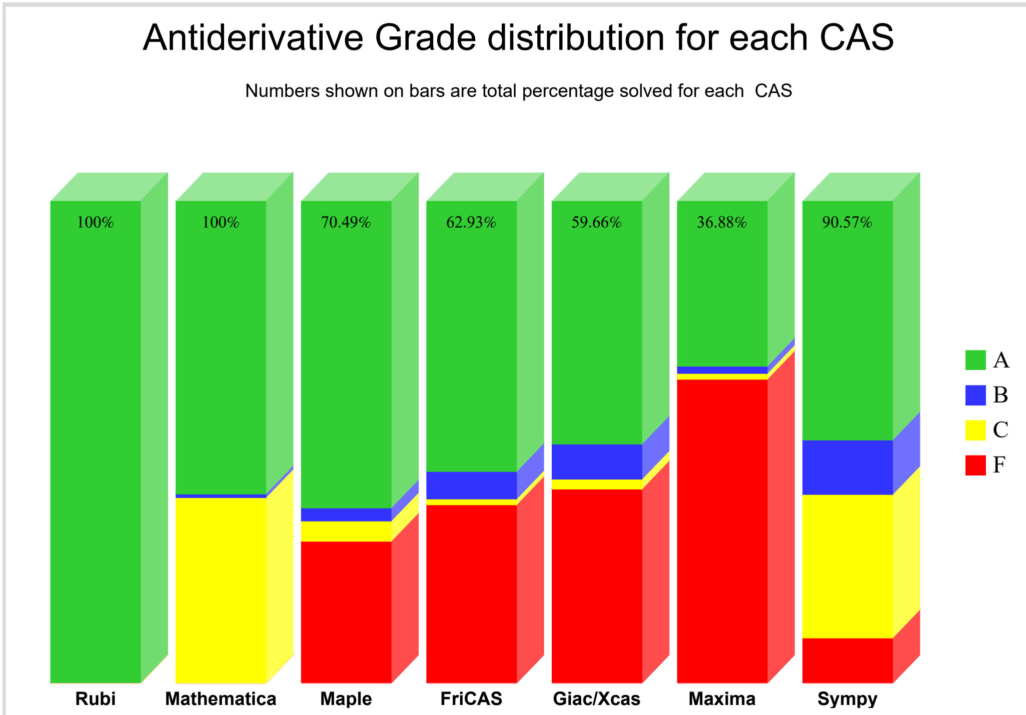
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

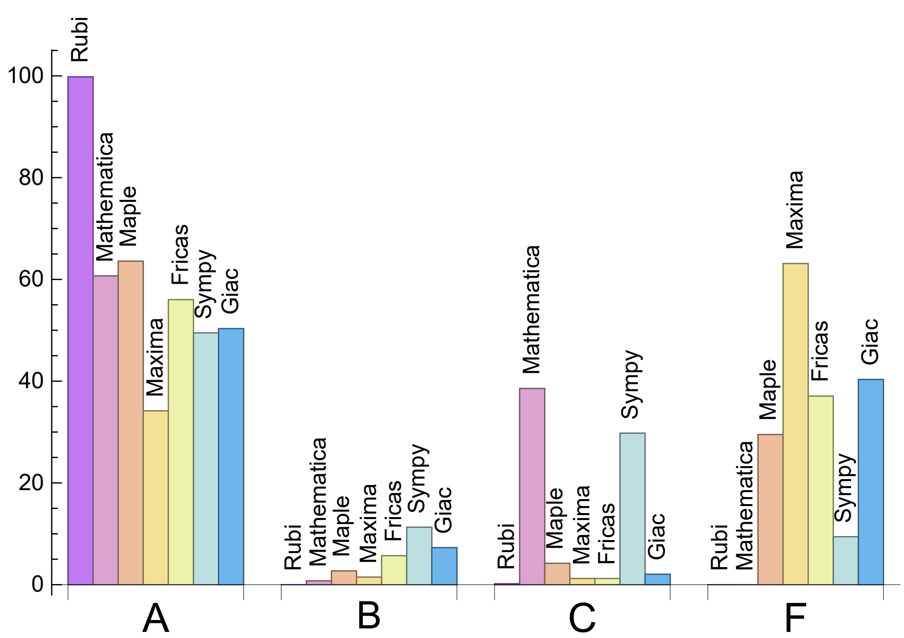
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.	0.19	0.
Mathematica	60.69	0.75	38.56	0.
Maple	63.59	2.71	4.2	29.51
Maxima	34.17	1.49	1.21	63.12
Fricas	56.02	5.7	1.21	37.07
Sympy	49.49	11.3	29.79	9.43
Giac	50.33	7.28	2.05	40.34

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	113.	1.03	78.	1.
Mathematica	0.02	55.7	0.76	51.	0.75
Maple	0.01	69.55	0.91	49.	0.84
Maxima	2.34	69.91	1.32	50.	1.17
Fricas	1.44	261.03	3.43	157.	2.6
Sympy	7.38	127.12	1.74	46.	0.94
Giac	2.34	111.98	1.61	77.	1.19

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception ValueError then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

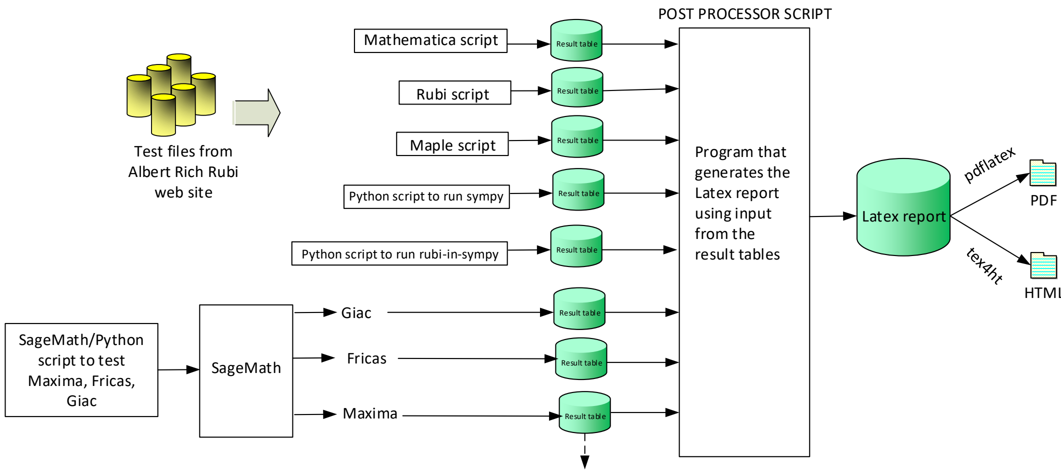
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

B grade: { }

C grade: { 662, 664 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 298, 300, 304, 306, 308, 312, 313, 315, 317, 321, 323, 325, 329, 331, 333, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 378, 379, 380, 383, 384, 385, 386, 387, 388, 389, 390, 391, 394, 397, 398, 399, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 422, 425, 426, 427, 428, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 503, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 665, 666, 667, 668, 669, 677, 678, 679, 680, 681, 689, 690, 691, 692, 693, 701, 702, 703, 704, 705, 706, 714, 715, 716, 717, 718, 726, 727, 728, 729, 742, 743, 744, 745, 751, 752, 753, 759, 760, 761, 768, 769, 770, 775, 776, 777, 778, 784, 785, 786, 884, 885, 886, 887, 928, 929, 930, 931, 940, 941, 942, 943, 944, 945, 946, 947, 948, 955, 956, 957, 958, 959, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 987, 988, 989, 990, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 559 }

C grade: { 293, 294, 295, 297, 299, 301, 302, 303, 305, 307, 309, 310, 311, 314, 316, 318, 319, 320, 322, 324, 326, 327, 328, 330, 332, 334, 335, 336, 338, 359, 360, 374, 376, 377, 381, 382, 392, 393, 395, 396, 400, 401, 402, 418, 419, 420, 421, 423, 424, 429, 430, 431, 432, 433, 451, 462, 473, 484, 495, 502, 504, 513, 515, 528, 530, 542, 553, 564, 575, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 661, 662, 663, 664, 670, 671, 672, 673, 674, 675, 676, 682, 683, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 710, 711, 712, 713, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 771, 772, 773, 774, 779, 780, 781, 782, 783, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 949, 950, 951, 952, 953, 954, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 975, 976, 977, 978, 979, 985, 986, 991, 992, 993, 994, 995, 996, 997, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1061, 1063 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 608, 610, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 642, 644, 646, 648, 650, 661, 663, 665, 666, 667, 668, 677, 678, 679, 680, 689, 690, 691, 692, 701, 702, 703, 704, 705, 714, 715, 716, 717, 726, 727, 728, 729, 742, 743, 744, 745, 759, 760, 761, 775, 776, 777, 778, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 196, 197, 198, 337, 339, 340, 341, 342, 422, 433, 607, 609, 611, 637, 639, 640, 641, 643, 645, 647, 649 }

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F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 706, 707, 708, 709, 710, 711, 712, 713, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 884, 885, 886, 888, 889, 890, 912, 913, 914, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 192, 193, 194, 195, 200, 204, 205, 206, 207, 208, 225, 227, 229, 231, 232, 234, 236, 238, 239, 241, 243, 245, 246, 248, 250, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 356, 372, 390, 416, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 483, 489, 500, 501, 509, 510, 511, 512, 520, 522, 524, 525, 526, 527, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 572, 574, 588, 661, 662, 663, 664, 665, 666, 667, 668, 677, 678, 679, 680, 689, 690, 691, 692, 701, 702, 703, 704, 705, 714, 715, 716, 717, 726, 727, 728, 729, 742, 745, 775, 778, 1039, 1040, 1041, 1061, 1063, 1065

B grade: { 38, 65, 90, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 337, 521, 523 }

C grade: { 475, 477, 479, 480, 481, 482, 484, 566, 568, 570, 571, 573, 575 }

F grade: { 125, 127, 129, 131, 133, 135, 137, 139, 141, 143, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 228, 230, 233, 235, 237, 240, 242, 244, 247, 249, 251, 257, 258, 261, 262, 263, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 504, 505, 506, 507, 508, 513, 514, 515, 516, 517, 518, 519, 528, 529, 530, 531, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 706, 707, 708, 709, 710, 711, 712, 713, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 525, 527, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 572, 574, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 720, 726, 727, 728, 729, 730, 732, 742, 743, 744, 745, 759, 760, 761, 775, 776, 777, 778, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 955, 956, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 221, 248, 250, 252, 263, 312, 335, 337, 338, 339, 340, 341, 372, 383, 390, 403, 415, 416, 434, 435, 510, 511, 521, 524, 526, 528, 537, 538, 548, 576, 577, 692, 701, 718, 719, 731, 944, 945, 985, 986 }

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2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 136, 138, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 225, 226, 227, 229, 231, 232, 233, 234, 235, 236, 237, 238, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 299, 312, 313, 314, 315, 316, 317, 318, 319, 320, 338, 339, 340, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 441, 442, 443, 444, 446, 447, 448, 449, 451, 452, 453, 454, 455, 457, 458, 459, 460, 462, 463, 464, 465, 466, 468, 470, 471, 473, 474, 476, 479, 481, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 507, 509, 511, 518, 520, 522, 524, 526, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 576, 577, 578, 580, 581, 582, 584, 585, 586, 588, 608, 610, 612, 638, 640, 642, 644, 646, 648, 668, 672, 673, 674, 675, 676, 679, 680, 684, 685, 686, 687, 688, 689, 690, 691, 692, 696, 697, 698, 699, 700, 705, 709, 710, 711, 712, 713, 717, 721, 722, 723, 724, 725, 728, 729, 733, 734, 735, 736, 737, 775, 946, 957, 972, 982, 987, 988, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042 }

B grade: { 17, 33, 38, 57, 58, 65, 89, 90, 91, 101, 102, 131, 133, 135, 137, 139, 141, 156, 158, 174, 186, 196, 197, 198, 199, 201, 202, 203, 204, 228, 230, 239, 240, 241, 248, 257, 258, 261, 262, 263, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 365, 367, 368, 383, 384, 385, 386, 403, 404, 405, 406, 407, 408, 434, 435, 436, 437, 438, 439, 440, 445, 450, 456, 461, 467, 472, 478, 502, 505, 506, 508, 510, 512, 513, 514, 515, 516, 517, 519, 521, 523, 525, 527, 528, 529, 530, 531, 649, 665, 666, 667, 677, 678, 701, 702, 703, 704, 714, 715, 716, 726, 727, 928, 940 }

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F grade: { 207, 208, 223, 224, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 589, 597, 605, 606, 628, 636, 738, 739, 742, 743, 744, 745, 746, 750, 754, 755, 758, 759, 760, 761, 762, 766, 767, 771, 772, 776, 777, 778, 779, 783, 919, 923, 924, 929, 930, 931, 935, 936, 941, 942, 943, 947, 948, 949, 954, 958, 959, 964, 968, 969, 973, 974, 978, 979, 983, 984, 985, 989, 990, 991, 992, 996, 997, 1050, 1051, 1052, 1053, 1055, 1056, 1057, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1069, 1070, 1071 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 576, 578, 579, 580, 582, 583, 584, 585, 586, 587, 588, 665, 666, 667, 668, 669, 671, 677, 678, 679, 680, 681, 682, 683, 692, 693, 694, 695, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 928, 937, 938, 939, 940, 1042 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 312, 337, 339, 340, 341, 342, 365, 366, 367, 368, 369, 370, 371, 382, 383, 384, 385, 386, 387, 388, 389, 390, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 432, 433, 434, 435, 436, 437, 438, 439, 440, 450, 461, 472, 537, 538, 549, 550, 552, 577, 581, 689, 690, 691, 925, 926, 927, 929, 930, 931, 941, 942, 943, 1039, 1040, 1041 }

C grade: { 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575 }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 670, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 932, 933, 934, 935, 936, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.005	0.001	0.	1.174	1.489	0.057	1.368

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.005	0.001	0.001	1.633	1.403	0.056	1.977

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.005	0.001	0.001	1.218	1.367	0.056	2.387

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	19	31	12	18
normalized size	1	1.	1.	0.82	1.12	1.82	0.71	1.06
time (sec)	N/A	0.005	0.001	0.001	2.319	1.246	0.054	2.108

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.002	0.	0.	1.692	1.263	0.08	2.036

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	30	10	19
normalized size	1	1.	1.	0.92	1.46	2.31	0.77	1.46
time (sec)	N/A	0.004	0.001	0.003	2.228	1.468	0.081	2.511

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	20	5	14
normalized size	1	1.	1.	1.1	1.4	2.	0.5	1.4
time (sec)	N/A	0.004	0.001	0.003	1.713	1.382	0.234	2.61

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	41	10	27
normalized size	1	1.	1.	0.92	1.46	3.15	0.77	2.08
time (sec)	N/A	0.004	0.002	0.005	1.214	1.432	0.256	2.62

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	14	18
normalized size	1	1.	1.	0.93	1.2	2.13	0.93	1.2
time (sec)	N/A	0.005	0.002	0.004	1.576	1.388	0.258	2.513

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	32	14	18
normalized size	1	1.	1.	0.82	1.06	1.88	0.82	1.06
time (sec)	N/A	0.005	0.002	0.004	1.493	1.415	0.269	2.424

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	20	36	15	20
normalized size	1	1.	1.	0.82	1.18	2.12	0.88	1.18
time (sec)	N/A	0.005	0.002	0.004	2.003	1.407	0.274	2.493

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	20	36	15	20
normalized size	1	1.	1.	0.82	1.18	2.12	0.88	1.18
time (sec)	N/A	0.005	0.002	0.004	1.607	1.401	0.271	2.64

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	58	24	32
normalized size	1	1.	1.	0.83	1.07	1.93	0.8	1.07
time (sec)	N/A	0.018	0.001	0.001	1.688	1.27	0.059	2.103

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.011	0.001	0.	2.408	1.284	0.06	3.284

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.018	0.001	0.001	2.324	1.262	0.06	1.347

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.01	0.001	0.002	1.867	1.289	0.059	2.269

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	19	55	24	19
normalized size	1	1.	1.	1.56	1.19	3.44	1.5	1.19
time (sec)	N/A	0.002	0.002	0.	2.244	1.247	0.06	2.277

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.007	0.001	0.	2.296	1.283	0.062	2.053

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	32	49	20	32
normalized size	1	1.	1.	0.96	1.39	2.13	0.87	1.39
time (sec)	N/A	0.013	0.001	0.002	1.91	1.485	0.244	1.503

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	50	19	30
normalized size	1	1.	1.	0.96	1.25	2.08	0.79	1.25
time (sec)	N/A	0.01	0.001	0.003	1.544	1.412	0.243	1.68

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	32	59	24	43
normalized size	1	1.	1.	0.89	1.19	2.19	0.89	1.59
time (sec)	N/A	0.014	0.001	0.005	1.338	1.4	0.27	1.554

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	30	53	20	30
normalized size	1	1.	1.	0.96	1.3	2.3	0.87	1.3
time (sec)	N/A	0.009	0.001	0.003	2.325	1.382	0.278	1.914

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	35	62	22	46
normalized size	1	1.	1.	0.96	1.46	2.58	0.92	1.92
time (sec)	N/A	0.013	0.001	0.005	2.403	1.445	0.313	2.706

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	35	61	27	35
normalized size	1	1.	1.	0.89	1.25	2.18	0.96	1.25
time (sec)	N/A	0.01	0.001	0.004	1.178	1.368	0.317	2.259

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	32	54	26	32
normalized size	1	1.	1.58	1.32	1.68	2.84	1.37	1.68
time (sec)	N/A	0.003	0.001	0.004	1.737	1.422	0.353	2.447

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	63	27	35
normalized size	1	1.	1.	0.83	1.17	2.1	0.9	1.17
time (sec)	N/A	0.01	0.001	0.006	2.477	1.395	0.345	2.096

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	58	27	35
normalized size	1	1.	1.	0.83	1.17	1.93	0.9	1.17
time (sec)	N/A	0.014	0.001	0.004	2.049	1.379	0.391	2.636

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	63	27	35
normalized size	1	1.	1.	0.83	1.17	2.1	0.9	1.17
time (sec)	N/A	0.01	0.001	0.004	1.264	1.395	0.435	1.357

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	89	37	47
normalized size	1	1.	1.	0.84	1.09	2.07	0.86	1.09
time (sec)	N/A	0.028	0.002	0.001	2.529	1.281	0.065	1.716

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	86	37	47
normalized size	1	1.	1.	0.84	1.09	2.	0.86	1.09
time (sec)	N/A	0.027	0.002	0.001	1.194	1.319	0.064	2.014

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	85	39	47
normalized size	1	1.	1.	0.84	1.09	1.98	0.91	1.09
time (sec)	N/A	0.025	0.002	0.001	1.843	1.546	0.065	2.022

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	47	82	37	47
normalized size	1	1.	1.26	1.06	1.38	2.41	1.09	1.38
time (sec)	N/A	0.032	0.002	0.001	1.849	1.785	0.063	1.594

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	19	80	37	19
normalized size	1	1.	1.	2.25	1.19	5.	2.31	1.19
time (sec)	N/A	0.002	0.002	0.001	1.784	1.813	0.062	2.702

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	49	78	37	49
normalized size	1	1.	1.	0.87	1.26	2.	0.95	1.26
time (sec)	N/A	0.018	0.003	0.002	2.059	1.985	0.252	1.904

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	49	85	37	62
normalized size	1	1.	1.	0.88	1.22	2.12	0.92	1.55
time (sec)	N/A	0.021	0.006	0.004	1.287	1.309	0.287	2.595

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	50	85	36	62
normalized size	1	1.	1.	0.88	1.25	2.12	0.9	1.55
time (sec)	N/A	0.019	0.004	0.006	2.361	1.289	0.339	2.351

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	53	90	36	63
normalized size	1	1.	1.	0.87	1.36	2.31	0.92	1.62
time (sec)	N/A	0.021	0.004	0.005	2.424	1.236	0.377	1.563

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	47	76	37	47
normalized size	1	1.	2.26	1.89	2.47	4.	1.95	2.47
time (sec)	N/A	0.003	0.006	0.004	1.086	1.2	0.404	2.641

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	50	85	39	50
normalized size	1	1.	1.08	0.9	1.25	2.12	0.98	1.25
time (sec)	N/A	0.019	0.004	0.004	2.088	1.285	0.44	2.526

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	88	39	50
normalized size	1	1.	1.	0.84	1.16	2.05	0.91	1.16
time (sec)	N/A	0.019	0.004	0.003	2.317	1.312	0.459	1.797

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	88	39	50
normalized size	1	1.	1.	0.84	1.16	2.05	0.91	1.16
time (sec)	N/A	0.019	0.006	0.004	2.638	1.272	0.501	2.013

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	85	37	47
normalized size	1	1.	1.	0.84	1.09	1.98	0.86	1.09
time (sec)	N/A	0.014	0.002	0.	2.069	1.114	0.064	1.91

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	82	37	47
normalized size	1	1.	1.	0.84	1.09	1.91	0.86	1.09
time (sec)	N/A	0.013	0.002	0.001	1.996	1.087	0.069	2.54

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	80	39	47
normalized size	1	1.	1.	0.84	1.09	1.86	0.91	1.09
time (sec)	N/A	0.013	0.002	0.001	1.8	1.021	0.098	2.011

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	66	32	42
normalized size	1	1.	1.	0.91	1.2	1.89	0.91	1.2
time (sec)	N/A	0.011	0.001	0.	1.657	1.184	0.062	2.81

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	43	73	29	43
normalized size	1	1.	1.	0.97	1.26	2.15	0.85	1.26
time (sec)	N/A	0.013	0.003	0.003	1.01	1.163	0.257	2.363

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	46	72	34	46
normalized size	1	1.	1.	0.92	1.24	1.95	0.92	1.24
time (sec)	N/A	0.013	0.003	0.006	1.246	1.202	0.294	2.786

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	45	76	32	45
normalized size	1	1.	1.	0.97	1.32	2.24	0.94	1.32
time (sec)	N/A	0.013	0.005	0.004	1.679	1.197	0.341	2.331

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	50	84	39	50
normalized size	1	1.	1.	0.92	1.28	2.15	1.	1.28
time (sec)	N/A	0.013	0.004	0.004	1.928	1.267	0.387	2.532

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	90	39	50
normalized size	1	1.	1.	0.84	1.16	2.09	0.91	1.16
time (sec)	N/A	0.013	0.004	0.005	2.286	1.269	0.449	1.817

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	95	39	50
normalized size	1	1.	1.	0.84	1.16	2.21	0.91	1.16
time (sec)	N/A	0.015	0.006	0.003	2.532	1.273	0.441	2.5

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	142	65	77
normalized size	1	1.	1.	0.84	1.12	2.06	0.94	1.12
time (sec)	N/A	0.049	0.003	0.002	2.12	1.115	0.073	2.587

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	140	65	77
normalized size	1	1.	1.	0.84	1.12	2.03	0.94	1.12
time (sec)	N/A	0.044	0.002	0.	1.327	1.071	0.071	2.518

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	142	66	77
normalized size	1	1.	1.	0.84	1.12	2.06	0.96	1.12
time (sec)	N/A	0.043	0.002	0.001	2.358	1.105	0.071	2.617

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	58	77	138	65	77
normalized size	1	1.	0.96	0.81	1.07	1.92	0.9	1.07
time (sec)	N/A	0.091	0.002	0.001	2.278	1.124	0.071	2.334

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	57	76	131	63	76
normalized size	1	1.	1.25	1.08	1.43	2.47	1.19	1.43
time (sec)	N/A	0.065	0.002	0.001	2.875	0.985	0.07	2.011

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	66	57	76	130	63	76
normalized size	1	1.	1.94	1.68	2.24	3.82	1.85	2.24
time (sec)	N/A	0.036	0.002	0.001	2.038	1.199	0.073	1.311

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	58	19	132	65	19
normalized size	1	1.	1.	3.62	1.19	8.25	4.06	1.19
time (sec)	N/A	0.002	0.002	0.002	2.056	1.1	0.068	3.021

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	78	130	65	78
normalized size	1	1.	1.	0.86	1.2	2.	1.	1.2
time (sec)	N/A	0.034	0.004	0.001	1.736	1.28	0.304	2.153

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	78	142	63	92
normalized size	1	1.	1.	0.89	1.22	2.22	0.98	1.44
time (sec)	N/A	0.037	0.004	0.005	3.017	1.223	0.348	2.837

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	80	139	61	95
normalized size	1	1.	1.	0.89	1.25	2.17	0.95	1.48
time (sec)	N/A	0.036	0.006	0.005	2.058	1.371	0.487	2.165

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	82	139	63	97
normalized size	1	1.	1.	0.89	1.28	2.17	0.98	1.52
time (sec)	N/A	0.034	0.004	0.005	2.048	1.077	0.485	2.662

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	82	142	61	95
normalized size	1	1.	1.	0.89	1.28	2.22	0.95	1.48
time (sec)	N/A	0.033	0.004	0.006	1.586	1.297	0.492	2.114

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	82	149	60	93
normalized size	1	1.	1.	0.86	1.26	2.29	0.92	1.43
time (sec)	N/A	0.031	0.004	0.006	1.958	1.227	0.562	2.25

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	77	127	61	77
normalized size	1	1.	3.63	3.05	4.05	6.68	3.21	4.05
time (sec)	N/A	0.004	0.004	0.004	1.402	1.255	0.602	2.872

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	67	58	80	134	63	80
normalized size	1	1.	1.68	1.45	2.	3.35	1.58	2.
time (sec)	N/A	0.018	0.006	0.005	1.962	1.23	0.644	2.458

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	67	58	80	140	63	80
normalized size	1	1.	1.08	0.94	1.29	2.26	1.02	1.29
time (sec)	N/A	0.028	0.004	0.006	1.828	1.29	0.688	2.672

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	143	63	80
normalized size	1	1.	1.	0.84	1.16	2.07	0.91	1.16
time (sec)	N/A	0.032	0.004	0.006	2.145	1.225	0.773	1.603

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	149	63	80
normalized size	1	1.	1.	0.84	1.16	2.16	0.91	1.16
time (sec)	N/A	0.031	0.004	0.006	2.773	1.262	0.874	2.518

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	142	66	77
normalized size	1	1.	1.	0.84	1.12	2.06	0.96	1.12
time (sec)	N/A	0.025	0.002	0.001	2.218	1.184	0.087	1.484

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	140	65	77
normalized size	1	1.	1.	0.84	1.12	2.03	0.94	1.12
time (sec)	N/A	0.024	0.002	0.001	1.712	1.049	0.073	2.282

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	139	66	77
normalized size	1	1.	1.	0.84	1.12	2.01	0.96	1.12
time (sec)	N/A	0.022	0.002	0.001	2.066	1.018	0.072	2.336

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	76	131	63	76
normalized size	1	1.	1.	0.86	1.15	1.98	0.95	1.15
time (sec)	N/A	0.022	0.002	0.	1.583	1.132	0.071	1.392

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	73	122	61	73
normalized size	1	1.	1.	0.89	1.18	1.97	0.98	1.18
time (sec)	N/A	0.019	0.001	0.001	2.315	1.114	0.067	2.293

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	74	131	58	74
normalized size	1	1.	1.	0.92	1.21	2.15	0.95	1.21
time (sec)	N/A	0.021	0.004	0.003	1.76	1.181	0.279	2.722

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	74	131	58	74
normalized size	1	1.	1.	0.92	1.23	2.18	0.97	1.23
time (sec)	N/A	0.022	0.004	0.006	2.77	1.091	0.318	2.043

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	78	131	61	78
normalized size	1	1.	1.	0.89	1.24	2.08	0.97	1.24
time (sec)	N/A	0.022	0.004	0.006	1.8	1.215	0.374	1.915

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	78	131	60	78
normalized size	1	1.	1.	0.92	1.28	2.15	0.98	1.28
time (sec)	N/A	0.022	0.004	0.006	1.875	1.225	0.451	2.197

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	77	134	58	77
normalized size	1	1.	1.	0.92	1.28	2.23	0.97	1.28
time (sec)	N/A	0.022	0.005	0.006	1.975	1.291	0.514	2.808

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	80	144	63	80
normalized size	1	1.	1.	0.89	1.23	2.22	0.97	1.23
time (sec)	N/A	0.022	0.004	0.006	2.136	1.193	0.584	1.334

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	80	154	63	80
normalized size	1	1.	1.	0.87	1.19	2.3	0.94	1.19
time (sec)	N/A	0.023	0.004	0.005	2.49	1.323	0.645	2.812

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	159	63	80
normalized size	1	1.	1.	0.84	1.16	2.3	0.91	1.16
time (sec)	N/A	0.023	0.004	0.004	2.301	1.232	0.73	2.123

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	165	63	80
normalized size	1	1.	1.	0.84	1.16	2.39	0.91	1.16
time (sec)	N/A	0.022	0.004	0.007	1.389	1.292	0.738	2.477

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	169	63	80
normalized size	1	1.	1.	0.84	1.16	2.45	0.91	1.16
time (sec)	N/A	0.023	0.006	0.005	1.785	1.188	0.741	2.475

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	108	91	122	224	105	122
normalized size	1	1.	0.84	0.71	0.95	1.74	0.81	0.95
time (sec)	N/A	0.21	0.003	0.001	2.47	1.035	0.082	2.279

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	108	91	122	221	107	122
normalized size	1	1.	0.98	0.83	1.11	2.01	0.97	1.11
time (sec)	N/A	0.171	0.002	0.001	1.381	1.123	0.083	1.412

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	106	91	122	219	104	122
normalized size	1	1.	1.16	1.	1.34	2.41	1.14	1.34
time (sec)	N/A	0.142	0.002	0.	2.004	1.172	0.082	1.996

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	106	91	122	215	105	122
normalized size	1	1.	1.47	1.26	1.69	2.99	1.46	1.69
time (sec)	N/A	0.116	0.003	0.002	1.591	1.374	0.083	2.103

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	103	90	120	208	102	120
normalized size	1	1.	1.94	1.7	2.26	3.92	1.92	2.26
time (sec)	N/A	0.084	0.002	0.001	1.301	1.419	0.084	2.434

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	106	91	122	211	105	122
normalized size	1	1.	3.12	2.68	3.59	6.21	3.09	3.59
time (sec)	N/A	0.046	0.002	0.002	1.823	1.307	0.082	2.13

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	91	19	201	99	19
normalized size	1	1.	1.	5.69	1.19	12.56	6.19	1.19
time (sec)	N/A	0.002	0.002	0.	2.534	1.322	0.078	1.679

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	123	205	102	123
normalized size	1	1.	1.	0.89	1.23	2.05	1.02	1.23
time (sec)	N/A	0.059	0.004	0.003	2.709	1.486	0.325	2.4

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	123	232	100	136
normalized size	1	1.	1.	0.91	1.24	2.34	1.01	1.37
time (sec)	N/A	0.06	0.004	0.007	1.754	1.505	0.372	2.843

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	124	224	102	139
normalized size	1	1.	1.	0.89	1.23	2.22	1.01	1.38
time (sec)	N/A	0.061	0.004	0.005	1.946	1.263	0.424	1.893

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	89	123	220	95	138
normalized size	1	1.	1.	0.95	1.31	2.34	1.01	1.47
time (sec)	N/A	0.057	0.005	0.006	1.499	1.369	0.502	3.882

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	127	219	99	142
normalized size	1	1.	1.	0.93	1.31	2.26	1.02	1.46
time (sec)	N/A	0.057	0.005	0.007	2.334	1.178	0.596	2.474

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	90	127	221	97	142
normalized size	1	1.	1.	0.95	1.34	2.33	1.02	1.49
time (sec)	N/A	0.058	0.005	0.007	2.75	1.172	0.77	1.826

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	127	225	97	142
normalized size	1	1.	1.	0.89	1.26	2.23	0.96	1.41
time (sec)	N/A	0.054	0.005	0.008	1.635	1.255	1.01	2.827

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	127	234	97	139
normalized size	1	1.	1.	0.91	1.28	2.36	0.98	1.4
time (sec)	N/A	0.051	0.005	0.006	2.331	1.273	0.955	2.277

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	127	243	95	138
normalized size	1	1.	1.	0.89	1.27	2.43	0.95	1.38
time (sec)	N/A	0.054	0.005	0.008	1.482	1.359	1.027	2.512

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	100	91	122	203	97	122
normalized size	1	1.	5.26	4.79	6.42	10.68	5.11	6.42
time (sec)	N/A	0.003	0.005	0.006	2.82	1.234	1.13	2.097

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	106	91	124	217	99	124
normalized size	1	1.	2.65	2.28	3.1	5.42	2.48	3.1
time (sec)	N/A	0.017	0.004	0.005	2.652	1.276	1.175	2.136

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	104	91	124	225	99	124
normalized size	1	1.	1.68	1.47	2.	3.63	1.6	2.
time (sec)	N/A	0.029	0.004	0.005	2.036	1.259	1.286	1.862

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	106	91	124	235	99	124
normalized size	1	1.	1.26	1.08	1.48	2.8	1.18	1.48
time (sec)	N/A	0.041	0.004	0.006	2.77	1.236	1.323	2.863

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	124	240	99	124
normalized size	1	1.	1.	0.86	1.17	2.26	0.93	1.17
time (sec)	N/A	0.055	0.004	0.006	2.367	1.187	1.378	2.796

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	124	247	99	124
normalized size	1	1.	1.	0.84	1.15	2.29	0.92	1.15
time (sec)	N/A	0.054	0.004	0.007	1.972	1.295	1.492	2.087

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	124	250	99	124
normalized size	1	1.	1.	0.84	1.15	2.31	0.92	1.15
time (sec)	N/A	0.052	0.004	0.008	1.996	1.103	1.628	2.984

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	124	254	99	124
normalized size	1	1.	1.	0.86	1.17	2.4	0.93	1.17
time (sec)	N/A	0.052	0.004	0.006	1.35	1.257	1.833	1.866

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	122	227	107	122
normalized size	1	1.	1.	0.84	1.13	2.1	0.99	1.13
time (sec)	N/A	0.048	0.003	0.001	2.617	1.102	0.096	2.85

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	122	225	107	122
normalized size	1	1.	1.	0.84	1.13	2.08	0.99	1.13
time (sec)	N/A	0.039	0.002	0.001	2.557	1.178	0.084	1.746

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	122	224	107	122
normalized size	1	1.	1.	0.84	1.13	2.07	0.99	1.13
time (sec)	N/A	0.039	0.002	0.001	2.198	1.125	0.086	2.862

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	122	217	105	122
normalized size	1	1.	1.	0.86	1.15	2.05	0.99	1.15
time (sec)	N/A	0.04	0.002	0.001	2.38	1.012	0.088	1.363

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	88	117	207	102	117
normalized size	1	1.	1.	0.87	1.16	2.05	1.01	1.16
time (sec)	N/A	0.037	0.001	0.001	1.87	1.121	0.081	1.836

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	119	235	99	119
normalized size	1	1.	1.	0.89	1.19	2.35	0.99	1.19
time (sec)	N/A	0.039	0.015	0.003	1.15	1.176	0.349	2.817

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	120	232	99	120
normalized size	1	1.	1.	0.91	1.22	2.37	1.01	1.22
time (sec)	N/A	0.038	0.008	0.005	1.966	1.256	0.385	1.919

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	123	227	100	123
normalized size	1	1.	1.	0.89	1.23	2.27	1.	1.23
time (sec)	N/A	0.039	0.009	0.005	2.649	1.17	0.452	2.438

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	123	224	100	123
normalized size	1	1.	1.	0.87	1.21	2.2	0.98	1.21
time (sec)	N/A	0.037	0.006	0.005	1.425	1.251	0.511	2.627

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	123	224	99	123
normalized size	1	1.	1.	0.87	1.21	2.2	0.97	1.21
time (sec)	N/A	0.039	0.01	0.005	2.338	1.199	0.603	1.946

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	123	228	97	123
normalized size	1	1.	1.	0.89	1.23	2.28	0.97	1.23
time (sec)	N/A	0.04	0.01	0.007	2.004	1.282	0.76	2.581

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	123	234	95	123
normalized size	1	1.	1.	0.91	1.26	2.39	0.97	1.26
time (sec)	N/A	0.038	0.01	0.008	2.712	1.128	0.889	2.136

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	122	239	94	122
normalized size	1	1.	1.	0.89	1.23	2.41	0.95	1.23
time (sec)	N/A	0.039	0.005	0.007	1.334	1.261	0.977	2.159

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	91	124	257	99	124
normalized size	1	1.	1.	0.88	1.19	2.47	0.95	1.19
time (sec)	N/A	0.039	0.01	0.006	1.545	1.175	1.004	2.394

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	124	273	99	124
normalized size	1	1.	1.	0.86	1.17	2.58	0.93	1.17
time (sec)	N/A	0.04	0.011	0.007	2.649	1.176	1.053	1.571

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	92	153	68	93
normalized size	1	1.	1.	0.86	1.16	1.94	0.86	1.18
time (sec)	N/A	0.056	0.006	0.003	1.34	1.241	0.32	1.782

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	71	0	386	119	104
normalized size	1	1.	1.	0.88	0.	4.77	1.47	1.28
time (sec)	N/A	0.035	0.034	0.004	0.	1.348	0.341	2.261

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	77	123	56	78
normalized size	1	1.	1.	0.86	1.17	1.86	0.85	1.18
time (sec)	N/A	0.044	0.006	0.004	2.288	1.288	0.31	2.392

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	0	336	107	88
normalized size	1	1.	1.	0.88	0.	4.94	1.57	1.29
time (sec)	N/A	0.03	0.025	0.003	0.	1.329	0.329	2.509

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	62	99	44	63
normalized size	1	1.	1.	0.87	1.17	1.87	0.83	1.19
time (sec)	N/A	0.035	0.005	0.003	2.592	1.282	0.306	2.383

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	278	95	74
normalized size	1	1.	1.	0.89	0.	5.05	1.73	1.35
time (sec)	N/A	0.024	0.022	0.003	0.	1.174	0.328	2.638

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	46	73	32	47
normalized size	1	1.	1.	0.88	1.15	1.82	0.8	1.18
time (sec)	N/A	0.027	0.005	0.002	1.881	1.231	0.301	1.527

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	217	80	54
normalized size	1	1.	1.	0.9	0.	5.17	1.9	1.29
time (sec)	N/A	0.02	0.019	0.003	0.	1.291	0.31	1.448

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	49	20	32
normalized size	1	1.	1.	0.89	1.15	1.81	0.74	1.19
time (sec)	N/A	0.018	0.004	0.002	2.657	1.291	0.281	2.659

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	165	56	35
normalized size	1	1.	1.	0.87	0.	5.32	1.81	1.13
time (sec)	N/A	0.012	0.008	0.003	0.	1.319	0.286	2.139

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	19
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.27
time (sec)	N/A	0.003	0.002	0.002	1.97	1.255	0.101	1.736

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	53	20
normalized size	1	1.	1.	0.67	0.	6.29	2.21	0.83
time (sec)	N/A	0.005	0.004	0.001	0.	1.208	0.119	1.951

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	31	49	15	32
normalized size	1	1.	1.	0.95	1.41	2.23	0.68	1.45
time (sec)	N/A	0.011	0.005	0.003	1.337	1.344	0.179	2.599

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	173	65	39
normalized size	1	1.	1.	0.88	0.	5.09	1.91	1.15
time (sec)	N/A	0.012	0.012	0.004	0.	1.273	0.321	1.722

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	45	80	31	58
normalized size	1	1.	1.	0.91	1.29	2.29	0.89	1.66
time (sec)	N/A	0.022	0.006	0.003	1.355	1.208	0.398	2.041

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	0	234	87	54
normalized size	1	1.	1.	0.91	0.	5.44	2.02	1.26
time (sec)	N/A	0.017	0.019	0.006	0.	1.306	0.366	2.957

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	63	108	42	77
normalized size	1	1.	1.	0.9	1.29	2.2	0.86	1.57
time (sec)	N/A	0.028	0.006	0.007	2.118	1.297	0.452	1.631

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	0	296	100	70
normalized size	1	1.	1.	0.9	0.	5.1	1.72	1.21
time (sec)	N/A	0.025	0.023	0.007	0.	1.299	0.434	1.788

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	78	134	56	95
normalized size	1	1.	1.	0.89	1.24	2.13	0.89	1.51
time (sec)	N/A	0.035	0.007	0.006	2.083	1.273	0.538	2.874

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	61	0	352	112	84
normalized size	1	1.	1.	0.88	0.	5.1	1.62	1.22
time (sec)	N/A	0.035	0.025	0.005	0.	1.175	0.535	1.995

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	66	93	159	68	109
normalized size	1	1.	1.	0.88	1.24	2.12	0.91	1.45
time (sec)	N/A	0.041	0.007	0.005	2.47	1.252	0.684	2.742

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	83	85	119	221	88	139
normalized size	1	1.	0.88	0.9	1.27	2.35	0.94	1.48
time (sec)	N/A	0.076	0.03	0.008	2.174	1.276	0.439	2.451

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	90	0	528	151	128
normalized size	1	1.	0.89	0.86	0.	5.03	1.44	1.22
time (sec)	N/A	0.046	0.059	0.007	0.	1.279	0.469	2.989

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	74	104	198	80	124
normalized size	1	1.	0.87	0.89	1.25	2.39	0.96	1.49
time (sec)	N/A	0.066	0.023	0.009	1.379	1.249	0.423	1.515

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	0	459	134	113
normalized size	1	1.	0.89	0.85	0.	4.99	1.46	1.23
time (sec)	N/A	0.037	0.054	0.008	0.	1.287	0.479	3.232

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	88	166	66	108
normalized size	1	1.	0.86	0.9	1.26	2.37	0.94	1.54
time (sec)	N/A	0.054	0.023	0.009	2.63	1.185	0.404	2.489

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	0	409	124	99
normalized size	1	1.	0.9	0.86	0.	5.18	1.57	1.25
time (sec)	N/A	0.032	0.05	0.008	0.	1.322	0.437	2.566

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	73	143	53	90
normalized size	1	1.	0.86	0.91	1.28	2.51	0.93	1.58
time (sec)	N/A	0.042	0.02	0.008	1.881	1.205	0.392	2.579

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	0	348	107	82
normalized size	1	1.	0.91	0.86	0.	5.27	1.62	1.24
time (sec)	N/A	0.027	0.044	0.008	0.	1.281	0.418	2.578

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	58	113	39	66
normalized size	1	1.	0.86	0.93	1.32	2.57	0.89	1.5
time (sec)	N/A	0.032	0.022	0.009	1.411	1.22	0.368	3.293

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	0	285	83	57
normalized size	1	1.	0.93	0.78	0.	5.18	1.51	1.04
time (sec)	N/A	0.017	0.031	0.007	0.	1.331	0.387	2.847

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	43	76	29	65
normalized size	1	1.	0.82	0.91	1.3	2.3	0.88	1.97
time (sec)	N/A	0.024	0.008	0.008	1.957	1.292	0.332	2.693

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	263	78	47
normalized size	1	1.	1.	0.8	0.	5.84	1.73	1.04
time (sec)	N/A	0.012	0.02	0.007	0.	1.212	0.342	3.151

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	30	15	19
normalized size	1	1.	1.	0.94	1.19	1.88	0.94	1.19
time (sec)	N/A	0.003	0.002	0.001	2.073	1.285	0.297	3.563

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	261	78	47
normalized size	1	1.	1.	0.8	0.	5.8	1.73	1.04
time (sec)	N/A	0.01	0.024	0.004	0.	1.33	0.357	3.358

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	50	108	34	63
normalized size	1	1.	0.87	0.92	1.32	2.84	0.89	1.66
time (sec)	N/A	0.027	0.012	0.01	1.966	1.271	0.426	1.785

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	288	90	63
normalized size	1	1.	0.95	0.81	0.	5.05	1.58	1.11
time (sec)	N/A	0.017	0.035	0.008	0.	1.271	0.444	2.563

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	70	157	49	69
normalized size	1	1.	0.84	0.94	1.43	3.2	1.	1.41
time (sec)	N/A	0.036	0.038	0.011	1.292	1.297	0.53	2.635

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	359	114	80
normalized size	1	1.	0.99	0.87	0.	5.28	1.68	1.18
time (sec)	N/A	0.026	0.039	0.01	0.	1.326	0.588	1.9

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	95	184	68	116
normalized size	1	1.	0.86	0.92	1.44	2.79	1.03	1.76
time (sec)	N/A	0.044	0.049	0.012	1.15	1.241	0.69	2.418

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	0	423	126	95
normalized size	1	1.	0.99	0.86	0.	5.22	1.56	1.17
time (sec)	N/A	0.033	0.044	0.011	0.	1.272	0.676	2.147

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	73	107	209	78	134
normalized size	1	1.	0.85	0.91	1.34	2.61	0.98	1.68
time (sec)	N/A	0.054	0.054	0.012	2.209	1.368	0.834	2.766

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	81	0	470	138	109
normalized size	1	1.	0.97	0.86	0.	5.	1.47	1.16
time (sec)	N/A	0.043	0.053	0.01	0.	1.177	1.006	2.33

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	79	84	124	242	94	149
normalized size	1	1.	0.85	0.9	1.33	2.6	1.01	1.6
time (sec)	N/A	0.065	0.071	0.013	2.109	1.144	1.293	2.675

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	97	101	150	298	117	154
normalized size	1	1.	0.85	0.89	1.32	2.61	1.03	1.35
time (sec)	N/A	0.103	0.029	0.01	1.665	1.231	0.669	2.401

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	85	91	134	261	104	138
normalized size	1	1.	0.85	0.91	1.34	2.61	1.04	1.38
time (sec)	N/A	0.082	0.027	0.009	2.317	1.204	0.598	1.816

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	75	80	120	240	90	124
normalized size	1	1.	0.86	0.92	1.38	2.76	1.03	1.43
time (sec)	N/A	0.072	0.023	0.01	1.254	1.177	0.579	1.809

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	69	104	211	78	108
normalized size	1	1.	0.85	0.93	1.41	2.85	1.05	1.46
time (sec)	N/A	0.058	0.023	0.01	2.066	1.133	0.562	2.633

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	89	186	66	84
normalized size	1	1.	0.74	0.89	1.37	2.86	1.02	1.29
time (sec)	N/A	0.046	0.054	0.009	2.335	1.191	0.539	1.716

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	74	143	53	57
normalized size	1	1.	0.8	0.94	1.51	2.92	1.08	1.16
time (sec)	N/A	0.038	0.018	0.009	2.113	1.153	0.469	2.026

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	73	36	30
normalized size	1	1.	1.26	1.63	2.58	3.84	1.89	1.58
time (sec)	N/A	0.004	0.008	0.007	2.158	1.242	0.418	1.369

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	51	27	19
normalized size	1	1.	1.	0.94	1.19	3.19	1.69	1.19
time (sec)	N/A	0.003	0.002	0.001	2.115	1.114	0.42	2.005

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	81	196	56	80
normalized size	1	1.	0.8	0.91	1.5	3.63	1.04	1.48
time (sec)	N/A	0.038	0.034	0.011	2.501	1.229	0.632	1.939

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	104	247	78	111
normalized size	1	1.	0.88	0.93	1.55	3.69	1.16	1.66
time (sec)	N/A	0.047	0.054	0.012	1.952	1.248	0.781	2.914

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	124	274	90	108
normalized size	1	1.	0.86	0.92	1.44	3.19	1.05	1.26
time (sec)	N/A	0.06	0.047	0.013	2.15	1.273	1.	2.203

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	90	139	308	104	149
normalized size	1	1.	0.89	0.95	1.46	3.24	1.09	1.57
time (sec)	N/A	0.068	0.067	0.012	1.808	1.26	1.719	2.11

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	96	101	154	329	116	161
normalized size	1	1.	0.86	0.9	1.38	2.94	1.04	1.44
time (sec)	N/A	0.08	0.058	0.014	2.72	1.244	3.171	1.607

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	99	99	0	614	160	130
normalized size	1	1.	0.89	0.89	0.	5.53	1.44	1.17
time (sec)	N/A	0.048	0.059	0.01	0.	1.266	0.619	2.768

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	88	88	0	547	144	113
normalized size	1	1.	0.9	0.9	0.	5.58	1.47	1.15
time (sec)	N/A	0.041	0.047	0.008	0.	1.284	0.611	2.733

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	0	493	131	99
normalized size	1	1.	0.91	0.91	0.	5.8	1.54	1.16
time (sec)	N/A	0.035	0.045	0.01	0.	1.291	0.579	1.535

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	0	425	107	73
normalized size	1	1.	0.89	0.85	0.	5.74	1.45	0.99
time (sec)	N/A	0.025	0.046	0.008	0.	1.321	0.546	3.039

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	0	404	109	61
normalized size	1	1.	0.86	0.73	0.	6.31	1.7	0.95
time (sec)	N/A	0.02	0.038	0.007	0.	1.289	0.483	1.369

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	0	394	110	68
normalized size	1	1.	0.89	0.75	0.	6.06	1.69	1.05
time (sec)	N/A	0.018	0.027	0.007	0.	1.255	0.494	1.752

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	0	401	105	61
normalized size	1	1.	0.89	0.82	0.	6.47	1.69	0.98
time (sec)	N/A	0.016	0.035	0.003	0.	1.249	0.508	2.869

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	0	428	114	77
normalized size	1	1.	0.89	0.87	0.	5.63	1.5	1.01
time (sec)	N/A	0.026	0.039	0.01	0.	1.328	0.636	2.101

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	0	504	138	96
normalized size	1	1.	0.91	0.91	0.	5.79	1.59	1.1
time (sec)	N/A	0.034	0.041	0.012	0.	1.25	0.852	2.127

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	89	0	560	150	108
normalized size	1	1.	0.9	0.89	0.	5.6	1.5	1.08
time (sec)	N/A	0.042	0.051	0.012	0.	1.259	1.212	2.518

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	101	0	630	162	126
normalized size	1	1.	0.89	0.89	0.	5.58	1.43	1.12
time (sec)	N/A	0.056	0.056	0.011	0.	1.278	2.25	2.986

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	169	199	327	842	258	227
normalized size	1	1.	0.78	0.92	1.51	3.9	1.19	1.05
time (sec)	N/A	0.255	0.045	0.019	2.877	1.278	8.692	2.45

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	158	188	312	817	245	212
normalized size	1	1.	0.77	0.92	1.52	3.99	1.2	1.03
time (sec)	N/A	0.214	0.028	0.018	2.692	1.22	8.461	3.204

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	145	177	297	767	231	188
normalized size	1	1.	0.77	0.94	1.58	4.08	1.23	1.
time (sec)	N/A	0.189	0.034	0.017	2.57	1.289	8.49	2.479

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	116	166	282	714	219	161
normalized size	1	1.	0.65	0.93	1.58	3.99	1.22	0.9
time (sec)	N/A	0.169	0.032	0.011	2.323	1.272	7.946	1.312

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	101	150	257	423	202	134
normalized size	1	1.	5.32	7.89	13.53	22.26	10.63	7.05
time (sec)	N/A	0.003	0.019	0.01	2.019	1.207	7.608	2.715

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	90	133	242	400	190	119
normalized size	1	1.	2.31	3.41	6.21	10.26	4.87	3.05
time (sec)	N/A	0.017	0.015	0.008	2.724	1.227	7.671	2.312

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	79	116	227	374	178	104
normalized size	1	1.	1.36	2.	3.91	6.45	3.07	1.79
time (sec)	N/A	0.027	0.017	0.01	2.767	1.163	7.591	1.844

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	68	99	212	350	167	89
normalized size	1	1.	0.88	1.29	2.75	4.55	2.17	1.16
time (sec)	N/A	0.039	0.021	0.009	2.364	1.24	7.254	2.577

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	57	82	197	323	155	74
normalized size	1	1.	0.63	0.9	2.16	3.55	1.7	0.81
time (sec)	N/A	0.068	0.015	0.008	2.252	1.214	7.25	1.844

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	65	182	297	143	59
normalized size	1	1.	0.64	0.9	2.53	4.12	1.99	0.82
time (sec)	N/A	0.053	0.011	0.008	2.818	1.193	7.121	2.532

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	167	271	131	45
normalized size	1	1.	0.66	0.91	3.15	5.11	2.47	0.85
time (sec)	N/A	0.039	0.013	0.007	2.341	1.186	7.055	2.327

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	153	247	119	30
normalized size	1	1.	0.71	0.91	4.5	7.26	3.5	0.88
time (sec)	N/A	0.025	0.008	0.007	2.608	1.199	6.963	1.694

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	223	110	19
normalized size	1	1.	1.	0.94	1.19	13.94	6.88	1.19
time (sec)	N/A	0.003	0.002	0.	2.249	1.236	7.013	2.066

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	120	147	289	936	223	184
normalized size	1	1.	0.72	0.89	1.74	5.64	1.34	1.11
time (sec)	N/A	0.128	0.082	0.018	1.358	1.348	77.24	2.274

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	136	167	312	996	243	215
normalized size	1	1.	0.74	0.91	1.7	5.41	1.32	1.17
time (sec)	N/A	0.186	0.121	0.019	1.267	1.389	168.877	1.949

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	151	198	332	1053	0	235
normalized size	1	1.	0.7	0.91	1.53	4.85	0.	1.08
time (sec)	N/A	0.22	0.093	0.021	3.036	1.327	0.	2.696

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	162	209	347	1087	0	252
normalized size	1	1.	0.72	0.92	1.54	4.81	0.	1.12
time (sec)	N/A	0.233	0.114	0.019	2.58	1.392	0.	2.025

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	166	228	0	1877	314	219
normalized size	1	1.	0.72	0.99	0.	8.13	1.36	0.95
time (sec)	N/A	0.167	0.089	0.018	0.	1.224	8.651	2.658

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	155	217	0	1760	298	203
normalized size	1	1.	0.71	1.	0.	8.07	1.37	0.93
time (sec)	N/A	0.141	0.076	0.017	0.	1.227	8.378	2.714

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	144	203	0	1683	274	177
normalized size	1	1.	0.7	0.98	0.	8.13	1.32	0.86
time (sec)	N/A	0.124	0.071	0.018	0.	1.282	8.209	2.538

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	134	124	0	1615	275	165
normalized size	1	1.	0.68	0.63	0.	8.2	1.4	0.84
time (sec)	N/A	0.113	0.074	0.014	0.	1.538	7.778	2.551

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	138	124	0	1580	289	173
normalized size	1	1.	0.7	0.63	0.	7.98	1.46	0.87
time (sec)	N/A	0.122	0.069	0.013	0.	1.325	7.66	2.979

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	138	122	0	1581	291	173
normalized size	1	1.	0.69	0.61	0.	7.94	1.46	0.87
time (sec)	N/A	0.123	0.059	0.014	0.	1.29	7.47	2.594

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	138	122	0	1551	291	173
normalized size	1	1.	0.69	0.61	0.	7.76	1.46	0.86
time (sec)	N/A	0.12	0.069	0.013	0.	1.321	7.418	1.899

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	138	122	0	1523	291	173
normalized size	1	1.	0.69	0.61	0.	7.58	1.45	0.86
time (sec)	N/A	0.119	0.069	0.013	0.	1.31	7.455	1.841

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	138	122	0	1523	291	173
normalized size	1	1.	0.68	0.6	0.	7.54	1.44	0.86
time (sec)	N/A	0.116	0.056	0.011	0.	1.26	7.21	1.457

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	138	122	0	1551	291	173
normalized size	1	1.	0.68	0.6	0.	7.64	1.43	0.85
time (sec)	N/A	0.111	0.061	0.012	0.	1.349	6.999	2.23

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	138	122	0	1581	291	173
normalized size	1	1.	0.68	0.6	0.	7.75	1.43	0.85
time (sec)	N/A	0.108	0.062	0.013	0.	1.368	7.151	3.375

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	138	124	0	1580	286	173
normalized size	1	1.	0.67	0.6	0.	7.71	1.4	0.84
time (sec)	N/A	0.106	0.055	0.011	0.	1.295	7.632	2.214

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	131	156	0	1612	272	165
normalized size	1	1.	0.72	0.86	0.	8.91	1.5	0.91
time (sec)	N/A	0.1	0.097	0.005	0.	1.332	7.302	2.797

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	147	206	0	1686	280	181
normalized size	1	1.	0.7	0.99	0.	8.07	1.34	0.87
time (sec)	N/A	0.131	0.094	0.019	0.	1.548	118.924	2.369

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	157	219	0	1777	0	200
normalized size	1	1.	0.71	1.	0.	8.08	0.	0.91
time (sec)	N/A	0.141	0.083	0.02	0.	1.323	0.	3.05

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	169	230	0	1901	0	215
normalized size	1	1.	0.73	0.99	0.	8.16	0.	0.92
time (sec)	N/A	0.158	0.088	0.022	0.	1.44	0.	2.091

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	34	50	22	35
normalized size	1	1.	1.	0.93	1.21	1.79	0.79	1.25
time (sec)	N/A	0.022	0.006	0.002	1.521	1.211	0.296	2.587

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	165	49	39
normalized size	1	1.	1.	0.87	0.	5.32	1.58	1.26
time (sec)	N/A	0.012	0.009	0.003	0.	1.336	0.294	1.759

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	20	31	12	22
normalized size	1	1.	1.	1.	1.25	1.94	0.75	1.38
time (sec)	N/A	0.003	0.002	0.	1.556	1.245	0.109	1.694

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	46	24
normalized size	1	1.	1.	0.67	0.	6.29	1.92	1.
time (sec)	N/A	0.007	0.004	0.002	0.	1.314	0.13	1.554

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	34	49	15	35
normalized size	1	1.	1.	1.	1.48	2.13	0.65	1.52
time (sec)	N/A	0.012	0.006	0.005	2.509	1.262	0.193	1.852

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	173	58	42
normalized size	1	1.	1.	0.88	0.	5.24	1.76	1.27
time (sec)	N/A	0.012	0.011	0.004	0.	1.216	0.33	1.93

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	47	81	31	58
normalized size	1	1.	1.	0.94	1.34	2.31	0.89	1.66
time (sec)	N/A	0.024	0.008	0.005	1.585	1.265	0.417	2.966

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	34	47	76	29	72
normalized size	1	1.	0.83	0.97	1.34	2.17	0.83	2.06
time (sec)	N/A	0.027	0.013	0.007	1.093	1.245	0.353	2.063

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	38	0	263	71	53
normalized size	1	1.	1.02	0.83	0.	5.72	1.54	1.15
time (sec)	N/A	0.014	0.029	0.006	0.	1.303	0.352	2.566

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	30	15	22
normalized size	1	1.	1.	1.	1.29	1.76	0.88	1.29
time (sec)	N/A	0.003	0.002	0.	2.087	1.214	0.306	1.585

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	38	0	263	71	53
normalized size	1	1.	1.02	0.83	0.	5.72	1.54	1.15
time (sec)	N/A	0.012	0.016	0.003	0.	1.253	0.376	2.054

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	35	39	55	108	34	69
normalized size	1	1.	0.88	0.98	1.38	2.7	0.85	1.72
time (sec)	N/A	0.028	0.016	0.008	1.207	1.241	0.448	2.763

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	47	0	288	83	68
normalized size	1	1.	0.97	0.81	0.	4.97	1.43	1.17
time (sec)	N/A	0.018	0.036	0.01	0.	1.324	0.484	1.971

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	44	51	77	157	49	76
normalized size	1	1.	0.85	0.98	1.48	3.02	0.94	1.46
time (sec)	N/A	0.038	0.031	0.011	1.909	1.239	0.592	1.865

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	25	35	51	72	34	35
normalized size	1	1.	1.25	1.75	2.55	3.6	1.7	1.75
time (sec)	N/A	0.004	0.009	0.006	2.677	1.225	0.633	2.219

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	52	0	394	104	72
normalized size	1	1.	0.84	0.78	0.	5.88	1.55	1.07
time (sec)	N/A	0.02	0.029	0.006	0.	1.299	0.476	2.145

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	50	26	22
normalized size	1	1.	1.	1.	1.29	2.94	1.53	1.29
time (sec)	N/A	0.003	0.002	0.001	1.515	1.187	0.39	2.898

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	61	0	404	99	66
normalized size	1	1.	0.88	0.95	0.	6.31	1.55	1.03
time (sec)	N/A	0.018	0.038	0.004	0.	1.288	0.49	1.613

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	55	84	197	56	85
normalized size	1	1.	0.79	0.96	1.47	3.46	0.98	1.49
time (sec)	N/A	0.038	0.032	0.011	2.54	1.217	0.589	2.571

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	56	0	428	107	82
normalized size	1	1.	0.88	0.72	0.	5.49	1.37	1.05
time (sec)	N/A	0.026	0.043	0.01	0.	1.26	0.706	1.935

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	68	107	247	78	113
normalized size	1	1.	0.87	0.99	1.55	3.58	1.13	1.64
time (sec)	N/A	0.051	0.053	0.012	2.054	1.263	0.831	1.715

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	35	81	116	58	53
normalized size	1	1.	0.69	0.97	2.25	3.22	1.61	1.47
time (sec)	N/A	0.03	0.01	0.007	1.908	1.215	0.732	2.339

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	81	72	0	684	160	104
normalized size	1	1.	0.74	0.66	0.	6.28	1.47	0.95
time (sec)	N/A	0.037	0.048	0.009	0.	1.29	0.851	2.575

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	93	49	22
normalized size	1	1.	1.	1.	1.29	5.47	2.88	1.29
time (sec)	N/A	0.003	0.003	0.	2.105	1.169	0.73	2.821

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	107	0	694	146	96
normalized size	1	1.	0.79	1.07	0.	6.94	1.46	0.96
time (sec)	N/A	0.034	0.044	0.004	0.	1.267	0.982	2.526

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	87	143	379	104	115
normalized size	1	1.	0.74	0.96	1.57	4.16	1.14	1.26
time (sec)	N/A	0.064	0.032	0.013	1.365	1.257	1.408	2.944

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	92	78	0	721	155	112
normalized size	1	1.	0.78	0.66	0.	6.11	1.31	0.95
time (sec)	N/A	0.049	0.053	0.013	0.	1.282	1.847	2.37

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	83	102	166	440	126	143
normalized size	1	1.	0.78	0.96	1.57	4.15	1.19	1.35
time (sec)	N/A	0.087	0.063	0.015	2.125	1.275	2.811	2.555

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	41	12	24
normalized size	1	1.	1.	0.93	1.53	2.73	0.8	1.6
time (sec)	N/A	0.009	0.004	0.006	2.057	1.249	0.121	2.172

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	23	39	12	24
normalized size	1	1.	1.	0.89	1.28	2.17	0.67	1.33
time (sec)	N/A	0.01	0.005	0.004	2.8	1.259	0.124	2.663

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	32	72	22	43
normalized size	1	1.	1.	0.88	1.23	2.77	0.85	1.65
time (sec)	N/A	0.017	0.005	0.004	2.814	1.213	0.344	1.799

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	32	72	22	43
normalized size	1	1.	1.	0.85	1.19	2.67	0.81	1.59
time (sec)	N/A	0.018	0.004	0.007	2.296	1.246	0.194	1.855

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	20	0	189	83	31
normalized size	1	1.	0.93	0.67	0.	6.3	2.77	1.03
time (sec)	N/A	0.027	0.01	0.005	0.	1.18	0.149	2.23

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	44	33	0	217	87	45
normalized size	1	1.	1.19	0.89	0.	5.86	2.35	1.22
time (sec)	N/A	0.033	0.016	0.007	0.	1.293	0.213	2.757

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	38	100	22	49
normalized size	1	1.	0.89	0.89	1.36	3.57	0.79	1.75
time (sec)	N/A	0.02	0.013	0.009	2.587	1.248	0.344	2.266

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	38	100	22	49
normalized size	1	1.	0.87	0.83	1.27	3.33	0.73	1.63
time (sec)	N/A	0.019	0.012	0.01	2.482	1.404	0.354	2.778

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	219	60	50
normalized size	1	1.	1.06	1.	0.	6.44	1.76	1.47
time (sec)	N/A	0.025	0.015	0.005	0.	1.53	0.236	2.779

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	219	66	49
normalized size	1	1.	1.06	1.	0.	6.44	1.94	1.44
time (sec)	N/A	0.011	0.01	0.003	0.	1.466	0.256	2.481

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	38	0	370	104	78
normalized size	1	1.	1.	0.76	0.	7.4	2.08	1.56
time (sec)	N/A	0.053	0.02	0.007	0.	1.525	0.352	2.471

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	49	19	18
normalized size	1	1.	1.	0.76	0.86	2.33	0.9	0.86
time (sec)	N/A	0.004	0.005	0.003	2.043	1.463	5.797	2.728

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	47	19	18
normalized size	1	1.	1.	0.76	0.86	2.24	0.9	0.86
time (sec)	N/A	0.004	0.005	0.002	1.254	1.475	2.725	2.776

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	46	19	18
normalized size	1	1.	1.	0.76	0.86	2.19	0.9	0.86
time (sec)	N/A	0.004	0.004	0.002	1.898	1.485	0.94	2.039

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	43	19	18
normalized size	1	1.	1.	0.76	0.86	2.05	0.9	0.86
time (sec)	N/A	0.004	0.004	0.002	2.461	1.431	1.143	1.406

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	18	36	17	18
normalized size	1	1.	1.	0.79	0.95	1.89	0.89	0.95
time (sec)	N/A	0.004	0.004	0.002	1.899	1.485	0.226	3.12

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	18	36	17	18
normalized size	1	1.	1.	0.84	0.95	1.89	0.89	0.95
time (sec)	N/A	0.004	0.005	0.003	1.746	1.564	0.507	2.493

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	36	17	18
normalized size	1	1.	1.	0.74	0.95	1.89	0.89	0.95
time (sec)	N/A	0.004	0.006	0.002	2.021	1.443	0.639	2.635

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	38	19	18
normalized size	1	1.	1.	0.74	0.95	2.	1.	0.95
time (sec)	N/A	0.004	0.005	0.002	2.622	1.488	1.297	2.765

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	78	34	32
normalized size	1	1.	0.83	0.75	0.89	2.17	0.94	0.89
time (sec)	N/A	0.009	0.009	0.004	2.73	1.43	11.476	1.516

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	77	34	32
normalized size	1	1.	0.83	0.75	0.89	2.14	0.94	0.89
time (sec)	N/A	0.008	0.007	0.004	1.504	1.213	5.698	2.759

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	76	34	32
normalized size	1	1.	0.83	0.75	0.89	2.11	0.94	0.89
time (sec)	N/A	0.009	0.007	0.004	1.988	1.242	2.561	1.793

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	70	34	32
normalized size	1	1.	0.83	0.75	0.89	1.94	0.94	0.89
time (sec)	N/A	0.008	0.007	0.004	2.458	1.308	1.62	3.118

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	32	65	32	32
normalized size	1	1.	0.88	0.79	0.94	1.91	0.94	0.94
time (sec)	N/A	0.009	0.008	0.004	2.218	1.167	0.73	2.62

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	32	65	32	32
normalized size	1	1.	0.88	0.79	0.94	1.91	0.94	0.94
time (sec)	N/A	0.008	0.009	0.005	2.222	1.225	0.974	3.035

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	32	63	32	32
normalized size	1	1.	0.88	0.79	0.94	1.85	0.94	0.94
time (sec)	N/A	0.008	0.009	0.004	1.223	1.198	1.214	2.349

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	34	63	32	34
normalized size	1	1.	0.88	0.79	1.	1.85	0.94	1.
time (sec)	N/A	0.008	0.009	0.004	1.935	1.302	1.828	2.651

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	109	49	47
normalized size	1	1.	0.8	0.75	0.92	2.14	0.96	0.92
time (sec)	N/A	0.013	0.01	0.004	1.992	1.194	21.01	1.477

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	107	49	47
normalized size	1	1.	0.8	0.75	0.92	2.1	0.96	0.92
time (sec)	N/A	0.012	0.009	0.004	2.39	1.168	11.271	2.976

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	104	49	47
normalized size	1	1.	0.8	0.75	0.92	2.04	0.96	0.92
time (sec)	N/A	0.013	0.01	0.003	2.174	1.308	5.723	2.257

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	99	49	47
normalized size	1	1.	0.8	0.75	0.92	1.94	0.96	0.92
time (sec)	N/A	0.013	0.01	0.004	2.303	1.276	2.111	2.926

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	47	93	48	47
normalized size	1	1.	0.84	0.78	0.96	1.9	0.98	0.96
time (sec)	N/A	0.012	0.01	0.003	2.336	1.255	2.203	2.156

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	47	88	46	47
normalized size	1	1.	0.87	0.81	1.	1.87	0.98	1.
time (sec)	N/A	0.012	0.011	0.006	1.945	1.15	2.405	2.233

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	47	89	48	47
normalized size	1	1.	0.84	0.78	0.96	1.82	0.98	0.96
time (sec)	N/A	0.012	0.011	0.004	1.889	1.212	3.039	2.89

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	49	88	46	49
normalized size	1	1.	0.87	0.81	1.04	1.87	0.98	1.04
time (sec)	N/A	0.012	0.011	0.004	2.46	1.222	3.845	1.615

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	0	393	192	265
normalized size	1	1.	0.94	0.71	0.	1.83	0.89	1.23
time (sec)	N/A	0.198	0.065	0.012	0.	1.378	166.208	1.739

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	78	143	0	377	180	240
normalized size	1	1.	0.38	0.7	0.	1.85	0.88	1.18
time (sec)	N/A	0.152	0.025	0.006	0.	1.412	25.563	2.242

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	0	312	177	240
normalized size	1	1.	0.94	0.69	0.	1.54	0.88	1.19
time (sec)	N/A	0.163	0.039	0.006	0.	1.353	9.493	1.866

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	54	132	0	332	170	246
normalized size	1	1.	0.28	0.69	0.	1.73	0.89	1.28
time (sec)	N/A	0.144	0.024	0.006	0.	1.351	4.467	3.017

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	0	329	170	246
normalized size	1	1.	0.76	0.69	0.	1.71	0.89	1.28
time (sec)	N/A	0.146	0.034	0.004	0.	1.334	8.714	2.604

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	27	140	0	343	180	257
normalized size	1	1.	0.13	0.69	0.	1.7	0.89	1.27
time (sec)	N/A	0.167	0.005	0.007	0.	1.327	17.57	3.067

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	29	143	0	387	184	240
normalized size	1	1.	0.14	0.7	0.	1.9	0.9	1.18
time (sec)	N/A	0.161	0.006	0.007	0.	1.37	76.112	2.208

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	29	152	0	433	0	270
normalized size	1	1.	0.13	0.71	0.	2.01	0.	1.26
time (sec)	N/A	0.175	0.007	0.008	0.	1.434	0.	3.08

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	221	158	0	443	0	265
normalized size	1	1.	0.96	0.69	0.	1.93	0.	1.15
time (sec)	N/A	0.166	0.102	0.011	0.	1.343	0.	2.553

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	43	149	0	447	0	269
normalized size	1	1.	0.2	0.68	0.	2.05	0.	1.23
time (sec)	N/A	0.148	0.012	0.009	0.	1.373	0.	1.76

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	0	467	0	269
normalized size	1	1.	0.91	0.72	0.	2.14	0.	1.23
time (sec)	N/A	0.148	0.095	0.008	0.	1.455	0.	1.985

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	29	158	0	467	619	269
normalized size	1	1.	0.13	0.72	0.	2.14	2.84	1.23
time (sec)	N/A	0.147	0.005	0.007	0.	1.399	125.801	2.164

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	0	441	0	269
normalized size	1	1.	0.91	0.68	0.	2.02	0.	1.23
time (sec)	N/A	0.151	0.098	0.008	0.	1.436	0.	1.673

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	27	158	0	512	0	284
normalized size	1	1.	0.12	0.69	0.	2.23	0.	1.23
time (sec)	N/A	0.188	0.005	0.013	0.	1.389	0.	1.827

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	29	161	0	518	0	265
normalized size	1	1.	0.13	0.7	0.	2.25	0.	1.15
time (sec)	N/A	0.177	0.006	0.011	0.	1.45	0.	2.939

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	29	172	0	605	0	297
normalized size	1	1.	0.12	0.71	0.	2.49	0.	1.22
time (sec)	N/A	0.191	0.007	0.015	0.	1.433	0.	2.324

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	242	170	0	599	0	282
normalized size	1	1.	1.01	0.71	0.	2.51	0.	1.18
time (sec)	N/A	0.177	0.106	0.012	0.	1.473	0.	2.435

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	45	169	0	613	0	286
normalized size	1	1.	0.19	0.7	0.	2.53	0.	1.18
time (sec)	N/A	0.17	0.015	0.013	0.	1.356	0.	2.29

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	0	608	0	285
normalized size	1	1.	0.92	0.7	0.	2.51	0.	1.18
time (sec)	N/A	0.164	0.108	0.011	0.	1.441	0.	2.388

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	29	175	0	602	0	282
normalized size	1	1.	0.12	0.73	0.	2.52	0.	1.18
time (sec)	N/A	0.164	0.005	0.007	0.	1.455	0.	2.493

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	220	166	0	576	0	282
normalized size	1	1.	0.92	0.69	0.	2.41	0.	1.18
time (sec)	N/A	0.172	0.084	0.007	0.	1.455	0.	2.452

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	27	178	0	672	0	297
normalized size	1	1.	0.11	0.71	0.	2.68	0.	1.18
time (sec)	N/A	0.191	0.006	0.017	0.	1.416	0.	2.142

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	29	181	0	645	0	281
normalized size	1	1.	0.12	0.72	0.	2.57	0.	1.12
time (sec)	N/A	0.185	0.007	0.016	0.	1.484	0.	2.118

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	29	192	0	778	0	313
normalized size	1	1.	0.11	0.73	0.	2.95	0.	1.19
time (sec)	N/A	0.212	0.006	0.017	0.	1.472	0.	2.291

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	48	66	0	319	128	262
normalized size	1	1.	0.83	1.14	0.	5.5	2.21	4.52
time (sec)	N/A	0.034	0.016	0.007	0.	1.383	4.397	1.887

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	72	113	382	105	113
normalized size	1	1.	1.	0.67	1.05	3.54	0.97	1.05
time (sec)	N/A	0.061	0.023	0.007	3.189	1.335	5.348	2.952

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	24	67	107	369	99	107
normalized size	1	1.	0.24	0.66	1.06	3.65	0.98	1.06
time (sec)	N/A	0.055	0.006	0.006	4.833	1.417	1.972	1.513

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	67	107	366	97	107
normalized size	1	1.	1.	0.68	1.08	3.7	0.98	1.08
time (sec)	N/A	0.055	0.014	0.004	3.239	1.373	0.923	2.447

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	22	62	100	351	90	100
normalized size	1	1.	0.24	0.67	1.09	3.82	0.98	1.09
time (sec)	N/A	0.055	0.004	0.004	2.175	1.348	0.57	2.615

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	76	62	100	351	90	100
normalized size	1	1.	0.83	0.67	1.09	3.82	0.98	1.09
time (sec)	N/A	0.053	0.012	0.003	1.969	1.36	0.786	2.089

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	20	67	107	382	97	107
normalized size	1	1.	0.2	0.68	1.08	3.86	0.98	1.08
time (sec)	N/A	0.056	0.005	0.005	2.227	1.411	1.691	2.675

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	22	67	107	405	99	107
normalized size	1	1.	0.22	0.66	1.06	4.01	0.98	1.06
time (sec)	N/A	0.054	0.005	0.006	3.337	1.362	2.988	2.318

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	22	72	116	423	105	116
normalized size	1	1.	0.2	0.67	1.07	3.92	0.97	1.07
time (sec)	N/A	0.057	0.005	0.007	4.313	1.315	7.111	1.691

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	121	79	123	462	277	123
normalized size	1	1.	0.99	0.65	1.01	3.79	2.27	1.01
time (sec)	N/A	0.061	0.046	0.01	2.351	1.394	13.795	2.354

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	30	74	116	447	264	116
normalized size	1	1.	0.27	0.65	1.03	3.96	2.34	1.03
time (sec)	N/A	0.059	0.011	0.008	4.392	1.371	7.907	2.443

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	106	74	116	439	257	116
normalized size	1	1.	0.94	0.65	1.03	3.88	2.27	1.03
time (sec)	N/A	0.059	0.044	0.009	4.546	1.37	4.662	2.332

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	22	74	116	439	257	116
normalized size	1	1.	0.19	0.65	1.03	3.88	2.27	1.03
time (sec)	N/A	0.06	0.004	0.005	3.218	1.465	2.856	1.889

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	74	116	447	264	116
normalized size	1	1.	0.95	0.65	1.03	3.96	2.34	1.03
time (sec)	N/A	0.059	0.041	0.006	3.685	1.412	3.789	2.102

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	20	79	124	462	366	124
normalized size	1	1.	0.16	0.65	1.02	3.79	3.	1.02
time (sec)	N/A	0.065	0.005	0.011	3.729	1.345	6.462	1.987

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	22	79	124	478	366	123
normalized size	1	1.	0.18	0.65	1.02	3.92	3.	1.01
time (sec)	N/A	0.061	0.005	0.009	3.286	1.35	13.644	2.658

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	22	84	131	495	384	132
normalized size	1	1.	0.17	0.64	1.	3.78	2.93	1.01
time (sec)	N/A	0.065	0.006	0.011	3.846	1.354	32.298	2.03

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	135	82	134	518	481	127
normalized size	1	1.	1.05	0.64	1.04	4.02	3.73	0.98
time (sec)	N/A	0.068	0.044	0.009	2.939	1.371	44.692	2.066

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	32	82	134	518	481	127
normalized size	1	1.	0.25	0.64	1.04	4.02	3.73	0.98
time (sec)	N/A	0.07	0.012	0.009	2.542	1.674	29.08	1.576

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	121	82	131	516	481	124
normalized size	1	1.	0.94	0.64	1.02	4.	3.73	0.96
time (sec)	N/A	0.068	0.03	0.007	2.931	1.51	17.485	1.779

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	22	86	134	521	481	127
normalized size	1	1.	0.17	0.67	1.04	4.04	3.73	0.98
time (sec)	N/A	0.069	0.004	0.006	3.8	1.387	10.729	1.722

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	121	86	134	522	481	127
normalized size	1	1.	0.94	0.67	1.04	4.05	3.73	0.98
time (sec)	N/A	0.068	0.031	0.004	2.961	1.472	14.412	3.144

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	20	87	138	537	653	134
normalized size	1	1.	0.14	0.63	1.	3.89	4.73	0.97
time (sec)	N/A	0.07	0.005	0.01	4.071	1.34	22.549	2.474

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	22	87	138	555	653	134
normalized size	1	1.	0.16	0.63	1.	4.02	4.73	0.97
time (sec)	N/A	0.071	0.005	0.01	3.777	1.417	43.67	2.67

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	22	92	144	575	678	143
normalized size	1	1.	0.15	0.63	0.98	3.91	4.61	0.97
time (sec)	N/A	0.075	0.005	0.011	1.938	1.663	97.584	2.158

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	24	31	86	26	32
normalized size	1	1.	1.	1.6	2.07	5.73	1.73	2.13
time (sec)	N/A	0.007	0.004	0.007	2.091	1.665	0.299	2.824

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	100	22	69	92	381	94	92
normalized size	1	1.37	0.3	0.95	1.26	5.22	1.29	1.26
time (sec)	N/A	0.257	0.005	0.024	2.031	1.744	2.342	1.502

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	432	0	868	1999	729
normalized size	1	1.	0.91	4.45	0.	8.95	20.61	7.52
time (sec)	N/A	0.043	0.05	0.004	0.	1.593	4.061	2.893

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	72	291	0	571	1221	493
normalized size	1	1.	0.91	3.68	0.	7.23	15.46	6.24
time (sec)	N/A	0.032	0.032	0.005	0.	1.625	2.533	2.947

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	178	0	347	683	302
normalized size	1	1.	0.92	2.92	0.	5.69	11.2	4.95
time (sec)	N/A	0.022	0.03	0.004	0.	1.664	1.54	2.114

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	93	0	181	306	158
normalized size	1	1.	0.93	2.16	0.	4.21	7.12	3.67
time (sec)	N/A	0.015	0.019	0.003	0.	1.577	0.842	2.97

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	35	0	72	94	58
normalized size	1	1.	1.	1.4	0.	2.88	3.76	2.32
time (sec)	N/A	0.007	0.012	0.002	0.	1.533	0.375	2.58

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0
normalized size	1	1.	1.05	0.	0.	0.	2.26	0.
time (sec)	N/A	0.008	0.007	0.028	0.	0.	1.28	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	374	0
normalized size	1	1.	1.05	0.	0.	0.	9.59	0.
time (sec)	N/A	0.008	0.007	0.035	0.	0.	7.129	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	1556	0
normalized size	1	1.	1.05	0.	0.	0.	39.9	0.
time (sec)	N/A	0.008	0.007	0.047	0.	0.	25.388	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	92	0
normalized size	1	1.	1.02	0.	0.	0.	2.09	0.
time (sec)	N/A	0.012	0.014	0.03	0.	0.	5.379	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	95	0
normalized size	1	1.	0.95	0.	0.	0.	2.16	0.
time (sec)	N/A	0.01	0.005	0.03	0.	0.	1.285	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	39	0
normalized size	1	1.	1.	0.	0.	0.	1.03	0.
time (sec)	N/A	0.009	0.007	0.028	0.	0.	18.756	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	0	0	102	0
normalized size	1	1.	0.94	0.	0.	0.	2.17	0.
time (sec)	N/A	0.012	0.011	0.029	0.	0.	51.957	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	92	0
normalized size	1	1.	0.98	0.	0.	0.	2.04	0.
time (sec)	N/A	0.012	0.011	0.03	0.	0.	141.816	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	92	0	0	343	0
normalized size	1	1.	1.06	2.56	0.	0.	9.53	0.
time (sec)	N/A	0.008	0.009	0.05	0.	0.	7.308	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	126	110	77
normalized size	1	1.	0.62	0.59	0.	1.58	1.38	0.96
time (sec)	N/A	0.045	0.026	0.005	0.	1.613	1.296	1.864

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	103	87	58
normalized size	1	1.	0.66	0.61	0.	1.75	1.47	0.98
time (sec)	N/A	0.033	0.018	0.005	0.	1.517	0.672	2.283

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	76	63	39
normalized size	1	1.	0.74	0.66	0.	2.	1.66	1.03
time (sec)	N/A	0.022	0.014	0.003	0.	1.588	0.304	1.97

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	34	39	19
normalized size	1	1.	1.	0.83	1.06	1.89	2.17	1.06
time (sec)	N/A	0.003	0.003	0.002	2.261	1.471	0.161	2.48

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	0	200	56	45
normalized size	1	1.	1.	1.05	0.	5.41	1.51	1.22
time (sec)	N/A	0.024	0.008	0.004	0.	1.596	1.409	2.443

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	59	63	0	257	42	58
normalized size	1	1.	1.26	1.34	0.	5.47	0.89	1.23
time (sec)	N/A	0.025	0.031	0.004	0.	1.598	1.991	2.888

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	39	85	0	313	92	84
normalized size	1	1.	0.55	1.2	0.	4.41	1.3	1.18
time (sec)	N/A	0.039	0.008	0.005	0.	1.608	3.627	2.667

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	39	105	0	367	117	108
normalized size	1	1.	0.41	1.11	0.	3.86	1.23	1.14
time (sec)	N/A	0.052	0.008	0.007	0.	1.637	5.58	1.85

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	77	0	344	117	86
normalized size	1	1.	0.82	0.82	0.	3.66	1.24	0.91
time (sec)	N/A	0.033	0.032	0.007	0.	1.653	5.297	2.553

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	64	57	0	288	92	68
normalized size	1	1.	0.91	0.81	0.	4.11	1.31	0.97
time (sec)	N/A	0.021	0.023	0.004	0.	1.628	3.407	2.511

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	0	232	41	50
normalized size	1	1.	1.07	0.78	0.	5.04	0.89	1.09
time (sec)	N/A	0.01	0.013	0.003	0.	1.562	1.813	2.855

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	63	54	0	217	56	77
normalized size	1	1.	1.5	1.29	0.	5.17	1.33	1.83
time (sec)	N/A	0.012	0.068	0.005	0.	1.563	1.37	2.309

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	43	42	80
normalized size	1	1.	1.	0.86	0.	2.05	2.	3.81
time (sec)	N/A	0.004	0.005	0.003	0.	1.445	0.628	2.822

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	84	68	151
normalized size	1	1.	0.7	0.64	0.	1.91	1.55	3.43
time (sec)	N/A	0.011	0.009	0.003	0.	1.528	0.842	1.469

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	112	359	186
normalized size	1	1.	0.62	0.57	0.	1.65	5.28	2.74
time (sec)	N/A	0.018	0.01	0.003	0.	1.583	1.318	2.131

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	134	575	224
normalized size	1	1.	0.58	0.54	0.	1.46	6.25	2.43
time (sec)	N/A	0.029	0.012	0.003	0.	1.594	1.854	2.469

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	154	133	181
normalized size	1	1.	0.62	0.59	0.	1.92	1.66	2.26
time (sec)	N/A	0.048	0.025	0.005	0.	1.556	3.692	2.034

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	126	109	143
normalized size	1	1.	0.66	0.61	0.	2.14	1.85	2.42
time (sec)	N/A	0.035	0.018	0.005	0.	1.572	2.078	2.723

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	97	85	105
normalized size	1	1.	0.74	0.66	0.	2.55	2.24	2.76
time (sec)	N/A	0.024	0.014	0.003	0.	1.557	1.163	1.442

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	69	61	19
normalized size	1	1.	1.	0.83	1.06	3.83	3.39	1.06
time (sec)	N/A	0.004	0.003	0.002	2.172	1.518	0.575	3.161

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	52	0	251	78	65
normalized size	1	1.	0.93	0.96	0.	4.65	1.44	1.2
time (sec)	N/A	0.036	0.02	0.003	0.	1.587	1.864	1.373

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	37	75	0	284	88	77
normalized size	1	1.	0.59	1.19	0.	4.51	1.4	1.22
time (sec)	N/A	0.037	0.009	0.003	0.	1.615	2.343	1.599

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	76	102	0	317	71	82
normalized size	1	1.	1.12	1.5	0.	4.66	1.04	1.21
time (sec)	N/A	0.04	0.035	0.004	0.	1.542	3.006	2.619

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	39	122	0	371	119	108
normalized size	1	1.	0.42	1.33	0.	4.03	1.29	1.17
time (sec)	N/A	0.054	0.009	0.007	0.	1.683	5.288	1.566

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	39	142	0	419	148	127
normalized size	1	1.	0.34	1.22	0.	3.61	1.28	1.09
time (sec)	N/A	0.069	0.009	0.013	0.	1.653	8.186	2.948

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	95	0	396	148	103
normalized size	1	1.	0.82	0.83	0.	3.44	1.29	0.9
time (sec)	N/A	0.042	0.13	0.006	0.	1.664	7.566	2.001

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	83	75	0	346	119	85
normalized size	1	1.	0.91	0.82	0.	3.8	1.31	0.93
time (sec)	N/A	0.031	0.116	0.006	0.	1.613	4.977	2.667

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	0	294	70	66
normalized size	1	1.	1.	0.78	0.	4.52	1.08	1.02
time (sec)	N/A	0.015	0.087	0.001	0.	1.526	2.802	2.407

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	69	0	271	88	99
normalized size	1	1.	0.79	1.1	0.	4.3	1.4	1.57
time (sec)	N/A	0.018	0.008	0.004	0.	1.563	2.293	2.47

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	52	92	0	279	78	154
normalized size	1	1.	0.85	1.51	0.	4.57	1.28	2.52
time (sec)	N/A	0.018	0.009	0.005	0.	1.602	1.976	2.906

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	78	68	116
normalized size	1	1.	1.	0.86	0.	3.71	3.24	5.52
time (sec)	N/A	0.005	0.006	0.003	0.	1.49	0.968	2.678

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	105	94	224
normalized size	1	1.	0.7	0.64	0.	2.39	2.14	5.09
time (sec)	N/A	0.012	0.01	0.002	0.	1.575	1.375	2.038

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	135	420	259
normalized size	1	1.	0.62	0.57	0.	1.99	6.18	3.81
time (sec)	N/A	0.021	0.011	0.004	0.	1.592	1.98	2.511

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	163	648	297
normalized size	1	1.	0.58	0.54	0.	1.77	7.04	3.23
time (sec)	N/A	0.029	0.013	0.006	0.	1.639	2.662	1.871

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	180	158	301
normalized size	1	1.	0.62	0.59	0.	2.25	1.98	3.76
time (sec)	N/A	0.047	0.027	0.005	0.	1.546	8.75	1.628

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	153	133	243
normalized size	1	1.	0.66	0.61	0.	2.59	2.25	4.12
time (sec)	N/A	0.034	0.02	0.004	0.	1.58	5.514	2.966

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	122	109	186
normalized size	1	1.	0.74	0.66	0.	3.21	2.87	4.89
time (sec)	N/A	0.024	0.016	0.004	0.	1.561	3.533	2.651

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	90	85	95
normalized size	1	1.	1.	0.83	1.06	5.	4.72	5.28
time (sec)	N/A	0.003	0.005	0.002	1.941	1.581	1.948	1.678

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	62	66	0	311	105	84
normalized size	1	1.	0.86	0.92	0.	4.32	1.46	1.17
time (sec)	N/A	0.043	0.027	0.005	0.	1.589	3.407	2.772

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	37	88	0	342	112	99
normalized size	1	1.	0.46	1.1	0.	4.28	1.4	1.24
time (sec)	N/A	0.045	0.009	0.006	0.	1.627	3.238	2.077

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	39	116	0	342	117	103
normalized size	1	1.	0.45	1.35	0.	3.98	1.36	1.2
time (sec)	N/A	0.048	0.01	0.006	0.	1.607	3.844	3.23

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	87	139	0	370	99	101
normalized size	1	1.	0.98	1.56	0.	4.16	1.11	1.13
time (sec)	N/A	0.053	0.04	0.008	0.	1.612	4.756	1.725

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	39	159	0	433	150	127
normalized size	1	1.	0.35	1.41	0.	3.83	1.33	1.12
time (sec)	N/A	0.067	0.01	0.013	0.	1.617	8.152	2.533

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	39	179	0	489	175	146
normalized size	1	1.	0.28	1.31	0.	3.57	1.28	1.07
time (sec)	N/A	0.083	0.01	0.029	0.	1.722	11.764	1.811

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	105	113	0	460	175	123
normalized size	1	1.	0.77	0.83	0.	3.38	1.29	0.9
time (sec)	N/A	0.054	0.15	0.006	0.	1.711	10.927	2.342

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	94	93	0	408	150	104
normalized size	1	1.	0.84	0.83	0.	3.64	1.34	0.93
time (sec)	N/A	0.042	0.131	0.004	0.	1.682	7.125	2.607

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	76	66	0	347	97	85
normalized size	1	1.	0.9	0.79	0.	4.13	1.15	1.01
time (sec)	N/A	0.022	0.107	0.003	0.	1.594	4.238	1.712

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	52	85	0	329	117	117
normalized size	1	1.	0.63	1.02	0.	3.96	1.41	1.41
time (sec)	N/A	0.025	0.008	0.004	0.	1.576	3.673	2.867

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	54	110	0	340	112	178
normalized size	1	1.	0.63	1.28	0.	3.95	1.3	2.07
time (sec)	N/A	0.027	0.008	0.006	0.	1.604	3.222	2.51

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	130	0	342	105	227
normalized size	1	1.	0.66	1.59	0.	4.17	1.28	2.77
time (sec)	N/A	0.028	0.009	0.007	0.	1.589	3.645	2.739

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	100	95	153
normalized size	1	1.	1.	0.86	0.	4.76	4.52	7.29
time (sec)	N/A	0.005	0.008	0.002	0.	1.583	1.695	2.08

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	130	121	297
normalized size	1	1.	0.7	0.64	0.	2.95	2.75	6.75
time (sec)	N/A	0.011	0.011	0.003	0.	1.602	2.168	1.7

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	163	481	332
normalized size	1	1.	0.62	0.57	0.	2.4	7.07	4.88
time (sec)	N/A	0.019	0.011	0.003	0.	1.718	3.19	1.866

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	189	721	370
normalized size	1	1.	0.58	0.54	0.	2.05	7.84	4.02
time (sec)	N/A	0.028	0.014	0.004	0.	1.869	4.452	1.883

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	0	225	1012	405
normalized size	1	1.	0.55	0.53	0.	1.94	8.72	3.49
time (sec)	N/A	0.042	0.015	0.004	0.	2.057	5.812	2.69

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	75	72	0	254	1346	443
normalized size	1	1.	0.54	0.51	0.	1.81	9.61	3.16
time (sec)	N/A	0.056	0.018	0.005	0.	2.52	8.261	2.469

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	94	91	0	386	301	973
normalized size	1	1.	0.58	0.57	0.	2.4	1.87	6.04
time (sec)	N/A	0.1	0.057	0.006	0.	1.83	99.653	1.744

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	83	80	0	352	277	879
normalized size	1	1.	0.59	0.57	0.	2.51	1.98	6.28
time (sec)	N/A	0.08	0.044	0.005	0.	1.723	80.134	1.867

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	72	69	0	309	253	784
normalized size	1	1.	0.59	0.57	0.	2.53	2.07	6.43
time (sec)	N/A	0.071	0.041	0.005	0.	1.723	62.488	2.428

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	0	277	230	690
normalized size	1	1.	0.6	0.57	0.	2.74	2.28	6.83
time (sec)	N/A	0.058	0.033	0.006	0.	1.693	46.268	2.282

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	238	204	595
normalized size	1	1.	0.62	0.59	0.	2.98	2.55	7.44
time (sec)	N/A	0.046	0.03	0.005	0.	1.724	34.488	2.64

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	209	180	500
normalized size	1	1.	0.66	0.61	0.	3.54	3.05	8.47
time (sec)	N/A	0.035	0.022	0.005	0.	1.61	25.297	2.761

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	174	156	406
normalized size	1	1.	0.74	0.66	0.	4.58	4.11	10.68
time (sec)	N/A	0.024	0.018	0.004	0.	1.625	18.04	2.442

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	139	133	267
normalized size	1	1.	1.	0.83	1.06	7.72	7.39	14.83
time (sec)	N/A	0.003	0.006	0.002	2.13	1.643	12.273	2.592

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	84	94	0	419	160	122
normalized size	1	1.	0.78	0.87	0.	3.88	1.48	1.13
time (sec)	N/A	0.07	0.039	0.004	0.	1.691	9.372	1.587

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	37	118	0	452	167	136
normalized size	1	1.	0.31	1.	0.	3.83	1.42	1.15
time (sec)	N/A	0.071	0.012	0.004	0.	1.66	9.354	2.622

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	39	148	0	451	175	143
normalized size	1	1.	0.31	1.17	0.	3.58	1.39	1.13
time (sec)	N/A	0.076	0.012	0.007	0.	1.653	8.482	2.823

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	39	168	0	451	175	143
normalized size	1	1.	0.31	1.33	0.	3.58	1.39	1.13
time (sec)	N/A	0.076	0.012	0.01	0.	1.621	7.828	1.853

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	39	190	0	456	173	140
normalized size	1	1.	0.3	1.48	0.	3.56	1.35	1.09
time (sec)	N/A	0.079	0.012	0.018	0.	1.663	8.462	2.657

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	213	0	497	153	139
normalized size	1	1.	0.83	1.63	0.	3.79	1.17	1.06
time (sec)	N/A	0.083	0.048	0.046	0.	1.728	10.383	1.666

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	39	233	0	571	204	165
normalized size	1	1.	0.25	1.5	0.	3.68	1.32	1.06
time (sec)	N/A	0.101	0.012	0.113	0.	1.849	16.219	2.376

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	39	253	0	628	231	184
normalized size	1	1.	0.22	1.41	0.	3.51	1.29	1.03
time (sec)	N/A	0.122	0.012	0.29	0.	2.101	22.874	2.357

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	138	169	0	656	258	180
normalized size	1	1.	0.68	0.84	0.	3.25	1.28	0.89
time (sec)	N/A	0.114	0.214	0.011	0.	2.538	27.218	2.547

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	127	149	0	599	231	161
normalized size	1	1.	0.71	0.84	0.	3.37	1.3	0.9
time (sec)	N/A	0.086	0.185	0.008	0.	2.204	19.973	1.987

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	116	129	0	540	204	142
normalized size	1	1.	0.75	0.84	0.	3.51	1.32	0.92
time (sec)	N/A	0.07	0.168	0.007	0.	2.116	13.879	1.474

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	98	96	0	468	151	123
normalized size	1	1.	0.8	0.79	0.	3.84	1.24	1.01
time (sec)	N/A	0.042	0.131	0.002	0.	1.99	9.494	1.483

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	52	117	0	444	173	155
normalized size	1	1.	0.42	0.95	0.	3.61	1.41	1.26
time (sec)	N/A	0.044	0.01	0.004	0.	1.675	8.286	1.697

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	54	146	0	450	175	216
normalized size	1	1.	0.42	1.14	0.	3.52	1.37	1.69
time (sec)	N/A	0.049	0.01	0.006	0.	1.747	7.657	1.644

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	54	166	0	450	175	270
normalized size	1	1.	0.42	1.29	0.	3.49	1.36	2.09
time (sec)	N/A	0.05	0.01	0.008	0.	1.968	8.318	2.385

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	54	186	0	451	167	324
normalized size	1	1.	0.43	1.48	0.	3.58	1.33	2.57
time (sec)	N/A	0.051	0.01	0.014	0.	2.132	9.033	1.936

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	54	206	0	452	160	373
normalized size	1	1.	0.44	1.66	0.	3.65	1.29	3.01
time (sec)	N/A	0.052	0.01	0.03	0.	2.114	10.348	2.87

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	150	150	225
normalized size	1	1.	1.	0.86	0.	7.14	7.14	10.71
time (sec)	N/A	0.005	0.009	0.003	0.	1.833	4.848	1.596

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	184	175	443
normalized size	1	1.	0.7	0.64	0.	4.18	3.98	10.07
time (sec)	N/A	0.011	0.013	0.004	0.	2.121	6.517	1.682

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	220	604	478
normalized size	1	1.	0.62	0.57	0.	3.24	8.88	7.03
time (sec)	N/A	0.021	0.014	0.003	0.	2.37	10.058	3.067

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	247	867	516
normalized size	1	1.	0.58	0.54	0.	2.68	9.42	5.61
time (sec)	N/A	0.03	0.018	0.003	0.	2.767	13.089	2.859

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	0	288	1182	551
normalized size	1	1.	0.55	0.53	0.	2.48	10.19	4.75
time (sec)	N/A	0.045	0.017	0.003	0.	3.323	17.097	3.051

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	75	72	0	319	1540	589
normalized size	1	1.	0.54	0.51	0.	2.28	11.	4.21
time (sec)	N/A	0.056	0.02	0.005	0.	4.339	18.815	2.508

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	86	83	0	363	1950	624
normalized size	1	1.	0.52	0.51	0.	2.21	11.89	3.8
time (sec)	N/A	0.072	0.021	0.005	0.	5.688	23.785	3.058

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	76	61	46
normalized size	1	1.	0.59	0.52	1.17	1.65	1.33	1.
time (sec)	N/A	0.02	0.01	0.003	1.744	1.648	1.887	1.415

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	39	46	61	119	75	58
normalized size	1	1.	0.62	0.73	0.97	1.89	1.19	0.92
time (sec)	N/A	0.016	0.012	0.005	3.113	1.709	4.551	1.483

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	58	44	31
normalized size	1	1.	0.71	0.61	1.13	1.87	1.42	1.
time (sec)	N/A	0.015	0.007	0.003	3.284	1.661	0.584	2.613

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	36	32	42	97	54	49
normalized size	1	1.	0.8	0.71	0.93	2.16	1.2	1.09
time (sec)	N/A	0.01	0.014	0.004	2.788	1.449	2.663	2.485

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	32	27	15
normalized size	1	1.	1.	0.8	1.	2.13	1.8	1.
time (sec)	N/A	0.002	0.002	0.001	1.23	1.495	0.184	2.556

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	77	22	39
normalized size	1	1.	1.	0.74	0.96	2.85	0.81	1.44
time (sec)	N/A	0.004	0.006	0.002	2.29	1.445	0.193	1.338

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	120	39	51
normalized size	1	1.	1.	0.83	0.87	4.	1.3	1.7
time (sec)	N/A	0.016	0.004	0.003	3.551	1.442	1.224	2.472

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	28	84	19	54
normalized size	1	1.	1.	1.36	1.12	3.36	0.76	2.16
time (sec)	N/A	0.005	0.006	0.002	3.049	1.455	0.231	1.966

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	47	149	24	58
normalized size	1	1.	0.95	1.05	1.21	3.82	0.62	1.49
time (sec)	N/A	0.016	0.018	0.004	3.54	1.553	1.713	2.231

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	53	34	57
normalized size	1	1.	1.	0.83	1.06	2.94	1.89	3.17
time (sec)	N/A	0.003	0.003	0.003	3.611	1.514	1.003	2.455

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	66	166	63	74
normalized size	1	1.	0.56	0.96	1.16	2.91	1.11	1.3
time (sec)	N/A	0.022	0.005	0.006	3.952	1.512	3.473	2.685

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	77	61	70
normalized size	1	1.	0.59	0.74	1.17	1.67	1.33	1.52
time (sec)	N/A	0.021	0.012	0.002	3.888	1.495	1.928	2.339

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	39	46	61	132	167	45
normalized size	1	1.	0.62	0.73	0.97	2.1	2.65	0.71
time (sec)	N/A	0.016	0.012	0.006	4.078	1.542	4.52	2.558

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	59	44	43
normalized size	1	1.	0.71	0.94	1.13	1.9	1.42	1.39
time (sec)	N/A	0.015	0.009	0.003	3.107	1.491	0.591	2.693

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	36	32	42	111	124	35
normalized size	1	1.	0.8	0.71	0.93	2.47	2.76	0.78
time (sec)	N/A	0.01	0.016	0.003	3.074	1.548	2.652	2.259

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	47	27	15
normalized size	1	1.	1.	1.47	1.	3.13	1.8	1.
time (sec)	N/A	0.002	0.002	0.001	2.8	1.497	0.187	2.69

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	90	22	26
normalized size	1	1.	1.	0.74	0.96	3.33	0.81	0.96
time (sec)	N/A	0.003	0.006	0.002	3.625	1.535	0.193	2.566

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	47	70	76	54
normalized size	1	1.	1.	0.83	1.57	2.33	2.53	1.8
time (sec)	N/A	0.015	0.004	0.003	2.275	1.558	1.308	2.416

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	28	88	20	53
normalized size	1	1.	1.	1.36	1.12	3.52	0.8	2.12
time (sec)	N/A	0.005	0.006	0.002	3.75	1.514	0.231	2.339

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	69	93	97	61
normalized size	1	1.	0.95	1.05	1.77	2.38	2.49	1.56
time (sec)	N/A	0.016	0.025	0.004	3.903	1.508	1.807	2.565

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	53	76	99
normalized size	1	1.	1.	1.39	1.06	2.94	4.22	5.5
time (sec)	N/A	0.003	0.003	0.003	3.656	1.456	1.037	2.001

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	88	112	139	77
normalized size	1	1.	0.56	0.96	1.54	1.96	2.44	1.35
time (sec)	N/A	0.022	0.005	0.004	2.872	1.533	3.522	1.983

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	76	61	46
normalized size	1	1.	0.59	0.74	1.17	1.65	1.33	1.
time (sec)	N/A	0.02	0.009	0.003	3.409	1.431	1.917	2.216

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	49	61	77	119	167	59
normalized size	1	1.	0.68	0.85	1.07	1.65	2.32	0.82
time (sec)	N/A	0.019	0.011	0.006	2.721	1.474	4.59	2.538

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	58	44	31
normalized size	1	1.	0.71	0.94	1.13	1.87	1.42	1.
time (sec)	N/A	0.014	0.006	0.003	2.42	1.494	0.589	1.919

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	47	58	97	124	50
normalized size	1	1.	0.85	0.87	1.07	1.8	2.3	0.93
time (sec)	N/A	0.013	0.013	0.004	3.997	1.465	2.726	2.218

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	32	27	15
normalized size	1	1.	1.	1.47	1.	2.13	1.8	1.
time (sec)	N/A	0.002	0.002	0.003	2.649	1.431	0.186	1.398

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	42	77	22	41
normalized size	1	1.	1.03	0.97	1.17	2.14	0.61	1.14
time (sec)	N/A	0.006	0.006	0.003	3.191	1.487	0.196	1.588

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	78	82	32
normalized size	1	1.	1.	0.83	0.87	2.6	2.73	1.07
time (sec)	N/A	0.017	0.004	0.004	4.401	1.499	1.334	2.695

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	48	48	45	84	19	59
normalized size	1	1.	1.41	1.41	1.32	2.47	0.56	1.74
time (sec)	N/A	0.008	0.009	0.003	2.816	1.495	0.232	2.209

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	55	41	47	100	97	39
normalized size	1	1.	1.41	1.05	1.21	2.56	2.49	1.
time (sec)	N/A	0.016	0.007	0.003	3.939	1.562	1.781	2.561

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	51	76	57
normalized size	1	1.	1.	1.39	1.06	2.83	4.22	3.17
time (sec)	N/A	0.003	0.002	0.002	3.602	1.452	1.054	2.189

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	66	120	139	55
normalized size	1	1.	0.56	0.96	1.16	2.11	2.44	0.96
time (sec)	N/A	0.021	0.004	0.003	3.5	1.431	3.516	2.129

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	77	68	46
normalized size	1	1.	0.59	0.52	1.17	1.67	1.48	1.
time (sec)	N/A	0.022	0.012	0.003	3.089	1.275	1.919	2.181

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	55	61	204	83	45
normalized size	1	1.	0.67	0.76	0.85	2.83	1.15	0.62
time (sec)	N/A	0.021	0.015	0.007	3.455	1.336	4.554	2.395

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	59	49	31
normalized size	1	1.	0.71	0.61	1.13	1.9	1.58	1.
time (sec)	N/A	0.014	0.01	0.002	4.208	1.271	0.603	2.769

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	41	42	180	61	35
normalized size	1	1.	0.8	0.76	0.78	3.33	1.13	0.65
time (sec)	N/A	0.014	0.016	0.003	3.421	1.259	2.702	1.544

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	47	31	15
normalized size	1	1.	1.	0.8	1.	3.13	2.07	1.
time (sec)	N/A	0.002	0.002	0.002	2.097	1.261	0.203	1.583

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	26	154	34	26
normalized size	1	1.	1.	0.81	0.72	4.28	0.94	0.72
time (sec)	N/A	0.006	0.008	0.002	4.26	1.267	0.379	2.367

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	47	142	44	35
normalized size	1	1.	1.	0.83	1.57	4.73	1.47	1.17
time (sec)	N/A	0.015	0.004	0.004	3.304	1.289	1.241	2.428

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	49	43	28	147	32	58
normalized size	1	1.	1.44	1.26	0.82	4.32	0.94	1.71
time (sec)	N/A	0.008	0.007	0.003	3.476	1.345	0.42	1.339

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	55	41	69	170	27	39
normalized size	1	1.	1.41	1.05	1.77	4.36	0.69	1.
time (sec)	N/A	0.016	0.014	0.003	3.571	1.257	1.745	2.001

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	39	37	109
normalized size	1	1.	1.	0.83	1.06	2.17	2.06	6.06
time (sec)	N/A	0.003	0.002	0.003	3.314	1.235	1.027	1.893

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	88	192	68	58
normalized size	1	1.	0.56	0.96	1.54	3.37	1.19	1.02
time (sec)	N/A	0.021	0.005	0.004	4.017	1.319	3.537	2.925

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	39	36	0	78	68	58
normalized size	1	1.	0.7	0.64	0.	1.39	1.21	1.04
time (sec)	N/A	0.031	0.018	0.004	0.	1.275	0.781	2.498

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	59	0	300	95	73
normalized size	1	1.	0.85	0.81	0.	4.11	1.3	1.
time (sec)	N/A	0.02	0.023	0.005	0.	1.321	3.709	2.799

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	0	53	44	36
normalized size	1	1.	0.75	0.69	0.	1.47	1.22	1.
time (sec)	N/A	0.022	0.012	0.005	0.	1.279	0.467	1.432

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	39	0	238	42	54
normalized size	1	1.	1.	0.8	0.	4.86	0.86	1.1
time (sec)	N/A	0.012	0.017	0.004	0.	1.33	2.086	2.853

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	26	20	18
normalized size	1	1.	1.	0.93	1.2	1.73	1.33	1.2
time (sec)	N/A	0.003	0.002	0.002	2.233	1.188	0.375	1.413

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	153	17	31
normalized size	1	1.	1.	0.84	0.	6.12	0.68	1.24
time (sec)	N/A	0.006	0.004	0.003	0.	1.385	1.01	2.334

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	154	19	30
normalized size	1	1.	1.	1.16	0.	6.16	0.76	1.2
time (sec)	N/A	0.017	0.005	0.004	0.	1.334	1.053	2.32

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	0	32	19	41
normalized size	1	1.	1.	0.95	0.	1.68	1.	2.16
time (sec)	N/A	0.005	0.003	0.005	0.	1.278	0.568	2.403

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	61	48	0	263	42	65
normalized size	1	1.	1.22	0.96	0.	5.26	0.84	1.3
time (sec)	N/A	0.026	0.047	0.005	0.	1.351	2.265	2.884

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	0	61	46	74
normalized size	1	1.	0.66	0.59	0.	1.39	1.05	1.68
time (sec)	N/A	0.01	0.006	0.003	0.	1.331	0.792	1.673

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	37	68	0	323	97	89
normalized size	1	1.	0.5	0.92	0.	4.36	1.31	1.2
time (sec)	N/A	0.038	0.007	0.005	0.	1.313	4.099	2.359

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	36	0	93	68	55
normalized size	1	1.	0.69	0.65	0.	1.69	1.24	1.
time (sec)	N/A	0.033	0.016	0.004	0.	1.296	0.943	1.956

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	71	57	0	362	71	69
normalized size	1	1.	1.04	0.84	0.	5.32	1.04	1.01
time (sec)	N/A	0.023	0.033	0.004	0.	1.344	3.082	1.681

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	23	0	66	41	34
normalized size	1	1.	0.75	0.72	0.	2.06	1.28	1.06
time (sec)	N/A	0.022	0.011	0.003	0.	1.299	0.559	2.116

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	59	37	0	300	37	53
normalized size	1	1.	1.37	0.86	0.	6.98	0.86	1.23
time (sec)	N/A	0.013	0.051	0.005	0.	1.326	1.652	1.862

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	46	24	19
normalized size	1	1.	1.	0.94	1.19	2.88	1.5	1.19
time (sec)	N/A	0.003	0.003	0.002	2.861	1.309	0.54	1.998

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	47	17	19
normalized size	1	1.	1.	0.94	1.19	2.94	1.06	1.19
time (sec)	N/A	0.002	0.003	0.002	3.009	1.176	0.551	3.087

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	43	0	294	184	53
normalized size	1	1.	0.8	1.05	0.	7.17	4.49	1.29
time (sec)	N/A	0.027	0.006	0.004	0.	1.377	1.695	1.561

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	0	70	46	68
normalized size	1	1.	0.71	0.68	0.	1.84	1.21	1.79
time (sec)	N/A	0.008	0.006	0.003	0.	1.275	0.788	2.614

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	68	35	63	0	382	73	89
normalized size	1	0.99	0.51	0.91	0.	5.54	1.06	1.29
time (sec)	N/A	0.039	0.008	0.004	0.	1.355	3.193	1.726

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	40	37	0	99	233	143
normalized size	1	1.	0.61	0.56	0.	1.5	3.53	2.17
time (sec)	N/A	0.016	0.008	0.004	0.	1.215	1.196	2.169

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	75	0	506	367	88
normalized size	1	1.	0.99	0.82	0.	5.56	4.03	0.97
time (sec)	N/A	0.029	0.13	0.007	0.	1.396	4.717	2.709

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	39	36	0	119	138	58
normalized size	1	1.	0.72	0.67	0.	2.2	2.56	1.07
time (sec)	N/A	0.031	0.017	0.003	0.	1.27	1.076	2.353

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	80	54	0	444	303	69
normalized size	1	1.	1.25	0.84	0.	6.94	4.73	1.08
time (sec)	N/A	0.02	0.111	0.006	0.	1.322	2.774	2.563

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	45	97	92	32
normalized size	1	1.	0.78	0.69	1.25	2.69	2.56	0.89
time (sec)	N/A	0.022	0.011	0.005	2.361	1.301	1.017	2.551

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	46	77	44	23
normalized size	1	1.	1.	0.86	2.19	3.67	2.1	1.1
time (sec)	N/A	0.005	0.005	0.004	3.022	1.241	0.708	2.046

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	73	46	19
normalized size	1	1.	1.	0.83	1.06	4.06	2.56	1.06
time (sec)	N/A	0.003	0.003	0.002	3.31	1.349	0.969	2.805

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	99	95	36
normalized size	1	1.	0.74	0.67	1.08	2.54	2.44	0.92
time (sec)	N/A	0.005	0.007	0.003	2.986	1.296	0.777	2.301

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	36	57	0	440	740	68
normalized size	1	1.	0.61	0.97	0.	7.46	12.54	1.15
time (sec)	N/A	0.035	0.006	0.005	0.	1.348	2.733	1.555

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	39	0	123	165	86
normalized size	1	1.	0.7	0.65	0.	2.05	2.75	1.43
time (sec)	N/A	0.014	0.008	0.005	0.	1.34	1.311	1.881

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	92	37	78	0	527	864	100
normalized size	1	1.05	0.42	0.89	0.	5.99	9.82	1.14
time (sec)	N/A	0.05	0.007	0.005	0.	1.426	4.962	2.139

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	53	48	0	144	354	163
normalized size	1	1.	0.62	0.56	0.	1.67	4.12	1.9
time (sec)	N/A	0.022	0.01	0.005	0.	1.319	1.88	2.755

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	114	111	0	798	3181	123
normalized size	1	1.	0.87	0.85	0.	6.09	24.28	0.94
time (sec)	N/A	0.054	0.189	0.029	0.	1.432	11.999	1.888

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	61	58	0	217	454	96
normalized size	1	1.	0.65	0.62	0.	2.31	4.83	1.02
time (sec)	N/A	0.053	0.028	0.004	0.	1.344	5.563	1.727

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	101	88	0	736	2980	105
normalized size	1	1.	0.95	0.83	0.	6.94	28.11	0.99
time (sec)	N/A	0.042	0.127	0.015	0.	1.461	8.096	2.367

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	47	99	190	364	74
normalized size	1	1.	0.67	0.63	1.32	2.53	4.85	0.99
time (sec)	N/A	0.045	0.024	0.004	1.086	1.32	5.452	2.279

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	139	120	95	23
normalized size	1	1.	1.	0.86	6.62	5.71	4.52	1.1
time (sec)	N/A	0.005	0.006	0.004	1.318	1.327	2.004	3.004

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	72	167	272	55
normalized size	1	1.	0.66	0.61	1.22	2.83	4.61	0.93
time (sec)	N/A	0.033	0.018	0.006	1.491	1.313	5.39	2.458

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	115	146	199	39
normalized size	1	1.	0.7	0.64	2.61	3.32	4.52	0.89
time (sec)	N/A	0.011	0.012	0.005	1.851	1.271	2.041	3.01

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	45	142	180	32
normalized size	1	1.	0.74	0.66	1.18	3.74	4.74	0.84
time (sec)	N/A	0.023	0.014	0.005	2.013	1.336	5.252	2.391

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	95	171	517	58
normalized size	1	1.	0.62	0.57	1.4	2.51	7.6	0.85
time (sec)	N/A	0.021	0.013	0.005	2.704	1.36	2.319	2.578

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	116	90	19
normalized size	1	1.	1.	0.83	1.06	6.44	5.	1.06
time (sec)	N/A	0.004	0.004	0.003	2.299	1.372	5.127	1.518

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	82	192	1265	74
normalized size	1	1.	0.66	0.62	1.06	2.49	16.43	0.96
time (sec)	N/A	0.016	0.012	0.003	2.457	1.32	3.049	1.427

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	36	85	0	732	5250	109
normalized size	1	1.	0.38	0.89	0.	7.71	55.26	1.15
time (sec)	N/A	0.059	0.006	0.006	0.	1.483	7.725	3.135

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	61	0	221	400	122
normalized size	1	1.	0.64	0.61	0.	2.21	4.	1.22
time (sec)	N/A	0.026	0.011	0.004	0.	1.388	3.851	2.091

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	132	37	108	0	824	5540	142
normalized size	1	1.05	0.29	0.86	0.	6.54	43.97	1.13
time (sec)	N/A	0.078	0.009	0.006	0.	1.55	12.391	1.825

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	75	72	0	250	668	198
normalized size	1	1.	0.57	0.55	0.	1.89	5.06	1.5
time (sec)	N/A	0.041	0.013	0.004	0.	1.776	5.328	2.376

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	58	44	46
normalized size	1	1.	0.59	0.52	1.17	1.26	0.96	1.
time (sec)	N/A	0.019	0.008	0.003	3.06	1.542	1.132	2.68

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	34	45	103	39	49
normalized size	1	1.	0.76	0.76	1.	2.29	0.87	1.09
time (sec)	N/A	0.009	0.011	0.004	3.571	1.449	0.672	3.024

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	46	27	31
normalized size	1	1.	0.71	0.61	1.13	1.48	0.87	1.
time (sec)	N/A	0.014	0.006	0.003	2.965	1.291	0.339	2.177

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	78	22	39
normalized size	1	1.	1.	0.74	0.96	2.89	0.81	1.44
time (sec)	N/A	0.006	0.005	0.005	3.103	1.269	0.203	2.974

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	28	10	15
normalized size	1	1.	1.	0.8	1.	1.87	0.67	1.
time (sec)	N/A	0.002	0.001	0.002	2.951	1.29	0.133	2.462

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	46	7	22
normalized size	1	1.	1.	0.7	0.8	4.6	0.7	2.2
time (sec)	N/A	0.001	0.003	0.002	1.92	1.258	0.133	2.548

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	12	103	8	39
normalized size	1	1.	1.	0.75	0.6	5.15	0.4	1.95
time (sec)	N/A	0.01	0.002	0.004	2.99	1.242	0.998	2.201

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	43	15	31
normalized size	1	1.	1.	0.83	1.06	2.39	0.83	1.72
time (sec)	N/A	0.003	0.002	0.002	2.124	1.219	0.707	2.27

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	32	149	44	58
normalized size	1	1.	0.95	0.77	0.82	3.82	1.13	1.49
time (sec)	N/A	0.017	0.011	0.005	1.908	1.232	2.177	1.728

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	22	39	68	32	57
normalized size	1	1.	0.73	0.59	1.05	1.84	0.86	1.54
time (sec)	N/A	0.007	0.004	0.002	3.561	1.256	1.455	1.931

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	51	167	63	74
normalized size	1	1.	0.56	0.77	0.89	2.93	1.11	1.3
time (sec)	N/A	0.023	0.005	0.003	2.976	1.369	4.46	3.022

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	61	46	58
normalized size	1	1.	0.59	0.74	1.17	1.33	1.	1.26
time (sec)	N/A	0.019	0.008	0.003	1.806	1.27	1.16	2.936

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	34	45	117	39	35
normalized size	1	1.	0.76	0.76	1.	2.6	0.87	0.78
time (sec)	N/A	0.01	0.012	0.004	3.611	1.287	0.671	2.52

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	49	29	31
normalized size	1	1.	0.71	0.94	1.13	1.58	0.94	1.
time (sec)	N/A	0.014	0.006	0.003	3.204	1.251	0.344	2.569

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	92	22	26
normalized size	1	1.	1.	0.74	0.96	3.41	0.81	0.96
time (sec)	N/A	0.005	0.005	0.003	2.578	1.289	0.202	2.962

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	31	12	15
normalized size	1	1.	1.	1.47	1.	2.07	0.8	1.
time (sec)	N/A	0.002	0.001	0.002	2.501	1.197	0.135	2.812

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	53	7	8
normalized size	1	1.	1.	0.7	0.8	5.3	0.7	0.8
time (sec)	N/A	0.001	0.004	0.003	3.718	1.265	0.134	2.095

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	34	47	26	42
normalized size	1	1.	1.	0.75	1.7	2.35	1.3	2.1
time (sec)	N/A	0.01	0.003	0.004	2.489	1.274	1.058	2.648

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	34	41	45
normalized size	1	1.	1.	1.39	1.06	1.89	2.28	2.5
time (sec)	N/A	0.003	0.002	0.002	1.771	1.271	0.739	2.48

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	54	93	99	61
normalized size	1	1.	0.95	0.77	1.38	2.38	2.54	1.56
time (sec)	N/A	0.016	0.01	0.004	3.389	1.278	2.246	1.718

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	32	39	55	80	99
normalized size	1	1.	0.73	0.86	1.05	1.49	2.16	2.68
time (sec)	N/A	0.007	0.004	0.002	3.453	1.211	1.499	2.576

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	73	111	136	77
normalized size	1	1.	0.56	0.77	1.28	1.95	2.39	1.35
time (sec)	N/A	0.024	0.005	0.004	2.646	1.371	4.445	2.4

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	58	44	46
normalized size	1	1.	0.59	0.74	1.17	1.26	0.96	1.
time (sec)	N/A	0.019	0.009	0.003	2.524	1.324	1.123	1.413

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	61	103	39	50
normalized size	1	1.	0.8	0.91	1.13	1.91	0.72	0.93
time (sec)	N/A	0.013	0.013	0.005	3.979	1.312	0.686	1.901

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	46	27	31
normalized size	1	1.	0.71	0.94	1.13	1.48	0.87	1.
time (sec)	N/A	0.013	0.006	0.003	4.581	1.205	0.34	2.706

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	35	42	78	22	41
normalized size	1	1.	1.	0.97	1.17	2.17	0.61	1.14
time (sec)	N/A	0.008	0.005	0.005	1.849	1.283	0.205	2.159

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	28	10	15
normalized size	1	1.	1.	1.47	1.	1.87	0.67	1.
time (sec)	N/A	0.002	0.001	0.002	2.956	1.188	0.136	2.68

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	43	22	24	46	7	23
normalized size	1	1.	2.26	1.16	1.26	2.42	0.37	1.21
time (sec)	N/A	0.003	0.003	0.002	3.601	1.286	0.133	2.607

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	12	57	26	19
normalized size	1	1.	1.	0.75	0.6	2.85	1.3	0.95
time (sec)	N/A	0.01	0.003	0.003	1.982	1.303	1.054	2.143

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	42	37	31
normalized size	1	1.	1.	1.39	1.06	2.33	2.06	1.72
time (sec)	N/A	0.003	0.002	0.002	3.098	1.287	0.746	1.835

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	54	30	32	101	99	39
normalized size	1	1.	1.38	0.77	0.82	2.59	2.54	1.
time (sec)	N/A	0.016	0.021	0.003	1.939	1.341	2.233	2.464

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	32	39	68	76	57
normalized size	1	1.	0.68	0.86	1.05	1.84	2.05	1.54
time (sec)	N/A	0.007	0.004	0.003	3.329	1.218	1.515	2.174

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	51	120	136	55
normalized size	1	1.	0.56	0.77	0.89	2.11	2.39	0.96
time (sec)	N/A	0.023	0.005	0.004	3.161	1.251	4.397	2.492

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	61	49	46
normalized size	1	1.	0.59	0.52	1.17	1.33	1.07	1.
time (sec)	N/A	0.019	0.008	0.003	3.953	1.297	1.17	2.1

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	43	45	186	53	35
normalized size	1	1.	0.8	0.8	0.83	3.44	0.98	0.65
time (sec)	N/A	0.013	0.014	0.005	1.879	1.365	0.855	2.254

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	49	31	31
normalized size	1	1.	0.71	0.61	1.13	1.58	1.	1.
time (sec)	N/A	0.014	0.008	0.001	3.195	1.226	0.357	1.684

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	26	158	36	26
normalized size	1	1.	1.	0.81	0.72	4.39	1.	0.72
time (sec)	N/A	0.007	0.006	0.003	3.832	1.264	0.392	1.536

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	31	14	15
normalized size	1	1.	1.	0.8	1.	2.07	0.93	1.
time (sec)	N/A	0.002	0.001	0.002	1.897	1.2	0.147	2.402

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	8	120	17	8
normalized size	1	1.	1.	0.84	0.42	6.32	0.89	0.42
time (sec)	N/A	0.003	0.003	0.003	2.556	1.266	0.319	2.441

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	34	123	8	19
normalized size	1	1.	1.	0.75	1.7	6.15	0.4	0.95
time (sec)	N/A	0.01	0.002	0.004	1.897	1.274	1.029	2.676

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	32	15	50
normalized size	1	1.	1.	0.83	1.06	1.78	0.83	2.78
time (sec)	N/A	0.003	0.002	0.003	3.663	1.189	0.727	2.569

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	54	30	54	174	46	39
normalized size	1	1.	1.38	0.77	1.38	4.46	1.18	1.
time (sec)	N/A	0.016	0.022	0.004	2.957	1.192	2.208	2.677

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	39	55	36	109
normalized size	1	1.	0.68	0.59	1.05	1.49	0.97	2.95
time (sec)	N/A	0.007	0.006	0.003	3.6	1.267	1.477	2.575

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	C	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	73	192	65	58
normalized size	1	1.	0.56	0.77	1.28	3.37	1.14	1.02
time (sec)	N/A	0.023	0.005	0.004	3.634	1.329	4.467	2.878

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	0	157	14	30
normalized size	1	1.	1.	1.24	0.	9.24	0.82	1.76
time (sec)	N/A	0.002	0.007	0.002	0.	1.261	0.943	2.499

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	0	146	39	36
normalized size	1	1.	1.	1.24	0.	8.59	2.29	2.12
time (sec)	N/A	0.003	0.007	0.003	0.	1.276	1.	2.401

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	151	39	31
normalized size	1	1.	1.	0.84	0.	6.04	1.56	1.24
time (sec)	N/A	0.006	0.006	0.002	0.	1.402	1.006	1.622

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	0	173	17	38
normalized size	1	1.	1.	0.81	0.	6.65	0.65	1.46
time (sec)	N/A	0.005	0.006	0.003	0.	1.291	0.948	1.388

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	0	157	17	30
normalized size	1	1.	1.	1.11	0.	8.26	0.89	1.58
time (sec)	N/A	0.006	0.008	0.005	0.	1.353	0.935	3.006

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	0	159	46	38
normalized size	1	1.	1.	1.11	0.	8.37	2.42	2.
time (sec)	N/A	0.003	0.007	0.006	0.	1.285	1.011	2.153

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	178	46	34
normalized size	1	1.	1.	0.85	0.	6.59	1.7	1.26
time (sec)	N/A	0.006	0.006	0.003	0.	1.305	1.023	1.622

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	178	20	41
normalized size	1	1.	1.	0.82	0.	6.36	0.71	1.46
time (sec)	N/A	0.005	0.006	0.004	0.	1.432	0.947	1.508

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	153	17	31
normalized size	1	1.	1.	0.84	0.	6.12	0.68	1.24
time (sec)	N/A	0.005	0.005	0.001	0.	1.263	0.996	1.693

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	0	171	48	38
normalized size	1	1.	1.	0.81	0.	6.58	1.85	1.46
time (sec)	N/A	0.005	0.006	0.003	0.	1.201	1.054	2.083

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	151	48	34
normalized size	1	1.	1.	0.85	0.	5.59	1.78	1.26
time (sec)	N/A	0.006	0.006	0.002	0.	1.316	1.07	2.458

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	173	20	41
normalized size	1	1.	1.	0.82	0.	6.18	0.71	1.46
time (sec)	N/A	0.005	0.006	0.003	0.	1.368	1.006	1.675

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	11	50	20	12
normalized size	1	1.	1.	0.94	0.69	3.12	1.25	0.75
time (sec)	N/A	0.002	0.003	0.003	2.805	1.208	0.986	2.172

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	103	152	0	0	0	0
normalized size	1	1.	0.56	0.83	0.	0.	0.	0.
time (sec)	N/A	0.133	0.062	0.042	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	85	221	0	0	46	0
normalized size	1	1.	0.28	0.73	0.	0.	0.15	0.
time (sec)	N/A	0.23	0.046	0.02	0.	0.	52.401	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	85	138	0	0	46	0
normalized size	1	1.	0.56	0.9	0.	0.	0.3	0.
time (sec)	N/A	0.09	0.041	0.017	0.	0.	4.265	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	56	205	0	0	46	0
normalized size	1	1.	0.21	0.76	0.	0.	0.17	0.
time (sec)	N/A	0.194	0.012	0.018	0.	0.	0.961	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	54	119	0	0	46	0
normalized size	1	1.	0.43	0.94	0.	0.	0.37	0.
time (sec)	N/A	0.074	0.012	0.018	0.	0.	0.843	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	54	194	0	0	49	0
normalized size	1	1.	0.21	0.74	0.	0.	0.19	0.
time (sec)	N/A	0.194	0.014	0.026	0.	0.	1.371	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	56	120	0	0	49	0
normalized size	1	1.	0.44	0.95	0.	0.	0.39	0.
time (sec)	N/A	0.076	0.013	0.019	0.	0.	4.626	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	56	219	0	0	53	0
normalized size	1	1.	0.18	0.72	0.	0.	0.17	0.
time (sec)	N/A	0.222	0.013	0.025	0.	0.	62.677	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	102	163	0	0	0	0
normalized size	1	1.	0.48	0.77	0.	0.	0.	0.
time (sec)	N/A	0.13	0.077	0.019	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	89	232	0	0	46	0
normalized size	1	1.	0.27	0.71	0.	0.	0.14	0.
time (sec)	N/A	0.256	0.062	0.021	0.	0.	91.753	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	89	150	0	0	46	0
normalized size	1	1.	0.49	0.83	0.	0.	0.25	0.
time (sec)	N/A	0.105	0.054	0.011	0.	0.	15.549	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	57	218	0	0	46	0
normalized size	1	1.	0.19	0.73	0.	0.	0.15	0.
time (sec)	N/A	0.224	0.012	0.01	0.	0.	4.213	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	55	134	0	0	46	0
normalized size	1	1.	0.36	0.88	0.	0.	0.3	0.
time (sec)	N/A	0.09	0.01	0.01	0.	0.	2.839	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	55	208	0	0	49	0
normalized size	1	1.	0.19	0.7	0.	0.	0.17	0.
time (sec)	N/A	0.23	0.013	0.014	0.	0.	2.904	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	57	125	0	0	49	0
normalized size	1	1.	0.38	0.82	0.	0.	0.32	0.
time (sec)	N/A	0.093	0.013	0.012	0.	0.	7.814	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	57	216	0	0	53	0
normalized size	1	1.	0.19	0.73	0.	0.	0.18	0.
time (sec)	N/A	0.227	0.012	0.014	0.	0.	62.726	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	57	135	0	0	0	0
normalized size	1	1.	0.38	0.89	0.	0.	0.	0.
time (sec)	N/A	0.092	0.015	0.023	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	57	234	0	0	0	0
normalized size	1	1.	0.17	0.71	0.	0.	0.	0.
time (sec)	N/A	0.27	0.013	0.029	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	74	237	0	0	53	0
normalized size	1	1.	0.58	1.85	0.	0.	0.41	0.
time (sec)	N/A	0.064	0.029	0.045	0.	0.	43.454	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	74	133	0	0	53	0
normalized size	1	1.	0.63	1.14	0.	0.	0.45	0.
time (sec)	N/A	0.077	0.023	0.028	0.	0.	4.679	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	51	229	0	0	53	0
normalized size	1	1.	0.52	2.31	0.	0.	0.54	0.
time (sec)	N/A	0.043	0.013	0.03	0.	0.	0.902	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	51	124	0	0	53	0
normalized size	1	1.	0.54	1.32	0.	0.	0.56	0.
time (sec)	N/A	0.047	0.012	0.029	0.	0.	0.791	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	51	225	0	0	56	0
normalized size	1	1.	0.52	2.3	0.	0.	0.57	0.
time (sec)	N/A	0.042	0.012	0.034	0.	0.	1.346	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	51	129	0	0	49	0
normalized size	1	1.	0.53	1.34	0.	0.	0.51	0.
time (sec)	N/A	0.048	0.013	0.029	0.	0.	5.296	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	87	141	0	0	44	0
normalized size	1	1.	0.56	0.9	0.	0.	0.28	0.
time (sec)	N/A	0.09	0.031	0.02	0.	0.	132.12	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	69	210	0	0	44	0
normalized size	1	1.	0.25	0.77	0.	0.	0.16	0.
time (sec)	N/A	0.186	0.025	0.018	0.	0.	33.328	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	69	125	0	0	44	0
normalized size	1	1.	0.54	0.98	0.	0.	0.35	0.
time (sec)	N/A	0.071	0.023	0.008	0.	0.	2.593	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	56	132	0	0	44	0
normalized size	1	1.	0.24	0.56	0.	0.	0.19	0.
time (sec)	N/A	0.166	0.012	0.008	0.	0.	0.768	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	54	104	0	0	44	0
normalized size	1	1.	0.56	1.07	0.	0.	0.45	0.
time (sec)	N/A	0.058	0.013	0.013	0.	0.	0.989	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	54	196	0	0	48	0
normalized size	1	1.	0.2	0.73	0.	0.	0.18	0.
time (sec)	N/A	0.188	0.012	0.014	0.	0.	1.889	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	56	123	0	0	48	0
normalized size	1	1.	0.43	0.95	0.	0.	0.37	0.
time (sec)	N/A	0.073	0.013	0.013	0.	0.	8.111	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	56	219	0	0	51	0
normalized size	1	1.	0.18	0.72	0.	0.	0.17	0.
time (sec)	N/A	0.22	0.012	0.016	0.	0.	72.705	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	74	128	0	0	44	0
normalized size	1	1.	0.48	0.84	0.	0.	0.29	0.
time (sec)	N/A	0.087	0.031	0.031	0.	0.	159.68	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	60	197	0	0	44	0
normalized size	1	1.	0.23	0.74	0.	0.	0.17	0.
time (sec)	N/A	0.189	0.028	0.027	0.	0.	40.151	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	59	115	0	0	44	0
normalized size	1	1.	0.47	0.92	0.	0.	0.35	0.
time (sec)	N/A	0.071	0.022	0.012	0.	0.	2.945	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	59	197	0	0	44	0
normalized size	1	1.	0.22	0.74	0.	0.	0.17	0.
time (sec)	N/A	0.187	0.012	0.013	0.	0.	1.358	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	59	114	0	0	44	0
normalized size	1	1.	0.47	0.9	0.	0.	0.35	0.
time (sec)	N/A	0.07	0.018	0.017	0.	0.	2.112	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	57	197	0	0	48	0
normalized size	1	1.	0.19	0.67	0.	0.	0.16	0.
time (sec)	N/A	0.219	0.013	0.018	0.	0.	5.162	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	59	124	0	0	48	0
normalized size	1	1.	0.38	0.81	0.	0.	0.31	0.
time (sec)	N/A	0.09	0.013	0.017	0.	0.	46.897	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	59	219	0	0	0	0
normalized size	1	1.	0.18	0.66	0.	0.	0.	0.
time (sec)	N/A	0.254	0.013	0.018	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	80	219	0	0	44	0
normalized size	1	1.	0.52	1.41	0.	0.	0.28	0.
time (sec)	N/A	0.089	0.051	0.031	0.	0.	159.189	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	74	385	0	0	44	0
normalized size	1	1.	0.24	1.27	0.	0.	0.14	0.
time (sec)	N/A	0.216	0.039	0.03	0.	0.	40.143	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	79	218	0	0	44	0
normalized size	1	1.	0.51	1.4	0.	0.	0.28	0.
time (sec)	N/A	0.088	0.039	0.012	0.	0.	8.205	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	59	382	0	0	44	0
normalized size	1	1.	0.2	1.26	0.	0.	0.15	0.
time (sec)	N/A	0.224	0.012	0.013	0.	0.	4.783	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	79	216	0	0	44	0
normalized size	1	1.	0.5	1.38	0.	0.	0.28	0.
time (sec)	N/A	0.088	0.035	0.019	0.	0.	8.716	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	57	384	0	0	48	0
normalized size	1	1.	0.17	1.15	0.	0.	0.14	0.
time (sec)	N/A	0.255	0.013	0.02	0.	0.	45.923	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	59	227	0	0	48	0
normalized size	1	1.	0.32	1.23	0.	0.	0.26	0.
time (sec)	N/A	0.11	0.013	0.018	0.	0.	85.211	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	59	410	0	0	0	0
normalized size	1	1.	0.16	1.13	0.	0.	0.	0.
time (sec)	N/A	0.295	0.013	0.019	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	61	235	0	0	51	0
normalized size	1	1.	0.57	2.2	0.	0.	0.48	0.
time (sec)	N/A	0.043	0.032	0.031	0.	0.	27.617	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	61	131	0	0	51	0
normalized size	1	1.	0.69	1.49	0.	0.	0.58	0.
time (sec)	N/A	0.047	0.021	0.029	0.	0.	2.867	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	165	0	0	51	0
normalized size	1	1.	0.79	2.46	0.	0.	0.76	0.
time (sec)	N/A	0.03	0.015	0.016	0.	0.	0.736	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	117	0	0	51	0
normalized size	1	1.	0.89	1.86	0.	0.	0.81	0.
time (sec)	N/A	0.034	0.016	0.025	0.	0.	0.963	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	51	228	0	0	54	0
normalized size	1	1.	0.48	2.13	0.	0.	0.5	0.
time (sec)	N/A	0.043	0.016	0.021	0.	0.	1.947	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	53	132	0	0	54	0
normalized size	1	1.	0.54	1.35	0.	0.	0.55	0.
time (sec)	N/A	0.046	0.015	0.018	0.	0.	10.471	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	59	230	0	0	51	0
normalized size	1	1.	0.54	2.09	0.	0.	0.46	0.
time (sec)	N/A	0.042	0.023	0.04	0.	0.	33.531	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	59	126	0	0	51	0
normalized size	1	1.	0.63	1.34	0.	0.	0.54	0.
time (sec)	N/A	0.048	0.017	0.033	0.	0.	3.314	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	58	227	0	0	51	0
normalized size	1	1.	0.57	2.25	0.	0.	0.5	0.
time (sec)	N/A	0.039	0.014	0.02	0.	0.	1.35	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	59	122	0	0	51	0
normalized size	1	1.	0.61	1.27	0.	0.	0.53	0.
time (sec)	N/A	0.046	0.022	0.024	0.	0.	2.304	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	58	228	0	0	54	0
normalized size	1	1.	0.41	1.63	0.	0.	0.39	0.
time (sec)	N/A	0.056	0.02	0.023	0.	0.	6.012	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	58	133	0	0	54	0
normalized size	1	1.	0.44	1.01	0.	0.	0.41	0.
time (sec)	N/A	0.063	0.019	0.024	0.	0.	38.339	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	66	0	0	36	0
normalized size	1	1.	1.14	3.14	0.	0.	1.71	0.
time (sec)	N/A	0.009	0.007	0.045	0.	0.	0.674	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	23	73	0	0	32	0
normalized size	1	1.	0.34	1.09	0.	0.	0.48	0.
time (sec)	N/A	0.038	0.006	0.041	0.	0.	0.64	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	64	66	0	0	0	54	0
normalized size	1	1.28	1.32	0.	0.	0.	1.08	0.
time (sec)	N/A	0.019	0.02	0.02	0.	0.	4.392	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	54	0
normalized size	1	1.26	1.3	0.	0.	0.	1.08	0.
time (sec)	N/A	0.018	0.013	0.02	0.	0.	1.062	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	53	0
normalized size	1	1.26	1.3	0.	0.	0.	1.06	0.
time (sec)	N/A	0.018	0.015	0.022	0.	0.	0.873	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	66	68	0	0	0	53	0
normalized size	1	1.38	1.42	0.	0.	0.	1.1	0.
time (sec)	N/A	0.02	0.016	0.017	0.	0.	1.538	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	66	68	0	0	0	53	0
normalized size	1	1.32	1.36	0.	0.	0.	1.06	0.
time (sec)	N/A	0.02	0.017	0.018	0.	0.	5.086	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	54	0
normalized size	1	1.26	1.3	0.	0.	0.	1.08	0.
time (sec)	N/A	0.02	0.017	0.02	0.	0.	3.894	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	48	0
normalized size	1	1.26	1.3	0.	0.	0.	0.96	0.
time (sec)	N/A	0.02	0.017	0.022	0.	0.	2.145	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	53	0
normalized size	1	1.26	1.3	0.	0.	0.	1.06	0.
time (sec)	N/A	0.017	0.003	0.	0.	0.	0.861	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	57	57	0	0	0	41	0
normalized size	1	1.24	1.24	0.	0.	0.	0.89	0.
time (sec)	N/A	0.019	0.012	0.021	0.	0.	4.702	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	66	65	0	0	0	53	0
normalized size	1	1.29	1.27	0.	0.	0.	1.04	0.
time (sec)	N/A	0.02	0.018	0.024	0.	0.	26.473	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	104	16	22	39	202	0
normalized size	1	1.	6.12	0.94	1.29	2.29	11.88	0.
time (sec)	N/A	0.011	0.102	0.008	2.159	1.607	13.147	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	127	104	0	22	39	105	0
normalized size	1	7.47	6.12	0.	1.29	2.29	6.18	0.
time (sec)	N/A	0.066	0.051	0.034	1.561	1.579	9.516	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	103	14	18	39	97	0
normalized size	1	1.	6.87	0.93	1.2	2.6	6.47	0.
time (sec)	N/A	0.012	0.107	0.006	2.427	1.575	111.034	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	123	103	0	18	55	94	0
normalized size	1	8.2	6.87	0.	1.2	3.67	6.27	0.
time (sec)	N/A	0.065	0.04	0.033	2.345	1.591	11.068	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	135	1795	77
normalized size	1	1.	0.62	0.59	1.08	1.69	22.44	0.96
time (sec)	N/A	0.049	0.027	0.005	2.549	1.495	2.628	1.843

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	105	700	58
normalized size	1	1.	0.66	0.61	1.07	1.78	11.86	0.98
time (sec)	N/A	0.035	0.017	0.003	2.317	1.462	1.716	2.663

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	41	78	223	39
normalized size	1	1.	0.74	0.66	1.08	2.05	5.87	1.03
time (sec)	N/A	0.023	0.013	0.005	1.627	1.5	1.085	1.939

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	34	42	19
normalized size	1	1.	1.	0.83	1.06	1.89	2.33	1.06
time (sec)	N/A	0.003	0.003	0.003	1.147	1.419	0.184	2.353

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	126	0	0	313	46	132
normalized size	1	1.	1.25	0.	0.	3.1	0.46	1.31
time (sec)	N/A	0.079	0.052	0.028	0.	1.535	1.066	3.268

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	37	0	0	436	42	0
normalized size	1	1.	0.35	0.	0.	4.07	0.39	0.
time (sec)	N/A	0.071	0.007	0.031	0.	1.561	1.264	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	39	0	0	497	42	167
normalized size	1	1.	0.29	0.	0.	3.68	0.31	1.24
time (sec)	N/A	0.094	0.008	0.029	0.	1.532	1.571	3.356

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	94	0	0	0	29	0
normalized size	1	1.	0.3	0.	0.	0.	0.09	0.
time (sec)	N/A	0.283	0.051	0.028	0.	0.	0.812	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	62	0	0	0	29	0
normalized size	1	1.	0.21	0.	0.	0.	0.1	0.
time (sec)	N/A	0.161	0.045	0.025	0.	0.	0.709	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	46	0	0	0	26	0
normalized size	1	1.	0.17	0.	0.	0.	0.1	0.
time (sec)	N/A	0.124	0.005	0.035	0.	0.	0.664	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	49	0	0	0	29	0
normalized size	1	1.	0.19	0.	0.	0.	0.11	0.
time (sec)	N/A	0.125	0.008	0.027	0.	0.	0.712	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	51	0	0	0	34	0
normalized size	1	1.	0.18	0.	0.	0.	0.12	0.
time (sec)	N/A	0.158	0.009	0.03	0.	0.	0.828	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	135	1795	77
normalized size	1	1.	0.62	0.59	1.08	1.69	22.44	0.96
time (sec)	N/A	0.047	0.025	0.005	1.599	1.719	2.933	2.978

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	105	700	58
normalized size	1	1.	0.66	0.61	1.07	1.78	11.86	0.98
time (sec)	N/A	0.036	0.017	0.004	1.964	1.726	1.871	2.151

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	41	81	66	39
normalized size	1	1.	0.74	0.66	1.08	2.13	1.74	1.03
time (sec)	N/A	0.023	0.013	0.003	2.467	1.769	0.737	2.887

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	35	42	19
normalized size	1	1.	1.	0.83	1.06	1.94	2.33	1.06
time (sec)	N/A	0.004	0.003	0.002	2.068	1.736	0.359	2.451

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	93	0	0	358	46	132
normalized size	1	1.	0.92	0.	0.	3.54	0.46	1.31
time (sec)	N/A	0.066	0.035	0.023	0.	1.837	1.11	4.262

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	37	0	0	819	42	144
normalized size	1	1.	0.36	0.	0.	7.88	0.4	1.38
time (sec)	N/A	0.066	0.008	0.03	0.	1.853	1.311	5.314

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	39	0	0	994	42	170
normalized size	1	1.	0.29	0.	0.	7.36	0.31	1.26
time (sec)	N/A	0.088	0.008	0.03	0.	1.855	1.645	4.259

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	601	601	94	0	0	0	29	0
normalized size	1	1.	0.16	0.	0.	0.	0.05	0.
time (sec)	N/A	0.462	0.049	0.026	0.	0.	0.985	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	62	0	0	0	29	0
normalized size	1	1.	0.11	0.	0.	0.	0.05	0.
time (sec)	N/A	0.369	0.051	0.025	0.	0.	0.809	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	46	0	0	0	26	0
normalized size	1	1.	0.08	0.	0.	0.	0.05	0.
time (sec)	N/A	0.307	0.006	0.033	0.	0.	0.698	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	538	538	49	0	0	0	29	0
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.
time (sec)	N/A	0.303	0.01	0.024	0.	0.	0.754	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	575	51	0	0	0	34	0
normalized size	1	1.	0.09	0.	0.	0.	0.06	0.
time (sec)	N/A	0.373	0.009	0.028	0.	0.	0.905	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	162	136	181
normalized size	1	1.	0.62	0.59	1.08	2.02	1.7	2.26
time (sec)	N/A	0.049	0.026	0.005	1.853	1.741	5.786	2.535

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	128	112	143
normalized size	1	1.	0.66	0.61	1.07	2.17	1.9	2.42
time (sec)	N/A	0.038	0.018	0.004	1.967	1.671	3.722	2.734

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	41	103	88	105
normalized size	1	1.	0.74	0.66	1.08	2.71	2.32	2.76
time (sec)	N/A	0.024	0.014	0.003	2.111	1.595	2.328	1.789

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	73	65	19
normalized size	1	1.	1.	0.83	1.06	4.06	3.61	1.06
time (sec)	N/A	0.004	0.004	0.001	1.857	1.822	1.34	2.658

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	144	0	0	332	49	149
normalized size	1	1.	1.23	0.	0.	2.84	0.42	1.27
time (sec)	N/A	0.082	0.046	0.023	0.	2.083	1.394	4.927

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	37	0	0	359	46	161
normalized size	1	1.	0.32	0.	0.	3.09	0.4	1.39
time (sec)	N/A	0.084	0.009	0.036	0.	2.059	1.608	4.625

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	39	0	0	471	42	167
normalized size	1	1.	0.3	0.	0.	3.57	0.32	1.27
time (sec)	N/A	0.087	0.009	0.03	0.	1.749	1.847	4.58

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	79	0	0	0	29	0
normalized size	1	1.	0.24	0.	0.	0.	0.09	0.
time (sec)	N/A	0.228	0.075	0.028	0.	0.	1.517	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	67	0	0	0	29	0
normalized size	1	1.	0.22	0.	0.	0.	0.09	0.
time (sec)	N/A	0.185	0.063	0.025	0.	0.	1.165	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	47	0	0	0	26	0
normalized size	1	1.	0.16	0.	0.	0.	0.09	0.
time (sec)	N/A	0.166	0.006	0.033	0.	0.	0.922	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	50	0	0	0	29	0
normalized size	1	1.	0.18	0.	0.	0.	0.1	0.
time (sec)	N/A	0.154	0.009	0.026	0.	0.	1.099	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	52	0	0	0	34	0
normalized size	1	1.	0.18	0.	0.	0.	0.12	0.
time (sec)	N/A	0.15	0.009	0.027	0.	0.	1.117	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	12	65	56	12
normalized size	1	1.	1.	1.23	0.92	5.	4.31	0.92
time (sec)	N/A	0.002	0.004	0.001	1.849	1.657	2.756	2.708

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	109	1690	77
normalized size	1	1.	0.62	0.59	1.08	1.36	21.12	0.96
time (sec)	N/A	0.046	0.028	0.004	1.724	1.652	2.495	2.482

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	81	631	58
normalized size	1	1.	0.66	0.61	1.07	1.37	10.69	0.98
time (sec)	N/A	0.034	0.022	0.005	1.668	1.683	1.577	1.471

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	41	59	178	39
normalized size	1	1.	0.74	0.66	1.08	1.55	4.68	1.03
time (sec)	N/A	0.023	0.012	0.004	1.941	1.698	1.01	2.583

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	34	24	19
normalized size	1	1.	1.	0.83	1.06	1.89	1.33	1.06
time (sec)	N/A	0.003	0.003	0.003	1.723	1.67	0.388	2.077

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	0	0	679	41	117
normalized size	1	1.	0.81	0.	0.	7.9	0.48	1.36
time (sec)	N/A	0.051	0.029	0.018	0.	1.779	1.011	4.024

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	37	0	0	929	41	149
normalized size	1	1.	0.34	0.	0.	8.45	0.37	1.35
time (sec)	N/A	0.065	0.007	0.027	0.	1.898	1.263	4.556

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	39	0	0	899	41	171
normalized size	1	1.	0.28	0.	0.	6.51	0.3	1.24
time (sec)	N/A	0.093	0.007	0.028	0.	1.887	1.6	4.834

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	79	0	0	0	27	0
normalized size	1	1.	0.14	0.	0.	0.	0.05	0.
time (sec)	N/A	0.366	0.024	0.024	0.	0.	0.709	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	62	0	0	0	27	0
normalized size	1	1.	0.11	0.	0.	0.	0.05	0.
time (sec)	N/A	0.309	0.018	0.025	0.	0.	0.652	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	46	0	0	0	24	0
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.
time (sec)	N/A	0.25	0.005	0.031	0.	0.	0.637	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	49	0	0	0	27	0
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.
time (sec)	N/A	0.3	0.009	0.023	0.	0.	0.712	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	578	51	0	0	0	32	0
normalized size	1	1.	0.09	0.	0.	0.	0.06	0.
time (sec)	N/A	0.357	0.009	0.03	0.	0.	0.828	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	109	1690	77
normalized size	1	1.	0.62	0.59	1.08	1.36	21.12	0.96
time (sec)	N/A	0.046	0.024	0.006	2.12	1.693	2.528	1.795

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	81	631	58
normalized size	1	1.	0.66	0.61	1.07	1.37	10.69	0.98
time (sec)	N/A	0.034	0.017	0.006	1.195	1.645	1.612	2.749

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	25	41	55	178	36
normalized size	1	1.	0.71	0.66	1.08	1.45	4.68	0.95
time (sec)	N/A	0.024	0.013	0.005	1.735	1.741	1.018	2.122

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	34	24	19
normalized size	1	1.	1.	0.83	1.06	1.89	1.33	1.06
time (sec)	N/A	0.004	0.002	0.003	2.159	1.865	0.412	1.512

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	101	0	0	356	41	117
normalized size	1	1.	1.17	0.	0.	4.14	0.48	1.36
time (sec)	N/A	0.052	0.031	0.023	0.	1.786	1.06	4.045

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	37	0	0	466	41	147
normalized size	1	1.	0.35	0.	0.	4.36	0.38	1.37
time (sec)	N/A	0.066	0.007	0.026	0.	1.824	1.334	4.165

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	39	0	0	474	41	171
normalized size	1	1.	0.28	0.	0.	3.43	0.3	1.24
time (sec)	N/A	0.088	0.007	0.031	0.	1.78	1.736	4.091

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	79	0	0	0	27	0
normalized size	1	1.	0.27	0.	0.	0.	0.09	0.
time (sec)	N/A	0.158	0.025	0.025	0.	0.	0.74	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	62	0	0	0	27	0
normalized size	1	1.	0.23	0.	0.	0.	0.1	0.
time (sec)	N/A	0.123	0.019	0.024	0.	0.	0.666	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	46	0	0	0	24	0
normalized size	1	1.	0.19	0.	0.	0.	0.1	0.
time (sec)	N/A	0.096	0.007	0.031	0.	0.	0.644	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	49	0	0	0	27	0
normalized size	1	1.	0.18	0.	0.	0.	0.1	0.
time (sec)	N/A	0.121	0.009	0.025	0.	0.	0.773	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	51	0	0	0	32	0
normalized size	1	1.	0.17	0.	0.	0.	0.11	0.
time (sec)	N/A	0.15	0.009	0.027	0.	0.	0.952	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	124	1584	77
normalized size	1	1.	0.62	0.59	1.08	1.55	19.8	0.96
time (sec)	N/A	0.044	0.023	0.006	1.227	1.682	2.642	2.006

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	38	36	63	97	561	55
normalized size	1	1.	0.64	0.61	1.07	1.64	9.51	0.93
time (sec)	N/A	0.033	0.016	0.005	1.66	1.745	1.637	2.763

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	41	74	46	36
normalized size	1	1.	0.71	0.63	1.08	1.95	1.21	0.95
time (sec)	N/A	0.023	0.011	0.004	2.243	1.721	0.649	1.993

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	54	26	19
normalized size	1	1.	1.	0.83	1.06	3.	1.44	1.06
time (sec)	N/A	0.004	0.003	0.002	1.533	1.725	0.603	2.325

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	36	0	0	894	41	136
normalized size	1	1.	0.35	0.	0.	8.6	0.39	1.31
time (sec)	N/A	0.065	0.006	0.026	0.	1.815	1.182	4.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	37	0	0	1127	41	171
normalized size	1	1.02	0.3	0.	0.	9.16	0.33	1.39
time (sec)	N/A	0.08	0.007	0.052	0.	1.896	1.584	4.926

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	39	0	0	1107	41	190
normalized size	1	1.	0.25	0.	0.	6.96	0.26	1.19
time (sec)	N/A	0.106	0.007	0.053	0.	1.844	1.995	3.479

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	65	0	0	0	27	0
normalized size	1	1.	0.11	0.	0.	0.	0.05	0.
time (sec)	N/A	0.365	0.024	0.043	0.	0.	0.748	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	55	0	0	0	27	0
normalized size	1	1.	0.1	0.	0.	0.	0.05	0.
time (sec)	N/A	0.303	0.016	0.023	0.	0.	0.748	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	58	0	0	0	24	0
normalized size	1	1.	0.11	0.	0.	0.	0.04	0.
time (sec)	N/A	0.298	0.011	0.031	0.	0.	0.722	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	571	52	0	0	0	27	0
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.
time (sec)	N/A	0.352	0.01	0.044	0.	0.	0.945	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	599	599	54	0	0	0	32	0
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.
time (sec)	N/A	0.418	0.009	0.045	0.	0.	1.175	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	275	102	0	0	0	0	0
normalized size	1	1.41	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.391	0.07	0.025	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	244	85	0	0	0	0	0
normalized size	1	1.49	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	0.048	0.019	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	211	56	0	0	0	46	0
normalized size	1	1.59	0.42	0.	0.	0.	0.35	0.
time (sec)	N/A	0.274	0.013	0.016	0.	0.	1.748	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	208	56	0	0	0	49	0
normalized size	1	1.59	0.43	0.	0.	0.	0.37	0.
time (sec)	N/A	0.274	0.013	0.018	0.	0.	5.208	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	47	65	0	0
normalized size	1	1.	0.93	0.75	1.68	2.32	0.	0.
time (sec)	N/A	0.006	0.009	0.003	1.927	2.509	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	108	0	0
normalized size	1	1.	0.72	0.54	0.	1.89	0.	0.
time (sec)	N/A	0.015	0.017	0.004	0.	2.499	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	136	0	0
normalized size	1	1.	0.61	0.49	0.	1.6	0.	0.
time (sec)	N/A	0.025	0.018	0.004	0.	1.761	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	86	166	0	0
normalized size	1	1.	0.56	0.47	0.76	1.47	0.	0.
time (sec)	N/A	0.039	0.017	0.006	1.381	1.699	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	103	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.971	0.058	0.02	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	85	0	0	0	46	0
normalized size	1	1.	0.2	0.	0.	0.	0.11	0.
time (sec)	N/A	0.713	0.04	0.016	0.	0.	19.891	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	54	0	0	0	46	0
normalized size	1	1.	0.14	0.	0.	0.	0.12	0.
time (sec)	N/A	0.659	0.011	0.014	0.	0.	1.383	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	56	0	0	0	32	0
normalized size	1	1.	0.14	0.	0.	0.	0.08	0.
time (sec)	N/A	0.665	0.014	0.02	0.	0.	49.898	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	56	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.72	0.012	0.021	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0
normalized size	1	1.	0.97	0.	0.	0.	0.79	0.
time (sec)	N/A	0.017	0.011	0.019	0.	0.	2.8	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0
normalized size	1	1.	0.97	0.	0.	0.	0.79	0.
time (sec)	N/A	0.017	0.012	0.018	0.	0.	1.052	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	49	0
normalized size	1	1.	0.96	0.	0.	0.	0.88	0.
time (sec)	N/A	0.018	0.011	0.017	0.	0.	3.098	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	303	102	0	0	0	0	0
normalized size	1	1.36	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.366	0.078	0.017	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	272	89	0	0	0	0	0
normalized size	1	1.42	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.323	0.063	0.014	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	243	57	0	0	0	46	0
normalized size	1	1.49	0.35	0.	0.	0.	0.28	0.
time (sec)	N/A	0.29	0.013	0.015	0.	0.	20.347	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	233	57	0	0	0	49	0
normalized size	1	1.52	0.37	0.	0.	0.	0.32	0.
time (sec)	N/A	0.287	0.013	0.019	0.	0.	22.197	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	234	57	0	0	0	0	0
normalized size	1	1.49	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.291	0.015	0.023	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	104	0	0
normalized size	1	1.	0.93	0.75	0.	3.71	0.	0.
time (sec)	N/A	0.006	0.01	0.004	0.	2.177	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	132	0	0
normalized size	1	1.	0.72	0.54	0.	2.32	0.	0.
time (sec)	N/A	0.015	0.018	0.004	0.	2.118	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	161	0	0
normalized size	1	1.	0.61	0.49	0.	1.89	0.	0.
time (sec)	N/A	0.027	0.016	0.005	0.	2.083	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	102	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.83	0.07	0.016	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	89	0	0	0	46	0
normalized size	1	1.	0.2	0.	0.	0.	0.1	0.
time (sec)	N/A	0.746	0.048	0.016	0.	0.	64.575	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	55	0	0	0	46	0
normalized size	1	1.	0.13	0.	0.	0.	0.11	0.
time (sec)	N/A	0.695	0.011	0.014	0.	0.	14.358	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	57	0	0	0	32	0
normalized size	1	1.	0.14	0.	0.	0.	0.08	0.
time (sec)	N/A	0.698	0.013	0.02	0.	0.	71.727	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	57	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.702	0.014	0.02	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	57	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.767	0.013	0.026	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0
normalized size	1	1.	0.97	0.	0.	0.	0.78	0.
time (sec)	N/A	0.019	0.011	0.013	0.	0.	30.337	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0
normalized size	1	1.	0.97	0.	0.	0.	0.78	0.
time (sec)	N/A	0.019	0.012	0.017	0.	0.	11.661	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	49	0
normalized size	1	1.	0.96	0.	0.	0.	0.86	0.
time (sec)	N/A	0.019	0.013	0.018	0.	0.	14.406	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	278	87	0	0	0	0	0
normalized size	1	1.4	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.028	0.02	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	247	76	0	0	0	0	0
normalized size	1	1.48	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	0.021	0.017	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	209	58	0	0	0	44	0
normalized size	1	1.6	0.44	0.	0.	0.	0.34	0.
time (sec)	N/A	0.262	0.018	0.016	0.	0.	147.3	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	183	45	0	0	0	44	0
normalized size	1	1.73	0.42	0.	0.	0.	0.42	0.
time (sec)	N/A	0.245	0.009	0.021	0.	0.	1.688	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	47	62	36	0
normalized size	1	1.	0.93	0.75	1.68	2.21	1.29	0.
time (sec)	N/A	0.006	0.006	0.003	2.298	2.015	9.031	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	34	29	0	85	0	0
normalized size	1	1.	0.6	0.51	0.	1.49	0.	0.
time (sec)	N/A	0.015	0.018	0.003	0.	2.042	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	112	0	0
normalized size	1	1.	0.61	0.49	0.	1.32	0.	0.
time (sec)	N/A	0.025	0.024	0.004	0.	2.031	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	86	139	0	0
normalized size	1	1.	0.56	0.47	0.76	1.23	0.	0.
time (sec)	N/A	0.039	0.029	0.005	1.386	1.543	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	87	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.689	0.03	0.017	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	66	0	0	0	44	0
normalized size	1	1.	0.17	0.	0.	0.	0.11	0.
time (sec)	N/A	0.64	0.023	0.013	0.	0.	12.016	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	54	0	0	0	31	0
normalized size	1	1.	0.15	0.	0.	0.	0.09	0.
time (sec)	N/A	0.591	0.014	0.025	0.	0.	2.271	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	56	0	0	0	48	0
normalized size	1	1.	0.14	0.	0.	0.	0.12	0.
time (sec)	N/A	0.642	0.012	0.019	0.	0.	116.922	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	56	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.692	0.013	0.02	0.	0.	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.76	0.
time (sec)	N/A	0.018	0.012	0.022	0.	0.	2.033	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0
normalized size	1	1.	0.97	0.	0.	0.	0.79	0.
time (sec)	N/A	0.019	0.01	0.027	0.	0.	1.519	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0
normalized size	1	1.	0.96	0.	0.	0.	0.86	0.
time (sec)	N/A	0.019	0.011	0.018	0.	0.	5.384	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	93	0	0	0	29	0
normalized size	1	1.	0.77	0.	0.	0.	0.24	0.
time (sec)	N/A	0.049	0.051	0.03	0.	0.	0.889	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	62	0	0	0	29	0
normalized size	1	1.	0.64	0.	0.	0.	0.3	0.
time (sec)	N/A	0.033	0.045	0.025	0.	0.	0.766	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	46	0	0	0	26	0
normalized size	1	1.	0.61	0.	0.	0.	0.35	0.
time (sec)	N/A	0.018	0.005	0.033	0.	0.	0.713	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	0	0	29	0
normalized size	1	1.	0.68	0.	0.	0.	0.4	0.
time (sec)	N/A	0.021	0.009	0.025	0.	0.	0.75	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	51	0	0	0	34	0
normalized size	1	1.	0.52	0.	0.	0.	0.34	0.
time (sec)	N/A	0.031	0.009	0.026	0.	0.	0.914	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	51	0	0	0	34	0
normalized size	1	1.	0.41	0.	0.	0.	0.28	0.
time (sec)	N/A	0.046	0.009	0.029	0.	0.	1.189	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	31	0
normalized size	1	1.	0.75	0.	0.	0.	0.25	0.
time (sec)	N/A	0.046	0.056	0.023	0.	0.	0.916	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	31	0
normalized size	1	1.	0.63	0.	0.	0.	0.31	0.
time (sec)	N/A	0.033	0.051	0.013	0.	0.	0.788	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0
normalized size	1	1.	0.6	0.	0.	0.	0.35	0.
time (sec)	N/A	0.018	0.007	0.027	0.	0.	0.728	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0
normalized size	1	1.	0.66	0.	0.	0.	0.41	0.
time (sec)	N/A	0.02	0.009	0.016	0.	0.	0.776	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0
normalized size	1	1.	0.5	0.	0.	0.	0.35	0.
time (sec)	N/A	0.032	0.01	0.017	0.	0.	0.923	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	52	0	0	0	36	0
normalized size	1	1.	0.41	0.	0.	0.	0.28	0.
time (sec)	N/A	0.046	0.01	0.018	0.	0.	1.195	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	93	0	0	0	29	0
normalized size	1	1.	0.65	0.	0.	0.	0.2	0.
time (sec)	N/A	0.051	0.052	0.026	0.	0.	1.354	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	62	0	0	0	29	0
normalized size	1	1.	0.52	0.	0.	0.	0.24	0.
time (sec)	N/A	0.038	0.051	0.024	0.	0.	1.094	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	46	0	0	0	26	0
normalized size	1	1.	0.5	0.	0.	0.	0.28	0.
time (sec)	N/A	0.021	0.005	0.033	0.	0.	0.897	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	49	0	0	0	29	0
normalized size	1	1.	0.56	0.	0.	0.	0.33	0.
time (sec)	N/A	0.024	0.008	0.026	0.	0.	1.024	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	51	0	0	0	34	0
normalized size	1	1.	0.42	0.	0.	0.	0.28	0.
time (sec)	N/A	0.038	0.01	0.027	0.	0.	1.15	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	51	0	0	0	34	0
normalized size	1	1.	0.35	0.	0.	0.	0.23	0.
time (sec)	N/A	0.051	0.009	0.028	0.	0.	1.409	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	31	0
normalized size	1	1.	0.75	0.	0.	0.	0.25	0.
time (sec)	N/A	0.045	0.058	0.027	0.	0.	1.397	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	31	0
normalized size	1	1.	0.63	0.	0.	0.	0.31	0.
time (sec)	N/A	0.033	0.052	0.025	0.	0.	1.114	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0
normalized size	1	1.	0.6	0.	0.	0.	0.35	0.
time (sec)	N/A	0.019	0.006	0.033	0.	0.	0.887	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0
normalized size	1	1.	0.66	0.	0.	0.	0.41	0.
time (sec)	N/A	0.02	0.009	0.026	0.	0.	1.038	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0
normalized size	1	1.	0.5	0.	0.	0.	0.35	0.
time (sec)	N/A	0.032	0.009	0.026	0.	0.	1.188	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	52	0	0	0	36	0
normalized size	1	1.	0.41	0.	0.	0.	0.28	0.
time (sec)	N/A	0.045	0.009	0.029	0.	0.	1.46	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	47	0	0	0	26	0
normalized size	1	1.	0.51	0.	0.	0.	0.28	0.
time (sec)	N/A	0.024	0.009	0.032	0.	0.	1.337	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	48	0	0	0	27	0
normalized size	1	1.	0.5	0.	0.	0.	0.28	0.
time (sec)	N/A	0.025	0.01	0.023	0.	0.	1.32	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	47	0	0	0	26	0
normalized size	1	1.	0.42	0.	0.	0.	0.23	0.
time (sec)	N/A	0.029	0.007	0.033	0.	0.	2.21	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	48	0	0	0	27	0
normalized size	1	1.	0.5	0.	0.	0.	0.28	0.
time (sec)	N/A	0.026	0.008	0.034	0.	0.	2.26	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	90	0	0	0	27	0
normalized size	1	1.	0.62	0.	0.	0.	0.18	0.
time (sec)	N/A	0.052	0.033	0.025	0.	0.	0.917	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	79	0	0	0	27	0
normalized size	1	1.	0.65	0.	0.	0.	0.22	0.
time (sec)	N/A	0.036	0.021	0.024	0.	0.	0.765	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	0	0	0	27	0
normalized size	1	1.	0.63	0.	0.	0.	0.28	0.
time (sec)	N/A	0.024	0.016	0.023	0.	0.	0.691	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	46	0	0	0	24	0
normalized size	1	1.	0.65	0.	0.	0.	0.34	0.
time (sec)	N/A	0.014	0.006	0.029	0.	0.	0.678	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	49	0	0	0	27	0
normalized size	1	1.	0.53	0.	0.	0.	0.29	0.
time (sec)	N/A	0.025	0.008	0.024	0.	0.	0.762	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	51	0	0	0	32	0
normalized size	1	1.	0.41	0.	0.	0.	0.26	0.
time (sec)	N/A	0.036	0.009	0.026	0.	0.	0.922	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	51	0	0	0	32	0
normalized size	1	1.	0.34	0.	0.	0.	0.22	0.
time (sec)	N/A	0.051	0.009	0.027	0.	0.	1.184	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	29	0
normalized size	1	1.	0.69	0.	0.	0.	0.22	0.
time (sec)	N/A	0.045	0.03	0.024	0.	0.	0.95	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	79	0	0	0	29	0
normalized size	1	1.	0.76	0.	0.	0.	0.28	0.
time (sec)	N/A	0.034	0.022	0.024	0.	0.	0.788	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0
normalized size	1	1.	0.79	0.	0.	0.	0.36	0.
time (sec)	N/A	0.022	0.018	0.025	0.	0.	0.724	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0
normalized size	1	1.	0.81	0.	0.	0.	0.45	0.
time (sec)	N/A	0.011	0.007	0.03	0.	0.	0.689	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	50	0	0	0	29	0
normalized size	1	1.	0.63	0.	0.	0.	0.37	0.
time (sec)	N/A	0.02	0.009	0.025	0.	0.	0.791	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	34	0
normalized size	1	1.	0.49	0.	0.	0.	0.32	0.
time (sec)	N/A	0.031	0.01	0.027	0.	0.	0.954	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	52	0	0	0	34	0
normalized size	1	1.	0.4	0.	0.	0.	0.26	0.
time (sec)	N/A	0.044	0.01	0.028	0.	0.	1.191	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	90	0	0	0	27	0
normalized size	1	1.	0.73	0.	0.	0.	0.22	0.
time (sec)	N/A	0.043	0.035	0.024	0.	0.	0.902	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	0	27	0
normalized size	1	1.	0.78	0.	0.	0.	0.27	0.
time (sec)	N/A	0.031	0.027	0.024	0.	0.	0.771	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	62	0	0	0	27	0
normalized size	1	1.	0.79	0.	0.	0.	0.35	0.
time (sec)	N/A	0.02	0.017	0.024	0.	0.	0.718	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	0	0	24	0
normalized size	1	1.	0.82	0.	0.	0.	0.43	0.
time (sec)	N/A	0.01	0.006	0.03	0.	0.	0.681	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	49	0	0	0	27	0
normalized size	1	1.	0.64	0.	0.	0.	0.36	0.
time (sec)	N/A	0.019	0.009	0.024	0.	0.	0.879	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	51	0	0	0	32	0
normalized size	1	1.	0.5	0.	0.	0.	0.31	0.
time (sec)	N/A	0.031	0.009	0.026	0.	0.	1.095	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	51	0	0	0	32	0
normalized size	1	1.	0.4	0.	0.	0.	0.25	0.
time (sec)	N/A	0.043	0.009	0.027	0.	0.	1.358	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	91	0	0	0	29	0
normalized size	1	1.	0.71	0.	0.	0.	0.22	0.
time (sec)	N/A	0.046	0.035	0.014	0.	0.	0.91	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	77	0	0	0	29	0
normalized size	1	1.	0.74	0.	0.	0.	0.28	0.
time (sec)	N/A	0.032	0.027	0.014	0.	0.	0.776	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0
normalized size	1	1.	0.79	0.	0.	0.	0.36	0.
time (sec)	N/A	0.022	0.02	0.013	0.	0.	0.725	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0
normalized size	1	1.	0.81	0.	0.	0.	0.45	0.
time (sec)	N/A	0.011	0.007	0.032	0.	0.	0.693	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	50	0	0	0	29	0
normalized size	1	1.	0.64	0.	0.	0.	0.37	0.
time (sec)	N/A	0.021	0.009	0.016	0.	0.	0.884	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	34	0
normalized size	1	1.	0.49	0.	0.	0.	0.32	0.
time (sec)	N/A	0.032	0.01	0.017	0.	0.	1.108	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	52	0	0	0	34	0
normalized size	1	1.	0.4	0.	0.	0.	0.26	0.
time (sec)	N/A	0.047	0.01	0.018	0.	0.	1.369	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	27	0
normalized size	1	1.	0.63	0.	0.	0.	0.22	0.
time (sec)	N/A	0.043	0.028	0.045	0.	0.	0.903	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	65	0	0	0	27	0
normalized size	1	1.	0.65	0.	0.	0.	0.27	0.
time (sec)	N/A	0.031	0.02	0.048	0.	0.	0.819	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	53	0	0	0	27	0
normalized size	1	1.	0.72	0.	0.	0.	0.36	0.
time (sec)	N/A	0.019	0.017	0.026	0.	0.	0.803	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	24	0
normalized size	1	1.	0.98	0.	0.	0.	0.43	0.
time (sec)	N/A	0.01	0.011	0.035	0.	0.	0.783	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	52	0	0	0	27	0
normalized size	1	1.	0.68	0.	0.	0.	0.36	0.
time (sec)	N/A	0.02	0.009	0.045	0.	0.	1.089	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	54	0	0	0	32	0
normalized size	1	1.	0.53	0.	0.	0.	0.31	0.
time (sec)	N/A	0.03	0.009	0.046	0.	0.	1.368	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	54	0	0	0	32	0
normalized size	1	1.	0.43	0.	0.	0.	0.25	0.
time (sec)	N/A	0.042	0.009	0.047	0.	0.	1.769	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	29	0
normalized size	1	1.	0.63	0.	0.	0.	0.23	0.
time (sec)	N/A	0.047	0.029	0.054	0.	0.	0.938	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	66	0	0	0	29	0
normalized size	1	1.	0.65	0.	0.	0.	0.29	0.
time (sec)	N/A	0.031	0.02	0.049	0.	0.	0.848	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	29	0
normalized size	1	1.	0.73	0.	0.	0.	0.38	0.
time (sec)	N/A	0.022	0.015	0.025	0.	0.	0.827	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	26	0
normalized size	1	1.	0.73	0.	0.	0.	0.34	0.
time (sec)	N/A	0.019	0.013	0.035	0.	0.	0.801	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	53	0	0	0	29	0
normalized size	1	1.	0.54	0.	0.	0.	0.29	0.
time (sec)	N/A	0.032	0.01	0.05	0.	0.	1.139	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	55	0	0	0	34	0
normalized size	1	1.	0.44	0.	0.	0.	0.27	0.
time (sec)	N/A	0.045	0.011	0.05	0.	0.	1.387	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	0	0	0	34	0
normalized size	1	1.	0.36	0.	0.	0.	0.23	0.
time (sec)	N/A	0.061	0.01	0.053	0.	0.	1.782	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	24	0
normalized size	1	1.	0.71	0.	0.	0.	0.31	0.
time (sec)	N/A	0.017	0.022	0.033	0.	0.	1.103	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	24	0
normalized size	1	1.	0.92	0.	0.	0.	0.31	0.
time (sec)	N/A	0.017	0.031	0.035	0.	0.	1.779	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	0	24	0
normalized size	1	1.	0.77	0.	0.	0.	0.25	0.
time (sec)	N/A	0.024	0.031	0.033	0.	0.	2.948	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	26	0
normalized size	1	1.	0.69	0.	0.	0.	0.32	0.
time (sec)	N/A	0.018	0.02	0.036	0.	0.	1.105	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	0	0	26	0
normalized size	1	1.	0.73	0.	0.	0.	0.26	0.
time (sec)	N/A	0.027	0.033	0.036	0.	0.	1.789	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	0	0	0	26	0
normalized size	1	1.	0.76	0.	0.	0.	0.26	0.
time (sec)	N/A	0.026	0.034	0.036	0.	0.	2.937	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	54	43	0	0	27	0
normalized size	1	1.	0.55	0.43	0.	0.	0.27	0.
time (sec)	N/A	0.028	0.027	0.028	0.	0.	0.787	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	49	38	0	0	27	0
normalized size	1	1.	0.6	0.47	0.	0.	0.33	0.
time (sec)	N/A	0.02	0.017	0.02	0.	0.	0.664	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	41	31	0	0	27	0
normalized size	1	1.	0.65	0.49	0.	0.	0.43	0.
time (sec)	N/A	0.013	0.009	0.02	0.	0.	0.598	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	24	18	0	0	26	0
normalized size	1	1.	0.56	0.42	0.	0.	0.6	0.
time (sec)	N/A	0.007	0.003	0.01	0.	0.	0.585	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	27	33	0	0	29	0
normalized size	1	1.	0.43	0.52	0.	0.	0.46	0.
time (sec)	N/A	0.013	0.005	0.021	0.	0.	0.664	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	29	45	0	0	32	0
normalized size	1	1.	0.35	0.54	0.	0.	0.39	0.
time (sec)	N/A	0.019	0.006	0.021	0.	0.	0.815	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	29	50	0	0	32	0
normalized size	1	1.	0.29	0.5	0.	0.	0.32	0.
time (sec)	N/A	0.03	0.005	0.022	0.	0.	1.028	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	54	50	0	0	29	0
normalized size	1	1.	0.65	0.6	0.	0.	0.35	0.
time (sec)	N/A	0.022	0.024	0.031	0.	0.	0.803	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	45	0	0	29	0
normalized size	1	1.	0.75	0.69	0.	0.	0.45	0.
time (sec)	N/A	0.014	0.019	0.026	0.	0.	0.681	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	0	0	29	0
normalized size	1	1.	0.87	0.81	0.	0.	0.62	0.
time (sec)	N/A	0.008	0.008	0.026	0.	0.	0.611	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	24	18	0	0	27	0
normalized size	1	1.	0.86	0.64	0.	0.	0.96	0.
time (sec)	N/A	0.003	0.003	0.018	0.	0.	0.59	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	27	40	0	0	31	0
normalized size	1	1.	0.57	0.85	0.	0.	0.66	0.
time (sec)	N/A	0.008	0.004	0.027	0.	0.	0.677	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	29	45	0	0	34	0
normalized size	1	1.	0.43	0.67	0.	0.	0.51	0.
time (sec)	N/A	0.014	0.005	0.027	0.	0.	0.813	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	29	50	0	0	34	0
normalized size	1	1.	0.34	0.59	0.	0.	0.4	0.
time (sec)	N/A	0.021	0.004	0.028	0.	0.	1.008	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	54	43	0	0	27	0
normalized size	1	1.	0.65	0.52	0.	0.	0.33	0.
time (sec)	N/A	0.025	0.019	0.031	0.	0.	0.774	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	38	0	0	27	0
normalized size	1	1.	0.75	0.58	0.	0.	0.42	0.
time (sec)	N/A	0.015	0.017	0.02	0.	0.	0.651	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	31	0	0	27	0
normalized size	1	1.	0.87	0.66	0.	0.	0.57	0.
time (sec)	N/A	0.009	0.008	0.02	0.	0.	0.61	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	18	0	0	26	0
normalized size	1	1.	0.89	0.67	0.	0.	0.96	0.
time (sec)	N/A	0.003	0.004	0.013	0.	0.	0.589	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	27	33	0	0	29	0
normalized size	1	1.	0.55	0.67	0.	0.	0.59	0.
time (sec)	N/A	0.008	0.005	0.021	0.	0.	0.768	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	29	45	0	0	32	0
normalized size	1	1.	0.43	0.67	0.	0.	0.48	0.
time (sec)	N/A	0.015	0.005	0.021	0.	0.	0.963	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	29	50	0	0	32	0
normalized size	1	1.	0.34	0.59	0.	0.	0.38	0.
time (sec)	N/A	0.023	0.005	0.022	0.	0.	1.187	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	59	0	0	0	29	0
normalized size	1	1.	0.71	0.	0.	0.	0.35	0.
time (sec)	N/A	0.023	0.035	0.018	0.	0.	0.807	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	29	0
normalized size	1	1.	0.83	0.	0.	0.	0.45	0.
time (sec)	N/A	0.016	0.025	0.011	0.	0.	0.682	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	29	0
normalized size	1	1.	1.	0.	0.	0.	0.62	0.
time (sec)	N/A	0.009	0.015	0.01	0.	0.	0.636	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	0	0	27	0
normalized size	1	1.	1.	0.67	0.	0.	1.	0.
time (sec)	N/A	0.003	0.003	0.02	0.	0.	0.611	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	27	0	0	0	31	0
normalized size	1	1.	0.55	0.	0.	0.	0.63	0.
time (sec)	N/A	0.009	0.005	0.014	0.	0.	0.768	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	29	0	0	0	34	0
normalized size	1	1.	0.43	0.	0.	0.	0.51	0.
time (sec)	N/A	0.015	0.004	0.013	0.	0.	0.968	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	29	0	0	0	34	0
normalized size	1	1.	0.34	0.	0.	0.	0.4	0.
time (sec)	N/A	0.022	0.005	0.013	0.	0.	1.159	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	68	65	0	0	29	0
normalized size	1	1.	0.26	0.25	0.	0.	0.11	0.
time (sec)	N/A	0.144	0.017	0.038	0.	0.	0.81	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	63	60	0	0	29	0
normalized size	1	1.	0.26	0.25	0.	0.	0.12	0.
time (sec)	N/A	0.112	0.015	0.037	0.	0.	0.695	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	57	53	0	0	29	0
normalized size	1	1.	0.26	0.24	0.	0.	0.13	0.
time (sec)	N/A	0.091	0.011	0.035	0.	0.	0.62	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	43	40	0	0	27	0
normalized size	1	1.	0.22	0.2	0.	0.	0.14	0.
time (sec)	N/A	0.074	0.006	0.028	0.	0.	0.6	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	46	55	0	0	31	0
normalized size	1	1.	0.21	0.25	0.	0.	0.14	0.
time (sec)	N/A	0.09	0.006	0.037	0.	0.	0.685	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	48	67	0	0	34	0
normalized size	1	1.	0.2	0.28	0.	0.	0.14	0.
time (sec)	N/A	0.109	0.007	0.037	0.	0.	0.824	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	48	72	0	0	34	0
normalized size	1	1.	0.18	0.28	0.	0.	0.13	0.
time (sec)	N/A	0.126	0.007	0.037	0.	0.	1.044	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	68	53	0	0	34	0
normalized size	1	1.	0.26	0.2	0.	0.	0.13	0.
time (sec)	N/A	0.134	0.019	0.023	0.	0.	0.809	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	63	48	0	0	34	0
normalized size	1	1.	0.26	0.2	0.	0.	0.14	0.
time (sec)	N/A	0.11	0.014	0.02	0.	0.	0.693	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	58	41	0	0	34	0
normalized size	1	1.	0.26	0.18	0.	0.	0.15	0.
time (sec)	N/A	0.094	0.011	0.019	0.	0.	0.605	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	43	21	0	0	32	0
normalized size	1	1.	0.21	0.1	0.	0.	0.16	0.
time (sec)	N/A	0.078	0.006	0.01	0.	0.	0.598	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	46	43	0	0	36	0
normalized size	1	1.	0.21	0.19	0.	0.	0.16	0.
time (sec)	N/A	0.093	0.007	0.019	0.	0.	0.669	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	48	48	0	0	39	0
normalized size	1	1.	0.2	0.2	0.	0.	0.16	0.
time (sec)	N/A	0.111	0.007	0.021	0.	0.	0.811	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	48	53	0	0	39	0
normalized size	1	1.	0.18	0.2	0.	0.	0.15	0.
time (sec)	N/A	0.129	0.008	0.022	0.	0.	1.024	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	68	65	0	0	31	0
normalized size	1	1.	0.49	0.47	0.	0.	0.22	0.
time (sec)	N/A	0.057	0.019	0.042	0.	0.	0.788	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	63	60	0	0	31	0
normalized size	1	1.	0.52	0.5	0.	0.	0.26	0.
time (sec)	N/A	0.046	0.015	0.037	0.	0.	0.674	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	57	53	0	0	31	0
normalized size	1	1.	0.56	0.52	0.	0.	0.3	0.
time (sec)	N/A	0.037	0.012	0.037	0.	0.	0.651	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	43	40	0	0	29	0
normalized size	1	1.	0.52	0.49	0.	0.	0.35	0.
time (sec)	N/A	0.026	0.006	0.028	0.	0.	0.61	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	46	55	0	0	29	0
normalized size	1	1.	0.44	0.53	0.	0.	0.28	0.
time (sec)	N/A	0.035	0.007	0.038	0.	0.	0.768	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	48	67	0	0	32	0
normalized size	1	1.	0.39	0.55	0.	0.	0.26	0.
time (sec)	N/A	0.046	0.008	0.038	0.	0.	0.955	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	48	72	0	0	32	0
normalized size	1	1.	0.34	0.51	0.	0.	0.23	0.
time (sec)	N/A	0.057	0.007	0.039	0.	0.	1.207	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	68	0	0	0	36	0
normalized size	1	1.	0.49	0.	0.	0.	0.26	0.
time (sec)	N/A	0.058	0.02	0.018	0.	0.	0.802	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	36	0
normalized size	1	1.	0.52	0.	0.	0.	0.3	0.
time (sec)	N/A	0.046	0.015	0.011	0.	0.	0.686	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	58	0	0	0	36	0
normalized size	1	1.	0.56	0.	0.	0.	0.35	0.
time (sec)	N/A	0.036	0.012	0.011	0.	0.	0.62	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	43	21	0	0	34	0
normalized size	1	1.	0.51	0.25	0.	0.	0.4	0.
time (sec)	N/A	0.027	0.007	0.011	0.	0.	0.599	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	46	0	0	0	34	0
normalized size	1	1.	0.44	0.	0.	0.	0.32	0.
time (sec)	N/A	0.035	0.007	0.013	0.	0.	0.755	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	48	0	0	0	37	0
normalized size	1	1.	0.39	0.	0.	0.	0.3	0.
time (sec)	N/A	0.049	0.008	0.013	0.	0.	0.955	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	48	0	0	0	37	0
normalized size	1	1.	0.34	0.	0.	0.	0.26	0.
time (sec)	N/A	0.058	0.008	0.013	0.	0.	1.148	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	102	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.065	0.024	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	85	0	0	0	46	0
normalized size	1	1.	0.72	0.	0.	0.	0.39	0.
time (sec)	N/A	0.085	0.042	0.019	0.	0.	5.193	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	54	0	0	0	46	0
normalized size	1	1.	0.61	0.	0.	0.	0.52	0.
time (sec)	N/A	0.073	0.012	0.017	0.	0.	1.43	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	56	0	0	0	32	0
normalized size	1	1.	0.6	0.	0.	0.	0.34	0.
time (sec)	N/A	0.074	0.012	0.02	0.	0.	11.206	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	56	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.013	0.023	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	56	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.013	0.026	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	85	0	0	0	46	570
normalized size	1	1.	0.58	0.	0.	0.	0.31	3.88
time (sec)	N/A	0.092	0.044	0.021	0.	0.	43.465	3.052

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	56	0	0	0	46	498
normalized size	1	1.	0.48	0.	0.	0.	0.4	4.29
time (sec)	N/A	0.07	0.01	0.016	0.	0.	2.5	1.645

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	54	0	0	0	49	452
normalized size	1	1.	0.5	0.	0.	0.	0.46	4.22
time (sec)	N/A	0.069	0.012	0.019	0.	0.	3.927	1.858

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	62	78	58
normalized size	1	1.	0.93	0.75	0.	2.21	2.79	2.07
time (sec)	N/A	0.006	0.008	0.004	0.	1.964	50.312	2.016

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	105	0	144
normalized size	1	1.	0.72	0.54	0.	1.84	0.	2.53
time (sec)	N/A	0.015	0.015	0.005	0.	1.697	0.	2.255

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	135	0	244
normalized size	1	1.	0.61	0.49	0.	1.59	0.	2.87
time (sec)	N/A	0.024	0.017	0.004	0.	1.779	0.	3.141

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	0	165	0	359
normalized size	1	1.	0.56	0.47	0.	1.46	0.	3.18
time (sec)	N/A	0.038	0.016	0.005	0.	1.803	0.	1.843

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	88	0	0	0	48	0
normalized size	1	1.	0.72	0.	0.	0.	0.39	0.
time (sec)	N/A	0.092	0.053	0.025	0.	0.	5.092	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	55	0	0	0	39	0
normalized size	1	1.	0.6	0.	0.	0.	0.42	0.
time (sec)	N/A	0.079	0.014	0.02	0.	0.	1.442	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	57	0	0	0	36	0
normalized size	1	1.	0.59	0.	0.	0.	0.37	0.
time (sec)	N/A	0.079	0.015	0.024	0.	0.	11.077	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	57	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.015	0.027	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	57	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.015	0.03	0.	0.	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	88	0	0	0	48	533
normalized size	1	1.	0.26	0.	0.	0.	0.14	1.55
time (sec)	N/A	0.368	0.047	0.023	0.	0.	44.795	2.108

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	57	0	0	0	48	459
normalized size	1	1.	0.19	0.	0.	0.	0.16	1.5
time (sec)	N/A	0.268	0.011	0.02	0.	0.	2.656	2.246

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	55	0	0	0	51	429
normalized size	1	1.	0.19	0.	0.	0.	0.17	1.45
time (sec)	N/A	0.275	0.013	0.021	0.	0.	4.253	2.8

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	78	182	61
normalized size	1	1.	0.93	0.76	0.	2.69	6.28	2.1
time (sec)	N/A	0.006	0.008	0.004	0.	2.225	52.936	2.435

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	32	0	107	0	149
normalized size	1	1.	0.71	0.54	0.	1.81	0.	2.53
time (sec)	N/A	0.016	0.017	0.003	0.	2.06	0.	2.626

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	53	43	0	135	0	251
normalized size	1	1.	0.6	0.49	0.	1.53	0.	2.85
time (sec)	N/A	0.028	0.018	0.003	0.	1.569	0.	1.686

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	64	54	0	166	0	367
normalized size	1	1.	0.55	0.46	0.	1.42	0.	3.14
time (sec)	N/A	0.04	0.017	0.005	0.	1.569	0.	2.122

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	689	44	0
normalized size	1	1.	0.83	0.	0.	5.89	0.38	0.
time (sec)	N/A	0.065	0.036	0.039	0.	1.791	3.636	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	576	44	0
normalized size	1	1.	0.78	0.	0.	6.94	0.53	0.
time (sec)	N/A	0.05	0.011	0.028	0.	1.713	1.73	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	62	36	0
normalized size	1	1.	0.93	0.75	0.	2.21	1.29	0.
time (sec)	N/A	0.006	0.006	0.004	0.	1.544	11.228	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	86	0	0
normalized size	1	1.	0.72	0.54	0.	1.51	0.	0.
time (sec)	N/A	0.016	0.018	0.003	0.	1.578	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	115	0	0
normalized size	1	1.	0.61	0.49	0.	1.35	0.	0.
time (sec)	N/A	0.026	0.024	0.004	0.	1.448	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	87	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.031	0.036	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	69	0	0	0	44	0
normalized size	1	1.	0.55	0.	0.	0.	0.35	0.
time (sec)	N/A	0.047	0.024	0.031	0.	0.	30.375	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	56	0	0	0	44	0
normalized size	1	1.	0.67	0.	0.	0.	0.53	0.
time (sec)	N/A	0.032	0.01	0.025	0.	0.	1.056	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	54	0	0	0	31	0
normalized size	1	1.	0.6	0.	0.	0.	0.34	0.
time (sec)	N/A	0.035	0.012	0.033	0.	0.	3.993	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	56	0	0	0	34	0
normalized size	1	1.	0.44	0.	0.	0.	0.27	0.
time (sec)	N/A	0.049	0.012	0.036	0.	0.	96.643	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	56	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.013	0.038	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	241	0	0	709	46	0
normalized size	1	1.	0.78	0.	0.	2.3	0.15	0.
time (sec)	N/A	0.265	0.144	0.035	0.	1.814	3.829	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	197	0	0	594	46	0
normalized size	1	1.	0.72	0.	0.	2.18	0.17	0.
time (sec)	N/A	0.228	0.042	0.03	0.	1.686	1.82	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	63	92	0
normalized size	1	1.	0.93	0.76	0.	2.17	3.17	0.
time (sec)	N/A	0.006	0.006	0.004	0.	1.552	11.232	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	32	0	89	0	0
normalized size	1	1.	0.71	0.54	0.	1.51	0.	0.
time (sec)	N/A	0.015	0.018	0.003	0.	1.576	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	53	43	0	116	0	0
normalized size	1	1.	0.6	0.49	0.	1.32	0.	0.
time (sec)	N/A	0.027	0.025	0.005	0.	1.544	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	71	0	0	0	46	0
normalized size	1	1.	0.55	0.	0.	0.	0.36	0.
time (sec)	N/A	0.049	0.029	0.036	0.	0.	29.799	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	57	0	0	0	46	0
normalized size	1	1.	0.63	0.	0.	0.	0.51	0.
time (sec)	N/A	0.036	0.012	0.025	0.	0.	1.014	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	0	0	0	32	0
normalized size	1	1.	0.81	0.	0.	0.	0.47	0.
time (sec)	N/A	0.024	0.012	0.034	0.	0.	3.799	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	39	0
normalized size	1	1.	0.57	0.	0.	0.	0.39	0.
time (sec)	N/A	0.036	0.013	0.037	0.	0.	98.511	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	57	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.013	0.04	0.	0.	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	66	0	0	0	44	0
normalized size	1	1.	0.77	0.	0.	0.	0.51	0.
time (sec)	N/A	0.07	0.026	0.016	0.	0.	3.699	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	0	0	0	31	0
normalized size	1	1.	0.82	0.	0.	0.	0.47	0.
time (sec)	N/A	0.063	0.011	0.026	0.	0.	2.563	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	56	0	0	0	48	0
normalized size	1	1.	0.58	0.	0.	0.	0.49	0.
time (sec)	N/A	0.073	0.012	0.022	0.	0.	38.248	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	56	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.013	0.023	0.	0.	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	56	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.014	0.023	0.	0.	0.	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	44	0
normalized size	1	1.	0.83	0.	0.	0.	0.38	0.
time (sec)	N/A	0.067	0.037	0.018	0.	0.	33.598	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	67	0	0	0	44	0
normalized size	1	1.	0.8	0.	0.	0.	0.52	0.
time (sec)	N/A	0.056	0.013	0.02	0.	0.	1.849	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	57	36	0
normalized size	1	1.	0.92	0.81	0.	2.19	1.38	0.
time (sec)	N/A	0.006	0.005	0.004	0.	1.388	6.234	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	82	0	0
normalized size	1	1.	0.62	0.53	0.	1.49	0.	0.
time (sec)	N/A	0.014	0.017	0.004	0.	1.38	0.	0.

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	52	42	0	111	0	0
normalized size	1	1.	0.63	0.51	0.	1.34	0.	0.
time (sec)	N/A	0.025	0.023	0.006	0.	1.295	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	0	0	0	46	0
normalized size	1	1.	0.75	0.	0.	0.	0.51	0.
time (sec)	N/A	0.076	0.026	0.017	0.	0.	3.951	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	0	0	0	32	0
normalized size	1	1.	0.81	0.	0.	0.	0.47	0.
time (sec)	N/A	0.064	0.013	0.029	0.	0.	2.639	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	39	0
normalized size	1	1.	0.57	0.	0.	0.	0.39	0.
time (sec)	N/A	0.076	0.012	0.024	0.	0.	40.175	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	57	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.013	0.024	0.	0.	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	57	0	0	0	0	0
normalized size	1	1.	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.013	0.028	0.	0.	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	112	0	0	0	46	0
normalized size	1	1.	0.36	0.	0.	0.	0.15	0.
time (sec)	N/A	0.262	0.056	0.02	0.	0.	36.528	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	75	0	0	0	46	0
normalized size	1	1.	0.28	0.	0.	0.	0.17	0.
time (sec)	N/A	0.232	0.023	0.026	0.	0.	1.996	0.

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	22	0	58	94	0
normalized size	1	1.	0.93	0.81	0.	2.15	3.48	0.
time (sec)	N/A	0.007	0.005	0.004	0.	2.125	7.039	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	30	0	85	0	0
normalized size	1	1.	0.61	0.53	0.	1.49	0.	0.
time (sec)	N/A	0.014	0.018	0.004	0.	2.072	0.	0.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	53	43	0	112	0	0
normalized size	1	1.	0.62	0.5	0.	1.3	0.	0.
time (sec)	N/A	0.025	0.024	0.004	0.	2.12	0.	0.

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	63	0	0	841	0	0
normalized size	1	1.	0.43	0.	0.	5.76	0.	0.
time (sec)	N/A	0.076	0.037	0.06	0.	2.297	0.	0.

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	59	0	0	701	44	0
normalized size	1	1.	0.55	0.	0.	6.55	0.41	0.
time (sec)	N/A	0.059	0.012	0.027	0.	1.729	13.346	0.

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	69	34	0
normalized size	1	1.	0.92	0.81	0.	2.65	1.31	0.
time (sec)	N/A	0.006	0.006	0.005	0.	1.505	7.044	0.

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	105	78	0
normalized size	1	1.	0.62	0.53	0.	1.91	1.42	0.
time (sec)	N/A	0.015	0.01	0.004	0.	1.533	128.074	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	47	42	0	131	0	0
normalized size	1	1.	0.57	0.51	0.	1.58	0.	0.
time (sec)	N/A	0.024	0.011	0.004	0.	1.599	0.	0.

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	58	53	0	158	0	0
normalized size	1	1.	0.53	0.49	0.	1.45	0.	0.
time (sec)	N/A	0.037	0.012	0.004	0.	1.616	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	87	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.035	0.058	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	74	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.031	0.051	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	60	0	0	0	44	0
normalized size	1	1.	0.67	0.	0.	0.	0.49	0.
time (sec)	N/A	0.033	0.028	0.03	0.	0.	37.517	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	0	0	0	44	0
normalized size	1	1.	0.94	0.	0.	0.	0.7	0.
time (sec)	N/A	0.02	0.011	0.025	0.	0.	4.167	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	57	0	0	0	48	0
normalized size	1	1.	0.61	0.	0.	0.	0.52	0.
time (sec)	N/A	0.035	0.012	0.055	0.	0.	27.363	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	59	0	0	0	0	0
normalized size	1	1.	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.012	0.057	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	59	0	0	0	0	0
normalized size	1	1.	0.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.013	0.06	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.76	0.
time (sec)	N/A	0.018	0.011	0.039	0.	0.	53.32	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.76	0.
time (sec)	N/A	0.017	0.01	0.028	0.	0.	8.525	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.76	0.
time (sec)	N/A	0.017	0.011	0.025	0.	0.	1.395	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.76	0.
time (sec)	N/A	0.018	0.011	0.027	0.	0.	1.481	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	44	0
normalized size	1	1.	0.96	0.	0.	0.	0.79	0.
time (sec)	N/A	0.017	0.01	0.028	0.	0.	3.507	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0
normalized size	1	1.	0.96	0.	0.	0.	0.86	0.
time (sec)	N/A	0.017	0.011	0.034	0.	0.	10.096	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.72	0.
time (sec)	N/A	0.019	0.012	0.034	0.	0.	124.957	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.72	0.
time (sec)	N/A	0.018	0.011	0.033	0.	0.	61.727	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.72	0.
time (sec)	N/A	0.019	0.01	0.027	0.	0.	26.713	0.

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.72	0.
time (sec)	N/A	0.018	0.01	0.036	0.	0.	28.794	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	44	0
normalized size	1	1.	0.97	0.	0.	0.	0.75	0.
time (sec)	N/A	0.018	0.011	0.034	0.	0.	65.33	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	48	0
normalized size	1	1.	0.97	0.	0.	0.	0.81	0.
time (sec)	N/A	0.019	0.011	0.047	0.	0.	133.22	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	105	0	0	0	29	0
normalized size	1	1.	0.3	0.	0.	0.	0.08	0.
time (sec)	N/A	0.425	0.059	0.033	0.	0.	1.456	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	93	0	0	0	29	0
normalized size	1	1.	0.29	0.	0.	0.	0.09	0.
time (sec)	N/A	0.291	0.046	0.025	0.	0.	1.239	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	62	0	0	0	29	0
normalized size	1	1.	0.21	0.	0.	0.	0.1	0.
time (sec)	N/A	0.247	0.048	0.024	0.	0.	0.96	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	46	0	0	0	26	0
normalized size	1	1.	0.17	0.	0.	0.	0.1	0.
time (sec)	N/A	0.209	0.005	0.032	0.	0.	0.918	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	49	0	0	0	29	0
normalized size	1	1.	0.18	0.	0.	0.	0.11	0.
time (sec)	N/A	0.21	0.009	0.026	0.	0.	0.958	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	51	0	0	0	34	0
normalized size	1	1.	0.17	0.	0.	0.	0.11	0.
time (sec)	N/A	0.248	0.009	0.026	0.	0.	1.258	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	51	0	0	0	34	0
normalized size	1	1.	0.16	0.	0.	0.	0.11	0.
time (sec)	N/A	0.283	0.009	0.03	0.	0.	1.54	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	51	0	0	0	34	0
normalized size	1	1.	0.15	0.	0.	0.	0.1	0.
time (sec)	N/A	0.318	0.009	0.031	0.	0.	2.034	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	90	0	0	0	27	0
normalized size	1	1.	0.14	0.	0.	0.	0.04	0.
time (sec)	N/A	0.693	0.031	0.026	0.	0.	1.167	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	635	635	79	0	0	0	27	0
normalized size	1	1.	0.12	0.	0.	0.	0.04	0.
time (sec)	N/A	0.609	0.021	0.024	0.	0.	0.947	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	611	62	0	0	0	27	0
normalized size	1	1.	0.1	0.	0.	0.	0.04	0.
time (sec)	N/A	0.547	0.017	0.024	0.	0.	0.82	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	46	0	0	0	24	0
normalized size	1	1.	0.08	0.	0.	0.	0.04	0.
time (sec)	N/A	0.467	0.006	0.03	0.	0.	0.805	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	586	586	49	0	0	0	27	0
normalized size	1	1.	0.08	0.	0.	0.	0.05	0.
time (sec)	N/A	0.533	0.009	0.025	0.	0.	0.993	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	633	51	0	0	0	32	0
normalized size	1	1.	0.08	0.	0.	0.	0.05	0.
time (sec)	N/A	0.595	0.009	0.025	0.	0.	1.268	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	51	0	0	0	32	0
normalized size	1	1.	0.08	0.	0.	0.	0.05	0.
time (sec)	N/A	0.672	0.009	0.028	0.	0.	1.615	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	89	0	0	0	27	0
normalized size	1	1.	0.27	0.	0.	0.	0.08	0.
time (sec)	N/A	0.292	0.031	0.025	0.	0.	0.977	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	79	0	0	0	27	0
normalized size	1	1.	0.26	0.	0.	0.	0.09	0.
time (sec)	N/A	0.251	0.022	0.024	0.	0.	0.925	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	62	0	0	0	27	0
normalized size	1	1.	0.22	0.	0.	0.	0.1	0.
time (sec)	N/A	0.212	0.018	0.024	0.	0.	0.899	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	46	0	0	0	24	0
normalized size	1	1.	0.18	0.	0.	0.	0.1	0.
time (sec)	N/A	0.184	0.006	0.03	0.	0.	0.854	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	49	0	0	0	27	0
normalized size	1	1.	0.18	0.	0.	0.	0.1	0.
time (sec)	N/A	0.212	0.009	0.026	0.	0.	1.218	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	51	0	0	0	32	0
normalized size	1	1.	0.17	0.	0.	0.	0.11	0.
time (sec)	N/A	0.243	0.009	0.027	0.	0.	1.496	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	51	0	0	0	32	0
normalized size	1	1.	0.16	0.	0.	0.	0.1	0.
time (sec)	N/A	0.279	0.01	0.028	0.	0.	1.973	0.

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	654	654	79	0	0	0	27	0
normalized size	1	1.	0.12	0.	0.	0.	0.04	0.
time (sec)	N/A	0.669	0.023	0.045	0.	0.	1.142	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	630	65	0	0	0	27	0
normalized size	1	1.	0.1	0.	0.	0.	0.04	0.
time (sec)	N/A	0.592	0.02	0.044	0.	0.	1.111	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	583	583	58	0	0	0	27	0
normalized size	1	1.	0.1	0.	0.	0.	0.05	0.
time (sec)	N/A	0.522	0.014	0.023	0.	0.	1.099	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	555	49	0	0	0	24	0
normalized size	1	1.	0.09	0.	0.	0.	0.04	0.
time (sec)	N/A	0.394	0.007	0.033	0.	0.	1.039	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	52	0	0	0	27	0
normalized size	1	1.	0.08	0.	0.	0.	0.04	0.
time (sec)	N/A	0.574	0.009	0.044	0.	0.	1.55	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	652	54	0	0	0	32	0
normalized size	1	1.	0.08	0.	0.	0.	0.05	0.
time (sec)	N/A	0.654	0.009	0.045	0.	0.	2.009	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	54	0	0	0	32	0
normalized size	1	1.	0.08	0.	0.	0.	0.05	0.
time (sec)	N/A	0.741	0.01	0.05	0.	0.	2.774	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	132	143	302	2025	554
normalized size	1	1.	0.95	1.32	1.43	3.02	20.25	5.54
time (sec)	N/A	0.064	0.05	0.005	1.288	1.617	11.447	2.438

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	80	99	197	979	312
normalized size	1	1.	0.89	1.11	1.38	2.74	13.6	4.33
time (sec)	N/A	0.043	0.027	0.004	2.044	1.6	5.327	1.473

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	42	63	115	364	127
normalized size	1	1.	0.83	0.88	1.31	2.4	7.58	2.65
time (sec)	N/A	0.029	0.018	0.003	2.288	1.557	2.052	1.544

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	0	55	97	28
normalized size	1	1.	0.96	0.96	0.	2.39	4.22	1.22
time (sec)	N/A	0.005	0.003	0.001	0.	1.542	0.71	1.779

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	0	39	0
normalized size	1	1.	1.	0.	0.	0.	0.95	0.
time (sec)	N/A	0.022	0.011	0.024	0.	0.	3.203	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	42	0
normalized size	1	1.	1.	0.	0.	0.	1.	0.
time (sec)	N/A	0.023	0.007	0.03	0.	0.	7.096	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0
normalized size	1	1.22	1.22	0.	0.	0.	0.65	0.
time (sec)	N/A	0.013	0.008	0.046	0.	0.	15.374	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0
normalized size	1	1.22	1.22	0.	0.	0.	0.65	0.
time (sec)	N/A	0.013	0.006	0.033	0.	0.	8.942	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0
normalized size	1	1.22	1.22	0.	0.	0.	0.65	0.
time (sec)	N/A	0.014	0.006	0.028	0.	0.	4.758	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	44	44	0	0	0	22	0
normalized size	1	1.26	1.26	0.	0.	0.	0.63	0.
time (sec)	N/A	0.009	0.004	0.022	0.	0.	2.66	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	47	47	0	0	0	26	0
normalized size	1	1.24	1.24	0.	0.	0.	0.68	0.
time (sec)	N/A	0.013	0.006	0.029	0.	0.	4.606	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.007	0.017	0.	0.	0.	0.

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.006	0.02	0.	0.	0.	0.

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.007	0.02	0.	0.	0.	0.

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.007	0.019	0.	0.	0.	0.

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	37	0
normalized size	1	1.22	1.22	0.	0.	0.	0.92	0.
time (sec)	N/A	0.013	0.006	0.02	0.	0.	79.388	0.

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	0	0
normalized size	1	1.22	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.007	0.019	0.	0.	0.	0.

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.007	0.019	0.	0.	0.	0.

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.007	0.02	0.	0.	0.	0.

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	61	63	0	0	0	51	0
normalized size	1	1.15	1.19	0.	0.	0.	0.96	0.
time (sec)	N/A	0.017	0.01	0.053	0.	0.	33.195	0.

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0
normalized size	1	1.	0.97	0.	0.	0.	0.82	0.
time (sec)	N/A	0.019	0.009	0.061	0.	0.	33.297	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.015	0.055	0.	0.	0.	0.

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	62	81	113	219	0	0
normalized size	1	1.	0.59	0.77	1.08	2.09	0.	0.
time (sec)	N/A	0.057	0.014	0.004	1.659	1.62	0.	0.

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.015	0.053	0.	0.	0.	0.

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	62	45	80	135	0	0
normalized size	1	1.	0.93	0.67	1.19	2.01	0.	0.
time (sec)	N/A	0.02	0.014	0.003	1.498	1.612	0.	0.

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.014	0.054	0.	0.	0.	0.

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	50	77	0	0
normalized size	1	1.	0.97	0.97	1.67	2.57	0.	0.
time (sec)	N/A	0.006	0.012	0.002	1.834	1.612	0.	0.

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.016	0.053	0.	0.	0.	0.

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	56	56	0	0	0	0	0
normalized size	1	1.3	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.007	0.053	0.	0.	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	24	0
normalized size	1	1.33	1.25	0.	0.	0.	0.46	0.
time (sec)	N/A	0.019	0.016	0.051	0.	0.	24.117	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	64	61	0	0	0	0	0
normalized size	1	1.31	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.017	0.053	0.	0.	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	0	0
normalized size	1	1.33	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.017	0.055	0.	0.	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	64	61	0	0	0	0	0
normalized size	1	1.31	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.015	0.053	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [321] had the largest ratio of [0.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	11	0.091
2	A	2	1	1.	11	0.091
3	A	2	1	1.	11	0.091
4	A	2	1	1.	9	0.111
5	A	1	0	1.	7	0.
6	A	2	1	1.	11	0.091
7	A	2	1	1.	11	0.091
8	A	2	1	1.	11	0.091
9	A	2	1	1.	11	0.091
10	A	2	1	1.	11	0.091
11	A	2	1	1.	11	0.091
12	A	2	1	1.	11	0.091
13	A	3	2	1.	13	0.154
14	A	2	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	3	2	1.	13	0.154
16	A	2	1	1.	13	0.077
17	A	1	1	1.	11	0.091
18	A	2	1	1.	9	0.111
19	A	3	2	1.	13	0.154
20	A	2	1	1.	13	0.077
21	A	3	2	1.	13	0.154
22	A	2	1	1.	13	0.077
23	A	3	2	1.	13	0.154
24	A	2	1	1.	13	0.077
25	A	1	1	1.	13	0.077
26	A	2	1	1.	13	0.077
27	A	3	2	1.	13	0.154
28	A	2	1	1.	13	0.077
29	A	3	2	1.	13	0.154
30	A	3	2	1.	13	0.154
31	A	3	2	1.	13	0.154
32	A	3	2	1.	13	0.154
33	A	1	1	1.	11	0.091
34	A	3	2	1.	13	0.154
35	A	3	2	1.	13	0.154
36	A	3	2	1.	13	0.154
37	A	3	2	1.	13	0.154
38	A	1	1	1.	13	0.077
39	A	3	3	1.	13	0.231
40	A	3	2	1.	13	0.154
41	A	3	2	1.	13	0.154
42	A	2	1	1.	13	0.077
43	A	2	1	1.	13	0.077
44	A	2	1	1.	13	0.077
45	A	2	1	1.	9	0.111
46	A	2	1	1.	13	0.077
47	A	2	1	1.	13	0.077
48	A	2	1	1.	13	0.077
49	A	2	1	1.	13	0.077
50	A	2	1	1.	13	0.077
51	A	2	1	1.	13	0.077
52	A	3	2	1.	13	0.154
53	A	3	2	1.	13	0.154
54	A	3	2	1.	13	0.154
55	A	3	2	1.	13	0.154
56	A	3	2	1.	13	0.154
57	A	3	2	1.	13	0.154
58	A	1	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	3	2	1.	13	0.154
60	A	3	2	1.	13	0.154
61	A	3	2	1.	13	0.154
62	A	3	2	1.	13	0.154
63	A	3	2	1.	13	0.154
64	A	3	2	1.	13	0.154
65	A	1	1	1.	13	0.077
66	A	3	3	1.	13	0.231
67	A	4	3	1.	13	0.231
68	A	3	2	1.	13	0.154
69	A	3	2	1.	13	0.154
70	A	2	1	1.	13	0.077
71	A	2	1	1.	13	0.077
72	A	2	1	1.	13	0.077
73	A	2	1	1.	13	0.077
74	A	2	1	1.	9	0.111
75	A	2	1	1.	13	0.077
76	A	2	1	1.	13	0.077
77	A	2	1	1.	13	0.077
78	A	2	1	1.	13	0.077
79	A	2	1	1.	13	0.077
80	A	2	1	1.	13	0.077
81	A	2	1	1.	13	0.077
82	A	2	1	1.	13	0.077
83	A	2	1	1.	13	0.077
84	A	2	1	1.	13	0.077
85	A	3	2	1.	13	0.154
86	A	3	2	1.	13	0.154
87	A	3	2	1.	13	0.154
88	A	3	2	1.	13	0.154
89	A	3	2	1.	13	0.154
90	A	3	2	1.	13	0.154
91	A	1	1	1.	11	0.091
92	A	3	2	1.	13	0.154
93	A	3	2	1.	13	0.154
94	A	3	2	1.	13	0.154
95	A	3	2	1.	13	0.154
96	A	3	2	1.	13	0.154
97	A	3	2	1.	13	0.154
98	A	3	2	1.	13	0.154
99	A	3	2	1.	13	0.154
100	A	3	2	1.	13	0.154
101	A	1	1	1.	13	0.077
102	A	3	3	1.	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
103	A	4	3	1.	13	0.231
104	A	5	3	1.	13	0.231
105	A	6	3	1.	13	0.231
106	A	3	2	1.	13	0.154
107	A	3	2	1.	13	0.154
108	A	3	2	1.	13	0.154
109	A	2	1	1.	13	0.077
110	A	2	1	1.	13	0.077
111	A	2	1	1.	13	0.077
112	A	2	1	1.	13	0.077
113	A	2	1	1.	9	0.111
114	A	2	1	1.	13	0.077
115	A	2	1	1.	13	0.077
116	A	2	1	1.	13	0.077
117	A	2	1	1.	13	0.077
118	A	2	1	1.	13	0.077
119	A	2	1	1.	13	0.077
120	A	2	1	1.	13	0.077
121	A	2	1	1.	13	0.077
122	A	2	1	1.	13	0.077
123	A	2	1	1.	13	0.077
124	A	3	2	1.	13	0.154
125	A	3	2	1.	13	0.154
126	A	3	2	1.	13	0.154
127	A	3	2	1.	13	0.154
128	A	3	2	1.	13	0.154
129	A	3	2	1.	13	0.154
130	A	3	2	1.	13	0.154
131	A	3	2	1.	13	0.154
132	A	3	2	1.	13	0.154
133	A	2	2	1.	13	0.154
134	A	1	1	1.	11	0.091
135	A	1	1	1.	9	0.111
136	A	4	4	1.	13	0.308
137	A	2	2	1.	13	0.154
138	A	3	2	1.	13	0.154
139	A	3	2	1.	13	0.154
140	A	3	2	1.	13	0.154
141	A	4	2	1.	13	0.154
142	A	3	2	1.	13	0.154
143	A	5	2	1.	13	0.154
144	A	3	2	1.	13	0.154
145	A	3	2	1.	13	0.154
146	A	4	3	1.	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	3	2	1.	13	0.154
148	A	4	3	1.	13	0.231
149	A	3	2	1.	13	0.154
150	A	4	3	1.	13	0.231
151	A	3	2	1.	13	0.154
152	A	4	3	1.	13	0.231
153	A	3	2	1.	13	0.154
154	A	3	3	1.	13	0.231
155	A	3	2	1.	13	0.154
156	A	2	2	1.	13	0.154
157	A	1	1	1.	11	0.091
158	A	2	2	1.	9	0.222
159	A	3	2	1.	13	0.154
160	A	3	3	1.	13	0.231
161	A	3	2	1.	13	0.154
162	A	4	3	1.	13	0.231
163	A	3	2	1.	13	0.154
164	A	5	3	1.	13	0.231
165	A	3	2	1.	13	0.154
166	A	6	3	1.	13	0.231
167	A	3	2	1.	13	0.154
168	A	3	2	1.	13	0.154
169	A	3	2	1.	13	0.154
170	A	3	2	1.	13	0.154
171	A	3	2	1.	13	0.154
172	A	3	2	1.	13	0.154
173	A	3	2	1.	13	0.154
174	A	1	1	1.	13	0.077
175	A	1	1	1.	11	0.091
176	A	3	2	1.	13	0.154
177	A	3	2	1.	13	0.154
178	A	3	2	1.	13	0.154
179	A	3	2	1.	13	0.154
180	A	3	2	1.	13	0.154
181	A	5	3	1.	13	0.231
182	A	5	3	1.	13	0.231
183	A	5	3	1.	13	0.231
184	A	4	3	1.	13	0.231
185	A	3	2	1.	13	0.154
186	A	3	3	1.	13	0.231
187	A	3	2	1.	9	0.222
188	A	4	3	1.	13	0.231
189	A	5	3	1.	13	0.231
190	A	6	3	1.	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
191	A	7	3	1.	13	0.231
192	A	3	2	1.	13	0.154
193	A	3	2	1.	13	0.154
194	A	3	2	1.	13	0.154
195	A	3	2	1.	13	0.154
196	A	1	1	1.	13	0.077
197	A	3	3	1.	13	0.231
198	A	4	3	1.	13	0.231
199	A	5	3	1.	13	0.231
200	A	3	2	1.	13	0.154
201	A	3	2	1.	13	0.154
202	A	3	2	1.	13	0.154
203	A	3	2	1.	13	0.154
204	A	1	1	1.	11	0.091
205	A	3	2	1.	13	0.154
206	A	3	2	1.	13	0.154
207	A	3	2	1.	13	0.154
208	A	3	2	1.	13	0.154
209	A	12	3	1.	13	0.231
210	A	12	3	1.	13	0.231
211	A	11	3	1.	13	0.231
212	A	10	2	1.	13	0.154
213	A	10	3	1.	13	0.231
214	A	10	3	1.	13	0.231
215	A	10	3	1.	13	0.231
216	A	10	3	1.	13	0.231
217	A	10	3	1.	13	0.231
218	A	10	3	1.	13	0.231
219	A	10	3	1.	13	0.231
220	A	10	3	1.	13	0.231
221	A	10	2	1.	9	0.222
222	A	11	3	1.	13	0.231
223	A	12	3	1.	13	0.231
224	A	13	3	1.	13	0.231
225	A	3	2	1.	14	0.143
226	A	2	2	1.	14	0.143
227	A	1	1	1.	12	0.083
228	A	1	1	1.	10	0.1
229	A	4	4	1.	14	0.286
230	A	2	2	1.	14	0.143
231	A	3	2	1.	14	0.143
232	A	3	2	1.	14	0.143
233	A	2	2	1.	14	0.143
234	A	1	1	1.	12	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	2	2	1.	10	0.2
236	A	3	2	1.	14	0.143
237	A	3	3	1.	14	0.214
238	A	3	2	1.	14	0.143
239	A	1	1	1.	14	0.071
240	A	3	3	1.	14	0.214
241	A	1	1	1.	12	0.083
242	A	3	2	1.	10	0.2
243	A	3	2	1.	14	0.143
244	A	4	3	1.	14	0.214
245	A	3	2	1.	14	0.143
246	A	3	2	1.	14	0.143
247	A	5	3	1.	14	0.214
248	A	1	1	1.	12	0.083
249	A	5	2	1.	10	0.2
250	A	3	2	1.	14	0.143
251	A	6	3	1.	14	0.214
252	A	3	2	1.	14	0.143
253	A	4	4	1.	13	0.308
254	A	4	4	1.	13	0.308
255	A	3	2	1.	13	0.154
256	A	3	2	1.	13	0.154
257	A	1	1	1.	10	0.1
258	A	1	1	1.	18	0.056
259	A	3	2	1.	13	0.154
260	A	3	2	1.	13	0.154
261	A	1	1	1.	14	0.071
262	A	1	1	1.	15	0.067
263	A	1	1	1.	20	0.05
264	A	2	1	1.	13	0.077
265	A	2	1	1.	13	0.077
266	A	2	1	1.	13	0.077
267	A	2	1	1.	13	0.077
268	A	2	1	1.	13	0.077
269	A	2	1	1.	13	0.077
270	A	2	1	1.	13	0.077
271	A	2	1	1.	13	0.077
272	A	2	1	1.	15	0.067
273	A	2	1	1.	15	0.067
274	A	2	1	1.	15	0.067
275	A	2	1	1.	15	0.067
276	A	2	1	1.	15	0.067
277	A	2	1	1.	15	0.067
278	A	2	1	1.	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
279	A	2	1	1.	15	0.067
280	A	2	1	1.	15	0.067
281	A	2	1	1.	15	0.067
282	A	2	1	1.	15	0.067
283	A	2	1	1.	15	0.067
284	A	2	1	1.	15	0.067
285	A	2	1	1.	15	0.067
286	A	2	1	1.	15	0.067
287	A	2	1	1.	15	0.067
288	A	12	8	1.	15	0.533
289	A	11	8	1.	15	0.533
290	A	11	8	1.	15	0.533
291	A	10	7	1.	15	0.467
292	A	10	7	1.	15	0.467
293	A	11	8	1.	15	0.533
294	A	11	8	1.	15	0.533
295	A	12	8	1.	15	0.533
296	A	12	9	1.	15	0.6
297	A	11	8	1.	15	0.533
298	A	11	8	1.	15	0.533
299	A	11	8	1.	15	0.533
300	A	11	8	1.	15	0.533
301	A	12	9	1.	15	0.6
302	A	12	9	1.	15	0.6
303	A	13	9	1.	15	0.6
304	A	12	8	1.	15	0.533
305	A	12	9	1.	15	0.6
306	A	12	9	1.	15	0.6
307	A	12	8	1.	15	0.533
308	A	12	8	1.	15	0.533
309	A	13	9	1.	15	0.6
310	A	13	9	1.	15	0.6
311	A	14	9	1.	15	0.6
312	A	4	4	1.	16	0.25
313	A	12	8	1.	13	0.615
314	A	11	8	1.	13	0.615
315	A	11	8	1.	13	0.615
316	A	10	7	1.	13	0.538
317	A	10	7	1.	13	0.538
318	A	11	8	1.	13	0.615
319	A	11	8	1.	13	0.615
320	A	12	8	1.	13	0.615
321	A	12	9	1.	13	0.692
322	A	11	8	1.	13	0.615

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	11	8	1.	13	0.615
324	A	11	8	1.	13	0.615
325	A	11	8	1.	13	0.615
326	A	12	9	1.	13	0.692
327	A	12	9	1.	13	0.692
328	A	13	9	1.	13	0.692
329	A	12	8	1.	13	0.615
330	A	12	9	1.	13	0.692
331	A	12	9	1.	13	0.692
332	A	12	8	1.	13	0.615
333	A	12	8	1.	13	0.615
334	A	13	9	1.	13	0.692
335	A	13	9	1.	13	0.692
336	A	14	9	1.	13	0.692
337	A	4	4	1.	15	0.267
338	A	11	7	1.37	13	0.538
339	A	2	1	1.	13	0.077
340	A	2	1	1.	13	0.077
341	A	2	1	1.	13	0.077
342	A	2	1	1.	13	0.077
343	A	2	1	1.	11	0.091
344	A	1	1	1.	13	0.077
345	A	1	1	1.	13	0.077
346	A	1	1	1.	13	0.077
347	A	1	1	1.	17	0.059
348	A	1	1	1.	15	0.067
349	A	1	1	1.	17	0.059
350	A	1	1	1.	17	0.059
351	A	1	1	1.	17	0.059
352	A	1	1	1.	16	0.062
353	A	3	2	1.	15	0.133
354	A	3	2	1.	15	0.133
355	A	3	2	1.	15	0.133
356	A	1	1	1.	13	0.077
357	A	4	4	1.	15	0.267
358	A	4	4	1.	15	0.267
359	A	5	5	1.	15	0.333
360	A	6	5	1.	15	0.333
361	A	5	4	1.	15	0.267
362	A	4	4	1.	15	0.267
363	A	3	3	1.	11	0.273
364	A	3	3	1.	15	0.2
365	A	1	1	1.	15	0.067
366	A	2	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	3	2	1.	15	0.133
368	A	4	2	1.	15	0.133
369	A	3	2	1.	15	0.133
370	A	3	2	1.	15	0.133
371	A	3	2	1.	15	0.133
372	A	1	1	1.	13	0.077
373	A	5	4	1.	15	0.267
374	A	5	5	1.	15	0.333
375	A	5	4	1.	15	0.267
376	A	6	5	1.	15	0.333
377	A	7	5	1.	15	0.333
378	A	6	4	1.	15	0.267
379	A	5	4	1.	15	0.267
380	A	4	3	1.	11	0.273
381	A	4	4	1.	15	0.267
382	A	4	3	1.	15	0.2
383	A	1	1	1.	15	0.067
384	A	2	2	1.	15	0.133
385	A	3	2	1.	15	0.133
386	A	4	2	1.	15	0.133
387	A	3	2	1.	15	0.133
388	A	3	2	1.	15	0.133
389	A	3	2	1.	15	0.133
390	A	1	1	1.	13	0.077
391	A	6	4	1.	15	0.267
392	A	6	5	1.	15	0.333
393	A	6	5	1.	15	0.333
394	A	6	4	1.	15	0.267
395	A	7	5	1.	15	0.333
396	A	8	5	1.	15	0.333
397	A	7	4	1.	15	0.267
398	A	6	4	1.	15	0.267
399	A	5	3	1.	11	0.273
400	A	5	4	1.	15	0.267
401	A	5	4	1.	15	0.267
402	A	5	3	1.	15	0.2
403	A	1	1	1.	15	0.067
404	A	2	2	1.	15	0.133
405	A	3	2	1.	15	0.133
406	A	4	2	1.	15	0.133
407	A	5	2	1.	15	0.133
408	A	6	2	1.	15	0.133
409	A	3	2	1.	15	0.133
410	A	3	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	3	2	1.	15	0.133
412	A	3	2	1.	15	0.133
413	A	3	2	1.	15	0.133
414	A	3	2	1.	15	0.133
415	A	3	2	1.	15	0.133
416	A	1	1	1.	13	0.077
417	A	8	4	1.	15	0.267
418	A	8	5	1.	15	0.333
419	A	8	5	1.	15	0.333
420	A	8	5	1.	15	0.333
421	A	8	5	1.	15	0.333
422	A	8	4	1.	15	0.267
423	A	9	5	1.	15	0.333
424	A	10	5	1.	15	0.333
425	A	10	4	1.	15	0.267
426	A	9	4	1.	15	0.267
427	A	8	4	1.	15	0.267
428	A	7	3	1.	11	0.273
429	A	7	4	1.	15	0.267
430	A	7	4	1.	15	0.267
431	A	7	4	1.	15	0.267
432	A	7	4	1.	15	0.267
433	A	7	3	1.	15	0.2
434	A	1	1	1.	15	0.067
435	A	2	2	1.	15	0.133
436	A	3	2	1.	15	0.133
437	A	4	2	1.	15	0.133
438	A	5	2	1.	15	0.133
439	A	6	2	1.	15	0.133
440	A	7	2	1.	15	0.133
441	A	3	2	1.	15	0.133
442	A	4	3	1.	15	0.2
443	A	3	2	1.	15	0.133
444	A	3	3	1.	15	0.2
445	A	1	1	1.	13	0.077
446	A	2	2	1.	11	0.182
447	A	4	4	1.	15	0.267
448	A	2	2	1.	15	0.133
449	A	4	4	1.	15	0.267
450	A	1	1	1.	15	0.067
451	A	5	5	1.	15	0.333
452	A	3	2	1.	15	0.133
453	A	4	3	1.	15	0.2
454	A	3	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
455	A	3	3	1.	15	0.2
456	A	1	1	1.	13	0.077
457	A	2	2	1.	11	0.182
458	A	4	4	1.	15	0.267
459	A	2	2	1.	15	0.133
460	A	4	4	1.	15	0.267
461	A	1	1	1.	15	0.067
462	A	5	5	1.	15	0.333
463	A	3	2	1.	15	0.133
464	A	5	4	1.	15	0.267
465	A	3	2	1.	15	0.133
466	A	4	4	1.	15	0.267
467	A	1	1	1.	13	0.077
468	A	3	3	1.	11	0.273
469	A	4	4	1.	15	0.267
470	A	3	3	1.	15	0.2
471	A	4	4	1.	15	0.267
472	A	1	1	1.	15	0.067
473	A	5	5	1.	15	0.333
474	A	3	2	1.	15	0.133
475	A	5	4	1.	15	0.267
476	A	3	2	1.	15	0.133
477	A	4	4	1.	15	0.267
478	A	1	1	1.	13	0.077
479	A	3	3	1.	11	0.273
480	A	4	4	1.	15	0.267
481	A	3	3	1.	15	0.2
482	A	4	4	1.	15	0.267
483	A	1	1	1.	15	0.067
484	A	5	5	1.	15	0.333
485	A	3	2	1.	15	0.133
486	A	4	3	1.	15	0.2
487	A	3	2	1.	15	0.133
488	A	3	3	1.	15	0.2
489	A	1	1	1.	13	0.077
490	A	2	2	1.	11	0.182
491	A	3	3	1.	15	0.2
492	A	1	1	1.	15	0.067
493	A	4	4	1.	15	0.267
494	A	2	2	1.	15	0.133
495	A	5	4	1.	15	0.267
496	A	3	2	1.	15	0.133
497	A	4	4	1.	15	0.267
498	A	3	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	3	3	1.	15	0.2
500	A	1	1	1.	13	0.077
501	A	1	1	1.	11	0.091
502	A	4	4	1.	15	0.267
503	A	2	2	1.	15	0.133
504	A	5	4	0.99	15	0.267
505	A	3	2	1.	15	0.133
506	A	5	4	1.	15	0.267
507	A	3	2	1.	15	0.133
508	A	4	3	1.	15	0.2
509	A	3	2	1.	15	0.133
510	A	1	1	1.	15	0.067
511	A	1	1	1.	13	0.077
512	A	2	2	1.	11	0.182
513	A	5	4	1.	15	0.267
514	A	3	3	1.	15	0.2
515	A	6	4	1.05	15	0.267
516	A	4	3	1.	15	0.2
517	A	7	4	1.	15	0.267
518	A	3	2	1.	15	0.133
519	A	6	3	1.	15	0.2
520	A	3	2	1.	15	0.133
521	A	1	1	1.	15	0.067
522	A	3	2	1.	15	0.133
523	A	2	2	1.	15	0.133
524	A	3	2	1.	15	0.133
525	A	3	2	1.	15	0.133
526	A	1	1	1.	13	0.077
527	A	4	2	1.	11	0.182
528	A	7	4	1.	15	0.267
529	A	5	3	1.	15	0.2
530	A	8	4	1.05	15	0.267
531	A	6	3	1.	15	0.2
532	A	3	2	1.	15	0.133
533	A	3	2	1.	15	0.133
534	A	3	2	1.	15	0.133
535	A	2	2	1.	15	0.133
536	A	1	1	1.	13	0.077
537	A	1	1	1.	11	0.091
538	A	3	3	1.	15	0.2
539	A	1	1	1.	15	0.067
540	A	4	4	1.	15	0.267
541	A	2	2	1.	15	0.133
542	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
543	A	3	2	1.	15	0.133
544	A	3	2	1.	15	0.133
545	A	3	2	1.	15	0.133
546	A	2	2	1.	15	0.133
547	A	1	1	1.	13	0.077
548	A	1	1	1.	11	0.091
549	A	3	3	1.	15	0.2
550	A	1	1	1.	15	0.067
551	A	4	4	1.	15	0.267
552	A	2	2	1.	15	0.133
553	A	5	4	1.	15	0.267
554	A	3	2	1.	15	0.133
555	A	4	3	1.	15	0.2
556	A	3	2	1.	15	0.133
557	A	3	3	1.	15	0.2
558	A	1	1	1.	13	0.077
559	A	2	2	1.	11	0.182
560	A	3	3	1.	15	0.2
561	A	1	1	1.	15	0.067
562	A	4	4	1.	15	0.267
563	A	2	2	1.	15	0.133
564	A	5	4	1.	15	0.267
565	A	3	2	1.	15	0.133
566	A	4	3	1.	15	0.2
567	A	3	2	1.	15	0.133
568	A	3	3	1.	15	0.2
569	A	1	1	1.	13	0.077
570	A	2	2	1.	11	0.182
571	A	3	3	1.	15	0.2
572	A	1	1	1.	15	0.067
573	A	4	4	1.	15	0.267
574	A	2	2	1.	15	0.133
575	A	5	4	1.	15	0.267
576	A	1	1	1.	11	0.091
577	A	1	1	1.	12	0.083
578	A	2	2	1.	11	0.182
579	A	2	2	1.	12	0.167
580	A	1	1	1.	11	0.091
581	A	1	1	1.	12	0.083
582	A	2	2	1.	13	0.154
583	A	2	2	1.	14	0.143
584	A	2	2	1.	11	0.182
585	A	2	2	1.	12	0.167
586	A	2	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
587	A	2	2	1.	14	0.143
588	A	2	2	1.	13	0.154
589	A	5	4	1.	19	0.21
590	A	6	6	1.	19	0.316
591	A	4	4	1.	19	0.21
592	A	5	5	1.	19	0.263
593	A	3	3	1.	19	0.158
594	A	5	5	1.	19	0.263
595	A	3	3	1.	19	0.158
596	A	6	6	1.	19	0.316
597	A	6	4	1.	19	0.21
598	A	7	6	1.	19	0.316
599	A	5	4	1.	19	0.21
600	A	6	5	1.	19	0.263
601	A	4	3	1.	19	0.158
602	A	6	6	1.	19	0.316
603	A	4	4	1.	19	0.21
604	A	6	5	1.	19	0.263
605	A	4	3	1.	19	0.158
606	A	7	6	1.	19	0.316
607	A	6	6	1.	22	0.273
608	A	5	5	1.	22	0.227
609	A	5	5	1.	22	0.227
610	A	4	4	1.	22	0.182
611	A	5	5	1.	22	0.227
612	A	4	4	1.	22	0.182
613	A	4	3	1.	19	0.158
614	A	5	5	1.	19	0.263
615	A	3	3	1.	19	0.158
616	A	4	4	1.	19	0.21
617	A	2	2	1.	19	0.105
618	A	5	5	1.	19	0.263
619	A	3	3	1.	19	0.158
620	A	6	5	1.	19	0.263
621	A	4	4	1.	19	0.21
622	A	5	5	1.	19	0.263
623	A	3	3	1.	19	0.158
624	A	5	5	1.	19	0.263
625	A	3	3	1.	19	0.158
626	A	6	6	1.	19	0.316
627	A	4	4	1.	19	0.21
628	A	7	6	1.	19	0.316
629	A	4	3	1.	19	0.158
630	A	6	6	1.	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
631	A	4	4	1.	19	0.21
632	A	6	5	1.	19	0.263
633	A	4	3	1.	19	0.158
634	A	7	6	1.	19	0.316
635	A	5	4	1.	19	0.21
636	A	8	6	1.	19	0.316
637	A	5	5	1.	22	0.227
638	A	4	4	1.	22	0.182
639	A	4	4	1.	22	0.182
640	A	3	3	1.	22	0.136
641	A	5	5	1.	22	0.227
642	A	4	4	1.	22	0.182
643	A	5	5	1.	22	0.227
644	A	4	4	1.	22	0.182
645	A	5	5	1.	22	0.227
646	A	4	4	1.	22	0.182
647	A	6	6	1.	22	0.273
648	A	5	5	1.	22	0.227
649	A	2	2	1.	20	0.1
650	A	2	2	1.	17	0.118
651	A	2	2	1.28	15	0.133
652	A	2	2	1.26	15	0.133
653	A	2	2	1.26	15	0.133
654	A	2	2	1.38	15	0.133
655	A	2	2	1.32	15	0.133
656	A	2	2	1.26	17	0.118
657	A	2	2	1.26	17	0.118
658	A	2	2	1.26	15	0.133
659	A	2	2	1.24	17	0.118
660	A	2	2	1.29	17	0.118
661	A	1	1	1.	31	0.032
662	C	5	2	7.47	43	0.047
663	A	1	1	1.	29	0.034
664	C	5	2	8.2	38	0.053
665	A	3	2	1.	15	0.133
666	A	3	2	1.	15	0.133
667	A	3	2	1.	15	0.133
668	A	1	1	1.	13	0.077
669	A	6	6	1.	15	0.4
670	A	6	6	1.	15	0.4
671	A	7	7	1.	15	0.467
672	A	5	4	1.	15	0.267
673	A	4	4	1.	15	0.267
674	A	3	3	1.	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
675	A	3	3	1.	15	0.2
676	A	4	4	1.	15	0.267
677	A	3	2	1.	15	0.133
678	A	3	2	1.	15	0.133
679	A	3	2	1.	15	0.133
680	A	1	1	1.	13	0.077
681	A	6	6	1.	15	0.4
682	A	6	6	1.	15	0.4
683	A	7	7	1.	15	0.467
684	A	7	6	1.	15	0.4
685	A	6	6	1.	15	0.4
686	A	5	5	1.	11	0.454
687	A	5	5	1.	15	0.333
688	A	6	6	1.	15	0.4
689	A	3	2	1.	15	0.133
690	A	3	2	1.	15	0.133
691	A	3	2	1.	15	0.133
692	A	1	1	1.	13	0.077
693	A	7	6	1.	15	0.4
694	A	7	7	1.	15	0.467
695	A	7	6	1.	15	0.4
696	A	6	4	1.	15	0.267
697	A	5	4	1.	15	0.267
698	A	4	3	1.	11	0.273
699	A	4	4	1.	15	0.267
700	A	4	3	1.	15	0.2
701	A	1	1	1.	11	0.091
702	A	3	2	1.	15	0.133
703	A	3	2	1.	15	0.133
704	A	3	2	1.	15	0.133
705	A	1	1	1.	13	0.077
706	A	5	5	1.	15	0.333
707	A	6	6	1.	15	0.4
708	A	7	6	1.	15	0.4
709	A	6	5	1.	15	0.333
710	A	5	5	1.	15	0.333
711	A	4	4	1.	11	0.364
712	A	5	5	1.	15	0.333
713	A	6	5	1.	15	0.333
714	A	3	2	1.	15	0.133
715	A	3	2	1.	15	0.133
716	A	3	2	1.	15	0.133
717	A	1	1	1.	13	0.077
718	A	5	5	1.	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
719	A	6	6	1.	15	0.4
720	A	7	6	1.	15	0.4
721	A	4	3	1.	15	0.2
722	A	3	3	1.	15	0.2
723	A	2	2	1.	11	0.182
724	A	3	3	1.	15	0.2
725	A	4	3	1.	15	0.2
726	A	3	2	1.	15	0.133
727	A	3	2	1.	15	0.133
728	A	3	2	1.	15	0.133
729	A	1	1	1.	13	0.077
730	A	6	6	1.	15	0.4
731	A	7	6	1.02	15	0.4
732	A	8	6	1.	15	0.4
733	A	6	6	1.	15	0.4
734	A	5	5	1.	15	0.333
735	A	5	5	1.	11	0.454
736	A	6	6	1.	15	0.4
737	A	7	6	1.	15	0.4
738	A	12	11	1.41	19	0.579
739	A	11	11	1.49	19	0.579
740	A	10	10	1.59	19	0.526
741	A	10	10	1.59	19	0.526
742	A	1	1	1.	19	0.053
743	A	2	2	1.	19	0.105
744	A	3	2	1.	19	0.105
745	A	4	2	1.	19	0.105
746	A	6	5	1.	19	0.263
747	A	5	5	1.	19	0.263
748	A	4	4	1.	19	0.21
749	A	4	4	1.	19	0.21
750	A	5	5	1.	19	0.263
751	A	2	2	1.	19	0.105
752	A	2	2	1.	19	0.105
753	A	2	2	1.	19	0.105
754	A	13	11	1.36	19	0.579
755	A	12	11	1.42	19	0.579
756	A	11	10	1.49	19	0.526
757	A	11	11	1.52	19	0.579
758	A	11	10	1.49	19	0.526
759	A	1	1	1.	19	0.053
760	A	2	2	1.	19	0.105
761	A	3	2	1.	19	0.105
762	A	7	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
763	A	6	5	1.	19	0.263
764	A	5	4	1.	19	0.21
765	A	5	5	1.	19	0.263
766	A	5	4	1.	19	0.21
767	A	6	5	1.	19	0.263
768	A	2	2	1.	19	0.105
769	A	2	2	1.	19	0.105
770	A	2	2	1.	19	0.105
771	A	12	10	1.4	19	0.526
772	A	11	10	1.48	19	0.526
773	A	10	10	1.6	19	0.526
774	A	9	9	1.73	19	0.474
775	A	1	1	1.	19	0.053
776	A	2	2	1.	19	0.105
777	A	3	2	1.	19	0.105
778	A	4	2	1.	19	0.105
779	A	5	4	1.	19	0.21
780	A	4	4	1.	19	0.21
781	A	3	3	1.	19	0.158
782	A	4	4	1.	19	0.21
783	A	5	4	1.	19	0.21
784	A	2	2	1.	19	0.105
785	A	2	2	1.	19	0.105
786	A	2	2	1.	19	0.105
787	A	5	4	1.	15	0.267
788	A	4	4	1.	15	0.267
789	A	3	3	1.	11	0.273
790	A	3	3	1.	15	0.2
791	A	4	4	1.	15	0.267
792	A	5	4	1.	15	0.267
793	A	5	4	1.	16	0.25
794	A	4	4	1.	16	0.25
795	A	3	3	1.	12	0.25
796	A	3	3	1.	16	0.188
797	A	4	4	1.	16	0.25
798	A	5	4	1.	16	0.25
799	A	6	5	1.	15	0.333
800	A	5	5	1.	15	0.333
801	A	4	4	1.	11	0.364
802	A	4	4	1.	15	0.267
803	A	5	5	1.	15	0.333
804	A	6	5	1.	15	0.333
805	A	5	4	1.	16	0.25
806	A	4	4	1.	16	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
807	A	3	3	1.	12	0.25
808	A	3	3	1.	16	0.188
809	A	4	4	1.	16	0.25
810	A	5	4	1.	16	0.25
811	A	4	3	1.	11	0.273
812	A	4	3	1.	12	0.25
813	A	5	4	1.	11	0.364
814	A	4	3	1.	12	0.25
815	A	6	4	1.	15	0.267
816	A	5	4	1.	15	0.267
817	A	4	4	1.	15	0.267
818	A	3	3	1.	11	0.273
819	A	4	4	1.	15	0.267
820	A	5	4	1.	15	0.267
821	A	6	4	1.	15	0.267
822	A	5	3	1.	16	0.188
823	A	4	3	1.	16	0.188
824	A	3	3	1.	16	0.188
825	A	2	2	1.	12	0.167
826	A	3	3	1.	16	0.188
827	A	4	3	1.	16	0.188
828	A	5	3	1.	16	0.188
829	A	5	3	1.	15	0.2
830	A	4	3	1.	15	0.2
831	A	3	3	1.	15	0.2
832	A	2	2	1.	11	0.182
833	A	3	3	1.	15	0.2
834	A	4	3	1.	15	0.2
835	A	5	3	1.	15	0.2
836	A	5	3	1.	16	0.188
837	A	4	3	1.	16	0.188
838	A	3	3	1.	16	0.188
839	A	2	2	1.	12	0.167
840	A	3	3	1.	16	0.188
841	A	4	3	1.	16	0.188
842	A	5	3	1.	16	0.188
843	A	5	3	1.	15	0.2
844	A	4	3	1.	15	0.2
845	A	3	3	1.	15	0.2
846	A	2	2	1.	11	0.182
847	A	3	3	1.	15	0.2
848	A	4	3	1.	15	0.2
849	A	5	3	1.	15	0.2
850	A	5	4	1.	16	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
851	A	4	4	1.	16	0.25
852	A	3	3	1.	16	0.188
853	A	3	3	1.	12	0.25
854	A	4	4	1.	16	0.25
855	A	5	4	1.	16	0.25
856	A	6	4	1.	16	0.25
857	A	3	3	1.	11	0.273
858	A	3	3	1.	11	0.273
859	A	4	3	1.	11	0.273
860	A	3	3	1.	12	0.25
861	A	4	3	1.	12	0.25
862	A	4	3	1.	12	0.25
863	A	5	3	1.	15	0.2
864	A	4	3	1.	15	0.2
865	A	3	3	1.	15	0.2
866	A	2	2	1.	11	0.182
867	A	3	3	1.	15	0.2
868	A	4	3	1.	15	0.2
869	A	5	3	1.	15	0.2
870	A	4	2	1.	15	0.133
871	A	3	2	1.	15	0.133
872	A	2	2	1.	15	0.133
873	A	1	1	1.	11	0.091
874	A	2	2	1.	15	0.133
875	A	3	2	1.	15	0.133
876	A	4	2	1.	15	0.133
877	A	4	2	1.	15	0.133
878	A	3	2	1.	15	0.133
879	A	2	2	1.	15	0.133
880	A	1	1	1.	11	0.091
881	A	2	2	1.	15	0.133
882	A	3	2	1.	15	0.133
883	A	4	2	1.	15	0.133
884	A	4	2	1.	15	0.133
885	A	3	2	1.	15	0.133
886	A	2	2	1.	15	0.133
887	A	1	1	1.	11	0.091
888	A	2	2	1.	15	0.133
889	A	3	2	1.	15	0.133
890	A	4	2	1.	15	0.133
891	A	7	5	1.	15	0.333
892	A	6	5	1.	15	0.333
893	A	5	5	1.	15	0.333
894	A	4	4	1.	11	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
895	A	5	5	1.	15	0.333
896	A	6	5	1.	15	0.333
897	A	7	5	1.	15	0.333
898	A	7	5	1.	15	0.333
899	A	6	5	1.	15	0.333
900	A	5	5	1.	15	0.333
901	A	4	4	1.	11	0.364
902	A	5	5	1.	15	0.333
903	A	6	5	1.	15	0.333
904	A	7	5	1.	15	0.333
905	A	5	3	1.	15	0.2
906	A	4	3	1.	15	0.2
907	A	3	3	1.	15	0.2
908	A	2	2	1.	11	0.182
909	A	3	3	1.	15	0.2
910	A	4	3	1.	15	0.2
911	A	5	3	1.	15	0.2
912	A	5	3	1.	15	0.2
913	A	4	3	1.	15	0.2
914	A	3	3	1.	15	0.2
915	A	2	2	1.	11	0.182
916	A	3	3	1.	15	0.2
917	A	4	3	1.	15	0.2
918	A	5	3	1.	15	0.2
919	A	8	7	1.	19	0.368
920	A	7	7	1.	19	0.368
921	A	6	6	1.	19	0.316
922	A	6	6	1.	19	0.316
923	A	7	7	1.	19	0.368
924	A	8	7	1.	19	0.368
925	A	7	7	1.	19	0.368
926	A	6	6	1.	19	0.316
927	A	6	6	1.	19	0.316
928	A	1	1	1.	19	0.053
929	A	2	2	1.	19	0.105
930	A	3	2	1.	19	0.105
931	A	4	2	1.	19	0.105
932	A	7	7	1.	20	0.35
933	A	6	6	1.	20	0.3
934	A	6	6	1.	20	0.3
935	A	7	7	1.	20	0.35
936	A	8	7	1.	20	0.35
937	A	13	10	1.	20	0.5
938	A	12	9	1.	20	0.45

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
939	A	12	9	1.	20	0.45
940	A	1	1	1.	20	0.05
941	A	2	2	1.	20	0.1
942	A	3	2	1.	20	0.1
943	A	4	2	1.	20	0.1
944	A	6	6	1.	19	0.316
945	A	5	5	1.	19	0.263
946	A	1	1	1.	19	0.053
947	A	2	2	1.	19	0.105
948	A	3	2	1.	19	0.105
949	A	6	5	1.	19	0.263
950	A	5	5	1.	19	0.263
951	A	4	4	1.	19	0.21
952	A	4	4	1.	19	0.21
953	A	5	5	1.	19	0.263
954	A	6	5	1.	19	0.263
955	A	12	9	1.	20	0.45
956	A	11	8	1.	20	0.4
957	A	1	1	1.	20	0.05
958	A	2	2	1.	20	0.1
959	A	3	2	1.	20	0.1
960	A	5	5	1.	20	0.25
961	A	4	4	1.	20	0.2
962	A	3	3	1.	20	0.15
963	A	4	4	1.	20	0.2
964	A	5	4	1.	20	0.2
965	A	6	6	1.	19	0.316
966	A	5	5	1.	19	0.263
967	A	6	6	1.	19	0.316
968	A	7	6	1.	19	0.316
969	A	8	6	1.	19	0.316
970	A	6	6	1.	19	0.316
971	A	5	5	1.	19	0.263
972	A	1	1	1.	19	0.053
973	A	2	2	1.	19	0.105
974	A	3	2	1.	19	0.105
975	A	6	6	1.	20	0.3
976	A	5	5	1.	20	0.25
977	A	6	6	1.	20	0.3
978	A	7	6	1.	20	0.3
979	A	8	6	1.	20	0.3
980	A	12	9	1.	20	0.45
981	A	11	8	1.	20	0.4
982	A	1	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
983	A	2	2	1.	20	0.1
984	A	3	2	1.	20	0.1
985	A	7	7	1.	19	0.368
986	A	6	6	1.	19	0.316
987	A	1	1	1.	19	0.053
988	A	2	2	1.	19	0.105
989	A	3	2	1.	19	0.105
990	A	4	2	1.	19	0.105
991	A	6	4	1.	19	0.21
992	A	5	4	1.	19	0.21
993	A	4	4	1.	19	0.21
994	A	3	3	1.	19	0.158
995	A	4	4	1.	19	0.21
996	A	5	4	1.	19	0.21
997	A	6	4	1.	19	0.21
998	A	2	2	1.	19	0.105
999	A	2	2	1.	19	0.105
1000	A	2	2	1.	19	0.105
1001	A	2	2	1.	19	0.105
1002	A	2	2	1.	19	0.105
1003	A	2	2	1.	19	0.105
1004	A	2	2	1.	19	0.105
1005	A	2	2	1.	19	0.105
1006	A	2	2	1.	19	0.105
1007	A	2	2	1.	19	0.105
1008	A	2	2	1.	19	0.105
1009	A	2	2	1.	19	0.105
1010	A	7	5	1.	15	0.333
1011	A	6	5	1.	15	0.333
1012	A	5	5	1.	15	0.333
1013	A	4	4	1.	11	0.364
1014	A	4	4	1.	15	0.267
1015	A	5	5	1.	15	0.333
1016	A	6	5	1.	15	0.333
1017	A	7	5	1.	15	0.333
1018	A	9	7	1.	15	0.467
1019	A	8	7	1.	15	0.467
1020	A	7	7	1.	15	0.467
1021	A	6	6	1.	11	0.546
1022	A	7	7	1.	15	0.467
1023	A	8	7	1.	15	0.467
1024	A	9	7	1.	15	0.467
1025	A	6	4	1.	15	0.267
1026	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1027	A	4	4	1.	15	0.267
1028	A	3	3	1.	11	0.273
1029	A	4	4	1.	15	0.267
1030	A	5	4	1.	15	0.267
1031	A	6	4	1.	15	0.267
1032	A	9	8	1.	15	0.533
1033	A	8	8	1.	15	0.533
1034	A	7	7	1.	15	0.467
1035	A	5	5	1.	11	0.454
1036	A	8	8	1.	15	0.533
1037	A	9	8	1.	15	0.533
1038	A	10	8	1.	15	0.533
1039	A	3	2	1.	13	0.154
1040	A	3	2	1.	13	0.154
1041	A	3	2	1.	13	0.154
1042	A	1	1	1.	11	0.091
1043	A	2	2	1.	13	0.154
1044	A	2	2	1.	13	0.154
1045	A	2	2	1.22	13	0.154
1046	A	2	2	1.22	13	0.154
1047	A	2	2	1.22	13	0.154
1048	A	2	2	1.26	9	0.222
1049	A	2	2	1.24	13	0.154
1050	A	2	2	1.21	15	0.133
1051	A	2	2	1.21	15	0.133
1052	A	2	2	1.21	15	0.133
1053	A	2	2	1.21	15	0.133
1054	A	2	2	1.22	15	0.133
1055	A	2	2	1.22	15	0.133
1056	A	2	2	1.21	15	0.133
1057	A	2	2	1.21	15	0.133
1058	A	2	2	1.15	13	0.154
1059	A	2	2	1.	15	0.133
1060	A	2	2	1.32	17	0.118
1061	A	3	2	1.	17	0.118
1062	A	2	2	1.32	17	0.118
1063	A	2	2	1.	17	0.118
1064	A	2	2	1.32	17	0.118
1065	A	1	1	1.	17	0.059
1066	A	2	2	1.32	17	0.118
1067	A	2	2	1.3	17	0.118
1068	A	2	2	1.33	15	0.133
1069	A	2	2	1.31	17	0.118
1070	A	2	2	1.33	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1071	A	2	2	1.31	17	0.118

Chapter 3

Listing of integrals

3.1 $\int x^4 \left(a + bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Rubi [A] time = 0.0048247, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a + bx^2\right) dx &= \int \left(ax^4 + bx^6\right) dx\\ &= \frac{ax^5}{5} + \frac{bx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0013178, size = 17, normalized size = 1.

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a),x)
```

```
[Out] 1/5*a*x^5+1/7*b*x^7
```

Maxima [A] time = 1.1743, size = 18, normalized size = 1.06

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Fricas [A] time = 1.48903, size = 31, normalized size = 1.82

$$\frac{1}{7}x^7b+\frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b + 1/5*x^5*a
```

Sympy [A] time = 0.056525, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

Giac [A] time = 1.36762, size = 18, normalized size = 1.06

$$\frac{1}{7}bx^7 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

3.2 $\int x^3 \left(a + bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rubi [A] time = 0.0046979, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^3 \left(a + bx^2\right) dx &= \int \left(ax^3 + bx^5\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^6}{6} \end{aligned}$$

Mathematica [A] time = 0.0009729, size = 17, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Maxima [A] time = 1.63291, size = 18, normalized size = 1.06

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A] time = 1.40298, size = 31, normalized size = 1.82

$$\frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/4*x^4*a
```

Sympy [A] time = 0.056355, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

Giac [A] time = 1.97722, size = 18, normalized size = 1.06

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

3.3 $\int x^2 (a + bx^2) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rubi [A] time = 0.0048836, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^2) dx &= \int (ax^2 + bx^4) dx \\ &= \frac{ax^3}{3} + \frac{bx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.000975, size = 17, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Maxima [A] time = 1.21825, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 1.3667, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/3*x^3*a
```

Sympy [A] time = 0.055765, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

Giac [A] time = 2.38683, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

3.4 $\int x\left(a+bx^2\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rubi [A] time = 0.0050435, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\,dx &= \int \left(ax+bx^3\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.0009521, size = 17, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a),x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Maxima [A] time = 2.31886, size = 19, normalized size = 1.12

$$\frac{\left(bx^2+a\right)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^2/b
```

Fricas [A] time = 1.24602, size = 31, normalized size = 1.82

$$\frac{1}{4}x^4b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/2*x^2*a
```

Sympy [A] time = 0.054487, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Giac [A] time = 2.10797, size = 18, normalized size = 1.06

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

3.5 $\int \left(a+bx^2\right) dx$

Optimal. Leaf size=12

$$ax+\frac{bx^3}{3}$$

```
[Out] a*x + (b*x^3)/3
```

Rubi [A] time = 0.0022155, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2,x]
```

```
[Out] a*x + (b*x^3)/3
```

Rubi steps

$$\int \left(a+bx^2\right) dx=ax+\frac{bx^3}{3}$$

Mathematica [A] time = 0.0000288, size = 12, normalized size = 1.

$$ax+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2,x]
```

```
[Out] a*x + (b*x^3)/3
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$ax+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2+a,x)
```

```
[Out] a*x+1/3*b*x^3
```

Maxima [A] time = 1.69238, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

Fricas [A] time = 1.26278, size = 23, normalized size = 1.92

$$\frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b + x*a
```

Sympy [A] time = 0.080374, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3
```

Giac [A] time = 2.03595, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + a*x
```

3.6 $\int \frac{a+bx^2}{x} dx$

Optimal. Leaf size=13

$$a\log(x) + \frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + a*Log[x]
```

Rubi [A] time = 0.0038555, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$a\log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x,x]

[Out] (b*x^2)/2 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x} \, dx &= \int \left(\frac{a}{x}+bx\right) \, dx \\ &= \frac{bx^2}{2}+a\log(x) \end{aligned}$$

Mathematica [A] time = 0.0009411, size = 13, normalized size = 1.

$$a\log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x,x]

[Out] (b*x^2)/2 + a*Log[x]
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{bx^2}{2} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x,x)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

Maxima [A] time = 2.22765, size = 19, normalized size = 1.46

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

Fricas [A] time = 1.46844, size = 30, normalized size = 2.31

$$\frac{1}{2}\,bx^2 + a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.080684, size = 10, normalized size = 0.77

$$a\log\left(x\right) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

Giac [A] time = 2.511, size = 19, normalized size = 1.46

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

3.7 $\int \frac{a+bx^2}{x^2} \, dx$

Optimal. Leaf size=10

$$bx - \frac{a}{x}$$

[Out] -(a/x) + b*x

Rubi [A] time = 0.0041099, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x^2,x]

[Out] -(a/x) + b*x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^2} \, dx &= \int \left(b+\frac{a}{x^2}\right) \, dx \\ &= -\frac{a}{x}+bx \end{aligned}$$

Mathematica [A] time = 0.0007458, size = 10, normalized size = 1.

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x^2,x]

[Out] -(a/x) + b*x

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2,x)
```

```
[Out] -a/x+b*x
```

Maxima [A] time = 1.71266, size = 14, normalized size = 1.4

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*x - a/x
```

Fricas [A] time = 1.38232, size = 20, normalized size = 2.

$$\frac{bx^2 - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x^2 - a)/x
```

Sympy [A] time = 0.234187, size = 5, normalized size = 0.5

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x
```

Giac [A] time = 2.6104, size = 14, normalized size = 1.4

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] b*x - a/x
```

3.8 $\int \frac{a+bx^2}{x^3} dx$

Optimal. Leaf size=13

$$b\log(x) - \frac{a}{2x^2}$$

```
[Out] -a/(2*x^2) + b*Log[x]
```

Rubi [A] time = 0.0044884, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$b\log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^3,x]

[Out] -a/(2*x^2) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^3} dx &= \int \left(\frac{a}{x^3} + \frac{b}{x}\right) dx \\ &= -\frac{a}{2x^2} + b\log(x)\end{aligned}$$

Mathematica [A] time = 0.0024697, size = 13, normalized size = 1.

$$b\log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^3,x]

[Out] -a/(2*x^2) + b*Log[x]
```

Maple [A] time = 0.005, size = 12, normalized size = 0.9

$$-\frac{a}{2\,x^2} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3,x)
```

```
[Out] -1/2/x^2*a+b*ln(x)
```

Maxima [A] time = 1.21374, size = 19, normalized size = 1.46

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

Fricas [A] time = 1.4325, size = 41, normalized size = 3.15

$$\frac{2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A] time = 0.256186, size = 10, normalized size = 0.77

$$-\frac{a}{2x^2}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

Giac [A] time = 2.61954, size = 27, normalized size = 2.08

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{bx^2+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

3.9 $\int \frac{a+bx^2}{x^4}\,dx$

Optimal. Leaf size=15

$$-\frac{a}{3x^3}-\frac{b}{x}$$

```
[Out] -a/(3*x^3) - b/x
```

Rubi [A] time = 0.0047591, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^2}\right)dx\\ &= -\frac{a}{3x^3}-\frac{b}{x}\end{aligned}$$

Mathematica [A] time = 0.0017611, size = 15, normalized size = 1.

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$-\frac{a}{3\,x^3}-\frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4,x)
```

```
[Out] -1/3*a/x^3-b/x
```

Maxima [A] time = 1.57621, size = 18, normalized size = 1.2

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Fricas [A] time = 1.38826, size = 32, normalized size = 2.13

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Sympy [A] time = 0.258209, size = 14, normalized size = 0.93

$$-\frac{a+3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4,x)
```

```
[Out] -(a + 3*b*x**2)/(3*x**3)
```

Giac [A] time = 2.51309, size = 18, normalized size = 1.2

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

3.10 $\int \frac{a+bx^2}{x^5} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

```
[Out] -a/(4*x^4) - b/(2*x^2)
```

Rubi [A] time = 0.0046325, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^5,x]

[Out] -a/(4*x^4) - b/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^5} \, dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^3}\right) \, dx\\ &= -\frac{a}{4x^4}-\frac{b}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0018084, size = 17, normalized size = 1.

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^5,x]

[Out] -a/(4*x^4) - b/(2*x^2)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$-\frac{a}{4\,x^4}-\frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5,x)
```

```
[Out] -1/4*a/x^4-1/2/x^2*b
```

Maxima [A] time = 1.49268, size = 18, normalized size = 1.06

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Fricas [A] time = 1.4151, size = 32, normalized size = 1.88

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Sympy [A] time = 0.268826, size = 14, normalized size = 0.82

$$-\frac{a+2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5,x)
```

```
[Out] -(a + 2*b*x**2)/(4*x**4)
```

Giac [A] time = 2.42417, size = 18, normalized size = 1.06

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

3.11 $\int \frac{a+bx^2}{x^6}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

```
[Out] -a/(5*x^5) - b/(3*x^3)
```

Rubi [A] time = 0.0045444, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}\right)dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0019326, size = 17, normalized size = 1.

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$-\frac{a}{5\,x^5}-\frac{b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^6,x)

[Out] -1/5*a/x^5-1/3*b/x^3
```

Maxima [A] time = 2.00317, size = 20, normalized size = 1.18

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="maxima")

[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Fricas [A] time = 1.40729, size = 36, normalized size = 2.12

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="fricas")

[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [A] time = 0.273641, size = 15, normalized size = 0.88

$$-\frac{3a+5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**6,x)

[Out] -(3*a + 5*b*x**2)/(15*x**5)
```

Giac [A] time = 2.49323, size = 20, normalized size = 1.18

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="giac")

[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

3.12 $\int \frac{a+bx^2}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

```
[Out] -a/(6*x^6) - b/(4*x^4)
```

Rubi [A] time = 0.0047639, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^7} dx &= \int \left(\frac{a}{x^7} + \frac{b}{x^5}\right) dx \\ &= -\frac{a}{6x^6} - \frac{b}{4x^4} \end{aligned}$$

Mathematica [A] time = 0.002024, size = 17, normalized size = 1.

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$-\frac{a}{6\,x^6} - \frac{b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^7,x)
```

```
[Out] -1/6*a/x^6-1/4*b/x^4
```

Maxima [A] time = 1.60667, size = 20, normalized size = 1.18

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Fricas [A] time = 1.40074, size = 36, normalized size = 2.12

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [A] time = 0.271296, size = 15, normalized size = 0.88

$$-\frac{2a+3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**7,x)
```

```
[Out] -(2*a + 3*b*x**2)/(12*x**6)
```

Giac [A] time = 2.63978, size = 20, normalized size = 1.18

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

3.13 $\int x^5 \left(a+bx^2\right)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

```
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rubi [A] time = 0.0175574, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^2 dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^2x^2+2abx^3+b^2x^4\right)dx,x,x^2\right)\\ &= \frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.000908, size = 30, normalized size = 1.

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2,x)
```

```
[Out] 1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Maxima [A] time = 1.68759, size = 32, normalized size = 1.07

$$\frac{1}{10}\,b^2x^{10} + \frac{1}{4}\,abx^8 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Fricas [A] time = 1.26961, size = 58, normalized size = 1.93

$$\frac{1}{10}x^{10}b^2 + \frac{1}{4}x^8ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2 + 1/4*x^8*b*a + 1/6*x^6*a^2
```

Sympy [A] time = 0.058929, size = 24, normalized size = 0.8

$$\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10
```

Giac [A] time = 2.10335, size = 32, normalized size = 1.07

$$\frac{1}{10}\,b^2x^{10} + \frac{1}{4}\,abx^8 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

3.14 $\int x^4 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rubi [A] time = 0.0106896, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^4+2abx^6+b^2x^8\right) dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0007241, size = 30, normalized size = 1.

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Maple [A] time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{2\,abx^7}{7}+\frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2,x)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Maxima [A] time = 2.40818, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Fricas [A] time = 1.28446, size = 55, normalized size = 1.83

$$\frac{1}{9}x^9b^2 + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2
```

Sympy [A] time = 0.05982, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Giac [A] time = 3.2839, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

3.15 $\int x^3 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi [A] time = 0.0177522, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2,x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^2 \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^2 \, dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right) dx, x, x^2\right)\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0006755, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Maxima [A] time = 2.32416, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Fricas [A] time = 1.26203, size = 55, normalized size = 1.83

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

Sympy [A] time = 0.060431, size = 24, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Giac [A] time = 1.34731, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.16 $\int x^2 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi [A] time = 0.0095821, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^2+2abx^4+b^2x^6\right) \, dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0006403, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A] time = 0.002, size = 25, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{2\,abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Maxima [A] time = 1.86725, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Fricas [A] time = 1.28939, size = 55, normalized size = 1.83

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.058723, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Giac [A] time = 2.26912, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

3.17 $\int x\left(a+bx^2\right)^2\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^3}{6b}$$

[Out] (a + b*x^2)^3/(6*b)

Rubi [A] time = 0.0022917, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^2,x]

[Out] (a + b*x^2)^3/(6*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\,dx=\frac{\left(a+bx^2\right)^3}{6b}$$

Mathematica [A] time = 0.0017881, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^2,x]

[Out] (a + b*x^2)^3/(6*b)

Maple [A] time = 0., size = 25, normalized size = 1.6

$$\frac{b^2x^6}{6}+\frac{abx^4}{2}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2,x)
```

```
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

Maxima [A] time = 2.24358, size = 19, normalized size = 1.19

$$\frac{\left(bx^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

Fricas [A] time = 1.24651, size = 55, normalized size = 3.44

$$\frac{1}{6}x^6b^2+\frac{1}{2}x^4ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2
```

Sympy [B] time = 0.059924, size = 24, normalized size = 1.5

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Giac [A] time = 2.27716, size = 19, normalized size = 1.19

$$\frac{\left(bx^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

3.18 $\int \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] time = 0.0072177, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+bx^2\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2x^4\right) \, dx \\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.0008121, size = 25, normalized size = 1.

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] time = 0., size = 22, normalized size = 0.9

$$a^2x+\frac{2\,abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A] time = 2.29613, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A] time = 1.28335, size = 47, normalized size = 1.88

$$\frac{1}{5}x^5b^2 + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

Sympy [A] time = 0.061817, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A] time = 2.05256, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.19 $\int \frac{(a+bx^2)^2}{x} \, dx$

Optimal. Leaf size=23

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi [A] time = 0.0127247, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^2}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(2ab + \frac{a^2}{x} + b^2x\right) \, dx, x, x^2\right) \\
&= abx^2 + \frac{b^2x^4}{4} + a^2 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.0007734, size = 23, normalized size = 1.

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Maple [A] time = 0.002, size = 22, normalized size = 1.

$$abx^2 + \frac{b^2x^4}{4} + a^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x,x)
```

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Maxima [A] time = 1.90964, size = 32, normalized size = 1.39

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2 \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Fricas [A] time = 1.48527, size = 49, normalized size = 2.13

$$\frac{1}{4}\,b^2x^4 + abx^2 + a^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] time = 0.244421, size = 20, normalized size = 0.87

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Giac [A] time = 1.50345, size = 32, normalized size = 1.39

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2 \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

3.20 $\int \frac{(a+bx^2)^2}{x^2}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] time = 0.009647, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0006078, size = 24, normalized size = 1.

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Maple [A] time = 0.003, size = 23, normalized size = 1.

$$-\frac{a^2}{x}+2\,abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2,x)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Maxima [A] time = 1.54354, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A] time = 1.41159, size = 50, normalized size = 2.08

$$\frac{b^2x^4 + 6\,abx^2 - 3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] time = 0.242517, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Giac [A] time = 1.6803, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

3.21 $\int \frac{(a+bx^2)^2}{x^3} dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] time = 0.0135157, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^2 + \frac{a^2}{x^2} + \frac{2ab}{x}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0007728, size = 27, normalized size = 1.

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Maple [A] time = 0.005, size = 24, normalized size = 0.9

$$-\frac{a^2}{2\,x^2}+\frac{b^2x^2}{2}+2\,ab\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3,x)
```

```
[Out] -1/2/x^2*a^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Maxima [A] time = 1.33831, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Fricas [A] time = 1.39969, size = 59, normalized size = 2.19

$$\frac{b^2x^4+4\,abx^2\log\left(x\right)-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A] time = 0.270194, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2}+2ab\log\left(x\right)+\frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Giac [A] time = 1.55442, size = 43, normalized size = 1.59

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{2\,abx^2+a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

3.22 $\int \frac{(a+bx^2)^2}{x^4} dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rubi [A] time = 0.0090147, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^4,x]

[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^4}\,dx &= \int \left(b^2+\frac{a^2}{x^4}+\frac{2ab}{x^2}\right) dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x\end{aligned}$$

Mathematica [A] time = 0.0007299, size = 23, normalized size = 1.

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^4,x]

[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Maple [A] time = 0.003, size = 22, normalized size = 1.

$$-\frac{a^2}{3\,x^3}-2\,\frac{ab}{x}+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Maxima [A] time = 2.32474, size = 30, normalized size = 1.3

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A] time = 1.38166, size = 53, normalized size = 2.3

$$\frac{3\,b^2x^4 - 6\,abx^2 - a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A] time = 0.278253, size = 20, normalized size = 0.87

$$b^2x - \frac{a^2 + 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4,x)
```

```
[Out] b**2*x - (a**2 + 6*a*b*x**2)/(3*x**3)
```

Giac [A] time = 1.91432, size = 30, normalized size = 1.3

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

3.23 $\int \frac{(a+bx^2)^2}{x^5} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rubi [A] time = 0.01285, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx, x, x^2\right)\\ &= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0007085, size = 24, normalized size = 1.

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Maple [A] time = 0.005, size = 23, normalized size = 1.

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/x^2*a*b+b^2*ln(x)
```

Maxima [A] time = 2.40295, size = 35, normalized size = 1.46

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Fricas [A] time = 1.44523, size = 62, normalized size = 2.58

$$\frac{4\,b^2x^4\log\left(x\right)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A] time = 0.31295, size = 22, normalized size = 0.92

$$b^2\log\left(x\right)-\frac{a^2+4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5,x)
```

```
[Out] b**2*log(x) - (a**2 + 4*a*b*x**2)/(4*x**4)
```

Giac [A] time = 2.70646, size = 46, normalized size = 1.92

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{3\,b^2x^4+4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

3.24 $\int \frac{(a+bx^2)^2}{x^6} \, dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rubi [A] time = 0.0102216, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^6,x]

[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^2}{x^6} \, dx &= \int \left(\frac{a^2}{x^6} + \frac{2ab}{x^4} + \frac{b^2}{x^2}\right) dx \\ &= -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x} \end{aligned}$$

Mathematica [A] time = 0.00074, size = 28, normalized size = 1.

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^6,x]

[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Maple [A] time = 0.004, size = 25, normalized size = 0.9

$$-\frac{a^2}{5\,x^5} - \frac{2\,ab}{3\,x^3} - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6,x)
```

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Maxima [A] time = 1.17832, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Fricas [A] time = 1.36785, size = 61, normalized size = 2.18

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A] time = 0.317343, size = 27, normalized size = 0.96

$$-\frac{3a^2+10abx^2+15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6,x)
```

```
[Out] -(3*a**2 + 10*a*b*x**2 + 15*b**2*x**4)/(15*x**5)
```

Giac [A] time = 2.2589, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

3.25 $\int \frac{(a+bx^2)^2}{x^7} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^3}{6ax^6}$$

```
[Out] -(a + b*x^2)^3/(6*a*x^6)
```

Rubi [A] time = 0.0034544, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^7,x]

[Out] -(a + b*x^2)^3/(6*a*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^7} dx = -\frac{(a+bx^2)^3}{6ax^6}$$

Mathematica [A] time = 0.0007695, size = 30, normalized size = 1.58

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^7,x]

[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Maple [A] time = 0.004, size = 25, normalized size = 1.3

$$-\frac{ab}{2x^4}-\frac{b^2}{2x^2}-\frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7,x)
```

```
[Out] -1/2*a*b/x^4-1/2*b^2/x^2-1/6*a^2/x^6
```

Maxima [A] time = 1.73741, size = 32, normalized size = 1.68

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Fricas [A] time = 1.42218, size = 54, normalized size = 2.84

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A] time = 0.353072, size = 26, normalized size = 1.37

$$-\frac{a^2+3abx^2+3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7,x)
```

```
[Out] -(a**2 + 3*a*b*x**2 + 3*b**2*x**4)/(6*x**6)
```

Giac [A] time = 2.44744, size = 32, normalized size = 1.68

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

3.26 $\int \frac{(a+bx^2)^2}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rubi [A] time = 0.0104559, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0007667, size = 30, normalized size = 1.

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^8,x)
```

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Maxima [A] time = 2.477, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Fricas [A] time = 1.39536, size = 63, normalized size = 2.1

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A] time = 0.344517, size = 27, normalized size = 0.9

$$-\frac{15a^2+42abx^2+35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**8,x)
```

```
[Out] -(15*a**2 + 42*a*b*x**2 + 35*b**2*x**4)/(105*x**7)
```

Giac [A] time = 2.09605, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

3.27 $\int \frac{(a+bx^2)^2}{x^9} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rubi [A] time = 0.014229, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^9} dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^5} dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right) dx, x, x^2\right)\\ &= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0007617, size = 30, normalized size = 1.

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{a^2}{8\,x^8}-\frac{ab}{3\,x^6}-\frac{b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^9,x)
```

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Maxima [A] time = 2.04931, size = 35, normalized size = 1.17

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Fricas [A] time = 1.37859, size = 58, normalized size = 1.93

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Sympy [A] time = 0.39098, size = 27, normalized size = 0.9

$$-\frac{3a^2+8abx^2+6b^2x^4}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**9,x)
```

```
[Out] -(3*a**2 + 8*a*b*x**2 + 6*b**2*x**4)/(24*x**8)
```

Giac [A] time = 2.63579, size = 35, normalized size = 1.17

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

3.28 $\int \frac{(a+bx^2)^2}{x^{10}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Rubi [A] time = 0.0099824, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2}{x^6}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.0007196, size = 30, normalized size = 1.

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{a^2}{9\,x^9}-\frac{2\,ab}{7\,x^7}-\frac{b^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^10,x)
```

```
[Out] -1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

Maxima [A] time = 1.26352, size = 35, normalized size = 1.17

$$-\frac{63\, b^2 x^4 + 90\, a b x^2 + 35\, a^2}{315\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Fricas [A] time = 1.39523, size = 63, normalized size = 2.1

$$-\frac{63\, b^2 x^4 + 90\, a b x^2 + 35\, a^2}{315\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Sympy [A] time = 0.434991, size = 27, normalized size = 0.9

$$-\frac{35a^2 + 90abx^2 + 63b^2x^4}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**10,x)
```

```
[Out] -(35*a**2 + 90*a*b*x**2 + 63*b**2*x**4)/(315*x**9)
```

Giac [A] time = 1.35684, size = 35, normalized size = 1.17

$$-\frac{63\, b^2 x^4 + 90\, a b x^2 + 35\, a^2}{315\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

3.29 $\int x^9 \left(a + bx^2\right)^3 dx$

Optimal. Leaf size=43

$$\frac{1}{4}a^2bx^{12} + \frac{a^3x^{10}}{10} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{16}}{16}$$

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Rubi [A] time = 0.027875, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{4}a^2bx^{12} + \frac{a^3x^{10}}{10} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^9 \left(a + bx^2\right)^3 dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a + bx)^3\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^3x^4 + 3a^2bx^5 + 3ab^2x^6 + b^3x^7\right) dx, x, x^2\right) \\
&= \frac{a^3x^{10}}{10} + \frac{1}{4}a^2bx^{12} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{16}}{16}
\end{aligned}$$

Mathematica [A] time = 0.0017848, size = 43, normalized size = 1.

$$\frac{1}{4}a^2bx^{12} + \frac{a^3x^{10}}{10} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^{10}}{10}+\frac{a^2bx^{12}}{4}+\frac{3\,ab^2x^{14}}{14}+\frac{b^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^3,x)
```

```
[Out] 1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

Maxima [A] time = 2.52931, size = 47, normalized size = 1.09

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Fricas [A] time = 1.2806, size = 89, normalized size = 2.07

$$\frac{1}{16}x^{16}b^3+\frac{3}{14}x^{14}b^2a+\frac{1}{4}x^{12}ba^2+\frac{1}{10}x^{10}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^3 + 3/14*x^14*b^2*a + 1/4*x^12*b*a^2 + 1/10*x^10*a^3
```

Sympy [A] time = 0.065409, size = 37, normalized size = 0.86

$$\frac{a^3x^{10}}{10}+\frac{a^2bx^{12}}{4}+\frac{3ab^2x^{14}}{14}+\frac{b^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**10/10 + a**2*b*x**12/4 + 3*a*b**2*x**14/14 + b**3*x**16/16
```

Giac [A] time = 1.71588, size = 47, normalized size = 1.09

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

3.30 $\int x^7 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{3}{10}a^2bx^{10}+\frac{a^3x^8}{8}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Rubi [A] time = 0.0268245, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{10}a^2bx^{10}+\frac{a^3x^8}{8}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^3 \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^3 \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right) dx, x, x^2\right)\\
&= \frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}
\end{aligned}$$

Mathematica [A] time = 0.0015993, size = 43, normalized size = 1.

$$\frac{3}{10}a^2bx^{10}+\frac{a^3x^8}{8}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^8}{8} + \frac{3\,a^2bx^{10}}{10} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^3,x)
```

```
[Out] 1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

Maxima [A] time = 1.19351, size = 47, normalized size = 1.09

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Fricas [A] time = 1.31896, size = 86, normalized size = 2.

$$\frac{1}{14}x^{14}b^3 + \frac{1}{4}x^{12}b^2a + \frac{3}{10}x^{10}ba^2 + \frac{1}{8}x^8a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^3 + 1/4*x^12*b^2*a + 3/10*x^10*b*a^2 + 1/8*x^8*a^3
```

Sympy [A] time = 0.064422, size = 37, normalized size = 0.86

$$\frac{a^3x^8}{8} + \frac{3a^2bx^{10}}{10} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**8/8 + 3*a**2*b*x**10/10 + a*b**2*x**12/4 + b**3*x**14/14
```

Giac [A] time = 2.01414, size = 47, normalized size = 1.09

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

3.31 $\int x^5 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{3}{8}a^2bx^8+\frac{a^3x^6}{6}+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Rubi [A] time = 0.0251348, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{8}a^2bx^8+\frac{a^3x^6}{6}+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^3\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)\,dx,x,x^2\right)\\ &= \frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}\end{aligned}$$

Mathematica [A] time = 0.0016333, size = 43, normalized size = 1.

$$\frac{3}{8}a^2bx^8+\frac{a^3x^6}{6}+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^6}{6}+\frac{3\,a^2bx^8}{8}+\frac{3\,ab^2x^{10}}{10}+\frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^3,x)
```

```
[Out] 1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

Maxima [A] time = 1.8432, size = 47, normalized size = 1.09

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Fricas [A] time = 1.54648, size = 85, normalized size = 1.98

$$\frac{1}{12}x^{12}b^3+\frac{3}{10}x^{10}b^2a+\frac{3}{8}x^8ba^2+\frac{1}{6}x^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^3 + 3/10*x^10*b^2*a + 3/8*x^8*b*a^2 + 1/6*x^6*a^3
```

Sympy [A] time = 0.064796, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6}+\frac{3a^2bx^8}{8}+\frac{3ab^2x^{10}}{10}+\frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12
```

Giac [A] time = 2.02206, size = 47, normalized size = 1.09

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

3.32 $\int x^3 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^5}{10b^2}-\frac{a\left(a+bx^2\right)^4}{8b^2}$$

```
[Out] -(a*(a + b*x^2)^4)/(8*b^2) + (a + b*x^2)^5/(10*b^2)
```

Rubi [A] time = 0.0316579, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^5}{10b^2}-\frac{a\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3,x]
```

```
[Out] -(a*(a + b*x^2)^4)/(8*b^2) + (a + b*x^2)^5/(10*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^3 \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^3 \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^4}{8b^2}+\frac{\left(a+bx^2\right)^5}{10b^2}
\end{aligned}$$

Mathematica [A] time = 0.0016854, size = 43, normalized size = 1.26

$$\frac{1}{2}a^2bx^6+\frac{a^3x^4}{4}+\frac{3}{8}ab^2x^8+\frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^4)/4 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8 + (b^3*x^10)/10
```

Maple [A] time = 0.001, size = 36, normalized size = 1.1

$$\frac{b^3x^{10}}{10} + \frac{3\,ab^2x^8}{8} + \frac{a^2bx^6}{2} + \frac{a^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3,x)
```

```
[Out] 1/10*b^3*x^10+3/8*a*b^2*x^8+1/2*a^2*b*x^6+1/4*a^3*x^4
```

Maxima [A] time = 1.84903, size = 47, normalized size = 1.38

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{8}\,ab^2x^8 + \frac{1}{2}\,a^2bx^6 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Fricas [A] time = 1.78459, size = 82, normalized size = 2.41

$$\frac{1}{10}x^{10}b^3 + \frac{3}{8}x^8b^2a + \frac{1}{2}x^6ba^2 + \frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^3 + 3/8*x^8*b^2*a + 1/2*x^6*b*a^2 + 1/4*x^4*a^3
```

Sympy [A] time = 0.063058, size = 37, normalized size = 1.09

$$\frac{a^3x^4}{4} + \frac{a^2bx^6}{2} + \frac{3ab^2x^8}{8} + \frac{b^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**4/4 + a**2*b*x**6/2 + 3*a*b**2*x**8/8 + b**3*x**10/10
```

Giac [A] time = 1.59351, size = 47, normalized size = 1.38

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{8}\,ab^2x^8 + \frac{1}{2}\,a^2bx^6 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

3.33 $\int x\left(a+bx^2\right)^3 dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^4}{8b}$$

```
[Out] (a + b*x^2)^4/(8*b)
```

Rubi [A] time = 0.0023568, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^3,x]
```

```
[Out] (a + b*x^2)^4/(8*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^3 dx=\frac{\left(a+bx^2\right)^4}{8b}$$

Mathematica [A] time = 0.0018186, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^3,x]
```

```
[Out] (a + b*x^2)^4/(8*b)
```

Maple [B] time = 0.001, size = 36, normalized size = 2.3

$$\frac{b^3x^8}{8}+\frac{ab^2x^6}{2}+\frac{3\,a^2bx^4}{4}+\frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3,x)

[Out] 1/8*b^3*x^8+1/2*a*b^2*x^6+3/4*a^2*b*x^4+1/2*a^3*x^2
```

Maxima [A] time = 1.78371, size = 19, normalized size = 1.19

$$\frac{\left(bx^2 + a\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/8*(b*x^2 + a)^4/b
```

Fricas [B] time = 1.81334, size = 80, normalized size = 5.

$$\frac{1}{8}x^8b^3 + \frac{1}{2}x^6b^2a + \frac{3}{4}x^4ba^2 + \frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/8*x^8*b^3 + 1/2*x^6*b^2*a + 3/4*x^4*b*a^2 + 1/2*x^2*a^3
```

Sympy [B] time = 0.06199, size = 37, normalized size = 2.31

$$\frac{a^3x^2}{2} + \frac{3a^2bx^4}{4} + \frac{ab^2x^6}{2} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3,x)

[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + a*b**2*x**6/2 + b**3*x**8/8
```

Giac [A] time = 2.70175, size = 19, normalized size = 1.19

$$\frac{\left(bx^2 + a\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="giac")

[Out] 1/8*(b*x^2 + a)^4/b
```

3.34 $\int \frac{\left(a+bx^2\right)^3}{x}\,dx$

Optimal. Leaf size=39

$$\frac{3}{2}a^2bx^2+a^3\log(x)+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}$$

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Rubi [A] time = 0.0184897, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{2}a^2bx^2+a^3\log(x)+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^3}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)dx,x,x^2\right)\\
&= \frac{3}{2}a^2bx^2+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}+a^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0034781, size = 39, normalized size = 1.

$$\frac{3}{2}a^2bx^2+a^3\log(x)+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Maple [A] time = 0.002, size = 34, normalized size = 0.9

$$\frac{3\,a^{2}bx^{2}}{2}+\frac{3\,ab^{2}x^{4}}{4}+\frac{b^{3}x^{6}}{6}+a^{3}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x,x)
```

```
[Out] 3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

Maxima [A] time = 2.0592, size = 49, normalized size = 1.26

$$\frac{1}{6}\,b^{3}x^{6}+\frac{3}{4}\,ab^{2}x^{4}+\frac{3}{2}\,a^{2}bx^{2}+\frac{1}{2}\,a^{3}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

Fricas [A] time = 1.98528, size = 78, normalized size = 2.

$$\frac{1}{6}\,b^{3}x^{6}+\frac{3}{4}\,ab^{2}x^{4}+\frac{3}{2}\,a^{2}bx^{2}+a^{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [A] time = 0.251546, size = 37, normalized size = 0.95

$$a^{3}\log\left(x\right)+\frac{3a^{2}bx^{2}}{2}+\frac{3ab^{2}x^{4}}{4}+\frac{b^{3}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/4 + b**3*x**6/6
```

Giac [A] time = 1.90448, size = 49, normalized size = 1.26

$$\frac{1}{6}\,b^{3}x^{6}+\frac{3}{4}\,ab^{2}x^{4}+\frac{3}{2}\,a^{2}bx^{2}+\frac{1}{2}\,a^{3}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

3.35 $\int \frac{(a+bx^2)^3}{x^3} \, dx$

Optimal. Leaf size=40

$$3a^2b\log(x) - \frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rubi [A] time = 0.0206277, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$3a^2b\log(x) - \frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(3ab^2 + \frac{a^3}{x^2} + \frac{3a^2b}{x} + b^3x\right) dx, x, x^2\right) \\
&= -\frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4} + 3a^2b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0061097, size = 40, normalized size = 1.

$$3a^2b\log(x) - \frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-\frac{a^{3}}{2\,x^{2}}+\frac{3\,ab^{2}x^{2}}{2}+\frac{b^{3}x^{4}}{4}+3\,a^{2}b\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Maxima [A] time = 1.28704, size = 49, normalized size = 1.22

$$\frac{1}{4}\,b^{3}x^{4}+\frac{3}{2}\,ab^{2}x^{2}+\frac{3}{2}\,a^{2}b\log\left(x^{2}\right)-\frac{a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*a^3/x^2
```

Fricas [A] time = 1.30871, size = 85, normalized size = 2.12

$$\frac{b^{3}x^{6}+6\,ab^{2}x^{4}+12\,a^{2}bx^{2}\log\left(x\right)-2\,a^{3}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A] time = 0.287121, size = 37, normalized size = 0.92

$$-\frac{a^{3}}{2x^{2}}+3a^{2}b\log\left(x\right)+\frac{3ab^{2}x^{2}}{2}+\frac{b^{3}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

Giac [A] time = 2.59501, size = 62, normalized size = 1.55

$$\frac{1}{4}\,b^{3}x^{4}+\frac{3}{2}\,ab^{2}x^{2}+\frac{3}{2}\,a^{2}b\log\left(x^{2}\right)-\frac{3\,a^{2}bx^{2}+a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

3.36 $\int \frac{(a+bx^2)^3}{x^5} dx$

Optimal. Leaf size=40

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{4x^4}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

```
[Out] -a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rubi [A] time = 0.0186338, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{4x^4}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^5} dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^3} dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right) dx, x, x^2\right) \\
&= -\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+\frac{b^3x^2}{2}+3ab^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0039348, size = 40, normalized size = 1.

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{4x^4}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$-\frac{a^3}{4\,x^4}-\frac{3\,a^2b}{2\,x^2}+\frac{b^3x^2}{2}+3\,ab^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^5,x)
```

```
[Out] -1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

Maxima [A] time = 2.36099, size = 50, normalized size = 1.25

$$\frac{1}{2}\,b^3x^2+\frac{3}{2}\,ab^2\log\left(x^2\right)-\frac{6\,a^2bx^2+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(6*a^2*b*x^2 + a^3)/x^4
```

Fricas [A] time = 1.28901, size = 85, normalized size = 2.12

$$\frac{2\,b^3x^6+12\,ab^2x^4\log(x)-6\,a^2bx^2-a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [A] time = 0.339119, size = 36, normalized size = 0.9

$$3ab^2\log(x)+\frac{b^3x^2}{2}-\frac{a^3+6a^2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**5,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x**2/2 - (a**3 + 6*a**2*b*x**2)/(4*x**4)
```

Giac [A] time = 2.3507, size = 62, normalized size = 1.55

$$\frac{1}{2}\,b^3x^2+\frac{3}{2}\,ab^2\log\left(x^2\right)-\frac{9\,ab^2x^4+6\,a^2bx^2+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(9*a*b^2*x^4 + 6*a^2*b*x^2 + a^3)/x^
4
```

3.37 $\int \frac{(a+bx^2)^3}{x^7} dx$

Optimal. Leaf size=39

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Rubi [A] time = 0.0211947, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx, x, x^2\right)\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0038464, size = 39, normalized size = 1.

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Maple [A] time = 0.005, size = 34, normalized size = 0.9

$$-\frac{a^{3}}{6\,x^{6}}-\frac{3\,a^{2}b}{4\,x^{4}}-\frac{3\,a b^{2}}{2\,x^{2}}+b^{3}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

Maxima [A] time = 2.42442, size = 53, normalized size = 1.36

$$\frac{1}{2}\,b^{3}\log\left(x^{2}\right)-\frac{18\,a b^{2}x^{4}+9\,a^{2}b x^{2}+2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6
```

Fricas [A] time = 1.23594, size = 90, normalized size = 2.31

$$\frac{12\,b^{3}x^{6}\log\left(x\right)-18\,a b^{2}x^{4}-9\,a^{2}b x^{2}-2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [A] time = 0.377119, size = 36, normalized size = 0.92

$$b^{3}\log\left(x\right)-\frac{2a^{3}+9a^{2}bx^{2}+18ab^{2}x^{4}}{12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**7,x)
```

```
[Out] b**3*log(x) - (2*a**3 + 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)
```

Giac [A] time = 1.56305, size = 63, normalized size = 1.62

$$\frac{1}{2}\,b^{3}\log\left(x^{2}\right)-\frac{11\,b^{3}x^{6}+18\,a b^{2}x^{4}+9\,a^{2}b x^{2}+2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2) - 1/12*(11*b^3*x^6 + 18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x
^6
```

3.38 $\int \frac{(a+bx^2)^3}{x^9} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^4}{8ax^8}$$

[Out] -(a + b*x^2)^4/(8*a*x^8)

Rubi [A] time = 0.0030151, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^3/x^9,x]

[Out] -(a + b*x^2)^4/(8*a*x^8)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{x^9} dx = -\frac{(a+bx^2)^4}{8ax^8}$$

Mathematica [B] time = 0.0062467, size = 43, normalized size = 2.26

$$-\frac{a^2b}{2x^6} - \frac{a^3}{8x^8} - \frac{3ab^2}{4x^4} - \frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^3/x^9,x]

[Out] -a^3/(8*x^8) - (a^2*b)/(2*x^6) - (3*a*b^2)/(4*x^4) - b^3/(2*x^2)

Maple [B] time = 0.004, size = 36, normalized size = 1.9

$$-\frac{a^3}{8x^8} - \frac{3ab^2}{4x^4} - \frac{b^3}{2x^2} - \frac{a^2b}{2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^9,x)
```

```
[Out] -1/8*a^3/x^8-3/4*a*b^2/x^4-1/2*b^3/x^2-1/2*a^2*b/x^6
```

Maxima [B] time = 1.08615, size = 47, normalized size = 2.47

$$-\frac{4\,b^3x^6 + 6\,ab^2x^4 + 4\,a^2bx^2 + a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Fricas [B] time = 1.20025, size = 76, normalized size = 4.

$$-\frac{4\,b^3x^6 + 6\,ab^2x^4 + 4\,a^2bx^2 + a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [B] time = 0.403944, size = 37, normalized size = 1.95

$$-\frac{a^3 + 4a^2bx^2 + 6ab^2x^4 + 4b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**9,x)
```

```
[Out] -(a**3 + 4*a**2*b*x**2 + 6*a*b**2*x**4 + 4*b**3*x**6)/(8*x**8)
```

Giac [B] time = 2.64129, size = 47, normalized size = 2.47

$$-\frac{4\,b^3x^6 + 6\,ab^2x^4 + 4\,a^2bx^2 + a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

3.39 $\int \frac{\left(a+bx^2\right)^3}{x^{11}} \, dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^4}{40a^2x^8}-\frac{\left(a+bx^2\right)^4}{10ax^{10}}$$

```
[Out] -(a + b*x^2)^4/(10*a*x^10) + (b*(a + b*x^2)^4)/(40*a^2*x^8)
```

Rubi [A] time = 0.0193902, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^2\right)^4}{40a^2x^8}-\frac{\left(a+bx^2\right)^4}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^11,x]
```

```
[Out] -(a + b*x^2)^4/(10*a*x^10) + (b*(a + b*x^2)^4)/(40*a^2*x^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^3}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^6}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^4}{10ax^{10}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^5}\,dx,x,x^2\right)}{10a}\\
&= -\frac{\left(a+bx^2\right)^4}{10ax^{10}} + \frac{b\left(a+bx^2\right)^4}{40a^2x^8}
\end{aligned}
$$

Mathematica [A] time = 0.0035637, size = 43, normalized size = 1.08

$$-\frac{3a^2b}{8x^8}-\frac{a^3}{10x^{10}}-\frac{ab^2}{2x^6}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^11,x]
```

```
[Out] -a^3/(10*x^10) - (3*a^2*b)/(8*x^8) - (a*b^2)/(2*x^6) - b^3/(4*x^4)
```

Maple [A] time = 0.004, size = 36, normalized size = 0.9

$$-\frac{3\,a^2b}{8\,x^8}-\frac{a^3}{10\,x^{10}}-\frac{b^3}{4\,x^4}-\frac{ab^2}{2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^11,x)
```

```
[Out] -3/8*a^2*b/x^8-1/10*a^3/x^10-1/4*b^3/x^4-1/2*a*b^2/x^6
```

Maxima [A] time = 2.088, size = 50, normalized size = 1.25

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="maxima")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Fricas [A] time = 1.28533, size = 85, normalized size = 2.12

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [A] time = 0.439568, size = 39, normalized size = 0.98

$$-\frac{4a^3+15a^2bx^2+20ab^2x^4+10b^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**11,x)
```

```
[Out] -(4*a**3 + 15*a**2*b*x**2 + 20*a*b**2*x**4 + 10*b**3*x**6)/(40*x**10)
```

Giac [A] time = 2.52558, size = 50, normalized size = 1.25

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

3.40 $\int \frac{(a+bx^2)^3}{x^{13}} dx$

Optimal. Leaf size=43

$$-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

```
[Out] -a^3/(12*x^12) - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Rubi [A] time = 0.0185851, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^13,x]
```

```
[Out] -a^3/(12*x^12) - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{13}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^7} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right) dx, x, x^2\right)\\ &= -\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.0037502, size = 43, normalized size = 1.

$$-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^13,x]
```

```
[Out] -a^3/(12*x^12) - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$-\frac{a^3}{12\,x^{12}}-\frac{3\,a^2b}{10\,x^{10}}-\frac{3\,ab^2}{8\,x^8}-\frac{b^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^13,x)
```

```
[Out] -1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

Maxima [A] time = 2.317, size = 50, normalized size = 1.16

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Fricas [A] time = 1.31164, size = 88, normalized size = 2.05

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [A] time = 0.459102, size = 39, normalized size = 0.91

$$-\frac{10a^3+36a^2bx^2+45ab^2x^4+20b^3x^6}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**13,x)
```

```
[Out] -(10*a**3 + 36*a**2*b*x**2 + 45*a*b**2*x**4 + 20*b**3*x**6)/(120*x**12)
```

Giac [A] time = 1.79705, size = 50, normalized size = 1.16

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

3.41 $\int \frac{(a+bx^2)^3}{x^{15}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

```
[Out] -a^3/(14*x^14) - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Rubi [A] time = 0.0185127, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^15,x]
```

```
[Out] -a^3/(14*x^14) - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{15}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^8}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx, x, x^2\right)\\
&= -\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}
\end{aligned}$$

Mathematica [A] time = 0.0060612, size = 43, normalized size = 1.

$$-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^15,x]
```

```
[Out] -a^3/(14*x^14) - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$-\frac{a^3}{14\,x^{14}}-\frac{a^2b}{4\,x^{12}}-\frac{3\,ab^2}{10\,x^{10}}-\frac{b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^15,x)
```

```
[Out] -1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

Maxima [A] time = 2.63808, size = 50, normalized size = 1.16

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Fricas [A] time = 1.27243, size = 88, normalized size = 2.05

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [A] time = 0.501027, size = 39, normalized size = 0.91

$$-\frac{20a^3+70a^2bx^2+84ab^2x^4+35b^3x^6}{280x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**15,x)
```

```
[Out] -(20*a**3 + 70*a**2*b*x**2 + 84*a*b**2*x**4 + 35*b**3*x**6)/(280*x**14)
```

Giac [A] time = 2.01264, size = 50, normalized size = 1.16

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

3.42 $\int x^6 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{1}{3}a^2bx^9+\frac{a^3x^7}{7}+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Rubi [A] time = 0.0141882, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{1}{3}a^2bx^9+\frac{a^3x^7}{7}+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^6+3a^2bx^8+3ab^2x^{10}+b^3x^{12}\right) dx\\ &= \frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.0018126, size = 43, normalized size = 1.

$$\frac{1}{3}a^2bx^9+\frac{a^3x^7}{7}+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Maple [A] time = 0., size = 36, normalized size = 0.8

$$\frac{a^3x^7}{7}+\frac{a^2bx^9}{3}+\frac{3\,ab^2x^{11}}{11}+\frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^3,x)
```

```
[Out] 1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

Maxima [A] time = 2.06904, size = 47, normalized size = 1.09

$$\frac{1}{13}\,b^3x^{13} + \frac{3}{11}\,ab^2x^{11} + \frac{1}{3}\,a^2bx^9 + \frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Fricas [A] time = 1.11366, size = 85, normalized size = 1.98

$$\frac{1}{13}x^{13}b^3 + \frac{3}{11}x^{11}b^2a + \frac{1}{3}x^9ba^2 + \frac{1}{7}x^7a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^3 + 3/11*x^11*b^2*a + 1/3*x^9*b*a^2 + 1/7*x^7*a^3
```

Sympy [A] time = 0.063923, size = 37, normalized size = 0.86

$$\frac{a^3x^7}{7} + \frac{a^2bx^9}{3} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**7/7 + a**2*b*x**9/3 + 3*a*b**2*x**11/11 + b**3*x**13/13
```

Giac [A] time = 1.91001, size = 47, normalized size = 1.09

$$\frac{1}{13}\,b^3x^{13} + \frac{3}{11}\,ab^2x^{11} + \frac{1}{3}\,a^2bx^9 + \frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

3.43 $\int x^4 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{3}{7}a^2bx^7+\frac{a^3x^5}{5}+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Rubi [A] time = 0.0134052, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3}{7}a^2bx^7+\frac{a^3x^5}{5}+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^4+3a^2bx^6+3ab^2x^8+b^3x^{10}\right) dx\\ &= \frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.0016709, size = 43, normalized size = 1.

$$\frac{3}{7}a^2bx^7+\frac{a^3x^5}{5}+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^5}{5}+\frac{3\,a^2bx^7}{7}+\frac{ab^2x^9}{3}+\frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^3,x)
```

```
[Out] 1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

Maxima [A] time = 1.99563, size = 47, normalized size = 1.09

$$\frac{1}{11}\,b^3x^{11} + \frac{1}{3}\,ab^2x^9 + \frac{3}{7}\,a^2bx^7 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Fricas [A] time = 1.08659, size = 82, normalized size = 1.91

$$\frac{1}{11}x^{11}b^3 + \frac{1}{3}x^9b^2a + \frac{3}{7}x^7ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^3 + 1/3*x^9*b^2*a + 3/7*x^7*b*a^2 + 1/5*x^5*a^3
```

Sympy [A] time = 0.069345, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5} + \frac{3a^2bx^7}{7} + \frac{ab^2x^9}{3} + \frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**5/5 + 3*a**2*b*x**7/7 + a*b**2*x**9/3 + b**3*x**11/11
```

Giac [A] time = 2.54028, size = 47, normalized size = 1.09

$$\frac{1}{11}\,b^3x^{11} + \frac{1}{3}\,ab^2x^9 + \frac{3}{7}\,a^2bx^7 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

3.44 $\int x^2 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Rubi [A] time = 0.0133414, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^2+3a^2bx^4+3ab^2x^6+b^3x^8\right) dx\\
&= \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}
\end{aligned}$$

Mathematica [A] time = 0.0017451, size = 43, normalized size = 1.

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^3}{3}+\frac{3\,a^2bx^5}{5}+\frac{3\,ab^2x^7}{7}+\frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3,x)
```

```
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

Maxima [A] time = 1.80046, size = 47, normalized size = 1.09

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Fricas [A] time = 1.02123, size = 80, normalized size = 1.86

$$\frac{1}{9}x^9b^3+\frac{3}{7}x^7b^2a+\frac{3}{5}x^5ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^3 + 3/7*x^7*b^2*a + 3/5*x^5*b*a^2 + 1/3*x^3*a^3
```

Sympy [A] time = 0.098189, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3ab^2x^7}{7}+\frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*b**2*x**7/7 + b**3*x**9/9
```

Giac [A] time = 2.01069, size = 47, normalized size = 1.09

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

3.45 $\int (a+bx^2)^3 \, dx$

Optimal. Leaf size=35

$$a^2bx^3 + a^3x + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Rubi [A] time = 0.0105425, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2bx^3 + a^3x + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^3 \, dx &= \int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right) dx\\ &= a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0008498, size = 35, normalized size = 1.

$$a^2bx^3 + a^3x + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Maple [A] time = 0., size = 32, normalized size = 0.9

$$a^3x + a^2bx^3 + \frac{3\,ab^2x^5}{5} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3,x)
```

```
[Out] a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Maxima [A] time = 1.65652, size = 42, normalized size = 1.2

$$\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Fricas [A] time = 1.18378, size = 66, normalized size = 1.89

$$\frac{1}{7}x^7b^3+\frac{3}{5}x^5b^2a+x^3ba^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^3 + 3/5*x^5*b^2*a + x^3*b*a^2 + x*a^3
```

Sympy [A] time = 0.062115, size = 32, normalized size = 0.91

$$a^3x+a^2bx^3+\frac{3ab^2x^5}{5}+\frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7
```

Giac [A] time = 2.81029, size = 42, normalized size = 1.2

$$\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

3.46 $\int \frac{(a+bx^2)^3}{x^2} dx$

Optimal. Leaf size=34

$$3a^2bx - \frac{a^3}{x} + ab^2x^3 + \frac{b^3x^5}{5}$$

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Rubi [A] time = 0.0129008, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$3a^2bx - \frac{a^3}{x} + ab^2x^3 + \frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^3}{x^2} dx &= \int \left(3a^2b + \frac{a^3}{x^2} + 3ab^2x^2 + b^3x^4\right) dx \\ &= -\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.003336, size = 34, normalized size = 1.

$$3a^2bx - \frac{a^3}{x} + ab^2x^3 + \frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Maple [A] time = 0.003, size = 33, normalized size = 1.

$$-\frac{a^3}{x} + 3\,a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^2,x)
```

```
[Out] -a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

Maxima [A] time = 1.00988, size = 43, normalized size = 1.26

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Fricas [A] time = 1.16299, size = 73, normalized size = 2.15

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [A] time = 0.256705, size = 29, normalized size = 0.85

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5
```

Giac [A] time = 2.36333, size = 43, normalized size = 1.26

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

3.47 $\int \frac{(a+bx^2)^3}{x^4} \, dx$

Optimal. Leaf size=37

$$-\frac{3a^2b}{x} - \frac{a^3}{3x^3} + 3ab^2x + \frac{b^3x^3}{3}$$

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Rubi [A] time = 0.0125237, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{x} - \frac{a^3}{3x^3} + 3ab^2x + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^3}{x^4} \, dx &= \int \left(3ab^2 + \frac{a^3}{x^4} + \frac{3a^2b}{x^2} + b^3x^2\right) dx \\ &= -\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0033825, size = 37, normalized size = 1.

$$-\frac{3a^2b}{x} - \frac{a^3}{3x^3} + 3ab^2x + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Maple [A] time = 0.006, size = 34, normalized size = 0.9

$$-\frac{a^3}{3x^3} - 3\frac{a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^4,x)
```

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

Maxima [A] time = 1.24575, size = 46, normalized size = 1.24

$$\frac{1}{3}\,b^3x^3 + 3\,ab^2x - \frac{9\,a^2bx^2 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Fricas [A] time = 1.20161, size = 72, normalized size = 1.95

$$\frac{b^3x^6 + 9\,ab^2x^4 - 9\,a^2bx^2 - a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [A] time = 0.293818, size = 34, normalized size = 0.92

$$3ab^2x + \frac{b^3x^3}{3} - \frac{a^3 + 9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**4,x)
```

```
[Out] 3*a*b**2*x + b**3*x**3/3 - (a**3 + 9*a**2*b*x**2)/(3*x**3)
```

Giac [A] time = 2.78575, size = 46, normalized size = 1.24

$$\frac{1}{3}\,b^3x^3 + 3\,ab^2x - \frac{9\,a^2bx^2 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

3.48 $\int \frac{(a+bx^2)^3}{x^6} dx$

Optimal. Leaf size=34

$$-\frac{a^2b}{x^3}-\frac{a^3}{5x^5}-\frac{3ab^2}{x}+b^3x$$

```
[Out] -a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Rubi [A] time = 0.0128706, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2b}{x^3}-\frac{a^3}{5x^5}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^6,x]
```

```
[Out] -a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^6}\,dx &= \int \left(b^3+\frac{a^3}{x^6}+\frac{3a^2b}{x^4}+\frac{3ab^2}{x^2}\right)dx\\ &= -\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x\end{aligned}$$

Mathematica [A] time = 0.0047768, size = 34, normalized size = 1.

$$-\frac{a^2b}{x^3}-\frac{a^3}{5x^5}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^6,x]
```

```
[Out] -a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Maple [A] time = 0.004, size = 33, normalized size = 1.

$$-\frac{a^3}{5\,x^5}-\frac{a^2b}{x^3}-3\,\frac{ab^2}{x}+b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^6,x)
```

```
[Out] -1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

Maxima [A] time = 1.6791, size = 45, normalized size = 1.32

$$b^3x - \frac{15\,ab^2x^4 + 5\,a^2bx^2 + a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="maxima")
```

```
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

Fricas [A] time = 1.19691, size = 76, normalized size = 2.24

$$\frac{5\,b^3x^6 - 15\,ab^2x^4 - 5\,a^2bx^2 - a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [A] time = 0.341248, size = 32, normalized size = 0.94

$$b^3x - \frac{a^3 + 5a^2bx^2 + 15ab^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**6,x)
```

```
[Out] b**3*x - (a**3 + 5*a**2*b*x**2 + 15*a*b**2*x**4)/(5*x**5)
```

Giac [A] time = 2.33084, size = 45, normalized size = 1.32

$$b^3x - \frac{15\,ab^2x^4 + 5\,a^2bx^2 + a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="giac")
```

```
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

3.49 $\int \frac{(a+bx^2)^3}{x^8} dx$

Optimal. Leaf size=39

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{7x^7}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

```
[Out] -a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Rubi [A] time = 0.0134047, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{7x^7}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^8} dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^4}+\frac{b^3}{x^2}\right) dx\\ &= -\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}\end{aligned}$$

Mathematica [A] time = 0.0035532, size = 39, normalized size = 1.

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{7x^7}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Maple [A] time = 0.004, size = 36, normalized size = 0.9

$$-\frac{a^3}{7\,x^7}-\frac{3\,a^2b}{5\,x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^8,x)
```

```
[Out] -1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

Maxima [A] time = 1.9276, size = 50, normalized size = 1.28

$$-\frac{35\,b^3x^6 + 35\,ab^2x^4 + 21\,a^2bx^2 + 5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Fricas [A] time = 1.26732, size = 84, normalized size = 2.15

$$-\frac{35\,b^3x^6 + 35\,ab^2x^4 + 21\,a^2bx^2 + 5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [A] time = 0.386656, size = 39, normalized size = 1.

$$-\frac{5a^3 + 21a^2bx^2 + 35ab^2x^4 + 35b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**8,x)
```

```
[Out] -(5*a**3 + 21*a**2*b*x**2 + 35*a*b**2*x**4 + 35*b**3*x**6)/(35*x**7)
```

Giac [A] time = 2.53245, size = 50, normalized size = 1.28

$$-\frac{35\,b^3x^6 + 35\,ab^2x^4 + 21\,a^2bx^2 + 5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

3.50 $\int \frac{\left(a+bx^2\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=43

$$-\frac{3a^2b}{7x^7}-\frac{a^3}{9x^9}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Rubi [A] time = 0.0134588, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{7x^7}-\frac{a^3}{9x^9}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{x^{10}}\,dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^6}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0039683, size = 43, normalized size = 1.

$$-\frac{3a^2b}{7x^7}-\frac{a^3}{9x^9}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$-\frac{a^3}{9\,x^9}-\frac{3\,a^2b}{7\,x^7}-\frac{3\,ab^2}{5\,x^5}-\frac{b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^10,x)
```

```
[Out] -1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

Maxima [A] time = 2.28645, size = 50, normalized size = 1.16

$$-\frac{105\,b^3x^6 + 189\,ab^2x^4 + 135\,a^2bx^2 + 35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Fricas [A] time = 1.26863, size = 90, normalized size = 2.09

$$-\frac{105\,b^3x^6 + 189\,ab^2x^4 + 135\,a^2bx^2 + 35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [A] time = 0.449378, size = 39, normalized size = 0.91

$$-\frac{35a^3 + 135a^2bx^2 + 189ab^2x^4 + 105b^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**10,x)
```

```
[Out] -(35*a**3 + 135*a**2*b*x**2 + 189*a*b**2*x**4 + 105*b**3*x**6)/(315*x**9)
```

Giac [A] time = 1.81699, size = 50, normalized size = 1.16

$$-\frac{105\,b^3x^6 + 189\,ab^2x^4 + 135\,a^2bx^2 + 35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

3.51 $\int \frac{(a+bx^2)^3}{x^{12}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^2b}{3x^9}-\frac{a^3}{11x^{11}}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

```
[Out] -a^3/(11*x^11) - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Rubi [A] time = 0.014571, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2b}{3x^9}-\frac{a^3}{11x^{11}}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^12,x]
```

```
[Out] -a^3/(11*x^11) - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{12}} \, dx &= \int \left(\frac{a^3}{x^{12}}+\frac{3a^2b}{x^{10}}+\frac{3ab^2}{x^8}+\frac{b^3}{x^6}\right) dx\\ &= -\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.0061936, size = 43, normalized size = 1.

$$-\frac{a^2b}{3x^9}-\frac{a^3}{11x^{11}}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^12,x]
```

```
[Out] -a^3/(11*x^11) - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$-\frac{a^3}{11\,x^{11}}-\frac{a^2b}{3\,x^9}-\frac{3\,ab^2}{7\,x^7}-\frac{b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^12,x)
```

```
[Out] -1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

Maxima [A] time = 2.53168, size = 50, normalized size = 1.16

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="maxima")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Fricas [A] time = 1.27293, size = 95, normalized size = 2.21

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="fricas")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Sympy [A] time = 0.441176, size = 39, normalized size = 0.91

$$-\frac{105a^3+385a^2bx^2+495ab^2x^4+231b^3x^6}{1155x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**12,x)
```

```
[Out] -(105*a**3 + 385*a**2*b*x**2 + 495*a*b**2*x**4 + 231*b**3*x**6)/(1155*x**11
)
```

Giac [A] time = 2.50031, size = 50, normalized size = 1.16

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="giac")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

3.52 $\int x^{13}\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4bx^{16}+\frac{a^5x^{14}}{14}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Rubi [A] time = 0.0488556, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4bx^{16}+\frac{a^5x^{14}}{14}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{13}\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^6(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx,x,x^2\right)\\&=\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}\end{aligned}$$

Mathematica [A] time = 0.002849, size = 69, normalized size = 1.

$$\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4bx^{16}+\frac{a^5x^{14}}{14}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Maple [A] time = 0.002, size = 58, normalized size = 0.8

$$\frac{a^{5}x^{14}}{14}+\frac{5\,a^{4}bx^{16}}{16}+\frac{5\,a^{3}b^{2}x^{18}}{9}+\frac{a^{2}b^{3}x^{20}}{2}+\frac{5\,ab^{4}x^{22}}{22}+\frac{b^{5}x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^5,x)
```

```
[Out] 1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4*
x^22+1/24*b^5*x^24
```

Maxima [A] time = 2.1205, size = 77, normalized size = 1.12

$$\frac{1}{24}\,b^{5}x^{24}+\frac{5}{22}\,ab^{4}x^{22}+\frac{1}{2}\,a^{2}b^{3}x^{20}+\frac{5}{9}\,a^{3}b^{2}x^{18}+\frac{5}{16}\,a^{4}bx^{16}+\frac{1}{14}\,a^{5}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Fricas [A] time = 1.11462, size = 142, normalized size = 2.06

$$\frac{1}{24}x^{24}b^{5}+\frac{5}{22}x^{22}b^{4}a+\frac{1}{2}x^{20}b^{3}a^{2}+\frac{5}{9}x^{18}b^{2}a^{3}+\frac{5}{16}x^{16}ba^{4}+\frac{1}{14}x^{14}a^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5 + 5/22*x^22*b^4*a + 1/2*x^20*b^3*a^2 + 5/9*x^18*b^2*a^3 + 5/1
6*x^16*b*a^4 + 1/14*x^14*a^5
```

Sympy [A] time = 0.072784, size = 65, normalized size = 0.94

$$\frac{a^{5}x^{14}}{14}+\frac{5a^{4}bx^{16}}{16}+\frac{5a^{3}b^{2}x^{18}}{9}+\frac{a^{2}b^{3}x^{20}}{2}+\frac{5ab^{4}x^{22}}{22}+\frac{b^{5}x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**14/14 + 5*a**4*b*x**16/16 + 5*a**3*b**2*x**18/9 + a**2*b**3*x**20/2
 + 5*a*b**4*x**22/22 + b**5*x**24/24
```

Giac [A] time = 2.58715, size = 77, normalized size = 1.12

$$\frac{1}{24}b^5x^{24} + \frac{5}{22}ab^4x^{22} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{9}a^3b^2x^{18} + \frac{5}{16}a^4bx^{16} + \frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

3.53 $\int x^{11}\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=69

$$\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4bx^{14}+\frac{a^5x^{12}}{12}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Rubi [A] time = 0.044137, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4bx^{14}+\frac{a^5x^{12}}{12}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx,x,x^2\right)\\&=\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}\end{aligned}$$

Mathematica [A] time = 0.002101, size = 69, normalized size = 1.

$$\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4bx^{14}+\frac{a^5x^{12}}{12}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Maple [A] time = 0., size = 58, normalized size = 0.8

$$\frac{a^5x^{12}}{12}+\frac{5\,a^4bx^{14}}{14}+\frac{5\,a^3b^2x^{16}}{8}+\frac{5\,a^2b^3x^{18}}{9}+\frac{ab^4x^{20}}{4}+\frac{b^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^5,x)
```

```
[Out] 1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*x
^20+1/22*b^5*x^22
```

Maxima [A] time = 1.3269, size = 77, normalized size = 1.12

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Fricas [A] time = 1.07144, size = 140, normalized size = 2.03

$$\frac{1}{22}x^{22}b^5+\frac{1}{4}x^{20}b^4a+\frac{5}{9}x^{18}b^3a^2+\frac{5}{8}x^{16}b^2a^3+\frac{5}{14}x^{14}ba^4+\frac{1}{12}x^{12}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5 + 1/4*x^20*b^4*a + 5/9*x^18*b^3*a^2 + 5/8*x^16*b^2*a^3 + 5/14
*x^14*b*a^4 + 1/12*x^12*a^5
```

Sympy [A] time = 0.070696, size = 65, normalized size = 0.94

$$\frac{a^5x^{12}}{12}+\frac{5a^4bx^{14}}{14}+\frac{5a^3b^2x^{16}}{8}+\frac{5a^2b^3x^{18}}{9}+\frac{ab^4x^{20}}{4}+\frac{b^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**12/12 + 5*a**4*b*x**14/14 + 5*a**3*b**2*x**16/8 + 5*a**2*b**3*x**18
/9 + a*b**4*x**20/4 + b**5*x**22/22
```

Giac [A] time = 2.51753, size = 77, normalized size = 1.12

$$\frac{1}{22}b^5x^{22} + \frac{1}{4}ab^4x^{20} + \frac{5}{9}a^2b^3x^{18} + \frac{5}{8}a^3b^2x^{16} + \frac{5}{14}a^4bx^{14} + \frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

3.54 $\int x^9 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{5}{8}a^2b^3x^{16}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{12}a^4bx^{12}+\frac{a^5x^{10}}{10}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Rubi [A] time = 0.0431, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{8}a^2b^3x^{16}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{12}a^4bx^{12}+\frac{a^5x^{10}}{10}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^5\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^4(a+bx)^5\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx,x,x^2\right)\\ &= \frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}\end{aligned}$$

Mathematica [A] time = 0.0021335, size = 69, normalized size = 1.

$$\frac{5}{8}a^2b^3x^{16}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{12}a^4bx^{12}+\frac{a^5x^{10}}{10}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{a^5x^{10}}{10}+\frac{5\,a^4bx^{12}}{12}+\frac{5\,a^3b^2x^{14}}{7}+\frac{5\,a^2b^3x^{16}}{8}+\frac{5\,ab^4x^{18}}{18}+\frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5,x)
```

```
[Out] 1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4*
x^18+1/20*b^5*x^20
```

Maxima [A] time = 2.35845, size = 77, normalized size = 1.12

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Fricas [A] time = 1.10531, size = 142, normalized size = 2.06

$$\frac{1}{20}x^{20}b^5+\frac{5}{18}x^{18}b^4a+\frac{5}{8}x^{16}b^3a^2+\frac{5}{7}x^{14}b^2a^3+\frac{5}{12}x^{12}ba^4+\frac{1}{10}x^{10}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5 + 5/18*x^18*b^4*a + 5/8*x^16*b^3*a^2 + 5/7*x^14*b^2*a^3 + 5/1
2*x^12*b*a^4 + 1/10*x^10*a^5
```

Sympy [A] time = 0.07128, size = 66, normalized size = 0.96

$$\frac{a^5x^{10}}{10}+\frac{5a^4bx^{12}}{12}+\frac{5a^3b^2x^{14}}{7}+\frac{5a^2b^3x^{16}}{8}+\frac{5ab^4x^{18}}{18}+\frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**10/10 + 5*a**4*b*x**12/12 + 5*a**3*b**2*x**14/7 + 5*a**2*b**3*x**16
/8 + 5*a*b**4*x**18/18 + b**5*x**20/20
```

Giac [A] time = 2.61746, size = 77, normalized size = 1.12

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

3.55 $\int x^7 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=72

$$\frac{3a^2\left(a+bx^2\right)^7}{14b^4}-\frac{a^3\left(a+bx^2\right)^6}{12b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}-\frac{3a\left(a+bx^2\right)^8}{16b^4}$$

```
[Out] -(a^3*(a + b*x^2)^6)/(12*b^4) + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a +
b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)
```

Rubi [A] time = 0.0905814, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^7}{14b^4}-\frac{a^3\left(a+bx^2\right)^6}{12b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}-\frac{3a\left(a+bx^2\right)^8}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5,x]
```

```
[Out] -(a^3*(a + b*x^2)^6)/(12*b^4) + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a +
b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^5 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^6}{12b^4}+\frac{3a^2\left(a+bx^2\right)^7}{14b^4}-\frac{3a\left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}
\end{aligned}$$

Mathematica [A] time = 0.0021167, size = 69, normalized size = 0.96

$$\frac{5}{7}a^2b^3x^{14}+\frac{5}{6}a^3b^2x^{12}+\frac{1}{2}a^4bx^{10}+\frac{a^5x^8}{8}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^8)/8 + (a^4*b*x^10)/2 + (5*a^3*b^2*x^12)/6 + (5*a^2*b^3*x^14)/7 + (5
*a*b^4*x^16)/16 + (b^5*x^18)/18
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{b^5x^{18}}{18} + \frac{5\,ab^4x^{16}}{16} + \frac{5\,a^2b^3x^{14}}{7} + \frac{5\,a^3b^2x^{12}}{6} + \frac{a^4bx^{10}}{2} + \frac{a^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5,x)
```

```
[Out] 1/18*b^5*x^18+5/16*a*b^4*x^16+5/7*a^2*b^3*x^14+5/6*a^3*b^2*x^12+1/2*a^4*b*x
^10+1/8*a^5*x^8
```

Maxima [A] time = 2.27767, size = 77, normalized size = 1.07

$$\frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Fricas [A] time = 1.12387, size = 138, normalized size = 1.92

$$\frac{1}{18}x^{18}b^5 + \frac{5}{16}x^{16}b^4a + \frac{5}{7}x^{14}b^3a^2 + \frac{5}{6}x^{12}b^2a^3 + \frac{1}{2}x^{10}ba^4 + \frac{1}{8}x^8a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5 + 5/16*x^16*b^4*a + 5/7*x^14*b^3*a^2 + 5/6*x^12*b^2*a^3 + 1/2
*x^10*b*a^4 + 1/8*x^8*a^5
```

Sympy [A] time = 0.071009, size = 65, normalized size = 0.9

$$\frac{a^5x^8}{8} + \frac{a^4bx^{10}}{2} + \frac{5a^3b^2x^{12}}{6} + \frac{5a^2b^3x^{14}}{7} + \frac{5ab^4x^{16}}{16} + \frac{b^5x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 +
5*a*b**4*x**16/16 + b**5*x**18/18
```

Giac [A] time = 2.33403, size = 77, normalized size = 1.07

$$\frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

3.56 $\int x^5 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^6}{12b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}-\frac{a\left(a+bx^2\right)^7}{7b^3}$$

```
[Out] (a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(1
6*b^3)
```

Rubi [A] time = 0.0647074, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^6}{12b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}-\frac{a\left(a+bx^2\right)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(1
6*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx,x,x^2\right)\\&=\frac{a^2\left(a+bx^2\right)^6}{12b^3}-\frac{a\left(a+bx^2\right)^7}{7b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}\end{aligned}$$

Mathematica [A] time = 0.0020264, size = 66, normalized size = 1.25

$$\frac{5}{6}a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}a^4bx^8+\frac{a^5x^6}{6}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^8)/8 + a^3*b^2*x^10 + (5*a^2*b^3*x^12)/6 + (5*a*b^
4*x^14)/14 + (b^5*x^16)/16
```

Maple [A] time = 0.001, size = 57, normalized size = 1.1

$$\frac{b^5x^{16}}{16}+\frac{5\,ab^4x^{14}}{14}+\frac{5\,a^2b^3x^{12}}{6}+a^3b^2x^{10}+\frac{5\,a^4bx^8}{8}+\frac{a^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5,x)
```

```
[Out] 1/16*b^5*x^16+5/14*a*b^4*x^14+5/6*a^2*b^3*x^12+a^3*b^2*x^10+5/8*a^4*b*x^8+1
/6*a^5*x^6
```

Maxima [A] time = 2.87543, size = 76, normalized size = 1.43

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Fricas [A] time = 0.984943, size = 131, normalized size = 2.47

$$\frac{1}{16}x^{16}b^5+\frac{5}{14}x^{14}b^4a+\frac{5}{6}x^{12}b^3a^2+x^{10}b^2a^3+\frac{5}{8}x^8ba^4+\frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5 + 5/14*x^14*b^4*a + 5/6*x^12*b^3*a^2 + x^10*b^2*a^3 + 5/8*x^8
*b*a^4 + 1/6*x^6*a^5
```

Sympy [A] time = 0.069944, size = 63, normalized size = 1.19

$$\frac{a^5x^6}{6}+\frac{5a^4bx^8}{8}+a^3b^2x^{10}+\frac{5a^2b^3x^{12}}{6}+\frac{5ab^4x^{14}}{14}+\frac{b^5x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**8/8 + a**3*b**2*x**10 + 5*a**2*b**3*x**12/6 + 5*a
*b**4*x**14/14 + b**5*x**16/16
```

Giac [A] time = 2.01114, size = 76, normalized size = 1.43

$$\frac{1}{16}\,b^5x^{16} + \frac{5}{14}\,ab^4x^{14} + \frac{5}{6}\,a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}\,a^4bx^8 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

3.57 $\int x^3 \left(a + bx^2\right)^5 dx$

Optimal. Leaf size=34

$$\frac{\left(a + bx^2\right)^7}{14b^2} - \frac{a\left(a + bx^2\right)^6}{12b^2}$$

```
[Out] -(a*(a + b*x^2)^6)/(12*b^2) + (a + b*x^2)^7/(14*b^2)
```

Rubi [A] time = 0.036031, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a + bx^2\right)^7}{14b^2} - \frac{a\left(a + bx^2\right)^6}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5,x]
```

```
[Out] -(a*(a + b*x^2)^6)/(12*b^2) + (a + b*x^2)^7/(14*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a + bx^2\right)^5 dx &= \frac{1}{2} \text{Subst}\left(\int x(a + bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{a(a + bx)^5}{b} + \frac{(a + bx)^6}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a + bx^2\right)^6}{12b^2} + \frac{\left(a + bx^2\right)^7}{14b^2}
\end{aligned}$$

Mathematica [A] time = 0.0020954, size = 66, normalized size = 1.94

$$a^2b^3x^{10} + \frac{5}{4}a^3b^2x^8 + \frac{5}{6}a^4bx^6 + \frac{a^5x^4}{4} + \frac{5}{12}ab^4x^{12} + \frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^4)/4 + (5*a^4*b*x^6)/6 + (5*a^3*b^2*x^8)/4 + a^2*b^3*x^10 + (5*a*b^4
*x^12)/12 + (b^5*x^14)/14
```

Maple [A] time = 0.001, size = 57, normalized size = 1.7

$$\frac{b^5x^{14}}{14} + \frac{5\,ab^4x^{12}}{12} + a^2b^3x^{10} + \frac{5\,a^3b^2x^8}{4} + \frac{5\,a^4bx^6}{6} + \frac{a^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5,x)
```

```
[Out] 1/14*b^5*x^14+5/12*a*b^4*x^12+a^2*b^3*x^10+5/4*a^3*b^2*x^8+5/6*a^4*b*x^6+1/
4*a^5*x^4
```

Maxima [A] time = 2.03753, size = 76, normalized size = 2.24

$$\frac{1}{14}\,b^5x^{14} + \frac{5}{12}\,ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{6}\,a^4bx^6 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Fricas [A] time = 1.1988, size = 130, normalized size = 3.82

$$\frac{1}{14}x^{14}b^5 + \frac{5}{12}x^{12}b^4a + x^{10}b^3a^2 + \frac{5}{4}x^8b^2a^3 + \frac{5}{6}x^6ba^4 + \frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^5 + 5/12*x^12*b^4*a + x^10*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/6*x^6*
b*a^4 + 1/4*x^4*a^5
```

Sympy [B] time = 0.072762, size = 63, normalized size = 1.85

$$\frac{a^5x^4}{4} + \frac{5a^4bx^6}{6} + \frac{5a^3b^2x^8}{4} + a^2b^3x^{10} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**4/4 + 5*a**4*b*x**6/6 + 5*a**3*b**2*x**8/4 + a**2*b**3*x**10 + 5*a*
b**4*x**12/12 + b**5*x**14/14
```

Giac [A] time = 1.31139, size = 76, normalized size = 2.24

$$\frac{1}{14}b^5x^{14} + \frac{5}{12}ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}a^3b^2x^8 + \frac{5}{6}a^4bx^6 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

3.58 $\int x\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^6}{12b}$$

```
[Out] (a + b*x^2)^6/(12*b)
```

Rubi [A] time = 0.0024507, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5,x]
```

```
[Out] (a + b*x^2)^6/(12*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^5\,dx=\frac{\left(a+bx^2\right)^6}{12b}$$

Mathematica [A] time = 0.0023003, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5,x]
```

```
[Out] (a + b*x^2)^6/(12*b)
```

Maple [B] time = 0.002, size = 58, normalized size = 3.6

$$\frac{b^5x^{12}}{12}+\frac{ab^4x^{10}}{2}+\frac{5\,a^2b^3x^8}{4}+\frac{5\,a^3b^2x^6}{3}+\frac{5\,a^4bx^4}{4}+\frac{a^5x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5,x)
```

```
[Out] 1/12*b^5*x^12+1/2*a*b^4*x^10+5/4*a^2*b^3*x^8+5/3*a^3*b^2*x^6+5/4*a^4*b*x^4+
1/2*a^5*x^2
```

Maxima [A] time = 2.05634, size = 19, normalized size = 1.19

$$\frac{(bx^2+a)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

Fricas [B] time = 1.09957, size = 132, normalized size = 8.25

$$\frac{1}{12}x^{12}b^5+\frac{1}{2}x^{10}b^4a+\frac{5}{4}x^8b^3a^2+\frac{5}{3}x^6b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^5 + 1/2*x^10*b^4*a + 5/4*x^8*b^3*a^2 + 5/3*x^6*b^2*a^3 + 5/4*x^
4*b*a^4 + 1/2*x^2*a^5
```

Sympy [B] time = 0.067715, size = 65, normalized size = 4.06

$$\frac{a^5x^2}{2}+\frac{5a^4bx^4}{4}+\frac{5a^3b^2x^6}{3}+\frac{5a^2b^3x^8}{4}+\frac{ab^4x^{10}}{2}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**4/4 + 5*a**3*b**2*x**6/3 + 5*a**2*b**3*x**8/4 + a
*b**4*x**10/2 + b**5*x**12/12
```

Giac [A] time = 3.02084, size = 19, normalized size = 1.19

$$\frac{(bx^2+a)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

3.59 $\int \frac{(a+bx^2)^5}{x}\,dx$

Optimal. Leaf size=65

$$\frac{5}{3}a^2b^3x^6+\frac{5}{2}a^3b^2x^4+\frac{5}{2}a^4bx^2+a^5\log(x)+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}$$

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Rubi [A] time = 0.0339468, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{3}a^2b^3x^6+\frac{5}{2}a^3b^2x^4+\frac{5}{2}a^4bx^2+a^5\log(x)+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx, x, x^2\right)\\ &= \frac{5}{2}a^4bx^2+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^2b^3x^6+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.0038627, size = 65, normalized size = 1.

$$\frac{5}{3}a^2b^3x^6+\frac{5}{2}a^3b^2x^4+\frac{5}{2}a^4bx^2+a^5\log(x)+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Maple [A] time = 0.001, size = 56, normalized size = 0.9

$$\frac{5\,a^4bx^2}{2}+\frac{5\,a^3b^2x^4}{2}+\frac{5\,a^2b^3x^6}{3}+\frac{5\,ab^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x,x)
```

```
[Out] 5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+a
^5*ln(x)
```

Maxima [A] time = 1.73627, size = 78, normalized size = 1.2

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+\frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

Fricas [A] time = 1.27972, size = 130, normalized size = 2.

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

Sympy [A] time = 0.304247, size = 65, normalized size = 1.

$$a^5\log(x)+\frac{5a^4bx^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^8}{8}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5
*a*b**4*x**8/8 + b**5*x**10/10
```

Giac [A] time = 2.15303, size = 78, normalized size = 1.2

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+\frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

3.60 $\int \frac{(a+bx^2)^5}{x^3}\,dx$

Optimal. Leaf size=64

$$\frac{5}{2}a^2b^3x^4+5a^3b^2x^2+5a^4b\log(x)-\frac{a^5}{2x^2}+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

```
[Out] -a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Rubi [A] time = 0.0366834, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{2}a^2b^3x^4+5a^3b^2x^2+5a^4b\log(x)-\frac{a^5}{2x^2}+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx, x, x^2\right)\\
&= -\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}+5a^4b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0043696, size = 64, normalized size = 1.

$$\frac{5}{2}a^2b^3x^4+5a^3b^2x^2+5a^4b\log(x)-\frac{a^5}{2x^2}+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Maple [A] time = 0.005, size = 57, normalized size = 0.9

$$-\frac{a^5}{2\,x^2}+5\,a^3b^2x^2+\frac{5\,a^2b^3x^4}{2}+\frac{5\,ab^4x^6}{6}+\frac{b^5x^8}{8}+5\,a^4b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*
b*ln(x)
```

Maxima [A] time = 3.017, size = 78, normalized size = 1.22

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+\frac{5}{2}\,a^4b\log\left(x^2\right)-\frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*a^5/x^2
```

Fricas [A] time = 1.22321, size = 142, normalized size = 2.22

$$\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log(x)-12\,a^5}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [A] time = 0.348286, size = 63, normalized size = 0.98

$$-\frac{a^5}{2x^2}+5a^4b\log(x)+5a^3b^2x^2+\frac{5a^2b^3x^4}{2}+\frac{5ab^4x^6}{6}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 +
5*a*b**4*x**6/6 + b**5*x**8/8
```

Giac [A] time = 2.83745, size = 92, normalized size = 1.44

$$\frac{1}{8}\,b^5x^8 + \frac{5}{6}\,ab^4x^6 + \frac{5}{2}\,a^2b^3x^4 + 5\,a^3b^2x^2 + \frac{5}{2}\,a^4b\log\left(x^2\right) - \frac{5\,a^4bx^2 + a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*(5*a^4*b*x^2 + a^5)/x^2
```

3.61 $\int \frac{(a+bx^2)^5}{x^5} \, dx$

Optimal. Leaf size=64

$$5a^2b^3x^2 + 10a^3b^2\log(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{4x^4} + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6}$$

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Rubi [A] time = 0.0361221, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$5a^2b^3x^2 + 10a^3b^2\log(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{4x^4} + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^5} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^3} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(10a^2b^3 + \frac{a^5}{x^3} + \frac{5a^4b}{x^2} + \frac{10a^3b^2}{x} + 5ab^4x + b^5x^2\right) dx, x, x^2\right)\\
&= -\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 5a^2b^3x^2 + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6} + 10a^3b^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.006366, size = 64, normalized size = 1.

$$5a^2b^3x^2 + 10a^3b^2\log(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{4x^4} + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Maple [A] time = 0.005, size = 57, normalized size = 0.9

$$-\frac{a^5}{4\,x^4}-\frac{5\,a^4b}{2\,x^2}+5\,a^2b^3x^2+\frac{5\,ab^4x^4}{4}+\frac{b^5x^6}{6}+10\,a^3b^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b
^2*ln(x)
```

Maxima [A] time = 2.05834, size = 80, normalized size = 1.25

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+5\,a^3b^2\log\left(x^2\right)-\frac{10\,a^4bx^2+a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(10*
a^4*b*x^2 + a^5)/x^4
```

Fricas [A] time = 1.37146, size = 139, normalized size = 2.17

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log\left(x\right)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) -
 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [A] time = 0.486586, size = 61, normalized size = 0.95

$$10a^3b^2\log\left(x\right)+5a^2b^3x^2+\frac{5ab^4x^4}{4}+\frac{b^5x^6}{6}-\frac{a^5+10a^4bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**5,x)
```

```
[Out] 10*a**3*b**2*log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6 - (a
**5 + 10*a**4*b*x**2)/(4*x**4)
```

Giac [A] time = 2.1646, size = 95, normalized size = 1.48

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+5\,a^3b^2\log\left(x^2\right)-\frac{30\,a^3b^2x^4+10\,a^4bx^2+a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(30*
a^3*b^2*x^4 + 10*a^4*b*x^2 + a^5)/x^4
```

3.62 $\int \frac{(a+bx^2)^5}{x^7} dx$

Optimal. Leaf size=64

$$-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

```
[Out] -a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Rubi [A] time = 0.0340468, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^4}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx, x, x^2\right)\\ &= -\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0043543, size = 64, normalized size = 1.

$$-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Maple [A] time = 0.005, size = 57, normalized size = 0.9

$$-\frac{a^5}{6\,x^6}-\frac{5\,a^4b}{4\,x^4}-5\,\frac{a^3b^2}{x^2}+\frac{5\,ab^4x^2}{2}+\frac{b^5x^4}{4}+10\,a^2b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b
^3*ln(x)
```

Maxima [A] time = 2.04804, size = 82, normalized size = 1.28

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(60*a^3*b^2*x^4 + 1
5*a^4*b*x^2 + 2*a^5)/x^6
```

Fricas [A] time = 1.07728, size = 139, normalized size = 2.17

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log(x)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [A] time = 0.485417, size = 63, normalized size = 0.98

$$10a^2b^3\log(x)+\frac{5ab^4x^2}{2}+\frac{b^5x^4}{4}-\frac{2a^5+15a^4bx^2+60a^3b^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**7,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4 - (2*a**5 + 15*a**4*b*x
**2 + 60*a**3*b**2*x**4)/(12*x**6)
```

Giac [A] time = 2.66172, size = 97, normalized size = 1.52

$$\frac{1}{4}\,b^5x^4 + \frac{5}{2}\,ab^4x^2 + 5\,a^2b^3\log\left(x^2\right) - \frac{110\,a^2b^3x^6 + 60\,a^3b^2x^4 + 15\,a^4bx^2 + 2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(110*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6
```

3.63 $\int \frac{(a+bx^2)^5}{x^9} dx$

Optimal. Leaf size=64

$$-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{6x^6}-\frac{a^5}{8x^8}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

```
[Out] -a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Rubi [A] time = 0.0328172, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{6x^6}-\frac{a^5}{8x^8}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^9,x]
```

```
[Out] -a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx, x, x^2\right)\\
&= -\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5ab^4\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0042871, size = 64, normalized size = 1.

$$-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{6x^6}-\frac{a^5}{8x^8}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^9,x]
```

```
[Out] -a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Maple [A] time = 0.006, size = 57, normalized size = 0.9

$$-\frac{a^5}{8\,x^8}-\frac{5\,a^4b}{6\,x^6}-\frac{5\,a^3b^2}{2\,x^4}-5\,\frac{a^2b^3}{x^2}+\frac{b^5x^2}{2}+5\,ab^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^9,x)
```

```
[Out] -1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^
4*ln(x)
```

Maxima [A] time = 1.58576, size = 82, normalized size = 1.28

$$\frac{1}{2}\,b^5x^2+\frac{5}{2}\,ab^4\log\left(x^2\right)-\frac{120\,a^2b^3x^6+60\,a^3b^2x^4+20\,a^4bx^2+3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(120*a^2*b^3*x^6 + 60*a^3*b^2*x^4 +
 20*a^4*b*x^2 + 3*a^5)/x^8
```

Fricas [A] time = 1.29682, size = 142, normalized size = 2.22

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log(x)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [A] time = 0.492037, size = 61, normalized size = 0.95

$$5ab^4\log(x)+\frac{b^5x^2}{2}-\frac{3a^5+20a^4bx^2+60a^3b^2x^4+120a^2b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**9,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x**2/2 - (3*a**5 + 20*a**4*b*x**2 + 60*a**3*b**2*x**
4 + 120*a**2*b**3*x**6)/(24*x**8)
```

Giac [A] time = 2.11403, size = 95, normalized size = 1.48

$$\frac{1}{2}\,b^{5}x^{2} + \frac{5}{2}\,ab^{4}\log\left(x^{2}\right) - \frac{125\,ab^{4}x^{8} + 120\,a^{2}b^{3}x^{6} + 60\,a^{3}b^{2}x^{4} + 20\,a^{4}bx^{2} + 3\,a^{5}}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(125*a*b^4*x^8 + 120*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8
```

3.64 $\int \frac{(a+bx^2)^5}{x^{11}} dx$

Optimal. Leaf size=65

$$-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5a^4b}{8x^8}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Rubi [A] time = 0.0310673, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5a^4b}{8x^8}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{11}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^6} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right) dx, x, x^2\right)\\ &= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.0043998, size = 65, normalized size = 1.

$$-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5a^4b}{8x^8}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{a^5}{10\,x^{10}}-\frac{5\,a^4b}{8\,x^8}-\frac{5\,a^3b^2}{3\,x^6}-\frac{5\,a^2b^3}{2\,x^4}-\frac{5\,ab^4}{2\,x^2}+b^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^11,x)
```

```
[Out] -1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+
b^5*ln(x)
```

Maxima [A] time = 1.9581, size = 82, normalized size = 1.26

$$\frac{1}{2}\,b^5\log\left(x^2\right)-\frac{300\,ab^4x^8+300\,a^2b^3x^6+200\,a^3b^2x^4+75\,a^4bx^2+12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^4
 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

Fricas [A] time = 1.22703, size = 149, normalized size = 2.29

$$\frac{120\,b^5x^{10}\log(x)-300\,ab^4x^8-300\,a^2b^3x^6-200\,a^3b^2x^4-75\,a^4bx^2-12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [A] time = 0.561579, size = 60, normalized size = 0.92

$$b^5\log(x)-\frac{12a^5+75a^4bx^2+200a^3b^2x^4+300a^2b^3x^6+300ab^4x^8}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**11,x)
```

```
[Out] b**5*log(x) - (12*a**5 + 75*a**4*b*x**2 + 200*a**3*b**2*x**4 + 300*a**2*b**
3*x**6 + 300*a*b**4*x**8)/(120*x**10)
```

Giac [A] time = 2.24977, size = 93, normalized size = 1.43

$$\frac{1}{2}\,b^5\log\left(x^2\right)-\frac{137\,b^5x^{10}+300\,ab^4x^8+300\,a^2b^3x^6+200\,a^3b^2x^4+75\,a^4bx^2+12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(137*b^5*x^10 + 300*a*b^4*x^8 + 300*a^2*b^3*x^6 +
200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

3.65 $\int \frac{(a+bx^2)^5}{x^{13}} \, dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

```
[Out] -(a + b*x^2)^6/(12*a*x^12)
```

Rubi [A] time = 0.0035702, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^13,x]
```

```
[Out] -(a + b*x^2)^6/(12*a*x^12)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{13}} \, dx = -\frac{(a+bx^2)^6}{12ax^{12}}$$

Mathematica [B] time = 0.0040731, size = 69, normalized size = 3.63

$$-\frac{5a^3b^2}{4x^8} - \frac{5a^2b^3}{3x^6} - \frac{a^4b}{2x^{10}} - \frac{a^5}{12x^{12}} - \frac{5ab^4}{4x^4} - \frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^13,x]
```

```
[Out] -a^5/(12*x^12) - (a^4*b)/(2*x^10) - (5*a^3*b^2)/(4*x^8) - (5*a^2*b^3)/(3*x^
6) - (5*a*b^4)/(4*x^4) - b^5/(2*x^2)
```

Maple [B] time = 0.004, size = 58, normalized size = 3.1

$$-\frac{5\,a^3b^2}{4\,x^8} - \frac{a^4b}{2\,x^{10}} - \frac{a^5}{12\,x^{12}} - \frac{5\,ab^4}{4\,x^4} - \frac{b^5}{2\,x^2} - \frac{5\,a^2b^3}{3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^13,x)
```

```
[Out] -5/4*a^3*b^2/x^8-1/2*a^4*b/x^10-1/12*a^5/x^12-5/4*a*b^4/x^4-1/2*b^5/x^2-5/3
*a^2*b^3/x^6
```

Maxima [B] time = 1.40163, size = 77, normalized size = 4.05

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Fricas [B] time = 1.25478, size = 127, normalized size = 6.68

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Sympy [B] time = 0.6015, size = 61, normalized size = 3.21

$$-\frac{a^5+6a^4bx^2+15a^3b^2x^4+20a^2b^3x^6+15ab^4x^8+6b^5x^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**13,x)
```

```
[Out] -(a**5 + 6*a**4*b*x**2 + 15*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 15*a*b**4*
x**8 + 6*b**5*x**10)/(12*x**12)
```

Giac [B] time = 2.87239, size = 77, normalized size = 4.05

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

3.66 $\int \frac{(a+bx^2)^5}{x^{15}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^6}{84a^2x^{12}}-\frac{\left(a+bx^2\right)^6}{14ax^{14}}$$

```
[Out] -(a + b*x^2)^6/(14*a*x^14) + (b*(a + b*x^2)^6)/(84*a^2*x^12)
```

Rubi [A] time = 0.0176447, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^2\right)^6}{84a^2x^{12}}-\frac{\left(a+bx^2\right)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^15,x]
```

```
[Out] -(a + b*x^2)^6/(14*a*x^14) + (b*(a + b*x^2)^6)/(84*a^2*x^12)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx,x,x^2\right)$$
$$= -\frac{(a+bx^2)^6}{14ax^{14}} - \frac{b\,\mathrm{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^2\right)}{14a}$$
$$= -\frac{(a+bx^2)^6}{14ax^{14}} + \frac{b(a+bx^2)^6}{84a^2x^{12}}$$

Mathematica [A] time = 0.005927, size = 67, normalized size = 1.68

$$-\frac{a^3b^2}{x^{10}} - \frac{5a^2b^3}{4x^8} - \frac{5a^4b}{12x^{12}} - \frac{a^5}{14x^{14}} - \frac{5ab^4}{6x^6} - \frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^15,x]
```

```
[Out] -a^5/(14*x^14) - (5*a^4*b)/(12*x^12) - (a^3*b^2)/x^10 - (5*a^2*b^3)/(4*x^8)
 - (5*a*b^4)/(6*x^6) - b^5/(4*x^4)
```

Maple [A] time = 0.005, size = 58, normalized size = 1.5

$$-\frac{5\,a^2b^3}{4\,x^8} - \frac{5\,a^4b}{12\,x^{12}} - \frac{a^3b^2}{x^{10}} - \frac{b^5}{4\,x^4} - \frac{5\,ab^4}{6\,x^6} - \frac{a^5}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^15,x)
```

```
[Out] -5/4*a^2*b^3/x^8-5/12*a^4*b/x^12-a^3*b^2/x^10-1/4*b^5/x^4-5/6*a*b^4/x^6-1/1
4*a^5/x^14
```

Maxima [A] time = 1.96214, size = 80, normalized size = 2.

$$-\frac{21\,b^5x^{10} + 70\,ab^4x^8 + 105\,a^2b^3x^6 + 84\,a^3b^2x^4 + 35\,a^4bx^2 + 6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Fricas [A] time = 1.23035, size = 134, normalized size = 3.35

$$-\frac{21\,b^5x^{10} + 70\,ab^4x^8 + 105\,a^2b^3x^6 + 84\,a^3b^2x^4 + 35\,a^4bx^2 + 6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Sympy [A] time = 0.644176, size = 63, normalized size = 1.58

$$-\frac{6a^5 + 35a^4bx^2 + 84a^3b^2x^4 + 105a^2b^3x^6 + 70ab^4x^8 + 21b^5x^{10}}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**15,x)
```

```
[Out] -(6*a**5 + 35*a**4*b*x**2 + 84*a**3*b**2*x**4 + 105*a**2*b**3*x**6 + 70*a*b
**4*x**8 + 21*b**5*x**10)/(84*x**14)
```

Giac [A] time = 2.45779, size = 80, normalized size = 2.

$$-\frac{21\,b^5x^{10} + 70\,ab^4x^8 + 105\,a^2b^3x^6 + 84\,a^3b^2x^4 + 35\,a^4bx^2 + 6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

3.67 $\int \frac{(a+bx^2)^5}{x^{17}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}}+\frac{b(a+bx^2)^6}{56a^2x^{14}}-\frac{(a+bx^2)^6}{16ax^{16}}$$

```
[Out] -(a + b*x^2)^6/(16*a*x^16) + (b*(a + b*x^2)^6)/(56*a^2*x^14) - (b^2*(a + b*
x^2)^6)/(336*a^3*x^12)
```

Rubi [A] time = 0.0280627, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}}+\frac{b(a+bx^2)^6}{56a^2x^{14}}-\frac{(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^17,x]
```

```
[Out] -(a + b*x^2)^6/(16*a*x^16) + (b*(a + b*x^2)^6)/(56*a^2*x^14) - (b^2*(a + b*
x^2)^6)/(336*a^3*x^12)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5}{x^{17}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^6}{16ax^{16}} - \frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)}{8a}$$
$$= -\frac{\left(a+bx^2\right)^6}{16ax^{16}} + \frac{b\left(a+bx^2\right)^6}{56a^2x^{14}} + \frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{56a^2}$$
$$= -\frac{\left(a+bx^2\right)^6}{16ax^{16}} + \frac{b\left(a+bx^2\right)^6}{56a^2x^{14}} - \frac{b^2\left(a+bx^2\right)^6}{336a^3x^{12}}$$

Mathematica [A] time = 0.0042791, size = 67, normalized size = 1.08

$$-\frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5a^4b}{14x^{14}} - \frac{a^5}{16x^{16}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^17,x]
```

```
[Out] -a^5/(16*x^16) - (5*a^4*b)/(14*x^14) - (5*a^3*b^2)/(6*x^12) - (a^2*b^3)/x^1
0 - (5*a*b^4)/(8*x^8) - b^5/(6*x^6)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$-\frac{5\,ab^4}{8\,x^8} - \frac{a^2b^3}{x^{10}} - \frac{5\,a^3b^2}{6\,x^{12}} - \frac{a^5}{16\,x^{16}} - \frac{b^5}{6\,x^6} - \frac{5\,a^4b}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^17,x)
```

```
[Out] -5/8*a*b^4/x^8-a^2*b^3/x^10-5/6*a^3*b^2/x^12-1/16*a^5/x^16-1/6*b^5/x^6-5/14
*a^4*b/x^14
```

Maxima [A] time = 1.82797, size = 80, normalized size = 1.29

$$-\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Fricas [A] time = 1.28982, size = 140, normalized size = 2.26

$$-\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [A] time = 0.687703, size = 63, normalized size = 1.02

$$-\frac{21a^5 + 120a^4bx^2 + 280a^3b^2x^4 + 336a^2b^3x^6 + 210ab^4x^8 + 56b^5x^{10}}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**17,x)
```

```
[Out] -(21*a**5 + 120*a**4*b*x**2 + 280*a**3*b**2*x**4 + 336*a**2*b**3*x**6 + 210
*a*b**4*x**8 + 56*b**5*x**10)/(336*x**16)
```

Giac [A] time = 2.6719, size = 80, normalized size = 1.29

$$-\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

3.68 $\int \frac{(a+bx^2)^5}{x^{19}} \, dx$

Optimal. Leaf size=69

$$-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{5a^4b}{16x^{16}}-\frac{a^5}{18x^{18}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

```
[Out] -a^5/(18*x^18) - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Rubi [A] time = 0.0316203, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{5a^4b}{16x^{16}}-\frac{a^5}{18x^{18}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^19,x]
```

```
[Out] -a^5/(18*x^18) - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{19}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^{10}} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right) dx, x, x^2\right)\\ &= -\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.0040041, size = 69, normalized size = 1.

$$-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{5a^4b}{16x^{16}}-\frac{a^5}{18x^{18}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^19,x]
```

```
[Out] -a^5/(18*x^18) - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.8

$$-\frac{a^5}{18\,x^{18}}-\frac{5\,a^4b}{16\,x^{16}}-\frac{5\,a^3b^2}{7\,x^{14}}-\frac{5\,a^2b^3}{6\,x^{12}}-\frac{ab^4}{2\,x^{10}}-\frac{b^5}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^19,x)
```

```
[Out] -1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4/
x^10-1/8*b^5/x^8
```

Maxima [A] time = 2.14467, size = 80, normalized size = 1.16

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Fricas [A] time = 1.22489, size = 143, normalized size = 2.07

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [A] time = 0.772806, size = 63, normalized size = 0.91

$$-\frac{56a^5+315a^4bx^2+720a^3b^2x^4+840a^2b^3x^6+504ab^4x^8+126b^5x^{10}}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**19,x)
```

```
[Out] -(56*a**5 + 315*a**4*b*x**2 + 720*a**3*b**2*x**4 + 840*a**2*b**3*x**6 + 504
*a*b**4*x**8 + 126*b**5*x**10)/(1008*x**18)
```

Giac [A] time = 1.60327, size = 80, normalized size = 1.16

$$-\frac{126\,b^5x^{10} + 504\,ab^4x^8 + 840\,a^2b^3x^6 + 720\,a^3b^2x^4 + 315\,a^4bx^2 + 56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

3.69 $\int \frac{\left(a+bx^2\right)^5}{x^{21}} \, dx$

Optimal. Leaf size=69

$$-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a^4b}{18x^{18}}-\frac{a^5}{20x^{20}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

```
[Out] -a^5/(20*x^20) - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Rubi [A] time = 0.0305198, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a^4b}{18x^{18}}-\frac{a^5}{20x^{20}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^21,x]
```

```
[Out] -a^5/(20*x^20) - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x^{21}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^{11}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right) dx, x, x^2\right)\\
&= -\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}
\end{aligned}$$

Mathematica [A] time = 0.0042634, size = 69, normalized size = 1.

$$-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a^4b}{18x^{18}}-\frac{a^5}{20x^{20}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^21,x]
```

```
[Out] -a^5/(20*x^20) - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.8

$$-\frac{a^{5}}{20\,x^{20}}-\frac{5\,a^{4}b}{18\,x^{18}}-\frac{5\,a^{3}b^{2}}{8\,x^{16}}-\frac{5\,a^{2}b^{3}}{7\,x^{14}}-\frac{5\,ab^{4}}{12\,x^{12}}-\frac{b^{5}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^21,x)
```

```
[Out] -1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^4
/x^12-1/10*b^5/x^10
```

Maxima [A] time = 2.77342, size = 80, normalized size = 1.16

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Fricas [A] time = 1.26214, size = 149, normalized size = 2.16

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [A] time = 0.874056, size = 63, normalized size = 0.91

$$-\frac{126a^{5}+700a^{4}bx^{2}+1575a^{3}b^{2}x^{4}+1800a^{2}b^{3}x^{6}+1050ab^{4}x^{8}+252b^{5}x^{10}}{2520x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**21,x)
```

```
[Out] -(126*a**5 + 700*a**4*b*x**2 + 1575*a**3*b**2*x**4 + 1800*a**2*b**3*x**6 +
1050*a*b**4*x**8 + 252*b**5*x**10)/(2520*x**20)
```

Giac [A] time = 2.51817, size = 80, normalized size = 1.16

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

3.70 $\int x^8 \left(a+bx^2\right)^5 dx$

Optimal. Leaf size=69

$$\frac{2}{3}a^2b^3x^{15}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{11}a^4bx^{11}+\frac{a^5x^9}{9}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Rubi [A] time = 0.0248095, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{2}{3}a^2b^3x^{15}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{11}a^4bx^{11}+\frac{a^5x^9}{9}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^2\right)^5 dx &= \int \left(a^5x^8+5a^4bx^{10}+10a^3b^2x^{12}+10a^2b^3x^{14}+5ab^4x^{16}+b^5x^{18}\right) dx\\ &= \frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.0021935, size = 69, normalized size = 1.

$$\frac{2}{3}a^2b^3x^{15}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{11}a^4bx^{11}+\frac{a^5x^9}{9}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{a^5x^9}{9}+\frac{5\,a^4bx^{11}}{11}+\frac{10\,a^3b^2x^{13}}{13}+\frac{2\,a^2b^3x^{15}}{3}+\frac{5\,ab^4x^{17}}{17}+\frac{b^5x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5,x)
```

```
[Out] 1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4*
x^17+1/19*b^5*x^19
```

Maxima [A] time = 2.21767, size = 77, normalized size = 1.12

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Fricas [A] time = 1.18403, size = 142, normalized size = 2.06

$$\frac{1}{19}x^{19}b^5+\frac{5}{17}x^{17}b^4a+\frac{2}{3}x^{15}b^3a^2+\frac{10}{13}x^{13}b^2a^3+\frac{5}{11}x^{11}ba^4+\frac{1}{9}x^9a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5 + 5/17*x^17*b^4*a + 2/3*x^15*b^3*a^2 + 10/13*x^13*b^2*a^3 + 5
/11*x^11*b*a^4 + 1/9*x^9*a^5
```

Sympy [A] time = 0.086702, size = 66, normalized size = 0.96

$$\frac{a^5x^9}{9}+\frac{5a^4bx^{11}}{11}+\frac{10a^3b^2x^{13}}{13}+\frac{2a^2b^3x^{15}}{3}+\frac{5ab^4x^{17}}{17}+\frac{b^5x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**9/9 + 5*a**4*b*x**11/11 + 10*a**3*b**2*x**13/13 + 2*a**2*b**3*x**15
/3 + 5*a*b**4*x**17/17 + b**5*x**19/19
```

Giac [A] time = 1.48448, size = 77, normalized size = 1.12

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

3.71 $\int x^6 \left(a+bx^2\right)^5 dx$

Optimal. Leaf size=69

$$\frac{10}{13}a^2b^3x^{13}+\frac{10}{11}a^3b^2x^{11}+\frac{5}{9}a^4bx^9+\frac{a^5x^7}{7}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Rubi [A] time = 0.0238795, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{13}a^2b^3x^{13}+\frac{10}{11}a^3b^2x^{11}+\frac{5}{9}a^4bx^9+\frac{a^5x^7}{7}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^2\right)^5 dx &= \int \left(a^5x^6+5a^4bx^8+10a^3b^2x^{10}+10a^2b^3x^{12}+5ab^4x^{14}+b^5x^{16}\right) dx\\ &= \frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.0020469, size = 69, normalized size = 1.

$$\frac{10}{13}a^2b^3x^{13}+\frac{10}{11}a^3b^2x^{11}+\frac{5}{9}a^4bx^9+\frac{a^5x^7}{7}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{a^5x^7}{7}+\frac{5\,a^4bx^9}{9}+\frac{10\,a^3b^2x^{11}}{11}+\frac{10\,a^2b^3x^{13}}{13}+\frac{ab^4x^{15}}{3}+\frac{b^5x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5,x)
```

```
[Out] 1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*x
^15+1/17*b^5*x^17
```

Maxima [A] time = 1.71218, size = 77, normalized size = 1.12

$$\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Fricas [A] time = 1.04876, size = 140, normalized size = 2.03

$$\frac{1}{17}x^{17}b^{5}+\frac{1}{3}x^{15}b^{4}a+\frac{10}{13}x^{13}b^{3}a^{2}+\frac{10}{11}x^{11}b^{2}a^{3}+\frac{5}{9}x^{9}ba^{4}+\frac{1}{7}x^{7}a^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5 + 1/3*x^15*b^4*a + 10/13*x^13*b^3*a^2 + 10/11*x^11*b^2*a^3 +
5/9*x^9*b*a^4 + 1/7*x^7*a^5
```

Sympy [A] time = 0.072639, size = 65, normalized size = 0.94

$$\frac{a^{5}x^{7}}{7}+\frac{5a^{4}bx^{9}}{9}+\frac{10a^{3}b^{2}x^{11}}{11}+\frac{10a^{2}b^{3}x^{13}}{13}+\frac{ab^{4}x^{15}}{3}+\frac{b^{5}x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**9/9 + 10*a**3*b**2*x**11/11 + 10*a**2*b**3*x**13/
13 + a*b**4*x**15/3 + b**5*x**17/17
```

Giac [A] time = 2.28224, size = 77, normalized size = 1.12

$$\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

3.72 $\int x^4 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{10}{11}a^2b^3x^{11}+\frac{10}{9}a^3b^2x^9+\frac{5}{7}a^4bx^7+\frac{a^5x^5}{5}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Rubi [A] time = 0.0222111, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{11}a^2b^3x^{11}+\frac{10}{9}a^3b^2x^9+\frac{5}{7}a^4bx^7+\frac{a^5x^5}{5}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^5\,dx&=\int\left(a^5x^4+5a^4bx^6+10a^3b^2x^8+10a^2b^3x^{10}+5ab^4x^{12}+b^5x^{14}\right)dx\\&=\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.002035, size = 69, normalized size = 1.

$$\frac{10}{11}a^2b^3x^{11}+\frac{10}{9}a^3b^2x^9+\frac{5}{7}a^4bx^7+\frac{a^5x^5}{5}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{a^5x^5}{5}+\frac{5\,a^4bx^7}{7}+\frac{10\,a^3b^2x^9}{9}+\frac{10\,a^2b^3x^{11}}{11}+\frac{5\,ab^4x^{13}}{13}+\frac{b^5x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x^
13+1/15*b^5*x^15
```

Maxima [A] time = 2.06623, size = 77, normalized size = 1.12

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Fricas [A] time = 1.01834, size = 139, normalized size = 2.01

$$\frac{1}{15}x^{15}b^5+\frac{5}{13}x^{13}b^4a+\frac{10}{11}x^{11}b^3a^2+\frac{10}{9}x^9b^2a^3+\frac{5}{7}x^7ba^4+\frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^5 + 5/13*x^13*b^4*a + 10/11*x^11*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5
/7*x^7*b*a^4 + 1/5*x^5*a^5
```

Sympy [A] time = 0.072165, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^7}{7}+\frac{10a^3b^2x^9}{9}+\frac{10a^2b^3x^{11}}{11}+\frac{5ab^4x^{13}}{13}+\frac{b^5x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**7/7 + 10*a**3*b**2*x**9/9 + 10*a**2*b**3*x**11/11
 + 5*a*b**4*x**13/13 + b**5*x**15/15
```

Giac [A] time = 2.33608, size = 77, normalized size = 1.12

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

3.73 $\int x^2 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=66

$$\frac{10}{9}a^2b^3x^9+\frac{10}{7}a^3b^2x^7+a^4bx^5+\frac{a^5x^3}{3}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Rubi [A] time = 0.0218526, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{9}a^2b^3x^9+\frac{10}{7}a^3b^2x^7+a^4bx^5+\frac{a^5x^3}{3}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^2+5a^4bx^4+10a^3b^2x^6+10a^2b^3x^8+5ab^4x^{10}+b^5x^{12}\right) dx\\ &= \frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.0020046, size = 66, normalized size = 1.

$$\frac{10}{9}a^2b^3x^9+\frac{10}{7}a^3b^2x^7+a^4bx^5+\frac{a^5x^3}{3}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Maple [A] time = 0., size = 57, normalized size = 0.9

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10\,a^3b^2x^7}{7}+\frac{10\,a^2b^3x^9}{9}+\frac{5\,ab^4x^{11}}{11}+\frac{b^5x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5,x)
```

```
[Out] 1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/1
3*b^5*x^13
```

Maxima [A] time = 1.58331, size = 76, normalized size = 1.15

$$\frac{1}{13}b^5x^{13} + \frac{5}{11}ab^4x^{11} + \frac{10}{9}a^2b^3x^9 + \frac{10}{7}a^3b^2x^7 + a^4bx^5 + \frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

Fricas [A] time = 1.13154, size = 131, normalized size = 1.98

$$\frac{1}{13}x^{13}b^5 + \frac{5}{11}x^{11}b^4a + \frac{10}{9}x^9b^3a^2 + \frac{10}{7}x^7b^2a^3 + x^5ba^4 + \frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^5 + 5/11*x^11*b^4*a + 10/9*x^9*b^3*a^2 + 10/7*x^7*b^2*a^3 + x^5
*b*a^4 + 1/3*x^3*a^5
```

Sympy [A] time = 0.0711, size = 63, normalized size = 0.95

$$\frac{a^5x^3}{3} + a^4bx^5 + \frac{10a^3b^2x^7}{7} + \frac{10a^2b^3x^9}{9} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**3/3 + a**4*b*x**5 + 10*a**3*b**2*x**7/7 + 10*a**2*b**3*x**9/9 + 5*a
*b**4*x**11/11 + b**5*x**13/13
```

Giac [A] time = 1.39158, size = 76, normalized size = 1.15

$$\frac{1}{13}b^5x^{13} + \frac{5}{11}ab^4x^{11} + \frac{10}{9}a^2b^3x^9 + \frac{10}{7}a^3b^2x^7 + a^4bx^5 + \frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

3.74 $\int \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=62

$$\frac{10}{7}a^2b^3x^7+2a^3b^2x^5+\frac{5}{3}a^4bx^3+a^5x+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Rubi [A] time = 0.019159, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{10}{7}a^2b^3x^7+2a^3b^2x^5+\frac{5}{3}a^4bx^3+a^5x+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^5 \, dx &= \int \left(a^5+5a^4bx^2+10a^3b^2x^4+10a^2b^3x^6+5ab^4x^8+b^5x^{10}\right) dx\\ &= a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.0009757, size = 62, normalized size = 1.

$$\frac{10}{7}a^2b^3x^7+2a^3b^2x^5+\frac{5}{3}a^4bx^3+a^5x+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Maple [A] time = 0.001, size = 55, normalized size = 0.9

$$a^5x+\frac{5\,a^4bx^3}{3}+2\,a^3b^2x^5+\frac{10\,a^2b^3x^7}{7}+\frac{5\,ab^4x^9}{9}+\frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5,x)
```

```
[Out] a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*x
^11
```

Maxima [A] time = 2.31468, size = 73, normalized size = 1.18

$$\frac{1}{11}\,b^{5}x^{11} + \frac{5}{9}\,ab^{4}x^{9} + \frac{10}{7}\,a^{2}b^{3}x^{7} + 2\,a^{3}b^{2}x^{5} + \frac{5}{3}\,a^{4}bx^{3} + a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Fricas [A] time = 1.1136, size = 122, normalized size = 1.97

$$\frac{1}{11}x^{11}b^{5} + \frac{5}{9}x^{9}b^{4}a + \frac{10}{7}x^{7}b^{3}a^{2} + 2x^{5}b^{2}a^{3} + \frac{5}{3}x^{3}ba^{4} + xa^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^5 + 5/9*x^9*b^4*a + 10/7*x^7*b^3*a^2 + 2*x^5*b^2*a^3 + 5/3*x^3*
b*a^4 + x*a^5
```

Sympy [A] time = 0.067331, size = 61, normalized size = 0.98

$$a^{5}x + \frac{5a^{4}bx^{3}}{3} + 2a^{3}b^{2}x^{5} + \frac{10a^{2}b^{3}x^{7}}{7} + \frac{5ab^{4}x^{9}}{9} + \frac{b^{5}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b**
4*x**9/9 + b**5*x**11/11
```

Giac [A] time = 2.29265, size = 73, normalized size = 1.18

$$\frac{1}{11}\,b^{5}x^{11} + \frac{5}{9}\,ab^{4}x^{9} + \frac{10}{7}\,a^{2}b^{3}x^{7} + 2\,a^{3}b^{2}x^{5} + \frac{5}{3}\,a^{4}bx^{3} + a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

3.75 $\int \frac{(a+bx^2)^5}{x^2} dx$

Optimal. Leaf size=61

$$2a^2b^3x^5 + \frac{10}{3}a^3b^2x^3 + 5a^4bx - \frac{a^5}{x} + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}$$

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Rubi [A] time = 0.0210491, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$2a^2b^3x^5 + \frac{10}{3}a^3b^2x^3 + 5a^4bx - \frac{a^5}{x} + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^2} dx &= \int \left(5a^4b + \frac{a^5}{x^2} + 10a^3b^2x^2 + 10a^2b^3x^4 + 5ab^4x^6 + b^5x^8\right) dx \\ &= -\frac{a^5}{x} + 5a^4bx + \frac{10}{3}a^3b^2x^3 + 2a^2b^3x^5 + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0036962, size = 61, normalized size = 1.

$$2a^2b^3x^5 + \frac{10}{3}a^3b^2x^3 + 5a^4bx - \frac{a^5}{x} + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Maple [A] time = 0.003, size = 56, normalized size = 0.9

$$-\frac{a^{5}}{x}+5\,a^{4}bx+\frac{10\,a^{3}b^{2}x^{3}}{3}+2\,a^{2}b^{3}x^{5}+\frac{5\,ab^{4}x^{7}}{7}+\frac{b^{5}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^2,x)
```

```
[Out] -a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

Maxima [A] time = 1.75965, size = 74, normalized size = 1.21

$$\frac{1}{9}\,b^{5}x^{9}+\frac{5}{7}\,ab^{4}x^{7}+2\,a^{2}b^{3}x^{5}+\frac{10}{3}\,a^{3}b^{2}x^{3}+5\,a^{4}bx-\frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

Fricas [A] time = 1.18136, size = 131, normalized size = 2.15

$$\frac{7\,b^{5}x^{10}+45\,ab^{4}x^{8}+126\,a^{2}b^{3}x^{6}+210\,a^{3}b^{2}x^{4}+315\,a^{4}bx^{2}-63\,a^{5}}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

Sympy [A] time = 0.279188, size = 58, normalized size = 0.95

$$-\frac{a^{5}}{x}+5a^{4}bx+\frac{10a^{3}b^{2}x^{3}}{3}+2a^{2}b^{3}x^{5}+\frac{5ab^{4}x^{7}}{7}+\frac{b^{5}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x*
*7/7 + b**5*x**9/9
```

Giac [A] time = 2.72171, size = 74, normalized size = 1.21

$$\frac{1}{9}\,b^{5}x^{9}+\frac{5}{7}\,ab^{4}x^{7}+2\,a^{2}b^{3}x^{5}+\frac{10}{3}\,a^{3}b^{2}x^{3}+5\,a^{4}bx-\frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

3.76 $\int \frac{(a+bx^2)^5}{x^4}\,dx$

Optimal. Leaf size=60

$$\frac{10}{3}a^2b^3x^3+10a^3b^2x-\frac{5a^4b}{x}-\frac{a^5}{3x^3}+ab^4x^5+\frac{b^5x^7}{7}$$

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Rubi [A] time = 0.0217368, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{3}a^2b^3x^3+10a^3b^2x-\frac{5a^4b}{x}-\frac{a^5}{3x^3}+ab^4x^5+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^4}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^4}+\frac{5a^4b}{x^2}+10a^2b^3x^2+5ab^4x^4+b^5x^6\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0038595, size = 60, normalized size = 1.

$$\frac{10}{3}a^2b^3x^3+10a^3b^2x-\frac{5a^4b}{x}-\frac{a^5}{3x^3}+ab^4x^5+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Maple [A] time = 0.006, size = 55, normalized size = 0.9

$$-\frac{a^5}{3\,x^3}-5\,\frac{a^4b}{x}+10\,a^3b^2x+\frac{10\,a^2b^3x^3}{3}+ab^4x^5+\frac{b^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

Maxima [A] time = 2.77038, size = 74, normalized size = 1.23

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{15\,a^4bx^2+a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

Fricas [A] time = 1.09077, size = 131, normalized size = 2.18

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

Sympy [A] time = 0.318195, size = 58, normalized size = 0.97

$$10a^3b^2x+\frac{10a^2b^3x^3}{3}+ab^4x^5+\frac{b^5x^7}{7}-\frac{a^5+15a^4bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**4,x)
```

```
[Out] 10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7 - (a**5 +
15*a**4*b*x**2)/(3*x**3)
```

Giac [A] time = 2.04291, size = 74, normalized size = 1.23

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{15\,a^4bx^2+a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

3.77 $\int \frac{(a+bx^2)^5}{x^6} \, dx$

Optimal. Leaf size=63

$$-\frac{10a^3b^2}{x}+10a^2b^3x-\frac{5a^4b}{3x^3}-\frac{a^5}{5x^5}+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Rubi [A] time = 0.0224724, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{x}+10a^2b^3x-\frac{5a^4b}{3x^3}-\frac{a^5}{5x^5}+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^6} \, dx &= \int \left(10a^2b^3+\frac{a^5}{x^6}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^2}+5ab^4x^2+b^5x^4\right) dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0039618, size = 63, normalized size = 1.

$$-\frac{10a^3b^2}{x}+10a^2b^3x-\frac{5a^4b}{3x^3}-\frac{a^5}{5x^5}+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{a^5}{5\,x^5}-\frac{5\,a^4b}{3\,x^3}-10\,\frac{a^3b^2}{x}+10\,a^2b^3x+\frac{5\,ab^4x^3}{3}+\frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*
x^5
```

Maxima [A] time = 1.80023, size = 78, normalized size = 1.24

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{150\,a^3b^2x^4+25\,a^4bx^2+3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

Fricas [A] time = 1.21474, size = 131, normalized size = 2.08

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

Sympy [A] time = 0.373617, size = 61, normalized size = 0.97

$$10a^2b^3x+\frac{5ab^4x^3}{3}+\frac{b^5x^5}{5}-\frac{3a^5+25a^4bx^2+150a^3b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**6,x)
```

```
[Out] 10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5 - (3*a**5 + 25*a**4*b*x**2 +
 150*a**3*b**2*x**4)/(15*x**5)
```

Giac [A] time = 1.91505, size = 78, normalized size = 1.24

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{150\,a^3b^2x^4+25\,a^4bx^2+3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

3.78 $\int \frac{(a+bx^2)^5}{x^8} dx$

Optimal. Leaf size=61

$$-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}-\frac{a^4b}{x^5}-\frac{a^5}{7x^7}+5ab^4x+\frac{b^5x^3}{3}$$

```
[Out] -a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Rubi [A] time = 0.0216362, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}-\frac{a^4b}{x^5}-\frac{a^5}{7x^7}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^8,x]
```

```
[Out] -a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^8}\,dx &= \int \left(5ab^4+\frac{a^5}{x^8}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^2}+b^5x^2\right)dx\\ &= -\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0038928, size = 61, normalized size = 1.

$$-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}-\frac{a^4b}{x^5}-\frac{a^5}{7x^7}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^8,x]
```

```
[Out] -a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{a^5}{7\,x^7}-\frac{a^4b}{x^5}-\frac{10\,a^3b^2}{3\,x^3}-10\,\frac{a^2b^3}{x}+5\,ab^4x+\frac{b^5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^8,x)
```

```
[Out] -1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

Maxima [A] time = 1.87512, size = 78, normalized size = 1.28

$$\frac{1}{3}\,b^5x^3+5\,ab^4x-\frac{210\,a^2b^3x^6+70\,a^3b^2x^4+21\,a^4bx^2+3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

Fricas [A] time = 1.22507, size = 131, normalized size = 2.15

$$\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

Sympy [A] time = 0.451072, size = 60, normalized size = 0.98

$$5ab^4x+\frac{b^5x^3}{3}-\frac{3a^5+21a^4bx^2+70a^3b^2x^4+210a^2b^3x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**8,x)
```

```
[Out] 5*a*b**4*x + b**5*x**3/3 - (3*a**5 + 21*a**4*b*x**2 + 70*a**3*b**2*x**4 + 2
10*a**2*b**3*x**6)/(21*x**7)
```

Giac [A] time = 2.19656, size = 78, normalized size = 1.28

$$\frac{1}{3}\,b^5x^3+5\,ab^4x-\frac{210\,a^2b^3x^6+70\,a^3b^2x^4+21\,a^4bx^2+3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

3.79 $\int \frac{(a+bx^2)^5}{x^{10}} dx$

Optimal. Leaf size=60

$$-\frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5a^4b}{7x^7} - \frac{a^5}{9x^9} - \frac{5ab^4}{x} + b^5x$$

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Rubi [A] time = 0.0219145, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5a^4b}{7x^7} - \frac{a^5}{9x^9} - \frac{5ab^4}{x} + b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^5}{x^{10}} dx &= \int \left(b^5 + \frac{a^5}{x^{10}} + \frac{5a^4b}{x^8} + \frac{10a^3b^2}{x^6} + \frac{10a^2b^3}{x^4} + \frac{5ab^4}{x^2}\right) dx \\ &= -\frac{a^5}{9x^9} - \frac{5a^4b}{7x^7} - \frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{x} + b^5x \end{aligned}$$

Mathematica [A] time = 0.0052029, size = 60, normalized size = 1.

$$-\frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5a^4b}{7x^7} - \frac{a^5}{9x^9} - \frac{5ab^4}{x} + b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Maple [A] time = 0.006, size = 55, normalized size = 0.9

$$-\frac{a^5}{9\,x^9}-\frac{5\,a^4b}{7\,x^7}-2\,\frac{a^3b^2}{x^5}-\frac{10\,a^2b^3}{3\,x^3}-5\,\frac{ab^4}{x}+b^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

Maxima [A] time = 1.97457, size = 77, normalized size = 1.28

$$b^5x-\frac{315\,ab^4x^8+210\,a^2b^3x^6+126\,a^3b^2x^4+45\,a^4bx^2+7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

Fricas [A] time = 1.29122, size = 134, normalized size = 2.23

$$\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [A] time = 0.513978, size = 58, normalized size = 0.97

$$b^5x-\frac{7a^5+45a^4bx^2+126a^3b^2x^4+210a^2b^3x^6+315ab^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**10,x)
```

```
[Out] b**5*x - (7*a**5 + 45*a**4*b*x**2 + 126*a**3*b**2*x**4 + 210*a**2*b**3*x**6
 + 315*a*b**4*x**8)/(63*x**9)
```

Giac [A] time = 2.80845, size = 77, normalized size = 1.28

$$b^5x-\frac{315\,ab^4x^8+210\,a^2b^3x^6+126\,a^3b^2x^4+45\,a^4bx^2+7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="giac")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

3.80 $\int \frac{\left(a+bx^2\right)^5}{x^{12}} \, dx$

Optimal. Leaf size=65

$$-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{9x^9}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

```
[Out] -a^5/(11*x^11) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Rubi [A] time = 0.0223313, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{9x^9}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^{12}} \, dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^6}+\frac{5ab^4}{x^4}+\frac{b^5}{x^2}\right) dx\\ &= -\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}\end{aligned}$$

Mathematica [A] time = 0.003967, size = 65, normalized size = 1.

$$-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{9x^9}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$-\frac{a^{5}}{11\,x^{11}}-\frac{5\,a^{4}b}{9\,x^{9}}-\frac{10\,a^{3}b^{2}}{7\,x^{7}}-2\,\frac{a^{2}b^{3}}{x^{5}}-\frac{5\,ab^{4}}{3\,x^{3}}-\frac{b^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^12,x)
```

```
[Out] -1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b
^5/x
```

Maxima [A] time = 2.13639, size = 80, normalized size = 1.23

$$-\frac{693\,b^{5}x^{10}+1155\,ab^{4}x^{8}+1386\,a^{2}b^{3}x^{6}+990\,a^{3}b^{2}x^{4}+385\,a^{4}bx^{2}+63\,a^{5}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Fricas [A] time = 1.19324, size = 144, normalized size = 2.22

$$-\frac{693\,b^{5}x^{10}+1155\,ab^{4}x^{8}+1386\,a^{2}b^{3}x^{6}+990\,a^{3}b^{2}x^{4}+385\,a^{4}bx^{2}+63\,a^{5}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [A] time = 0.583955, size = 63, normalized size = 0.97

$$-\frac{63a^{5}+385a^{4}bx^{2}+990a^{3}b^{2}x^{4}+1386a^{2}b^{3}x^{6}+1155ab^{4}x^{8}+693b^{5}x^{10}}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**12,x)
```

```
[Out] -(63*a**5 + 385*a**4*b*x**2 + 990*a**3*b**2*x**4 + 1386*a**2*b**3*x**6 + 11
55*a*b**4*x**8 + 693*b**5*x**10)/(693*x**11)
```

Giac [A] time = 1.33366, size = 80, normalized size = 1.23

$$-\frac{693\,b^{5}x^{10}+1155\,ab^{4}x^{8}+1386\,a^{2}b^{3}x^{6}+990\,a^{3}b^{2}x^{4}+385\,a^{4}bx^{2}+63\,a^{5}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

3.81 $\int \frac{\left(a+bx^2\right)^5}{x^{14}}\,dx$

Optimal. Leaf size=67

$$-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{13x^{13}}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Rubi [A] time = 0.0230642, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{13x^{13}}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^6}+\frac{b^5}{x^4}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0039706, size = 67, normalized size = 1.

$$-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{13x^{13}}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Maple [A] time = 0.005, size = 58, normalized size = 0.9

$$-\frac{a^{5}}{13 x^{13}}-\frac{5 a^{4} b}{11 x^{11}}-\frac{10 a^{3} b^{2}}{9 x^{9}}-\frac{10 a^{2} b^{3}}{7 x^{7}}-\frac{a b^{4}}{x^{5}}-\frac{b^{5}}{3 x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^14,x)
```

```
[Out] -1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-
1/3*b^5/x^3
```

Maxima [A] time = 2.49029, size = 80, normalized size = 1.19

$$-\frac{3003 b^{5} x^{10}+9009 a b^{4} x^{8}+12870 a^{2} b^{3} x^{6}+10010 a^{3} b^{2} x^{4}+4095 a^{4} b x^{2}+693 a^{5}}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Fricas [A] time = 1.32342, size = 154, normalized size = 2.3

$$-\frac{3003 b^{5} x^{10}+9009 a b^{4} x^{8}+12870 a^{2} b^{3} x^{6}+10010 a^{3} b^{2} x^{4}+4095 a^{4} b x^{2}+693 a^{5}}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [A] time = 0.645281, size = 63, normalized size = 0.94

$$-\frac{693a^{5}+4095a^{4}bx^{2}+10010a^{3}b^{2}x^{4}+12870a^{2}b^{3}x^{6}+9009ab^{4}x^{8}+3003b^{5}x^{10}}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**14,x)
```

```
[Out] -(693*a**5 + 4095*a**4*b*x**2 + 10010*a**3*b**2*x**4 + 12870*a**2*b**3*x**6
 + 9009*a*b**4*x**8 + 3003*b**5*x**10)/(9009*x**13)
```

Giac [A] time = 2.81248, size = 80, normalized size = 1.19

$$-\frac{3003 b^{5} x^{10}+9009 a b^{4} x^{8}+12870 a^{2} b^{3} x^{6}+10010 a^{3} b^{2} x^{4}+4095 a^{4} b x^{2}+693 a^{5}}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

3.82 $\int \frac{(a+bx^2)^5}{x^{16}} dx$

Optimal. Leaf size=69

$$-\frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5a^4b}{13x^{13}} - \frac{a^5}{15x^{15}} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

```
[Out] -a^5/(15*x^15) - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Rubi [A] time = 0.0231295, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5a^4b}{13x^{13}} - \frac{a^5}{15x^{15}} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^16,x]
```

```
[Out] -a^5/(15*x^15) - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^5}{x^{16}} dx &= \int \left(\frac{a^5}{x^{16}} + \frac{5a^4b}{x^{14}} + \frac{10a^3b^2}{x^{12}} + \frac{10a^2b^3}{x^{10}} + \frac{5ab^4}{x^8} + \frac{b^5}{x^6}\right) dx \\ &= -\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5} \end{aligned}$$

Mathematica [A] time = 0.0040832, size = 69, normalized size = 1.

$$-\frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5a^4b}{13x^{13}} - \frac{a^5}{15x^{15}} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^16,x]
```

```
[Out] -a^5/(15*x^15) - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Maple [A] time = 0.004, size = 58, normalized size = 0.8

$$-\frac{a^5}{15\,x^{15}}-\frac{5\,a^4b}{13\,x^{13}}-\frac{10\,a^3b^2}{11\,x^{11}}-\frac{10\,a^2b^3}{9\,x^9}-\frac{5\,ab^4}{7\,x^7}-\frac{b^5}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^16,x)
```

```
[Out] -1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b^
4/x^7-1/5*b^5/x^5
```

Maxima [A] time = 2.30062, size = 80, normalized size = 1.16

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Fricas [A] time = 1.23189, size = 159, normalized size = 2.3

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [A] time = 0.729849, size = 63, normalized size = 0.91

$$-\frac{3003a^5+17325a^4bx^2+40950a^3b^2x^4+50050a^2b^3x^6+32175ab^4x^8+9009b^5x^{10}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**16,x)
```

```
[Out] -(3003*a**5 + 17325*a**4*b*x**2 + 40950*a**3*b**2*x**4 + 50050*a**2*b**3*x*
*6 + 32175*a*b**4*x**8 + 9009*b**5*x**10)/(45045*x**15)
```

Giac [A] time = 2.1227, size = 80, normalized size = 1.16

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

3.83 $\int \frac{\left(a+bx^2\right)^5}{x^{18}} \, dx$

Optimal. Leaf size=69

$$-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

```
[Out] -a^5/(17*x^17) - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Rubi [A] time = 0.0219601, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^18,x]
```

```
[Out] -a^5/(17*x^17) - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^{18}} \, dx &= \int \left(\frac{a^5}{x^{18}}+\frac{5a^4b}{x^{16}}+\frac{10a^3b^2}{x^{14}}+\frac{10a^2b^3}{x^{12}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^8}\right) dx\\ &= -\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.003997, size = 69, normalized size = 1.

$$-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^18,x]
```

```
[Out] -a^5/(17*x^17) - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Maple [A] time = 0.007, size = 58, normalized size = 0.8

$$-\frac{a^5}{17\,x^{17}}-\frac{a^4b}{3\,x^{15}}-\frac{10\,a^3b^2}{13\,x^{13}}-\frac{10\,a^2b^3}{11\,x^{11}}-\frac{5\,ab^4}{9\,x^9}-\frac{b^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^18,x)
```

```
[Out] -1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*b
^4/x^9-1/7*b^5/x^7
```

Maxima [A] time = 1.38934, size = 80, normalized size = 1.16

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="maxima")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Fricas [A] time = 1.292, size = 165, normalized size = 2.39

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Sympy [A] time = 0.737782, size = 63, normalized size = 0.91

$$-\frac{9009a^5+51051a^4bx^2+117810a^3b^2x^4+139230a^2b^3x^6+85085ab^4x^8+21879b^5x^{10}}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**18,x)
```

```
[Out] -(9009*a**5 + 51051*a**4*b*x**2 + 117810*a**3*b**2*x**4 + 139230*a**2*b**3*
x**6 + 85085*a*b**4*x**8 + 21879*b**5*x**10)/(153153*x**17)
```

Giac [A] time = 2.47684, size = 80, normalized size = 1.16

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

3.84 $\int \frac{(a+bx^2)^5}{x^{20}}\,dx$

Optimal. Leaf size=69

$$-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

```
[Out] -a^5/(19*x^19) - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Rubi [A] time = 0.0225537, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^20,x]
```

```
[Out] -a^5/(19*x^19) - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{20}}\,dx&=\int \left(\frac{a^5}{x^{20}}+\frac{5a^4b}{x^{18}}+\frac{10a^3b^2}{x^{16}}+\frac{10a^2b^3}{x^{14}}+\frac{5ab^4}{x^{12}}+\frac{b^5}{x^{10}}\right)dx\\&=-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}\end{aligned}$$

Mathematica [A] time = 0.0060701, size = 69, normalized size = 1.

$$-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^20,x]
```

```
[Out] -a^5/(19*x^19) - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Maple [A] time = 0.005, size = 58, normalized size = 0.8

$$-\frac{a^{5}}{19\,x^{19}}-\frac{5\,a^{4}b}{17\,x^{17}}-\frac{2\,a^{3}b^{2}}{3\,x^{15}}-\frac{10\,a^{2}b^{3}}{13\,x^{13}}-\frac{5\,ab^{4}}{11\,x^{11}}-\frac{b^{5}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^20,x)
```

```
[Out] -1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*b
^4/x^11-1/9*b^5/x^9
```

Maxima [A] time = 1.78504, size = 80, normalized size = 1.16

$$-\frac{46189\,b^{5}x^{10}+188955\,ab^{4}x^{8}+319770\,a^{2}b^{3}x^{6}+277134\,a^{3}b^{2}x^{4}+122265\,a^{4}bx^{2}+21879\,a^{5}}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="maxima")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Fricas [A] time = 1.18763, size = 169, normalized size = 2.45

$$-\frac{46189\,b^{5}x^{10}+188955\,ab^{4}x^{8}+319770\,a^{2}b^{3}x^{6}+277134\,a^{3}b^{2}x^{4}+122265\,a^{4}bx^{2}+21879\,a^{5}}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [A] time = 0.740657, size = 63, normalized size = 0.91

$$-\frac{21879a^{5}+122265a^{4}bx^{2}+277134a^{3}b^{2}x^{4}+319770a^{2}b^{3}x^{6}+188955ab^{4}x^{8}+46189b^{5}x^{10}}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**20,x)
```

```
[Out] -(21879*a**5 + 122265*a**4*b*x**2 + 277134*a**3*b**2*x**4 + 319770*a**2*b**
3*x**6 + 188955*a*b**4*x**8 + 46189*b**5*x**10)/(415701*x**19)
```

Giac [A] time = 2.47468, size = 80, normalized size = 1.16

$$-\frac{46189\,b^{5}x^{10}+188955\,ab^{4}x^{8}+319770\,a^{2}b^{3}x^{6}+277134\,a^{3}b^{2}x^{4}+122265\,a^{4}bx^{2}+21879\,a^{5}}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

3.85 $\int x^{13} \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=129

$$\frac{15a^2\left(a+bx^2\right)^{13}}{26b^7}-\frac{5a^3\left(a+bx^2\right)^{12}}{6b^7}+\frac{15a^4\left(a+bx^2\right)^{11}}{22b^7}-\frac{3a^5\left(a+bx^2\right)^{10}}{10b^7}+\frac{a^6\left(a+bx^2\right)^9}{18b^7}+\frac{\left(a+bx^2\right)^{15}}{30b^7}-\frac{3a\left(a+bx^2\right.}{14b^7}$$

```
[Out] (a^6*(a + b*x^2)^9)/(18*b^7) - (3*a^5*(a + b*x^2)^10)/(10*b^7) + (15*a^4*(a
 + b*x^2)^11)/(22*b^7) - (5*a^3*(a + b*x^2)^12)/(6*b^7) + (15*a^2*(a + b*x^
2)^13)/(26*b^7) - (3*a*(a + b*x^2)^14)/(14*b^7) + (a + b*x^2)^15/(30*b^7)
```

Rubi [A] time = 0.210454, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{15a^2\left(a+bx^2\right)^{13}}{26b^7}-\frac{5a^3\left(a+bx^2\right)^{12}}{6b^7}+\frac{15a^4\left(a+bx^2\right)^{11}}{22b^7}-\frac{3a^5\left(a+bx^2\right)^{10}}{10b^7}+\frac{a^6\left(a+bx^2\right)^9}{18b^7}+\frac{\left(a+bx^2\right)^{15}}{30b^7}-\frac{3a\left(a+bx^2\right.}{14b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^6*(a + b*x^2)^9)/(18*b^7) - (3*a^5*(a + b*x^2)^10)/(10*b^7) + (15*a^4*(a
 + b*x^2)^11)/(22*b^7) - (5*a^3*(a + b*x^2)^12)/(6*b^7) + (15*a^2*(a + b*x^
2)^13)/(26*b^7) - (3*a*(a + b*x^2)^14)/(14*b^7) + (a + b*x^2)^15/(30*b^7)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{13}\left(a+bx^2\right)^8 dx &= \frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^6(a+bx)^8}{b^6}-\frac{6a^5(a+bx)^9}{b^6}+\frac{15a^4(a+bx)^{10}}{b^6}-\frac{20a^3(a+bx)^{11}}{b^6}+\frac{15a^2(a+bx)^{12}}{b^6}\right.\right.\\ &= \frac{a^6\left(a+bx^2\right)^9}{18b^7}-\frac{3a^5\left(a+bx^2\right)^{10}}{10b^7}+\frac{15a^4\left(a+bx^2\right)^{11}}{22b^7}-\frac{5a^3\left(a+bx^2\right)^{12}}{6b^7}+\frac{15a^2\left(a+bx^2\right)^{13}}{26b^7}-\frac{3a}{}\end{aligned}$$

Mathematica [A] time = 0.0028276, size = 108, normalized size = 0.84

$$\frac{14}{13}a^2b^6x^{26}+\frac{7}{3}a^3b^5x^{24}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{5}a^5b^3x^{20}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{2}a^7bx^{16}+\frac{a^8x^{14}}{14}+\frac{2}{7}ab^7x^{28}+\frac{b^8x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^14)/14 + (a^7*b*x^16)/2 + (14*a^6*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5
+ (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24)/3 + (14*a^2*b^6*x^26)/13 + (2*a*b
^7*x^28)/7 + (b^8*x^30)/30
```

Maple [A] time = 0.001, size = 91, normalized size = 0.7

$$\frac{b^8x^{30}}{30}+\frac{2\,ab^7x^{28}}{7}+\frac{14\,b^6a^2x^{26}}{13}+\frac{7\,a^3b^5x^{24}}{3}+\frac{35\,a^4b^4x^{22}}{11}+\frac{14\,a^5b^3x^{20}}{5}+\frac{14\,a^6b^2x^{18}}{9}+\frac{a^7bx^{16}}{2}+\frac{a^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^8,x)
```

```
[Out] 1/30*b^8*x^30+2/7*a*b^7*x^28+14/13*b^6*a^2*x^26+7/3*a^3*b^5*x^24+35/11*a^4*
b^4*x^22+14/5*a^5*b^3*x^20+14/9*a^6*b^2*x^18+1/2*a^7*b*x^16+1/14*a^8*x^14
```

Maxima [A] time = 2.46955, size = 122, normalized size = 0.95

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

Fricas [A] time = 1.03541, size = 224, normalized size = 1.74

$$\frac{1}{30}x^{30}b^8+\frac{2}{7}x^{28}b^7a+\frac{14}{13}x^{26}b^6a^2+\frac{7}{3}x^{24}b^5a^3+\frac{35}{11}x^{22}b^4a^4+\frac{14}{5}x^{20}b^3a^5+\frac{14}{9}x^{18}b^2a^6+\frac{1}{2}x^{16}ba^7+\frac{1}{14}x^{14}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/30*x^30*b^8 + 2/7*x^28*b^7*a + 14/13*x^26*b^6*a^2 + 7/3*x^24*b^5*a^3 + 35
/11*x^22*b^4*a^4 + 14/5*x^20*b^3*a^5 + 14/9*x^18*b^2*a^6 + 1/2*x^16*b*a^7 +
 1/14*x^14*a^8
```

Sympy [A] time = 0.082409, size = 105, normalized size = 0.81

$$\frac{a^8x^{14}}{14}+\frac{a^7bx^{16}}{2}+\frac{14a^6b^2x^{18}}{9}+\frac{14a^5b^3x^{20}}{5}+\frac{35a^4b^4x^{22}}{11}+\frac{7a^3b^5x^{24}}{3}+\frac{14a^2b^6x^{26}}{13}+\frac{2ab^7x^{28}}{7}+\frac{b^8x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**14/14 + a**7*b*x**16/2 + 14*a**6*b**2*x**18/9 + 14*a**5*b**3*x**20/
5 + 35*a**4*b**4*x**22/11 + 7*a**3*b**5*x**24/3 + 14*a**2*b**6*x**26/13 + 2
*a*b**7*x**28/7 + b**8*x**30/30
```

Giac [A] time = 2.27867, size = 122, normalized size = 0.95

$$\frac{1}{30}b^8x^{30} + \frac{2}{7}ab^7x^{28} + \frac{14}{13}a^2b^6x^{26} + \frac{7}{3}a^3b^5x^{24} + \frac{35}{11}a^4b^4x^{22} + \frac{14}{5}a^5b^3x^{20} + \frac{14}{9}a^6b^2x^{18} + \frac{1}{2}a^7bx^{16} + \frac{1}{14}a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

3.86 $\int x^{11} \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=110

$$\frac{5a^2\left(a+bx^2\right)^{12}}{12b^6}-\frac{5a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{4b^6}-\frac{a^5\left(a+bx^2\right)^9}{18b^6}+\frac{\left(a+bx^2\right)^{14}}{28b^6}-\frac{5a\left(a+bx^2\right)^{13}}{26b^6}$$

```
[Out] -(a^5*(a + b*x^2)^9)/(18*b^6) + (a^4*(a + b*x^2)^10)/(4*b^6) - (5*a^3*(a +
b*x^2)^11)/(11*b^6) + (5*a^2*(a + b*x^2)^12)/(12*b^6) - (5*a*(a + b*x^2)^13
)/(26*b^6) + (a + b*x^2)^14/(28*b^6)
```

Rubi [A] time = 0.170966, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5a^2\left(a+bx^2\right)^{12}}{12b^6}-\frac{5a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{4b^6}-\frac{a^5\left(a+bx^2\right)^9}{18b^6}+\frac{\left(a+bx^2\right)^{14}}{28b^6}-\frac{5a\left(a+bx^2\right)^{13}}{26b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^8,x]
```

```
[Out] -(a^5*(a + b*x^2)^9)/(18*b^6) + (a^4*(a + b*x^2)^10)/(4*b^6) - (5*a^3*(a +
b*x^2)^11)/(11*b^6) + (5*a^2*(a + b*x^2)^12)/(12*b^6) - (5*a*(a + b*x^2)^13
)/(26*b^6) + (a + b*x^2)^14/(28*b^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^{11}\left(a+bx^2\right)^8\,dx &= \frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^8\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^8}{b^5}+\frac{5a^4(a+bx)^9}{b^5}-\frac{10a^3(a+bx)^{10}}{b^5}+\frac{10a^2(a+bx)^{11}}{b^5}-\frac{5a(a+bx)^{12}}{b^5}+\cdots\right.\right.\\
&= -\frac{a^5\left(a+bx^2\right)^9}{18b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{4b^6}-\frac{5a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{5a^2\left(a+bx^2\right)^{12}}{12b^6}-\frac{5a\left(a+bx^2\right)^{13}}{26b^6}+\frac{\left(a+bx^2\right.}{28b^6}
\end{aligned}$$

Mathematica [A] time = 0.0024858, size = 108, normalized size = 0.98

$$\frac{7}{6}a^2b^6x^{24}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{7}a^7bx^{14}+\frac{a^8x^{12}}{12}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^12)/12 + (4*a^7*b*x^14)/7 + (7*a^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9
 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^22)/11 + (7*a^2*b^6*x^24)/6 + (4*a*b^
7*x^26)/13 + (b^8*x^28)/28
```

Maple [A] time = 0.001, size = 91, normalized size = 0.8

$$\frac{b^8x^{28}}{28}+\frac{4\,ab^7x^{26}}{13}+\frac{7\,b^6a^2x^{24}}{6}+\frac{28\,a^3b^5x^{22}}{11}+\frac{7\,a^4b^4x^{20}}{2}+\frac{28\,a^5b^3x^{18}}{9}+\frac{7\,a^6b^2x^{16}}{4}+\frac{4\,a^7bx^{14}}{7}+\frac{a^8x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^8,x)
```

```
[Out] 1/28*b^8*x^28+4/13*a*b^7*x^26+7/6*b^6*a^2*x^24+28/11*a^3*b^5*x^22+7/2*a^4*b
^4*x^20+28/9*a^5*b^3*x^18+7/4*a^6*b^2*x^16+4/7*a^7*b*x^14+1/12*a^8*x^12
```

Maxima [A] time = 1.3813, size = 122, normalized size = 1.11

$$\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

Fricas [A] time = 1.1226, size = 221, normalized size = 2.01

$$\frac{1}{28}x^{28}b^8+\frac{4}{13}x^{26}b^7a+\frac{7}{6}x^{24}b^6a^2+\frac{28}{11}x^{22}b^5a^3+\frac{7}{2}x^{20}b^4a^4+\frac{28}{9}x^{18}b^3a^5+\frac{7}{4}x^{16}b^2a^6+\frac{4}{7}x^{14}ba^7+\frac{1}{12}x^{12}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^8 + 4/13*x^26*b^7*a + 7/6*x^24*b^6*a^2 + 28/11*x^22*b^5*a^3 + 7
/2*x^20*b^4*a^4 + 28/9*x^18*b^3*a^5 + 7/4*x^16*b^2*a^6 + 4/7*x^14*b*a^7 + 1
/12*x^12*a^8
```

Sympy [A] time = 0.082945, size = 107, normalized size = 0.97

$$\frac{a^8x^{12}}{12}+\frac{4a^7bx^{14}}{7}+\frac{7a^6b^2x^{16}}{4}+\frac{28a^5b^3x^{18}}{9}+\frac{7a^4b^4x^{20}}{2}+\frac{28a^3b^5x^{22}}{11}+\frac{7a^2b^6x^{24}}{6}+\frac{4ab^7x^{26}}{13}+\frac{b^8x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**12/12 + 4*a**7*b*x**14/7 + 7*a**6*b**2*x**16/4 + 28*a**5*b**3*x**18
/9 + 7*a**4*b**4*x**20/2 + 28*a**3*b**5*x**22/11 + 7*a**2*b**6*x**24/6 + 4*
a*b**7*x**26/13 + b**8*x**28/28
```

Giac [A] time = 1.4117, size = 122, normalized size = 1.11

$$\frac{1}{28}\,b^{8}x^{28}+\frac{4}{13}\,ab^{7}x^{26}+\frac{7}{6}\,a^{2}b^{6}x^{24}+\frac{28}{11}\,a^{3}b^{5}x^{22}+\frac{7}{2}\,a^{4}b^{4}x^{20}+\frac{28}{9}\,a^{5}b^{3}x^{18}+\frac{7}{4}\,a^{6}b^{2}x^{16}+\frac{4}{7}\,a^{7}bx^{14}+\frac{1}{12}\,a^{8}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

3.87 $\int x^9 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=91

$$\frac{3a^2\left(a+bx^2\right)^{11}}{11b^5}-\frac{a^3\left(a+bx^2\right)^{10}}{5b^5}+\frac{a^4\left(a+bx^2\right)^9}{18b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}-\frac{a\left(a+bx^2\right)^{12}}{6b^5}$$

```
[Out] (a^4*(a + b*x^2)^9)/(18*b^5) - (a^3*(a + b*x^2)^10)/(5*b^5) + (3*a^2*(a + b
*x^2)^11)/(11*b^5) - (a*(a + b*x^2)^12)/(6*b^5) + (a + b*x^2)^13/(26*b^5)
```

Rubi [A] time = 0.141735, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{11}}{11b^5}-\frac{a^3\left(a+bx^2\right)^{10}}{5b^5}+\frac{a^4\left(a+bx^2\right)^9}{18b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}-\frac{a\left(a+bx^2\right)^{12}}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^4*(a + b*x^2)^9)/(18*b^5) - (a^3*(a + b*x^2)^10)/(5*b^5) + (3*a^2*(a + b
*x^2)^11)/(11*b^5) - (a*(a + b*x^2)^12)/(6*b^5) + (a + b*x^2)^13/(26*b^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^8 dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^4(a+bx)^8}{b^4}-\frac{4a^3(a+bx)^9}{b^4}+\frac{6a^2(a+bx)^{10}}{b^4}-\frac{4a(a+bx)^{11}}{b^4}+\frac{(a+bx)^{12}}{b^4}\right)dx,x,x\right.\\ &= \frac{a^4\left(a+bx^2\right)^9}{18b^5}-\frac{a^3\left(a+bx^2\right)^{10}}{5b^5}+\frac{3a^2\left(a+bx^2\right)^{11}}{11b^5}-\frac{a\left(a+bx^2\right)^{12}}{6b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}\end{aligned}$$

Mathematica [A] time = 0.0024927, size = 106, normalized size = 1.16

$$\frac{14}{11}a^2b^6x^{22}+\frac{14}{5}a^3b^5x^{20}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{2}a^5b^3x^{16}+2a^6b^2x^{14}+\frac{2}{3}a^7bx^{12}+\frac{a^8x^{10}}{10}+\frac{1}{3}ab^7x^{24}+\frac{b^8x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^10)/10 + (2*a^7*b*x^12)/3 + 2*a^6*b^2*x^14 + (7*a^5*b^3*x^16)/2 + (3
5*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 + (14*a^2*b^6*x^22)/11 + (a*b^7*x^2
4)/3 + (b^8*x^26)/26
```

Maple [A] time = 0., size = 91, normalized size = 1.

$$\frac{b^8x^{26}}{26}+\frac{ab^7x^{24}}{3}+\frac{14\,a^2b^6x^{22}}{11}+\frac{14\,a^3b^5x^{20}}{5}+\frac{35\,a^4b^4x^{18}}{9}+\frac{7\,a^5b^3x^{16}}{2}+2\,a^6b^2x^{14}+\frac{2\,a^7bx^{12}}{3}+\frac{a^8x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^8,x)
```

```
[Out] 1/26*b^8*x^26+1/3*a*b^7*x^24+14/11*a^2*b^6*x^22+14/5*a^3*b^5*x^20+35/9*a^4*
b^4*x^18+7/2*a^5*b^3*x^16+2*a^6*b^2*x^14+2/3*a^7*b*x^12+1/10*a^8*x^10
```

Maxima [A] time = 2.0037, size = 122, normalized size = 1.34

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

Fricas [A] time = 1.17211, size = 219, normalized size = 2.41

$$\frac{1}{26}x^{26}b^8+\frac{1}{3}x^{24}b^7a+\frac{14}{11}x^{22}b^6a^2+\frac{14}{5}x^{20}b^5a^3+\frac{35}{9}x^{18}b^4a^4+\frac{7}{2}x^{16}b^3a^5+2x^{14}b^2a^6+\frac{2}{3}x^{12}ba^7+\frac{1}{10}x^{10}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^8 + 1/3*x^24*b^7*a + 14/11*x^22*b^6*a^2 + 14/5*x^20*b^5*a^3 + 3
5/9*x^18*b^4*a^4 + 7/2*x^16*b^3*a^5 + 2*x^14*b^2*a^6 + 2/3*x^12*b*a^7 + 1/1
0*x^10*a^8
```

Sympy [A] time = 0.082466, size = 104, normalized size = 1.14

$$\frac{a^8x^{10}}{10}+\frac{2a^7bx^{12}}{3}+2a^6b^2x^{14}+\frac{7a^5b^3x^{16}}{2}+\frac{35a^4b^4x^{18}}{9}+\frac{14a^3b^5x^{20}}{5}+\frac{14a^2b^6x^{22}}{11}+\frac{ab^7x^{24}}{3}+\frac{b^8x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**10/10 + 2*a**7*b*x**12/3 + 2*a**6*b**2*x**14 + 7*a**5*b**3*x**16/2
+ 35*a**4*b**4*x**18/9 + 14*a**3*b**5*x**20/5 + 14*a**2*b**6*x**22/11 + a*b
**7*x**24/3 + b**8*x**26/26
```

Giac [A] time = 1.99636, size = 122, normalized size = 1.34

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

3.88 $\int x^7 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=72

$$\frac{3a^2\left(a+bx^2\right)^{10}}{20b^4}-\frac{a^3\left(a+bx^2\right)^9}{18b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}-\frac{3a\left(a+bx^2\right)^{11}}{22b^4}$$

```
[Out] -(a^3*(a + b*x^2)^9)/(18*b^4) + (3*a^2*(a + b*x^2)^10)/(20*b^4) - (3*a*(a +
 b*x^2)^11)/(22*b^4) + (a + b*x^2)^12/(24*b^4)
```

Rubi [A] time = 0.115799, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{10}}{20b^4}-\frac{a^3\left(a+bx^2\right)^9}{18b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}-\frac{3a\left(a+bx^2\right)^{11}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^8,x]
```

```
[Out] -(a^3*(a + b*x^2)^9)/(18*b^4) + (3*a^2*(a + b*x^2)^10)/(20*b^4) - (3*a*(a +
 b*x^2)^11)/(22*b^4) + (a + b*x^2)^12/(24*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+bx^2\right)^8\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^3(a+bx)^8\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^8}{b^3}+\frac{3a^2(a+bx)^9}{b^3}-\frac{3a(a+bx)^{10}}{b^3}+\frac{(a+bx)^{11}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^9}{18b^4}+\frac{3a^2\left(a+bx^2\right)^{10}}{20b^4}-\frac{3a\left(a+bx^2\right)^{11}}{22b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}
\end{aligned}$$

Mathematica [A] time = 0.0028108, size = 106, normalized size = 1.47

$$\frac{7}{5}a^2b^6x^{20}+\frac{28}{9}a^3b^5x^{18}+\frac{35}{8}a^4b^4x^{16}+4a^5b^3x^{14}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7bx^{10}+\frac{a^8x^8}{8}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^8)/8 + (4*a^7*b*x^10)/5 + (7*a^6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*
a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 + (7*a^2*b^6*x^20)/5 + (4*a*b^7*x^22)
/11 + (b^8*x^24)/24
```

Maple [A] time = 0.002, size = 91, normalized size = 1.3

$$\frac{b^8x^{24}}{24}+\frac{4\,ab^7x^{22}}{11}+\frac{7\,a^2b^6x^{20}}{5}+\frac{28\,a^3b^5x^{18}}{9}+\frac{35\,a^4b^4x^{16}}{8}+4\,a^5b^3x^{14}+\frac{7\,a^6b^2x^{12}}{3}+\frac{4\,a^7bx^{10}}{5}+\frac{a^8x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^8,x)
```

```
[Out] 1/24*b^8*x^24+4/11*a*b^7*x^22+7/5*a^2*b^6*x^20+28/9*a^3*b^5*x^18+35/8*a^4*b
^4*x^16+4*a^5*b^3*x^14+7/3*a^6*b^2*x^12+4/5*a^7*b*x^10+1/8*a^8*x^8
```

Maxima [A] time = 1.59132, size = 122, normalized size = 1.69

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

Fricas [A] time = 1.37431, size = 215, normalized size = 2.99

$$\frac{1}{24}x^{24}b^8+\frac{4}{11}x^{22}b^7a+\frac{7}{5}x^{20}b^6a^2+\frac{28}{9}x^{18}b^5a^3+\frac{35}{8}x^{16}b^4a^4+4x^{14}b^3a^5+\frac{7}{3}x^{12}b^2a^6+\frac{4}{5}x^{10}ba^7+\frac{1}{8}x^8a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^8 + 4/11*x^22*b^7*a + 7/5*x^20*b^6*a^2 + 28/9*x^18*b^5*a^3 + 35
/8*x^16*b^4*a^4 + 4*x^14*b^3*a^5 + 7/3*x^12*b^2*a^6 + 4/5*x^10*b*a^7 + 1/8*
x^8*a^8
```

Sympy [A] time = 0.082515, size = 105, normalized size = 1.46

$$\frac{a^8x^8}{8}+\frac{4a^7bx^{10}}{5}+\frac{7a^6b^2x^{12}}{3}+4a^5b^3x^{14}+\frac{35a^4b^4x^{16}}{8}+\frac{28a^3b^5x^{18}}{9}+\frac{7a^2b^6x^{20}}{5}+\frac{4ab^7x^{22}}{11}+\frac{b^8x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**8/8 + 4*a**7*b*x**10/5 + 7*a**6*b**2*x**12/3 + 4*a**5*b**3*x**14 +
35*a**4*b**4*x**16/8 + 28*a**3*b**5*x**18/9 + 7*a**2*b**6*x**20/5 + 4*a*b**
7*x**22/11 + b**8*x**24/24
```

Giac [A] time = 2.10331, size = 122, normalized size = 1.69

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

3.89 $\int x^5 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^9}{18b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}$$

```
[Out] (a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^10)/(10*b^3) + (a + b*x^2)^11
/(22*b^3)
```

Rubi [A] time = 0.0839906, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^9}{18b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^10)/(10*b^3) + (a + b*x^2)^11
/(22*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^8 dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^8}{b^2}-\frac{2a(a+bx)^9}{b^2}+\frac{(a+bx)^{10}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a^2\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}\end{aligned}$$

Mathematica [A] time = 0.0024927, size = 103, normalized size = 1.94

$$\frac{14}{9}a^2b^6x^{18}+\frac{7}{2}a^3b^5x^{16}+5a^4b^4x^{14}+\frac{14}{3}a^5b^3x^{12}+\frac{14}{5}a^6b^2x^{10}+a^7bx^8+\frac{a^8x^6}{6}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^6)/6 + a^7*b*x^8 + (14*a^6*b^2*x^10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4
*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2*b^6*x^18)/9 + (2*a*b^7*x^20)/5 + (
b^8*x^22)/22
```

Maple [A] time = 0.001, size = 90, normalized size = 1.7

$$\frac{b^8x^{22}}{22}+\frac{2\,ab^7x^{20}}{5}+\frac{14\,a^2b^6x^{18}}{9}+\frac{7\,a^3b^5x^{16}}{2}+5\,a^4b^4x^{14}+\frac{14\,a^5b^3x^{12}}{3}+\frac{14\,a^6b^2x^{10}}{5}+a^7bx^8+\frac{a^8x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^8,x)
```

```
[Out] 1/22*b^8*x^22+2/5*a*b^7*x^20+14/9*a^2*b^6*x^18+7/2*a^3*b^5*x^16+5*a^4*b^4*x
^14+14/3*a^5*b^3*x^12+14/5*a^6*b^2*x^10+a^7*b*x^8+1/6*a^8*x^6
```

Maxima [A] time = 1.3005, size = 120, normalized size = 2.26

$$\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

Fricas [A] time = 1.41869, size = 208, normalized size = 3.92

$$\frac{1}{22}x^{22}b^8+\frac{2}{5}x^{20}b^7a+\frac{14}{9}x^{18}b^6a^2+\frac{7}{2}x^{16}b^5a^3+5x^{14}b^4a^4+\frac{14}{3}x^{12}b^3a^5+\frac{14}{5}x^{10}b^2a^6+x^8ba^7+\frac{1}{6}x^6a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^8 + 2/5*x^20*b^7*a + 14/9*x^18*b^6*a^2 + 7/2*x^16*b^5*a^3 + 5*x
^14*b^4*a^4 + 14/3*x^12*b^3*a^5 + 14/5*x^10*b^2*a^6 + x^8*b*a^7 + 1/6*x^6*a
^8
```

Sympy [B] time = 0.083992, size = 102, normalized size = 1.92

$$\frac{a^8x^6}{6}+a^7bx^8+\frac{14a^6b^2x^{10}}{5}+\frac{14a^5b^3x^{12}}{3}+5a^4b^4x^{14}+\frac{7a^3b^5x^{16}}{2}+\frac{14a^2b^6x^{18}}{9}+\frac{2ab^7x^{20}}{5}+\frac{b^8x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**6/6 + a**7*b*x**8 + 14*a**6*b**2*x**10/5 + 14*a**5*b**3*x**12/3 + 5
*a**4*b**4*x**14 + 7*a**3*b**5*x**16/2 + 14*a**2*b**6*x**18/9 + 2*a*b**7*x*
*20/5 + b**8*x**22/22
```

Giac [A] time = 2.43434, size = 120, normalized size = 2.26

$$\frac{1}{22}\,b^8x^{22} + \frac{2}{5}\,ab^7x^{20} + \frac{14}{9}\,a^2b^6x^{18} + \frac{7}{2}\,a^3b^5x^{16} + 5\,a^4b^4x^{14} + \frac{14}{3}\,a^5b^3x^{12} + \frac{14}{5}\,a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

3.90 $\int x^3 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^{10}}{20b^2}-\frac{a\left(a+bx^2\right)^9}{18b^2}$$

```
[Out] -(a*(a + b*x^2)^9)/(18*b^2) + (a + b*x^2)^10/(20*b^2)
```

Rubi [A] time = 0.0456613, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{10}}{20b^2}-\frac{a\left(a+bx^2\right)^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^8,x]
```

```
[Out] -(a*(a + b*x^2)^9)/(18*b^2) + (a + b*x^2)^10/(20*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^8 \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^8 \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^8}{b}+\frac{(a+bx)^9}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^9}{18b^2}+\frac{\left(a+bx^2\right)^{10}}{20b^2}
\end{aligned}$$

Mathematica [B] time = 0.0024039, size = 106, normalized size = 3.12

$$\frac{7}{4}a^2b^6x^{16}+4a^3b^5x^{14}+\frac{35}{6}a^4b^4x^{12}+\frac{28}{5}a^5b^3x^{10}+\frac{7}{2}a^6b^2x^8+\frac{4}{3}a^7bx^6+\frac{a^8x^4}{4}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^4)/4 + (4*a^7*b*x^6)/3 + (7*a^6*b^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (
35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7*a^2*b^6*x^16)/4 + (4*a*b^7*x^18)/9
 + (b^8*x^20)/20
```

Maple [B] time = 0.002, size = 91, normalized size = 2.7

$$\frac{b^{8}x^{20}}{20}+\frac{4\,a b^{7}x^{18}}{9}+\frac{7\,a^{2}b^{6}x^{16}}{4}+4\,a^{3}b^{5}x^{14}+\frac{35\,a^{4}b^{4}x^{12}}{6}+\frac{28\,a^{5}b^{3}x^{10}}{5}+\frac{7\,a^{6}b^{2}x^{8}}{2}+\frac{4\,a^{7}b x^{6}}{3}+\frac{a^{8}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^8,x)
```

```
[Out] 1/20*b^8*x^20+4/9*a*b^7*x^18+7/4*a^2*b^6*x^16+4*a^3*b^5*x^14+35/6*a^4*b^4*x
^12+28/5*a^5*b^3*x^10+7/2*a^6*b^2*x^8+4/3*a^7*b*x^6+1/4*a^8*x^4
```

Maxima [B] time = 1.82346, size = 122, normalized size = 3.59

$$\frac{1}{20}\,b^{8}x^{20}+\frac{4}{9}\,a b^{7}x^{18}+\frac{7}{4}\,a^{2}b^{6}x^{16}+4\,a^{3}b^{5}x^{14}+\frac{35}{6}\,a^{4}b^{4}x^{12}+\frac{28}{5}\,a^{5}b^{3}x^{10}+\frac{7}{2}\,a^{6}b^{2}x^{8}+\frac{4}{3}\,a^{7}b x^{6}+\frac{1}{4}\,a^{8}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

Fricas [B] time = 1.3071, size = 211, normalized size = 6.21

$$\frac{1}{20}x^{20}b^{8}+\frac{4}{9}x^{18}b^{7}a+\frac{7}{4}x^{16}b^{6}a^{2}+4x^{14}b^{5}a^{3}+\frac{35}{6}x^{12}b^{4}a^{4}+\frac{28}{5}x^{10}b^{3}a^{5}+\frac{7}{2}x^{8}b^{2}a^{6}+\frac{4}{3}x^{6}ba^{7}+\frac{1}{4}x^{4}a^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^8 + 4/9*x^18*b^7*a + 7/4*x^16*b^6*a^2 + 4*x^14*b^5*a^3 + 35/6*x
^12*b^4*a^4 + 28/5*x^10*b^3*a^5 + 7/2*x^8*b^2*a^6 + 4/3*x^6*b*a^7 + 1/4*x^4
*a^8
```

Sympy [B] time = 0.082124, size = 105, normalized size = 3.09

$$\frac{a^{8}x^{4}}{4}+\frac{4a^{7}bx^{6}}{3}+\frac{7a^{6}b^{2}x^{8}}{2}+\frac{28a^{5}b^{3}x^{10}}{5}+\frac{35a^{4}b^{4}x^{12}}{6}+4a^{3}b^{5}x^{14}+\frac{7a^{2}b^{6}x^{16}}{4}+\frac{4ab^{7}x^{18}}{9}+\frac{b^{8}x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**4/4 + 4*a**7*b*x**6/3 + 7*a**6*b**2*x**8/2 + 28*a**5*b**3*x**10/5 +
 35*a**4*b**4*x**12/6 + 4*a**3*b**5*x**14 + 7*a**2*b**6*x**16/4 + 4*a*b**7*
x**18/9 + b**8*x**20/20
```

Giac [B] time = 2.12975, size = 122, normalized size = 3.59

$$\frac{1}{20}\,b^8x^{20} + \frac{4}{9}\,ab^7x^{18} + \frac{7}{4}\,a^2b^6x^{16} + 4\,a^3b^5x^{14} + \frac{35}{6}\,a^4b^4x^{12} + \frac{28}{5}\,a^5b^3x^{10} + \frac{7}{2}\,a^6b^2x^8 + \frac{4}{3}\,a^7bx^6 + \frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

3.91 $\int x\left(a+bx^2\right)^8\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^9}{18b}$$

```
[Out] (a + b*x^2)^9/(18*b)
```

Rubi [A] time = 0.0023727, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^8,x]
```

```
[Out] (a + b*x^2)^9/(18*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^8\,dx=\frac{\left(a+bx^2\right)^9}{18b}$$

Mathematica [A] time = 0.0018185, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^8,x]
```

```
[Out] (a + b*x^2)^9/(18*b)
```

Maple [B] time = 0., size = 91, normalized size = 5.7

$$\frac{b^8x^{18}}{18}+\frac{ab^7x^{16}}{2}+2\,a^2b^6x^{14}+\frac{14\,a^3b^5x^{12}}{3}+7\,a^4b^4x^{10}+7\,a^5b^3x^8+\frac{14\,a^6b^2x^6}{3}+2\,a^7bx^4+\frac{a^8x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^8,x)
```

```
[Out] 1/18*b^8*x^18+1/2*a*b^7*x^16+2*a^2*b^6*x^14+14/3*a^3*b^5*x^12+7*a^4*b^4*x^1
0+7*a^5*b^3*x^8+14/3*a^6*b^2*x^6+2*a^7*b*x^4+1/2*a^8*x^2
```

Maxima [A] time = 2.53358, size = 19, normalized size = 1.19

$$\frac{\left(bx^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

Fricas [B] time = 1.32205, size = 201, normalized size = 12.56

$$\frac{1}{18}x^{18}b^8+\frac{1}{2}x^{16}b^7a+2x^{14}b^6a^2+\frac{14}{3}x^{12}b^5a^3+7x^{10}b^4a^4+7x^8b^3a^5+\frac{14}{3}x^6b^2a^6+2x^4ba^7+\frac{1}{2}x^2a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^8 + 1/2*x^16*b^7*a + 2*x^14*b^6*a^2 + 14/3*x^12*b^5*a^3 + 7*x^1
0*b^4*a^4 + 7*x^8*b^3*a^5 + 14/3*x^6*b^2*a^6 + 2*x^4*b*a^7 + 1/2*x^2*a^8
```

Sympy [B] time = 0.078476, size = 99, normalized size = 6.19

$$\frac{a^8x^2}{2}+2a^7bx^4+\frac{14a^6b^2x^6}{3}+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14a^3b^5x^{12}}{3}+2a^2b^6x^{14}+\frac{ab^7x^{16}}{2}+\frac{b^8x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**2/2 + 2*a**7*b*x**4 + 14*a**6*b**2*x**6/3 + 7*a**5*b**3*x**8 + 7*a*
*4*b**4*x**10 + 14*a**3*b**5*x**12/3 + 2*a**2*b**6*x**14 + a*b**7*x**16/2 +
 b**8*x**18/18
```

Giac [A] time = 1.67879, size = 19, normalized size = 1.19

$$\frac{\left(bx^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

3.92 $\int \frac{\left(a+bx^2\right)^8}{x}\,dx$

Optimal. Leaf size=100

$$\frac{7}{3}a^2b^6x^{12}+\frac{28}{5}a^3b^5x^{10}+\frac{35}{4}a^4b^4x^8+\frac{28}{3}a^5b^3x^6+7a^6b^2x^4+4a^7bx^2+a^8\log(x)+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Rubi [A] time = 0.058515, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{3}a^2b^6x^{12}+\frac{28}{5}a^3b^5x^{10}+\frac{35}{4}a^4b^4x^8+\frac{28}{3}a^5b^3x^6+7a^6b^2x^4+4a^7bx^2+a^8\log(x)+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(8a^7b+\frac{a^8}{x}+28a^6b^2x+56a^5b^3x^2+70a^4b^4x^3+56a^3b^5x^4+28a^2b^6x^5+8ab^7x^6+b^8\right.\right.\\ &= 4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\log(\end{aligned}$$

Mathematica [A] time = 0.0041288, size = 100, normalized size = 1.

$$\frac{7}{3}a^2b^6x^{12}+\frac{28}{5}a^3b^5x^{10}+\frac{35}{4}a^4b^4x^8+\frac{28}{3}a^5b^3x^6+7a^6b^2x^4+4a^7bx^2+a^8\log(x)+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Maple [A] time = 0.003, size = 89, normalized size = 0.9

$$4\,a^{7}bx^{2}+7\,a^{6}b^{2}x^{4}+\frac{28\,a^{5}b^{3}x^{6}}{3}+\frac{35\,a^{4}b^{4}x^{8}}{4}+\frac{28\,a^{3}b^{5}x^{10}}{5}+\frac{7\,a^{2}b^{6}x^{12}}{3}+\frac{4\,ab^{7}x^{14}}{7}+\frac{b^{8}x^{16}}{16}+a^{8}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x,x)
```

```
[Out] 4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^
10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

Maxima [A] time = 2.70948, size = 123, normalized size = 1.23

$$\frac{1}{16}\,b^{8}x^{16}+\frac{4}{7}\,ab^{7}x^{14}+\frac{7}{3}\,a^{2}b^{6}x^{12}+\frac{28}{5}\,a^{3}b^{5}x^{10}+\frac{35}{4}\,a^{4}b^{4}x^{8}+\frac{28}{3}\,a^{5}b^{3}x^{6}+7\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+\frac{1}{2}\,a^{8}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

Fricas [A] time = 1.48633, size = 205, normalized size = 2.05

$$\frac{1}{16}\,b^{8}x^{16}+\frac{4}{7}\,ab^{7}x^{14}+\frac{7}{3}\,a^{2}b^{6}x^{12}+\frac{28}{5}\,a^{3}b^{5}x^{10}+\frac{35}{4}\,a^{4}b^{4}x^{8}+\frac{28}{3}\,a^{5}b^{3}x^{6}+7\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+a^{8}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="fricas")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8*log(x)
```

Sympy [A] time = 0.324676, size = 102, normalized size = 1.02

$$a^{8}\log\left(x\right)+4a^{7}bx^{2}+7a^{6}b^{2}x^{4}+\frac{28a^{5}b^{3}x^{6}}{3}+\frac{35a^{4}b^{4}x^{8}}{4}+\frac{28a^{3}b^{5}x^{10}}{5}+\frac{7a^{2}b^{6}x^{12}}{3}+\frac{4ab^{7}x^{14}}{7}+\frac{b^{8}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x,x)
```

```
[Out] a**8*log(x) + 4*a**7*b*x**2 + 7*a**6*b**2*x**4 + 28*a**5*b**3*x**6/3 + 35*a
**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x**
14/7 + b**8*x**16/16
```

Giac [A] time = 2.4002, size = 123, normalized size = 1.23

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+\frac{1}{2}\,a^8\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="giac")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

3.93 $\int \frac{(a+bx^2)^8}{x^3}\,dx$

Optimal. Leaf size=99

$$\frac{14}{5}a^2b^6x^{10}+7a^3b^5x^8+\frac{35}{3}a^4b^4x^6+14a^5b^3x^4+14a^6b^2x^2+8a^7b\log(x)-\frac{a^8}{2x^2}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

```
[Out] -a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Rubi [A] time = 0.0599015, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{14}{5}a^2b^6x^{10}+7a^3b^5x^8+\frac{35}{3}a^4b^4x^6+14a^5b^3x^4+14a^6b^2x^2+8a^7b\log(x)-\frac{a^8}{2x^2}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^3,x]
```

```
[Out] -a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(28a^6b^2+\frac{a^8}{x^2}+\frac{8a^7b}{x}+56a^5b^3x+70a^4b^4x^2+56a^3b^5x^3+28a^2b^6x^4+8ab^7x^5+b^8x^6\right) d\\ &= -\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}+8a^7b\log(x)\end{aligned}$$

Mathematica [A] time = 0.0044414, size = 99, normalized size = 1.

$$\frac{14}{5}a^2b^6x^{10}+7a^3b^5x^8+\frac{35}{3}a^4b^4x^6+14a^5b^3x^4+14a^6b^2x^2+8a^7b\log(x)-\frac{a^8}{2x^2}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^3,x]
```

```
[Out] -a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Maple [A] time = 0.007, size = 90, normalized size = 0.9

$$-\frac{a^8}{2\,x^2}+14\,a^6b^2x^2+14\,a^5b^3x^4+\frac{35\,a^4b^4x^6}{3}+7\,a^3b^5x^8+\frac{14\,a^2b^6x^{10}}{5}+\frac{2\,ab^7x^{12}}{3}+\frac{b^8x^{14}}{14}+8\,a^7b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^3,x)
```

```
[Out] -1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+1
4/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

Maxima [A] time = 1.75426, size = 123, normalized size = 1.24

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*a^8/x
^2
```

Fricas [A] time = 1.50534, size = 232, normalized size = 2.34

$$\frac{15\,b^8x^{16}+140\,ab^7x^{14}+588\,a^2b^6x^{12}+1470\,a^3b^5x^{10}+2450\,a^4b^4x^8+2940\,a^5b^3x^6+2940\,a^6b^2x^4+1680\,a^7bx^2\log(x)}{210\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="fricas")
```

```
[Out] 1/210*(15*b^8*x^16 + 140*a*b^7*x^14 + 588*a^2*b^6*x^12 + 1470*a^3*b^5*x^10
+ 2450*a^4*b^4*x^8 + 2940*a^5*b^3*x^6 + 2940*a^6*b^2*x^4 + 1680*a^7*b*x^2*l
og(x) - 105*a^8)/x^2
```

Sympy [A] time = 0.371833, size = 100, normalized size = 1.01

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**3,x)
```

```
[Out] -a**8/(2*x**2) + 8*a**7*b*log(x) + 14*a**6*b**2*x**2 + 14*a**5*b**3*x**4 +
35*a**4*b**4*x**6/3 + 7*a**3*b**5*x**8 + 14*a**2*b**6*x**10/5 + 2*a*b**7*x*
*12/3 + b**8*x**14/14
```

Giac [A] time = 2.84298, size = 136, normalized size = 1.37

$$\frac{1}{14}b^8x^{14} + \frac{2}{3}ab^7x^{12} + \frac{14}{5}a^2b^6x^{10} + 7a^3b^5x^8 + \frac{35}{3}a^4b^4x^6 + 14a^5b^3x^4 + 14a^6b^2x^2 + 4a^7b\log\left(x^2\right) - \frac{8a^7bx^2 + a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="giac")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*(8*a^
7*b*x^2 + a^8)/x^2
```

3.94 $\int \frac{(a+bx^2)^8}{x^5} dx$

Optimal. Leaf size=101

$$\frac{7}{2}a^2b^6x^8+\frac{28}{3}a^3b^5x^6+\frac{35}{2}a^4b^4x^4+28a^5b^3x^2+28a^6b^2\log(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

```
[Out] -a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Rubi [A] time = 0.061354, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{2}a^2b^6x^8+\frac{28}{3}a^3b^5x^6+\frac{35}{2}a^4b^4x^4+28a^5b^3x^2+28a^6b^2\log(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^5,x]
```

```
[Out] -a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^5}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^3}dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(56a^5b^3+\frac{a^8}{x^3}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x}+70a^4b^4x+56a^3b^5x^2+28a^2b^6x^3+8ab^7x^4+b^8x^5\right)\right.\\ &= -\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}+28a^6b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0044516, size = 101, normalized size = 1.

$$\frac{7}{2}a^2b^6x^8+\frac{28}{3}a^3b^5x^6+\frac{35}{2}a^4b^4x^4+28a^5b^3x^2+28a^6b^2\log(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^5,x]
```

```
[Out] -a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Maple [A] time = 0.005, size = 90, normalized size = 0.9

$$-\frac{a^8}{4\,x^4}-4\,\frac{a^7b}{x^2}+28\,a^5b^3x^2+\frac{35\,a^4b^4x^4}{2}+\frac{28\,a^3b^5x^6}{3}+\frac{7\,a^2b^6x^8}{2}+\frac{4\,ab^7x^{10}}{5}+\frac{b^8x^{12}}{12}+28\,a^6b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^5,x)
```

```
[Out] -1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7
/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

Maxima [A] time = 1.94606, size = 124, normalized size = 1.23

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(16*a^7*b*x^2 + a^
8)/x^4
```

Fricas [A] time = 1.26282, size = 224, normalized size = 2.22

$$\frac{5\,b^8x^{16}+48\,ab^7x^{14}+210\,a^2b^6x^{12}+560\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6+1680\,a^6b^2x^4\log(x)-240\,a^7bx^2-15\,a^8}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(5*b^8*x^16 + 48*a*b^7*x^14 + 210*a^2*b^6*x^12 + 560*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6 + 1680*a^6*b^2*x^4*log(x) - 240*a^7*b*x^2
 - 15*a^8)/x^4
```

Sympy [A] time = 0.424023, size = 102, normalized size = 1.01

$$28a^6b^2\log(x)+28a^5b^3x^2+\frac{35a^4b^4x^4}{2}+\frac{28a^3b^5x^6}{3}+\frac{7a^2b^6x^8}{2}+\frac{4ab^7x^{10}}{5}+\frac{b^8x^{12}}{12}-\frac{a^8+16a^7bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**5,x)
```

```
[Out] 28*a**6*b**2*log(x) + 28*a**5*b**3*x**2 + 35*a**4*b**4*x**4/2 + 28*a**3*b**
5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12 - (a**8 +
16*a**7*b*x**2)/(4*x**4)
```

Giac [A] time = 1.89315, size = 139, normalized size = 1.38

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{84\,a^6b^2x^4+16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="giac")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(84*a^6*b^2*x^4 +
16*a^7*b*x^2 + a^8)/x^4
```

3.95 $\int \frac{(a+bx^2)^8}{x^7} dx$

Optimal. Leaf size=94

$$\frac{14}{3}a^2b^6x^6+14a^3b^5x^4+35a^4b^4x^2-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}+ab^7x^8+\frac{b^8x^{10}}{10}$$

```
[Out] -a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Rubi [A] time = 0.0570915, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{14}{3}a^2b^6x^6+14a^3b^5x^4+35a^4b^4x^2-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^7,x]
```

```
[Out] -a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(70a^4b^4+\frac{a^8}{x^4}+\frac{8a^7b}{x^3}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x}+56a^3b^5x+28a^2b^6x^2+8ab^7x^3+b^8x^4\right)dx,x\right.\\
&= -\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0046497, size = 94, normalized size = 1.

$$\frac{14}{3}a^2b^6x^6+14a^3b^5x^4+35a^4b^4x^2-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^7,x]
```

```
[Out] -a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Maple [A] time = 0.006, size = 89, normalized size = 1.

$$-\frac{a^{8}}{6\,x^{6}}-2\,\frac{a^{7}b}{x^{4}}-14\,\frac{a^{6}b^{2}}{x^{2}}+35\,a^{4}b^{4}x^{2}+14\,a^{3}b^{5}x^{4}+\frac{14\,a^{2}b^{6}x^{6}}{3}+ab^{7}x^{8}+\frac{b^{8}x^{10}}{10}+56\,a^{5}b^{3}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^7,x)
```

```
[Out] -1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*
a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

Maxima [A] time = 1.49857, size = 123, normalized size = 1.31

$$\frac{1}{10}\,b^{8}x^{10}+ab^{7}x^{8}+\frac{14}{3}\,a^{2}b^{6}x^{6}+14\,a^{3}b^{5}x^{4}+35\,a^{4}b^{4}x^{2}+28\,a^{5}b^{3}\log\left(x^{2}\right)-\frac{84\,a^{6}b^{2}x^{4}+12\,a^{7}bx^{2}+a^{8}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(84*a^6*b^2*x^4 + 12*a^7*b*x^2 + a^8)/x^6
```

Fricas [A] time = 1.36912, size = 220, normalized size = 2.34

$$\frac{3\,b^{8}x^{16}+30\,ab^{7}x^{14}+140\,a^{2}b^{6}x^{12}+420\,a^{3}b^{5}x^{10}+1050\,a^{4}b^{4}x^{8}+1680\,a^{5}b^{3}x^{6}\log\left(x\right)-420\,a^{6}b^{2}x^{4}-60\,a^{7}bx^{2}-5\,a^{8}}{30\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="fricas")
```

```
[Out] 1/30*(3*b^8*x^16 + 30*a*b^7*x^14 + 140*a^2*b^6*x^12 + 420*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6*log(x) - 420*a^6*b^2*x^4 - 60*a^7*b*x^2 -
 5*a^8)/x^6
```

Sympy [A] time = 0.501642, size = 95, normalized size = 1.01

$$56a^{5}b^{3}\log\left(x\right)+35a^{4}b^{4}x^{2}+14a^{3}b^{5}x^{4}+\frac{14a^{2}b^{6}x^{6}}{3}+ab^{7}x^{8}+\frac{b^{8}x^{10}}{10}-\frac{a^{8}+12a^{7}bx^{2}+84a^{6}b^{2}x^{4}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**7,x)
```

```
[Out] 56*a**5*b**3*log(x) + 35*a**4*b**4*x**2 + 14*a**3*b**5*x**4 + 14*a**2*b**6*
x**6/3 + a*b**7*x**8 + b**8*x**10/10 - (a**8 + 12*a**7*b*x**2 + 84*a**6*b**
2*x**4)/(6*x**6)
```

Giac [A] time = 3.88237, size = 138, normalized size = 1.47

$$\frac{1}{10}\,b^8x^{10}+ab^7x^8+\frac{14}{3}\,a^2b^6x^6+14\,a^3b^5x^4+35\,a^4b^4x^2+28\,a^5b^3\log\left(x^2\right)-\frac{308\,a^5b^3x^6+84\,a^6b^2x^4+12\,a^7bx^2+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="giac")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(308*a^5*b^3*x^6 + 84*a^6*b^2*x^4 + 12*a^7*
b*x^2 + a^8)/x^6
```

3.96 $\int \frac{(a+bx^2)^8}{x^9}\,dx$

Optimal. Leaf size=97

$$-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+70a^4b^4\log(x)-\frac{4a^7b}{3x^6}-\frac{a^8}{8x^8}+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

```
[Out] -a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Rubi [A] time = 0.0568999, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+70a^4b^4\log(x)-\frac{4a^7b}{3x^6}-\frac{a^8}{8x^8}+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^9,x]
```

```
[Out] -a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(56a^3b^5+\frac{a^8}{x^5}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x}+28a^2b^6x+8ab^7x^2+b^8x^3\right)dx,\right.\\ &= -\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.0046682, size = 97, normalized size = 1.

$$-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+70a^4b^4\log(x)-\frac{4a^7b}{3x^6}-\frac{a^8}{8x^8}+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^9,x]
```

```
[Out] -a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Maple [A] time = 0.007, size = 90, normalized size = 0.9

$$-\frac{a^8}{8\,x^8}-\frac{4\,a^7 b}{3\,x^6}-7\,\frac{a^6 b^2}{x^4}-28\,\frac{a^5 b^3}{x^2}+28\,a^3 b^5 x^2+7\,a^2 b^6 x^4+\frac{4\,a b^7 x^6}{3}+\frac{b^8 x^8}{8}+70\,a^4 b^4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^9,x)
```

```
[Out] -1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^
2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

Maxima [A] time = 2.33408, size = 127, normalized size = 1.31

$$\frac{1}{8}\,b^8 x^8+\frac{4}{3}\,a b^7 x^6+7\,a^2 b^6 x^4+28\,a^3 b^5 x^2+35\,a^4 b^4 \log\left(x^2\right)-\frac{672\,a^5 b^3 x^6+168\,a^6 b^2 x^4+32\,a^7 b x^2+3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)/x
^8
```

Fricas [A] time = 1.17767, size = 219, normalized size = 2.26

$$\frac{3\,b^8 x^{16}+32\,a b^7 x^{14}+168\,a^2 b^6 x^{12}+672\,a^3 b^5 x^{10}+1680\,a^4 b^4 x^8 \log(x)-672\,a^5 b^3 x^6-168\,a^6 b^2 x^4-32\,a^7 b x^2-3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^8*x^16 + 32*a*b^7*x^14 + 168*a^2*b^6*x^12 + 672*a^3*b^5*x^10 + 16
80*a^4*b^4*x^8*log(x) - 672*a^5*b^3*x^6 - 168*a^6*b^2*x^4 - 32*a^7*b*x^2 -
3*a^8)/x^8
```

Sympy [A] time = 0.596116, size = 99, normalized size = 1.02

$$70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4ab^7x^6}{3}+\frac{b^8x^8}{8}-\frac{3a^8+32a^7bx^2+168a^6b^2x^4+672a^5b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**9,x)
```

```
[Out] 70*a**4*b**4*log(x) + 28*a**3*b**5*x**2 + 7*a**2*b**6*x**4 + 4*a*b**7*x**6/
3 + b**8*x**8/8 - (3*a**8 + 32*a**7*b*x**2 + 168*a**6*b**2*x**4 + 672*a**5*
b**3*x**6)/(24*x**8)
```

Giac [A] time = 2.47352, size = 142, normalized size = 1.46

$$\frac{1}{8}\,b^8x^8 + \frac{4}{3}\,ab^7x^6 + 7\,a^2b^6x^4 + 28\,a^3b^5x^2 + 35\,a^4b^4\log\left(x^2\right) - \frac{1750\,a^4b^4x^8 + 672\,a^5b^3x^6 + 168\,a^6b^2x^4 + 32\,a^7bx^2 + 3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="giac")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(1750*a^4*b^4*x^8 + 672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a
^7*b*x^2 + 3*a^8)/x^8
```

3.97 $\int \frac{(a+bx^2)^8}{x^{11}}\,dx$

Optimal. Leaf size=95

$$-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+56a^3b^5\log(x)-\frac{a^7b}{x^8}-\frac{a^8}{10x^{10}}+2ab^7x^4+\frac{b^8x^6}{6}$$

```
[Out] -a^8/(10*x^10) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

Rubi [A] time = 0.0582053, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+56a^3b^5\log(x)-\frac{a^7b}{x^8}-\frac{a^8}{10x^{10}}+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^11,x]
```


```
[Out] -a^8/(10*x^10) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(28a^2b^6+\frac{a^8}{x^6}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x}+8ab^7x+b^8x^2\right)dx,x,x^2\right)\\ &= -\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+56a^3b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.0046885, size = 95, normalized size = 1.

$$-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+56a^3b^5\log(x)-\frac{a^7b}{x^8}-\frac{a^8}{10x^{10}}+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^11,x]
```

```
[Out] -a^8/(10*x^10) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

Maple [A] time = 0.007, size = 90, normalized size = 1.

$$-\frac{a^8}{10\,x^{10}}-\frac{a^7b}{x^8}-\frac{14\,a^6b^2}{3\,x^6}-14\,\frac{a^5b^3}{x^4}-35\,\frac{a^4b^4}{x^2}+14\,a^2b^6x^2+2\,ab^7x^4+\frac{b^8x^6}{6}+56\,a^3b^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^11,x)
```

```
[Out] -1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*
a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

Maxima [A] time = 2.75032, size = 127, normalized size = 1.34

$$\frac{1}{6}\,b^8x^6+2\,ab^7x^4+14\,a^2b^6x^2+28\,a^3b^5\log\left(x^2\right)-\frac{1050\,a^4b^4x^8+420\,a^5b^3x^6+140\,a^6b^2x^4+30\,a^7bx^2+3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="maxima")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(10
50*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30*a^7*b*x^2 + 3*a^8)/
x^10
```

Fricas [A] time = 1.1723, size = 221, normalized size = 2.33

$$\frac{5\,b^8x^{16}+60\,ab^7x^{14}+420\,a^2b^6x^{12}+1680\,a^3b^5x^{10}\log(x)-1050\,a^4b^4x^8-420\,a^5b^3x^6-140\,a^6b^2x^4-30\,a^7bx^2-3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="fricas")
```

```
[Out] 1/30*(5*b^8*x^16 + 60*a*b^7*x^14 + 420*a^2*b^6*x^12 + 1680*a^3*b^5*x^10*log
(x) - 1050*a^4*b^4*x^8 - 420*a^5*b^3*x^6 - 140*a^6*b^2*x^4 - 30*a^7*b*x^2 -
 3*a^8)/x^10
```

Sympy [A] time = 0.770149, size = 97, normalized size = 1.02

$$56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}-\frac{3a^8+30a^7bx^2+140a^6b^2x^4+420a^5b^3x^6+1050a^4b^4x^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**11,x)
```

```
[Out] 56*a**3*b**5*log(x) + 14*a**2*b**6*x**2 + 2*a*b**7*x**4 + b**8*x**6/6 - (3*
a**8 + 30*a**7*b*x**2 + 140*a**6*b**2*x**4 + 420*a**5*b**3*x**6 + 1050*a**4
*b**4*x**8)/(30*x**10)
```

Giac [A] time = 1.82553, size = 142, normalized size = 1.49

$$\frac{1}{6}\,b^{8}x^{6}+2\,ab^{7}x^{4}+14\,a^{2}b^{6}x^{2}+28\,a^{3}b^{5}\log\left(x^{2}\right)-\frac{1918\,a^{3}b^{5}x^{10}+1050\,a^{4}b^{4}x^{8}+420\,a^{5}b^{3}x^{6}+140\,a^{6}b^{2}x^{4}+30\,a^{7}bx^{2}+3\,a}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="giac")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(19
18*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30
*a^7*b*x^2 + 3*a^8)/x^10
```

3.98 $\int \frac{(a+bx^2)^8}{x^{13}} dx$

Optimal. Leaf size=101

$$-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)-\frac{4a^7b}{5x^{10}}-\frac{a^8}{12x^{12}}+4ab^7x^2+\frac{b^8x^4}{4}$$

```
[Out] -a^8/(12*x^12) - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Rubi [A] time = 0.0536426, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)-\frac{4a^7b}{5x^{10}}-\frac{a^8}{12x^{12}}+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^13,x]
```

```
[Out] -a^8/(12*x^12) - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{13}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^7}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(8ab^7+\frac{a^8}{x^7}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^5}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x}+b^8x\right)dx, x, x\right.\\ &= -\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4ab^7x^2+\frac{b^8x^4}{4}+28a^2b^6\log(x)\end{aligned}$$

Mathematica [A] time = 0.0045843, size = 101, normalized size = 1.

$$-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)-\frac{4a^7b}{5x^{10}}-\frac{a^8}{12x^{12}}+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^13,x]
```

```
[Out] -a^8/(12*x^12) - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Maple [A] time = 0.008, size = 90, normalized size = 0.9

$$-\frac{a^{8}}{12\,x^{12}}-\frac{4\,a^{7}b}{5\,x^{10}}-\frac{7\,a^{6}b^{2}}{2\,x^{8}}-\frac{28\,a^{5}b^{3}}{3\,x^{6}}-\frac{35\,a^{4}b^{4}}{2\,x^{4}}-28\,\frac{a^{3}b^{5}}{x^{2}}+4\,ab^{7}x^{2}+\frac{b^{8}x^{4}}{4}+28\,a^{2}b^{6}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^13,x)
```

```
[Out] -1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4
/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

Maxima [A] time = 1.63539, size = 127, normalized size = 1.26

$$\frac{1}{4}\,b^{8}x^{4}+4\,ab^{7}x^{2}+14\,a^{2}b^{6}\log\left(x^{2}\right)-\frac{1680\,a^{3}b^{5}x^{10}+1050\,a^{4}b^{4}x^{8}+560\,a^{5}b^{3}x^{6}+210\,a^{6}b^{2}x^{4}+48\,a^{7}bx^{2}+5\,a^{8}}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="maxima")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(1680*a^3*b^5*x^10 +
 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*a^
8)/x^12
```

Fricas [A] time = 1.25541, size = 225, normalized size = 2.23

$$\frac{15\,b^{8}x^{16}+240\,ab^{7}x^{14}+1680\,a^{2}b^{6}x^{12}\log\left(x\right)-1680\,a^{3}b^{5}x^{10}-1050\,a^{4}b^{4}x^{8}-560\,a^{5}b^{3}x^{6}-210\,a^{6}b^{2}x^{4}-48\,a^{7}bx^{2}-5\,a^{8}}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^8*x^16 + 240*a*b^7*x^14 + 1680*a^2*b^6*x^12*log(x) - 1680*a^3*b^
5*x^10 - 1050*a^4*b^4*x^8 - 560*a^5*b^3*x^6 - 210*a^6*b^2*x^4 - 48*a^7*b*x^
2 - 5*a^8)/x^12
```

Sympy [A] time = 1.00977, size = 97, normalized size = 0.96

$$28a^{2}b^{6}\log\left(x\right)+4ab^{7}x^{2}+\frac{b^{8}x^{4}}{4}-\frac{5a^{8}+48a^{7}bx^{2}+210a^{6}b^{2}x^{4}+560a^{5}b^{3}x^{6}+1050a^{4}b^{4}x^{8}+1680a^{3}b^{5}x^{10}}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**13,x)
```

```
[Out] 28*a**2*b**6*log(x) + 4*a*b**7*x**2 + b**8*x**4/4 - (5*a**8 + 48*a**7*b*x**
2 + 210*a**6*b**2*x**4 + 560*a**5*b**3*x**6 + 1050*a**4*b**4*x**8 + 1680*a*
*3*b**5*x**10)/(60*x**12)
```

Giac [A] time = 2.82687, size = 142, normalized size = 1.41

$$\frac{1}{4}\,b^8x^4+4\,ab^7x^2+14\,a^2b^6\log\left(x^2\right)-\frac{2058\,a^2b^6x^{12}+1680\,a^3b^5x^{10}+1050\,a^4b^4x^8+560\,a^5b^3x^6+210\,a^6b^2x^4+48\,a^7bx}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="giac")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(2058*a^2*b^6*x^12 +
 1680*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 +
 48*a^7*b*x^2 + 5*a^8)/x^12
```

3.99 $\int \frac{(a+bx^2)^8}{x^{15}} dx$

Optimal. Leaf size=99

$$-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

```
[Out] -a^8/(14*x^14) - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Rubi [A] time = 0.0514795, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^15,x]
```

```
[Out] -a^8/(14*x^14) - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{15}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^8}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^8+\frac{a^8}{x^8}+\frac{8a^7b}{x^7}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x}\right)dx, x, x^2\right)\\
&= -\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0045511, size = 99, normalized size = 1.

$$-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^15,x]
```

```
[Out] -a^8/(14*x^14) - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Maple [A] time = 0.006, size = 90, normalized size = 0.9

$$-\frac{a^8}{14\,x^{14}}-\frac{2\,a^7b}{3\,x^{12}}-\frac{14\,a^6b^2}{5\,x^{10}}-7\,\frac{a^5b^3}{x^8}-\frac{35\,a^4b^4}{3\,x^6}-14\,\frac{a^3b^5}{x^4}-14\,\frac{a^2b^6}{x^2}+\frac{b^8x^2}{2}+8\,ab^7\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^15,x)
```

```
[Out] -1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4/
x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

Maxima [A] time = 2.33088, size = 127, normalized size = 1.28

$$\frac{1}{2}\,b^8x^2+4\,ab^7\log\left(x^2\right)-\frac{2940\,a^2b^6x^{12}+2940\,a^3b^5x^{10}+2450\,a^4b^4x^8+1470\,a^5b^3x^6+588\,a^6b^2x^4+140\,a^7bx^2+15\,a^8}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2940*a^2*b^6*x^12 + 2940*a^3*b^5*x^
10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^2
+ 15*a^8)/x^14
```

Fricas [A] time = 1.27335, size = 234, normalized size = 2.36

$$\frac{105\,b^8x^{16}+1680\,ab^7x^{14}\log(x)-2940\,a^2b^6x^{12}-2940\,a^3b^5x^{10}-2450\,a^4b^4x^8-1470\,a^5b^3x^6-588\,a^6b^2x^4-140\,a^7bx^2}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="fricas")
```

```
[Out] 1/210*(105*b^8*x^16 + 1680*a*b^7*x^14*log(x) - 2940*a^2*b^6*x^12 - 2940*a^3
*b^5*x^10 - 2450*a^4*b^4*x^8 - 1470*a^5*b^3*x^6 - 588*a^6*b^2*x^4 - 140*a^7
*b*x^2 - 15*a^8)/x^14
```

Sympy [A] time = 0.955497, size = 97, normalized size = 0.98

$$8ab^7\log(x)+\frac{b^8x^2}{2}-\frac{15a^8+140a^7bx^2+588a^6b^2x^4+1470a^5b^3x^6+2450a^4b^4x^8+2940a^3b^5x^{10}+2940a^2b^6x^{12}}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**15,x)
```

```
[Out] 8*a*b**7*log(x) + b**8*x**2/2 - (15*a**8 + 140*a**7*b*x**2 + 588*a**6*b**2*
x**4 + 1470*a**5*b**3*x**6 + 2450*a**4*b**4*x**8 + 2940*a**3*b**5*x**10 + 2
940*a**2*b**6*x**12)/(210*x**14)
```

Giac [A] time = 2.27729, size = 139, normalized size = 1.4

$$\frac{1}{2}\,b^8x^2 + 4\,ab^7\log\left(x^2\right) - \frac{2178\,ab^7x^{14} + 2940\,a^2b^6x^{12} + 2940\,a^3b^5x^{10} + 2450\,a^4b^4x^8 + 1470\,a^5b^3x^6 + 588\,a^6b^2x^4 + 140\,a^7}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="giac")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2178*a*b^7*x^14 + 2940*a^2*b^6*x^12
 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^
4 + 140*a^7*b*x^2 + 15*a^8)/x^14
```

3.100 $\int \frac{(a+bx^2)^8}{x^{17}} dx$

Optimal. Leaf size=100

$$-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4a^7b}{7x^{14}}-\frac{a^8}{16x^{16}}-\frac{4ab^7}{x^2}+b^8\log(x)$$

```
[Out] -a^8/(16*x^16) - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Rubi [A] time = 0.0540685, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4a^7b}{7x^{14}}-\frac{a^8}{16x^{16}}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^17,x]
```

```
[Out] -a^8/(16*x^16) - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{17}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^9}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^7}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^2}+\frac{b^8}{x}\right)dx,x,x\\ &= -\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)\end{aligned}$$

Mathematica [A] time = 0.0045836, size = 100, normalized size = 1.

$$-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4a^7b}{7x^{14}}-\frac{a^8}{16x^{16}}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^17,x]
```

```
[Out] -a^8/(16*x^16) - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Maple [A] time = 0.008, size = 89, normalized size = 0.9

$$-\frac{a^8}{16\,x^{16}}-\frac{4\,a^7b}{7\,x^{14}}-\frac{7\,a^6b^2}{3\,x^{12}}-\frac{28\,a^5b^3}{5\,x^{10}}-\frac{35\,a^4b^4}{4\,x^8}-\frac{28\,a^3b^5}{3\,x^6}-7\,\frac{a^2b^6}{x^4}-4\,\frac{ab^7}{x^2}+b^8\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^17,x)
```

```
[Out] -1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*b
^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

Maxima [A] time = 1.48186, size = 127, normalized size = 1.27

$$\frac{1}{2}\,b^8\log\left(x^2\right)-\frac{6720\,ab^7x^{14}+11760\,a^2b^6x^{12}+15680\,a^3b^5x^{10}+14700\,a^4b^4x^8+9408\,a^5b^3x^6+3920\,a^6b^2x^4+960\,a^7bx^2+}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(6720*a*b^7*x^14 + 11760*a^2*b^6*x^12 + 15680*a^3
*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6*b^2*x^4 + 960*a
^7*b*x^2 + 105*a^8)/x^16
```

Fricas [A] time = 1.35895, size = 243, normalized size = 2.43

$$\frac{1680\,b^8x^{16}\log(x)-6720\,ab^7x^{14}-11760\,a^2b^6x^{12}-15680\,a^3b^5x^{10}-14700\,a^4b^4x^8-9408\,a^5b^3x^6-3920\,a^6b^2x^4-960\,a^7b}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*b^8*x^16*log(x) - 6720*a*b^7*x^14 - 11760*a^2*b^6*x^12 - 15680
*a^3*b^5*x^10 - 14700*a^4*b^4*x^8 - 9408*a^5*b^3*x^6 - 3920*a^6*b^2*x^4 - 9
60*a^7*b*x^2 - 105*a^8)/x^16
```

Sympy [A] time = 1.02748, size = 95, normalized size = 0.95

$$b^8\log(x)-\frac{105a^8+960a^7bx^2+3920a^6b^2x^4+9408a^5b^3x^6+14700a^4b^4x^8+15680a^3b^5x^{10}+11760a^2b^6x^{12}+6720ab^7x^{14}}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**17,x)
```

```
[Out] b**8*log(x) - (105*a**8 + 960*a**7*b*x**2 + 3920*a**6*b**2*x**4 + 9408*a**5
*b**3*x**6 + 14700*a**4*b**4*x**8 + 15680*a**3*b**5*x**10 + 11760*a**2*b**6
*x**12 + 6720*a*b**7*x**14)/(1680*x**16)
```

Giac [A] time = 2.51173, size = 138, normalized size = 1.38

$$\frac{1}{2}\,b^8\log\left(x^2\right) - \frac{2283\,b^8x^{16} + 6720\,ab^7x^{14} + 11760\,a^2b^6x^{12} + 15680\,a^3b^5x^{10} + 14700\,a^4b^4x^8 + 9408\,a^5b^3x^6 + 3920\,a^6b^2}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="giac")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(2283*b^8*x^16 + 6720*a*b^7*x^14 + 11760*a^2*b^6*
x^12 + 15680*a^3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6
*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^16
```

3.101 $\int \frac{(a+bx^2)^8}{x^{19}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

[Out] -(a + b*x^2)^9/(18*a*x^18)

Rubi [A] time = 0.0031426, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^8/x^19,x]

[Out] -(a + b*x^2)^9/(18*a*x^18)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx=-\frac{(a+bx^2)^9}{18ax^{18}}$$

Mathematica [B] time = 0.0045611, size = 100, normalized size = 5.26

$$-\frac{2a^6b^2}{x^{14}}-\frac{14a^5b^3}{3x^{12}}-\frac{7a^4b^4}{x^{10}}-\frac{7a^3b^5}{x^8}-\frac{14a^2b^6}{3x^6}-\frac{a^7b}{2x^{16}}-\frac{a^8}{18x^{18}}-\frac{2ab^7}{x^4}-\frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^8/x^19,x]

[Out] -a^8/(18*x^18) - (a^7*b)/(2*x^16) - (2*a^6*b^2)/x^14 - (14*a^5*b^3)/(3*x^12
) - (7*a^4*b^4)/x^10 - (7*a^3*b^5)/x^8 - (14*a^2*b^6)/(3*x^6) - (2*a*b^7)/x
^4 - b^8/(2*x^2)

Maple [B] time = 0.006, size = 91, normalized size = 4.8

$$-7\,\frac{a^3b^5}{x^8}-7\,\frac{a^4b^4}{x^{10}}-2\,\frac{ab^7}{x^4}-\frac{a^8}{18\,x^{18}}-\frac{14\,a^5b^3}{3\,x^{12}}-\frac{b^8}{2\,x^2}-\frac{14\,a^2b^6}{3\,x^6}-2\,\frac{a^6b^2}{x^{14}}-\frac{a^7b}{2\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^19,x)
```

```
[Out] -7*a^3*b^5/x^8-7*a^4*b^4/x^10-2*a*b^7/x^4-1/18*a^8/x^18-14/3*a^5*b^3/x^12-1
/2*b^8/x^2-14/3*a^2*b^6/x^6-2*a^6*b^2/x^14-1/2*a^7*b/x^16
```

Maxima [B] time = 2.8204, size = 122, normalized size = 6.42

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Fricas [B] time = 1.23415, size = 203, normalized size = 10.68

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Sympy [B] time = 1.13006, size = 97, normalized size = 5.11

$$-\frac{a^8+9a^7bx^2+36a^6b^2x^4+84a^5b^3x^6+126a^4b^4x^8+126a^3b^5x^{10}+84a^2b^6x^{12}+36ab^7x^{14}+9b^8x^{16}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**19,x)
```

```
[Out] -(a**8 + 9*a**7*b*x**2 + 36*a**6*b**2*x**4 + 84*a**5*b**3*x**6 + 126*a**4*b
**4*x**8 + 126*a**3*b**5*x**10 + 84*a**2*b**6*x**12 + 36*a*b**7*x**14 + 9*b
**8*x**16)/(18*x**18)
```

Giac [B] time = 2.09674, size = 122, normalized size = 6.42

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

3.102 $\int \frac{(a+bx^2)^8}{x^{21}} \, dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^9}{180a^2x^{18}} - \frac{(a+bx^2)^9}{20ax^{20}}$$

```
[Out] -(a + b*x^2)^9/(20*a*x^20) + (b*(a + b*x^2)^9)/(180*a^2*x^18)
```

Rubi [A] time = 0.0170586, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b(a+bx^2)^9}{180a^2x^{18}} - \frac{(a+bx^2)^9}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^21,x]
```

```
[Out] -(a + b*x^2)^9/(20*a*x^20) + (b*(a + b*x^2)^9)/(180*a^2*x^18)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{21}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)\\ &= -\frac{(a+bx^2)^9}{20ax^{20}} - \frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{20a}\\ &= -\frac{(a+bx^2)^9}{20ax^{20}} + \frac{b(a+bx^2)^9}{180a^2x^{18}}\end{aligned}$$

Mathematica [B] time = 0.0042056, size = 106, normalized size = 2.65

$$-\frac{7a^6b^2}{4x^{16}} - \frac{4a^5b^3}{x^{14}} - \frac{35a^4b^4}{6x^{12}} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^2b^6}{2x^8} - \frac{4a^7b}{9x^{18}} - \frac{a^8}{20x^{20}} - \frac{4ab^7}{3x^6} - \frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^21,x]
```

```
[Out] -a^8/(20*x^20) - (4*a^7*b)/(9*x^18) - (7*a^6*b^2)/(4*x^16) - (4*a^5*b^3)/x^
14 - (35*a^4*b^4)/(6*x^12) - (28*a^3*b^5)/(5*x^10) - (7*a^2*b^6)/(2*x^8) -
(4*a*b^7)/(3*x^6) - b^8/(4*x^4)
```

Maple [B] time = 0.005, size = 91, normalized size = 2.3

$$-\frac{7a^2b^6}{2x^8} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^6b^2}{4x^{16}} - \frac{35a^4b^4}{6x^{12}} - \frac{b^8}{4x^4} - \frac{4ab^7}{3x^6} - 4\frac{a^5b^3}{x^{14}} - \frac{a^8}{20x^{20}} - \frac{4a^7b}{9x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^21,x)
```

```
[Out] -7/2*a^2*b^6/x^8-28/5*a^3*b^5/x^10-7/4*a^6*b^2/x^16-35/6*a^4*b^4/x^12-1/4*b
^8/x^4-4/3*a*b^7/x^6-4*a^5*b^3/x^14-1/20*a^8/x^20-4/9*a^7*b/x^18
```

Maxima [B] time = 2.65241, size = 124, normalized size = 3.1

$$-\frac{45b^8x^{16} + 240ab^7x^{14} + 630a^2b^6x^{12} + 1008a^3b^5x^{10} + 1050a^4b^4x^8 + 720a^5b^3x^6 + 315a^6b^2x^4 + 80a^7bx^2 + 9a^8}{180x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="maxima")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

Fricas [B] time = 1.27565, size = 217, normalized size = 5.42

$$-\frac{45b^8x^{16} + 240ab^7x^{14} + 630a^2b^6x^{12} + 1008a^3b^5x^{10} + 1050a^4b^4x^8 + 720a^5b^3x^6 + 315a^6b^2x^4 + 80a^7bx^2 + 9a^8}{180x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="fricas")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

Sympy [B] time = 1.17487, size = 99, normalized size = 2.48

$$-\frac{9a^8 + 80a^7bx^2 + 315a^6b^2x^4 + 720a^5b^3x^6 + 1050a^4b^4x^8 + 1008a^3b^5x^{10} + 630a^2b^6x^{12} + 240ab^7x^{14} + 45b^8x^{16}}{180x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**21,x)
```

```
[Out] -(9*a**8 + 80*a**7*b*x**2 + 315*a**6*b**2*x**4 + 720*a**5*b**3*x**6 + 1050*
a**4*b**4*x**8 + 1008*a**3*b**5*x**10 + 630*a**2*b**6*x**12 + 240*a*b**7*x*
*14 + 45*b**8*x**16)/(180*x**20)
```

Giac [B] time = 2.13553, size = 124, normalized size = 3.1

$$-\frac{45\,b^8x^{16} + 240\,ab^7x^{14} + 630\,a^2b^6x^{12} + 1008\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 720\,a^5b^3x^6 + 315\,a^6b^2x^4 + 80\,a^7bx^2 + 9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="giac")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

3.103 $\int \frac{(a+bx^2)^8}{x^{23}} \, dx$

Optimal. Leaf size=62

$$-\frac{b^2 (a + bx^2)^9}{990a^3x^{18}} + \frac{b (a + bx^2)^9}{110a^2x^{20}} - \frac{(a + bx^2)^9}{22ax^{22}}$$

```
[Out] -(a + b*x^2)^9/(22*a*x^22) + (b*(a + b*x^2)^9)/(110*a^2*x^20) - (b^2*(a + b
*x^2)^9)/(990*a^3*x^18)
```

Rubi [A] time = 0.0290133, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^2 (a + bx^2)^9}{990a^3x^{18}} + \frac{b (a + bx^2)^9}{110a^2x^{20}} - \frac{(a + bx^2)^9}{22ax^{22}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^23,x]
```

```
[Out] -(a + b*x^2)^9/(22*a*x^22) + (b*(a + b*x^2)^9)/(110*a^2*x^20) - (b^2*(a + b
*x^2)^9)/(990*a^3*x^18)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^8}{x^{23}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^9}{22ax^{22}} - \frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{11a}$$
$$= -\frac{\left(a+bx^2\right)^9}{22ax^{22}} + \frac{b\left(a+bx^2\right)^9}{110a^2x^{20}} + \frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{110a^2}$$
$$= -\frac{\left(a+bx^2\right)^9}{22ax^{22}} + \frac{b\left(a+bx^2\right)^9}{110a^2x^{20}} - \frac{b^2\left(a+bx^2\right)^9}{990a^3x^{18}}$$

Mathematica [A] time = 0.0040393, size = 104, normalized size = 1.68

$$-\frac{14a^6b^2}{9x^{18}} - \frac{7a^5b^3}{2x^{16}} - \frac{5a^4b^4}{x^{14}} - \frac{14a^3b^5}{3x^{12}} - \frac{14a^2b^6}{5x^{10}} - \frac{2a^7b}{5x^{20}} - \frac{a^8}{22x^{22}} - \frac{ab^7}{x^8} - \frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^23,x]
```

```
[Out] -a^8/(22*x^22) - (2*a^7*b)/(5*x^20) - (14*a^6*b^2)/(9*x^18) - (7*a^5*b^3)/(
2*x^16) - (5*a^4*b^4)/x^14 - (14*a^3*b^5)/(3*x^12) - (14*a^2*b^6)/(5*x^10)
- (a*b^7)/x^8 - b^8/(6*x^6)
```

Maple [A] time = 0.005, size = 91, normalized size = 1.5

$$-\frac{ab^7}{x^8} - \frac{14\,a^6b^2}{9\,x^{18}} - \frac{14\,a^2b^6}{5\,x^{10}} - \frac{7\,a^5b^3}{2\,x^{16}} - \frac{14\,a^3b^5}{3\,x^{12}} - \frac{2\,a^7b}{5\,x^{20}} - \frac{a^8}{22\,x^{22}} - \frac{b^8}{6\,x^6} - 5\,\frac{a^4b^4}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^23,x)
```

```
[Out] -a*b^7/x^8-14/9*a^6*b^2/x^18-14/5*a^2*b^6/x^10-7/2*a^5*b^3/x^16-14/3*a^3*b^
5/x^12-2/5*a^7*b/x^20-1/22*a^8/x^22-1/6*b^8/x^6-5*a^4*b^4/x^14
```

Maxima [A] time = 2.03609, size = 124, normalized size = 2.

$$-\frac{165\,b^8x^{16} + 990\,ab^7x^{14} + 2772\,a^2b^6x^{12} + 4620\,a^3b^5x^{10} + 4950\,a^4b^4x^8 + 3465\,a^5b^3x^6 + 1540\,a^6b^2x^4 + 396\,a^7bx^2 + 45}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="maxima")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

Fricas [A] time = 1.25925, size = 225, normalized size = 3.63

$$-\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+396\,a^7bx^2+45\,a^8}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="fricas")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

Sympy [A] time = 1.28588, size = 99, normalized size = 1.6

$$-\frac{45a^8+396a^7bx^2+1540a^6b^2x^4+3465a^5b^3x^6+4950a^4b^4x^8+4620a^3b^5x^{10}+2772a^2b^6x^{12}+990ab^7x^{14}+165b^8x^{16}}{990x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**23,x)
```

```
[Out] -(45*a**8 + 396*a**7*b*x**2 + 1540*a**6*b**2*x**4 + 3465*a**5*b**3*x**6 + 4
950*a**4*b**4*x**8 + 4620*a**3*b**5*x**10 + 2772*a**2*b**6*x**12 + 990*a*b*
*7*x**14 + 165*b**8*x**16)/(990*x**22)
```

Giac [A] time = 1.86166, size = 124, normalized size = 2.

$$-\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+396\,a^7bx^2+45\,a^8}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="giac")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

3.104 $\int \frac{(a+bx^2)^8}{x^{25}} \, dx$

Optimal. Leaf size=84

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

```
[Out] -(a + b*x^2)^9/(24*a*x^24) + (b*(a + b*x^2)^9)/(88*a^2*x^22) - (b^2*(a + b*
x^2)^9)/(440*a^3*x^20) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^18)
```

Rubi [A] time = 0.0414562, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^25,x]
```

```
[Out] -(a + b*x^2)^9/(24*a*x^24) + (b*(a + b*x^2)^9)/(88*a^2*x^22) - (b^2*(a + b*
x^2)^9)/(440*a^3*x^20) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^18)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^8}{x^{25}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{8a}$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} + \frac{b\left(a+bx^2\right)^9}{88a^2x^{22}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{44a^2}$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} + \frac{b\left(a+bx^2\right)^9}{88a^2x^{22}} - \frac{b^2\left(a+bx^2\right)^9}{440a^3x^{20}} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{440a^3}$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} + \frac{b\left(a+bx^2\right)^9}{88a^2x^{22}} - \frac{b^2\left(a+bx^2\right)^9}{440a^3x^{20}} + \frac{b^3\left(a+bx^2\right)^9}{3960a^4x^{18}}$$

Mathematica [A] time = 0.0040744, size = 106, normalized size = 1.26

$$-\frac{7a^6b^2}{5x^{20}} - \frac{28a^5b^3}{9x^{18}} - \frac{35a^4b^4}{8x^{16}} - \frac{4a^3b^5}{x^{14}} - \frac{7a^2b^6}{3x^{12}} - \frac{4a^7b}{11x^{22}} - \frac{a^8}{24x^{24}} - \frac{4ab^7}{5x^{10}} - \frac{b^8}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^25,x]
```

```
[Out] -a^8/(24*x^24) - (4*a^7*b)/(11*x^22) - (7*a^6*b^2)/(5*x^20) - (28*a^5*b^3)/
(9*x^18) - (35*a^4*b^4)/(8*x^16) - (4*a^3*b^5)/x^14 - (7*a^2*b^6)/(3*x^12)
- (4*a*b^7)/(5*x^10) - b^8/(8*x^8)
```

Maple [A] time = 0.006, size = 91, normalized size = 1.1

$$-\frac{b^8}{8\,x^8} - \frac{a^8}{24\,x^{24}} - \frac{35\,a^4b^4}{8\,x^{16}} - \frac{4\,ab^7}{5\,x^{10}} - 4\,\frac{a^3b^5}{x^{14}} - \frac{7\,a^6b^2}{5\,x^{20}} - \frac{4\,a^7b}{11\,x^{22}} - \frac{7\,a^2b^6}{3\,x^{12}} - \frac{28\,a^5b^3}{9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^25,x)
```

```
[Out] -1/8*b^8/x^8-1/24*a^8/x^24-35/8*a^4*b^4/x^16-4/5*a*b^7/x^10-4*a^3*b^5/x^14-
7/5*a^6*b^2/x^20-4/11*a^7*b/x^22-7/3*a^2*b^6/x^12-28/9*a^5*b^3/x^18
```

Maxima [A] time = 2.7703, size = 124, normalized size = 1.48

$$-\frac{495\,b^8x^{16} + 3168\,ab^7x^{14} + 9240\,a^2b^6x^{12} + 15840\,a^3b^5x^{10} + 17325\,a^4b^4x^8 + 12320\,a^5b^3x^6 + 5544\,a^6b^2x^4 + 1440\,a^7bx^2 +}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="maxima")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

Fricas [A] time = 1.23595, size = 235, normalized size = 2.8

$$-\frac{495\,b^8x^{16}+3168\,ab^7x^{14}+9240\,a^2b^6x^{12}+15840\,a^3b^5x^{10}+17325\,a^4b^4x^8+12320\,a^5b^3x^6+5544\,a^6b^2x^4+1440\,a^7bx}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="fricas")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

Sympy [A] time = 1.32326, size = 99, normalized size = 1.18

$$-\frac{165a^8+1440a^7bx^2+5544a^6b^2x^4+12320a^5b^3x^6+17325a^4b^4x^8+15840a^3b^5x^{10}+9240a^2b^6x^{12}+3168ab^7x^{14}+49}{3960x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**25,x)
```

```
[Out] -(165*a**8 + 1440*a**7*b*x**2 + 5544*a**6*b**2*x**4 + 12320*a**5*b**3*x**6
+ 17325*a**4*b**4*x**8 + 15840*a**3*b**5*x**10 + 9240*a**2*b**6*x**12 + 316
8*a*b**7*x**14 + 495*b**8*x**16)/(3960*x**24)
```

Giac [A] time = 2.86287, size = 124, normalized size = 1.48

$$-\frac{495\,b^8x^{16}+3168\,ab^7x^{14}+9240\,a^2b^6x^{12}+15840\,a^3b^5x^{10}+17325\,a^4b^4x^8+12320\,a^5b^3x^6+5544\,a^6b^2x^4+1440\,a^7bx}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="giac")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

3.105 $\int \frac{(a+bx^2)^8}{x^{27}} dx$

Optimal. Leaf size=106

$$-\frac{b^4 (a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3 (a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2 (a+bx^2)^9}{286a^3x^{22}}+\frac{b (a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

```
[Out] -(a + b*x^2)^9/(26*a*x^26) + (b*(a + b*x^2)^9)/(78*a^2*x^24) - (b^2*(a + b*
x^2)^9)/(286*a^3*x^22) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^20) - (b^4*(a + b*
x^2)^9)/(12870*a^5*x^18)
```

Rubi [A] time = 0.054699, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^4 (a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3 (a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2 (a+bx^2)^9}{286a^3x^{22}}+\frac{b (a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^27,x]
```

```
[Out] -(a + b*x^2)^9/(26*a*x^26) + (b*(a + b*x^2)^9)/(78*a^2*x^24) - (b^2*(a + b*
x^2)^9)/(286*a^3*x^22) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^20) - (b^4*(a + b*
x^2)^9)/(12870*a^5*x^18)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{27}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{14}}\,dx,x,x^2\right)\\ &= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}-\frac{(2b)\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)}{13a}\\ &= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}+\frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{26a^2}\\ &= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}-\frac{b^3\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{143a^3}\\ &= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}+\frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}}+\frac{b^4\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{1430a^4}\\ &= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}+\frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}}-\frac{b^4\left(a+bx^2\right)^9}{12870a^5x^{18}}\end{aligned}$$

Mathematica [A] time = 0.0043688, size = 106, normalized size = 1.

$$-\frac{14a^6b^2}{11x^{22}}-\frac{14a^5b^3}{5x^{20}}-\frac{35a^4b^4}{9x^{18}}-\frac{7a^3b^5}{2x^{16}}-\frac{2a^2b^6}{x^{14}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{26x^{26}}-\frac{2ab^7}{3x^{12}}-\frac{b^8}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^27,x]
```

```
[Out] -a^8/(26*x^26) - (a^7*b)/(3*x^24) - (14*a^6*b^2)/(11*x^22) - (14*a^5*b^3)/(
5*x^20) - (35*a^4*b^4)/(9*x^18) - (7*a^3*b^5)/(2*x^16) - (2*a^2*b^6)/x^14 -
 (2*a*b^7)/(3*x^12) - b^8/(10*x^10)
```

Maple [A] time = 0.006, size = 91, normalized size = 0.9

$$-\frac{a^8}{26\,x^{26}}-\frac{a^7b}{3\,x^{24}}-\frac{35\,a^4b^4}{9\,x^{18}}-\frac{b^8}{10\,x^{10}}-\frac{2\,ab^7}{3\,x^{12}}-\frac{7\,a^3b^5}{2\,x^{16}}-\frac{14\,a^6b^2}{11\,x^{22}}-2\,\frac{a^2b^6}{x^{14}}-\frac{14\,a^5b^3}{5\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^27,x)
```

```
[Out] -1/26*a^8/x^26-1/3*a^7*b/x^24-35/9*a^4*b^4/x^18-1/10*b^8/x^10-2/3*a*b^7/x^1
2-7/2*a^3*b^5/x^16-14/11*a^6*b^2/x^22-2*a^2*b^6/x^14-14/5*a^5*b^3/x^20
```

Maxima [A] time = 2.36697, size = 124, normalized size = 1.17

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="maxima")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

Fricas [A] time = 1.18707, size = 240, normalized size = 2.26

$$-\frac{1287\,b^{8}x^{16} + 8580\,ab^{7}x^{14} + 25740\,a^{2}b^{6}x^{12} + 45045\,a^{3}b^{5}x^{10} + 50050\,a^{4}b^{4}x^{8} + 36036\,a^{5}b^{3}x^{6} + 16380\,a^{6}b^{2}x^{4} + 4290\,a^{7}b}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="fricas")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

Sympy [A] time = 1.37786, size = 99, normalized size = 0.93

$$-\frac{495a^{8} + 4290a^{7}bx^{2} + 16380a^{6}b^{2}x^{4} + 36036a^{5}b^{3}x^{6} + 50050a^{4}b^{4}x^{8} + 45045a^{3}b^{5}x^{10} + 25740a^{2}b^{6}x^{12} + 8580ab^{7}x^{14} + 128}{12870x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**27,x)
```

```
[Out] -(495*a**8 + 4290*a**7*b*x**2 + 16380*a**6*b**2*x**4 + 36036*a**5*b**3*x**6
 + 50050*a**4*b**4*x**8 + 45045*a**3*b**5*x**10 + 25740*a**2*b**6*x**12 + 8
580*a*b**7*x**14 + 1287*b**8*x**16)/(12870*x**26)
```

Giac [A] time = 2.79553, size = 124, normalized size = 1.17

$$-\frac{1287\,b^{8}x^{16} + 8580\,ab^{7}x^{14} + 25740\,a^{2}b^{6}x^{12} + 45045\,a^{3}b^{5}x^{10} + 50050\,a^{4}b^{4}x^{8} + 36036\,a^{5}b^{3}x^{6} + 16380\,a^{6}b^{2}x^{4} + 4290\,a^{7}b}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="giac")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

3.106 $\int \frac{\left(a+bx^2\right)^8}{x^{29}}\,dx$

Optimal. Leaf size=108

$$-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a^7b}{13x^{26}}-\frac{a^8}{28x^{28}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

```
[Out] -a^8/(28*x^28) - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Rubi [A] time = 0.0536761, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a^7b}{13x^{26}}-\frac{a^8}{28x^{28}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^29,x]
```

```
[Out] -a^8/(28*x^28) - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{29}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{15}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{15}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{13}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^9}+\frac{8ab^7}{x^8}+\frac{b^8}{x^7}\right)dx, x,\right.\\ &= -\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}\end{aligned}$$

Mathematica [A] time = 0.0043779, size = 108, normalized size = 1.

$$-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a^7b}{13x^{26}}-\frac{a^8}{28x^{28}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^29,x]
```

```
[Out] -a^8/(28*x^28) - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Maple [A] time = 0.007, size = 91, normalized size = 0.8

$$-\frac{a^8}{28\,x^{28}}-\frac{4\,a^7b}{13\,x^{26}}-\frac{7\,a^6b^2}{6\,x^{24}}-\frac{28\,a^5b^3}{11\,x^{22}}-\frac{7\,a^4b^4}{2\,x^{20}}-\frac{28\,a^3b^5}{9\,x^{18}}-\frac{7\,a^2b^6}{4\,x^{16}}-\frac{4\,ab^7}{7\,x^{14}}-\frac{b^8}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^29,x)
```

```
[Out] -1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4*
b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

Maxima [A] time = 1.97239, size = 124, normalized size = 1.15

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="maxima")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

Fricas [A] time = 1.29526, size = 247, normalized size = 2.29

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="fricas")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

Sympy [A] time = 1.49153, size = 99, normalized size = 0.92

$$-\frac{1287a^8+11088a^7bx^2+42042a^6b^2x^4+91728a^5b^3x^6+126126a^4b^4x^8+112112a^3b^5x^{10}+63063a^2b^6x^{12}+20592ab^7x^{14}}{36036x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**29,x)
```

```
[Out] -(1287*a**8 + 11088*a**7*b*x**2 + 42042*a**6*b**2*x**4 + 91728*a**5*b**3*x*
*6 + 126126*a**4*b**4*x**8 + 112112*a**3*b**5*x**10 + 63063*a**2*b**6*x**12
 + 20592*a*b**7*x**14 + 3003*b**8*x**16)/(36036*x**28)
```

Giac [A] time = 2.08714, size = 124, normalized size = 1.15

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+110}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="giac")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

3.107 $\int \frac{(a+bx^2)^8}{x^{31}} dx$

Optimal. Leaf size=108

$$-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{2a^7b}{7x^{28}}-\frac{a^8}{30x^{30}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

```
[Out] -a^8/(30*x^30) - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Rubi [A] time = 0.0522581, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{2a^7b}{7x^{28}}-\frac{a^8}{30x^{30}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^31,x]
```

```
[Out] -a^8/(30*x^30) - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{31}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{16}} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{15}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{11}}+\frac{28a^2b^6}{x^{10}}+\frac{8ab^7}{x^9}+\frac{b^8}{x^8}\right) dx, x, x^2\right)\\ &= -\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}\end{aligned}$$

Mathematica [A] time = 0.004014, size = 108, normalized size = 1.

$$-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{2a^7b}{7x^{28}}-\frac{a^8}{30x^{30}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^31,x]
```

```
[Out] -a^8/(30*x^30) - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Maple [A] time = 0.008, size = 91, normalized size = 0.8

$$-\frac{a^8}{30\,x^{30}}-\frac{2\,a^7b}{7\,x^{28}}-\frac{14\,a^6b^2}{13\,x^{26}}-\frac{7\,a^5b^3}{3\,x^{24}}-\frac{35\,a^4b^4}{11\,x^{22}}-\frac{14\,a^3b^5}{5\,x^{20}}-\frac{14\,a^2b^6}{9\,x^{18}}-\frac{ab^7}{2\,x^{16}}-\frac{b^8}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^31,x)
```

```
[Out] -1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^4
*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^14
```

Maxima [A] time = 1.99641, size = 124, normalized size = 1.15

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="maxima")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Fricas [A] time = 1.1025, size = 250, normalized size = 2.31

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="fricas")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Sympy [A] time = 1.6282, size = 99, normalized size = 0.92

$$-\frac{3003a^8+25740a^7bx^2+97020a^6b^2x^4+210210a^5b^3x^6+286650a^4b^4x^8+252252a^3b^5x^{10}+140140a^2b^6x^{12}+45045a}{90090x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**31,x)
```

```
[Out] -(3003*a**8 + 25740*a**7*b*x**2 + 97020*a**6*b**2*x**4 + 210210*a**5*b**3*x
**6 + 286650*a**4*b**4*x**8 + 252252*a**3*b**5*x**10 + 140140*a**2*b**6*x**
12 + 45045*a*b**7*x**14 + 6435*b**8*x**16)/(90090*x**30)
```

Giac [A] time = 2.98414, size = 124, normalized size = 1.15

$$-\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 97020\,a^6b^2x^4 + 257}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="giac")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

3.108 $\int \frac{(a+bx^2)^8}{x^{33}}\,dx$

Optimal. Leaf size=106

$$-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4a^7b}{15x^{30}}-\frac{a^8}{32x^{32}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

```
[Out] -a^8/(32*x^32) - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Rubi [A] time = 0.0517498, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4a^7b}{15x^{30}}-\frac{a^8}{32x^{32}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^33,x]
```

```
[Out] -a^8/(32*x^32) - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{33}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{17}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^8}{x^{17}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{15}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{13}}+\frac{56a^3b^5}{x^{12}}+\frac{28a^2b^6}{x^{11}}+\frac{8ab^7}{x^{10}}+\frac{b^8}{x^9}\right)dx,x,\right.\\ &= -\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}\end{aligned}$$

Mathematica [A] time = 0.0040941, size = 106, normalized size = 1.

$$-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4a^7b}{15x^{30}}-\frac{a^8}{32x^{32}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^33,x]
```

```
[Out] -a^8/(32*x^32) - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Maple [A] time = 0.006, size = 91, normalized size = 0.9

$$-\frac{a^8}{32\,x^{32}}-\frac{4\,a^7b}{15\,x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28\,a^5b^3}{13\,x^{26}}-\frac{35\,a^4b^4}{12\,x^{24}}-\frac{28\,a^3b^5}{11\,x^{22}}-\frac{7\,a^2b^6}{5\,x^{20}}-\frac{4\,ab^7}{9\,x^{18}}-\frac{b^8}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^33,x)
```

```
[Out] -1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^
4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

Maxima [A] time = 1.34983, size = 124, normalized size = 1.17

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+5}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="maxima")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Fricas [A] time = 1.25686, size = 254, normalized size = 2.4

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+5}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="fricas")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Sympy [A] time = 1.83298, size = 99, normalized size = 0.93

$$-\frac{6435a^8+54912a^7bx^2+205920a^6b^2x^4+443520a^5b^3x^6+600600a^4b^4x^8+524160a^3b^5x^{10}+288288a^2b^6x^{12}+91520ab}{205920x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**33,x)
```

```
[Out] -(6435*a**8 + 54912*a**7*b*x**2 + 205920*a**6*b**2*x**4 + 443520*a**5*b**3*
x**6 + 600600*a**4*b**4*x**8 + 524160*a**3*b**5*x**10 + 288288*a**2*b**6*x*
*12 + 91520*a*b**7*x**14 + 12870*b**8*x**16)/(205920*x**32)
```

Giac [A] time = 1.86592, size = 124, normalized size = 1.17

$$-\frac{12870\,b^8x^{16} + 91520\,ab^7x^{14} + 288288\,a^2b^6x^{12} + 524160\,a^3b^5x^{10} + 600600\,a^4b^4x^8 + 443520\,a^5b^3x^6 + 205920\,a^6b^2x^4}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="giac")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

3.109 $\int x^8 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=108

$$\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{a^8x^9}{9}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Rubi [A] time = 0.0481737, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{a^8x^9}{9}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^2\right)^8\,dx &= \int \left(a^8x^8+8a^7bx^{10}+28a^6b^2x^{12}+56a^5b^3x^{14}+70a^4b^4x^{16}+56a^3b^5x^{18}+28a^2b^6x^{20}+8ab^7x^{22}+b^8\right.\\ &= \frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b}{}\end{aligned}$$

Mathematica [A] time = 0.002528, size = 108, normalized size = 1.

$$\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{a^8x^9}{9}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Maple [A] time = 0.001, size = 91, normalized size = 0.8

$$\frac{a^8x^9}{9}+\frac{8\,a^7bx^{11}}{11}+\frac{28\,a^6b^2x^{13}}{13}+\frac{56\,a^5b^3x^{15}}{15}+\frac{70\,a^4b^4x^{17}}{17}+\frac{56\,a^3b^5x^{19}}{19}+\frac{4\,a^2b^6x^{21}}{3}+\frac{8\,ab^7x^{23}}{23}+\frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^8,x)
```

```
[Out] 1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^4
*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^25
```

Maxima [A] time = 2.61743, size = 122, normalized size = 1.13

$$\frac{1}{25}\,b^8x^{25}+\frac{8}{23}\,ab^7x^{23}+\frac{4}{3}\,a^2b^6x^{21}+\frac{56}{19}\,a^3b^5x^{19}+\frac{70}{17}\,a^4b^4x^{17}+\frac{56}{15}\,a^5b^3x^{15}+\frac{28}{13}\,a^6b^2x^{13}+\frac{8}{11}\,a^7bx^{11}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

Fricas [A] time = 1.1024, size = 227, normalized size = 2.1

$$\frac{1}{25}x^{25}b^8+\frac{8}{23}x^{23}b^7a+\frac{4}{3}x^{21}b^6a^2+\frac{56}{19}x^{19}b^5a^3+\frac{70}{17}x^{17}b^4a^4+\frac{56}{15}x^{15}b^3a^5+\frac{28}{13}x^{13}b^2a^6+\frac{8}{11}x^{11}ba^7+\frac{1}{9}x^9a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^8 + 8/23*x^23*b^7*a + 4/3*x^21*b^6*a^2 + 56/19*x^19*b^5*a^3 + 7
0/17*x^17*b^4*a^4 + 56/15*x^15*b^3*a^5 + 28/13*x^13*b^2*a^6 + 8/11*x^11*b*a
^7 + 1/9*x^9*a^8
```

Sympy [A] time = 0.096444, size = 107, normalized size = 0.99

$$\frac{a^8x^9}{9}+\frac{8a^7bx^{11}}{11}+\frac{28a^6b^2x^{13}}{13}+\frac{56a^5b^3x^{15}}{15}+\frac{70a^4b^4x^{17}}{17}+\frac{56a^3b^5x^{19}}{19}+\frac{4a^2b^6x^{21}}{3}+\frac{8ab^7x^{23}}{23}+\frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**9/9 + 8*a**7*b*x**11/11 + 28*a**6*b**2*x**13/13 + 56*a**5*b**3*x**1
5/15 + 70*a**4*b**4*x**17/17 + 56*a**3*b**5*x**19/19 + 4*a**2*b**6*x**21/3
+ 8*a*b**7*x**23/23 + b**8*x**25/25
```

Giac [A] time = 2.84994, size = 122, normalized size = 1.13

$$\frac{1}{25}\,b^{8}x^{25} + \frac{8}{23}\,ab^{7}x^{23} + \frac{4}{3}\,a^{2}b^{6}x^{21} + \frac{56}{19}\,a^{3}b^{5}x^{19} + \frac{70}{17}\,a^{4}b^{4}x^{17} + \frac{56}{15}\,a^{5}b^{3}x^{15} + \frac{28}{13}\,a^{6}b^{2}x^{13} + \frac{8}{11}\,a^{7}bx^{11} + \frac{1}{9}\,a^{8}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

3.110 $\int x^6 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=108

$$\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7bx^9+\frac{a^8x^7}{7}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Rubi [A] time = 0.0390258, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7bx^9+\frac{a^8x^7}{7}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6 \left(a+bx^2\right)^8 \, dx = \int \left(a^8x^6+8a^7bx^8+28a^6b^2x^{10}+56a^5b^3x^{12}+70a^4b^4x^{14}+56a^3b^5x^{16}+28a^2b^6x^{18}+8ab^7x^{20}+\right.$$
$$=\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+$$

Mathematica [A] time = 0.0024546, size = 108, normalized size = 1.

$$\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7bx^9+\frac{a^8x^7}{7}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Maple [A] time = 0.001, size = 91, normalized size = 0.8

$$\frac{a^{8}x^{7}}{7}+\frac{8\,a^{7}b\,x^{9}}{9}+\frac{28\,a^{6}b^{2}x^{11}}{11}+\frac{56\,a^{5}b^{3}x^{13}}{13}+\frac{14\,a^{4}b^{4}x^{15}}{3}+\frac{56\,a^{3}b^{5}x^{17}}{17}+\frac{28\,a^{2}b^{6}x^{19}}{19}+\frac{8\,a\,b^{7}x^{21}}{21}+\frac{b^{8}x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^8,x)
```

```
[Out] 1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b^
4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^23
```

Maxima [A] time = 2.5572, size = 122, normalized size = 1.13

$$\frac{1}{23}\,b^{8}x^{23}+\frac{8}{21}\,a b^{7}x^{21}+\frac{28}{19}\,a^{2}b^{6}x^{19}+\frac{56}{17}\,a^{3}b^{5}x^{17}+\frac{14}{3}\,a^{4}b^{4}x^{15}+\frac{56}{13}\,a^{5}b^{3}x^{13}+\frac{28}{11}\,a^{6}b^{2}x^{11}+\frac{8}{9}\,a^{7}b x^{9}+\frac{1}{7}\,a^{8}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

Fricas [A] time = 1.17847, size = 225, normalized size = 2.08

$$\frac{1}{23}x^{23}b^{8}+\frac{8}{21}x^{21}b^{7}a+\frac{28}{19}x^{19}b^{6}a^{2}+\frac{56}{17}x^{17}b^{5}a^{3}+\frac{14}{3}x^{15}b^{4}a^{4}+\frac{56}{13}x^{13}b^{3}a^{5}+\frac{28}{11}x^{11}b^{2}a^{6}+\frac{8}{9}x^{9}b a^{7}+\frac{1}{7}x^{7}a^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^8 + 8/21*x^21*b^7*a + 28/19*x^19*b^6*a^2 + 56/17*x^17*b^5*a^3 +
 14/3*x^15*b^4*a^4 + 56/13*x^13*b^3*a^5 + 28/11*x^11*b^2*a^6 + 8/9*x^9*b*a^
7 + 1/7*x^7*a^8
```

Sympy [A] time = 0.084061, size = 107, normalized size = 0.99

$$\frac{a^{8}x^{7}}{7}+\frac{8a^{7}bx^{9}}{9}+\frac{28a^{6}b^{2}x^{11}}{11}+\frac{56a^{5}b^{3}x^{13}}{13}+\frac{14a^{4}b^{4}x^{15}}{3}+\frac{56a^{3}b^{5}x^{17}}{17}+\frac{28a^{2}b^{6}x^{19}}{19}+\frac{8ab^{7}x^{21}}{21}+\frac{b^{8}x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**7/7 + 8*a**7*b*x**9/9 + 28*a**6*b**2*x**11/11 + 56*a**5*b**3*x**13/
13 + 14*a**4*b**4*x**15/3 + 56*a**3*b**5*x**17/17 + 28*a**2*b**6*x**19/19 +
 8*a*b**7*x**21/21 + b**8*x**23/23
```

Giac [A] time = 1.74634, size = 122, normalized size = 1.13

$$\frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

3.111 $\int x^4 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=108

$$\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7bx^7+\frac{a^8x^5}{5}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Rubi [A] time = 0.039408, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7bx^7+\frac{a^8x^5}{5}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^8 dx &= \int \left(a^8x^4+8a^7bx^6+28a^6b^2x^8+56a^5b^3x^{10}+70a^4b^4x^{12}+56a^3b^5x^{14}+28a^2b^6x^{16}+8ab^7x^{18}+b^8x^2\right.\\ &= \frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8}{2}\end{aligned}$$

Mathematica [A] time = 0.0023435, size = 108, normalized size = 1.

$$\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7bx^7+\frac{a^8x^5}{5}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Maple [A] time = 0.001, size = 91, normalized size = 0.8

$$\frac{a^8x^5}{5}+\frac{8\,a^7bx^7}{7}+\frac{28\,a^6b^2x^9}{9}+\frac{56\,a^5b^3x^{11}}{11}+\frac{70\,a^4b^4x^{13}}{13}+\frac{56\,a^3b^5x^{15}}{15}+\frac{28\,a^2b^6x^{17}}{17}+\frac{8\,ab^7x^{19}}{19}+\frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^8,x)
```

```
[Out] 1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^4
*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

Maxima [A] time = 2.19793, size = 122, normalized size = 1.13

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

Fricas [A] time = 1.12468, size = 224, normalized size = 2.07

$$\frac{1}{21}x^{21}b^8+\frac{8}{19}x^{19}b^7a+\frac{28}{17}x^{17}b^6a^2+\frac{56}{15}x^{15}b^5a^3+\frac{70}{13}x^{13}b^4a^4+\frac{56}{11}x^{11}b^3a^5+\frac{28}{9}x^9b^2a^6+\frac{8}{7}x^7ba^7+\frac{1}{5}x^5a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^8 + 8/19*x^19*b^7*a + 28/17*x^17*b^6*a^2 + 56/15*x^15*b^5*a^3 +
 70/13*x^13*b^4*a^4 + 56/11*x^11*b^3*a^5 + 28/9*x^9*b^2*a^6 + 8/7*x^7*b*a^7
 + 1/5*x^5*a^8
```

Sympy [A] time = 0.086483, size = 107, normalized size = 0.99

$$\frac{a^8x^5}{5}+\frac{8a^7bx^7}{7}+\frac{28a^6b^2x^9}{9}+\frac{56a^5b^3x^{11}}{11}+\frac{70a^4b^4x^{13}}{13}+\frac{56a^3b^5x^{15}}{15}+\frac{28a^2b^6x^{17}}{17}+\frac{8ab^7x^{19}}{19}+\frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**5/5 + 8*a**7*b*x**7/7 + 28*a**6*b**2*x**9/9 + 56*a**5*b**3*x**11/11
 + 70*a**4*b**4*x**13/13 + 56*a**3*b**5*x**15/15 + 28*a**2*b**6*x**17/17 +
8*a*b**7*x**19/19 + b**8*x**21/21
```

Giac [A] time = 2.86162, size = 122, normalized size = 1.13

$$\frac{1}{21}b^8x^{21}+\frac{8}{19}ab^7x^{19}+\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7bx^7+\frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

3.112 $\int x^2 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=106

$$\frac{28}{15}a^2b^6x^{15}+\frac{56}{13}a^3b^5x^{13}+\frac{70}{11}a^4b^4x^{11}+\frac{56}{9}a^5b^3x^9+4a^6b^2x^7+\frac{8}{5}a^7bx^5+\frac{a^8x^3}{3}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Rubi [A] time = 0.0402238, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{15}a^2b^6x^{15}+\frac{56}{13}a^3b^5x^{13}+\frac{70}{11}a^4b^4x^{11}+\frac{56}{9}a^5b^3x^9+4a^6b^2x^7+\frac{8}{5}a^7bx^5+\frac{a^8x^3}{3}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^8 dx &= \int \left(a^8x^2+8a^7bx^4+28a^6b^2x^6+56a^5b^3x^8+70a^4b^4x^{10}+56a^3b^5x^{12}+28a^2b^6x^{14}+8ab^7x^{16}+b^8\right.\\ &= \frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8}{1}\end{aligned}$$

Mathematica [A] time = 0.0023477, size = 106, normalized size = 1.

$$\frac{28}{15}a^2b^6x^{15}+\frac{56}{13}a^3b^5x^{13}+\frac{70}{11}a^4b^4x^{11}+\frac{56}{9}a^5b^3x^9+4a^6b^2x^7+\frac{8}{5}a^7bx^5+\frac{a^8x^3}{3}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Maple [A] time = 0.001, size = 91, normalized size = 0.9

$$\frac{a^8x^3}{3}+\frac{8\,a^7bx^5}{5}+4\,a^6b^2x^7+\frac{56\,a^5b^3x^9}{9}+\frac{70\,a^4b^4x^{11}}{11}+\frac{56\,a^3b^5x^{13}}{13}+\frac{28\,a^2b^6x^{15}}{15}+\frac{8\,ab^7x^{17}}{17}+\frac{b^8x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^8,x)
```

```
[Out] 1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^11
+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

Maxima [A] time = 2.37976, size = 122, normalized size = 1.15

$$\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

Fricas [A] time = 1.01202, size = 217, normalized size = 2.05

$$\frac{1}{19}x^{19}b^8+\frac{8}{17}x^{17}b^7a+\frac{28}{15}x^{15}b^6a^2+\frac{56}{13}x^{13}b^5a^3+\frac{70}{11}x^{11}b^4a^4+\frac{56}{9}x^9b^3a^5+4x^7b^2a^6+\frac{8}{5}x^5ba^7+\frac{1}{3}x^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^8 + 8/17*x^17*b^7*a + 28/15*x^15*b^6*a^2 + 56/13*x^13*b^5*a^3 +
 70/11*x^11*b^4*a^4 + 56/9*x^9*b^3*a^5 + 4*x^7*b^2*a^6 + 8/5*x^5*b*a^7 + 1/
3*x^3*a^8
```

Sympy [A] time = 0.087928, size = 105, normalized size = 0.99

$$\frac{a^8x^3}{3}+\frac{8a^7bx^5}{5}+4a^6b^2x^7+\frac{56a^5b^3x^9}{9}+\frac{70a^4b^4x^{11}}{11}+\frac{56a^3b^5x^{13}}{13}+\frac{28a^2b^6x^{15}}{15}+\frac{8ab^7x^{17}}{17}+\frac{b^8x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**3/3 + 8*a**7*b*x**5/5 + 4*a**6*b**2*x**7 + 56*a**5*b**3*x**9/9 + 70
*a**4*b**4*x**11/11 + 56*a**3*b**5*x**13/13 + 28*a**2*b**6*x**15/15 + 8*a*b
**7*x**17/17 + b**8*x**19/19
```

Giac [A] time = 1.36252, size = 122, normalized size = 1.15

$$\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

3.113 $\int \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=101

$$\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Rubi [A] time = 0.0373987, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^8 \, dx &= \int \left(a^8+8a^7bx^2+28a^6b^2x^4+56a^5b^3x^6+70a^4b^4x^8+56a^3b^5x^{10}+28a^2b^6x^{12}+8ab^7x^{14}+b^8x^{16}\right) dx\\ &= a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.0014038, size = 101, normalized size = 1.

$$\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Maple [A] time = 0.001, size = 88, normalized size = 0.9

$$a^8x+\frac{8\,a^7bx^3}{3}+\frac{28\,a^6b^2x^5}{5}+8\,a^5b^3x^7+\frac{70\,a^4b^4x^9}{9}+\frac{56\,a^3b^5x^{11}}{11}+\frac{28\,a^2b^6x^{13}}{13}+\frac{8\,ab^7x^{15}}{15}+\frac{b^8x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8,x)
```

```
[Out] a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*a
^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

Maxima [A] time = 1.86977, size = 117, normalized size = 1.16

$$\frac{1}{17}\,b^8x^{17}+\frac{8}{15}\,ab^7x^{15}+\frac{28}{13}\,a^2b^6x^{13}+\frac{56}{11}\,a^3b^5x^{11}+\frac{70}{9}\,a^4b^4x^9+8\,a^5b^3x^7+\frac{28}{5}\,a^6b^2x^5+\frac{8}{3}\,a^7bx^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

Fricas [A] time = 1.12055, size = 207, normalized size = 2.05

$$\frac{1}{17}x^{17}b^8+\frac{8}{15}x^{15}b^7a+\frac{28}{13}x^{13}b^6a^2+\frac{56}{11}x^{11}b^5a^3+\frac{70}{9}x^9b^4a^4+8x^7b^3a^5+\frac{28}{5}x^5b^2a^6+\frac{8}{3}x^3ba^7+xa^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^8 + 8/15*x^15*b^7*a + 28/13*x^13*b^6*a^2 + 56/11*x^11*b^5*a^3 +
 70/9*x^9*b^4*a^4 + 8*x^7*b^3*a^5 + 28/5*x^5*b^2*a^6 + 8/3*x^3*b*a^7 + x*a^
8
```

Sympy [A] time = 0.080738, size = 102, normalized size = 1.01

$$a^8x+\frac{8a^7bx^3}{3}+\frac{28a^6b^2x^5}{5}+8a^5b^3x^7+\frac{70a^4b^4x^9}{9}+\frac{56a^3b^5x^{11}}{11}+\frac{28a^2b^6x^{13}}{13}+\frac{8ab^7x^{15}}{15}+\frac{b^8x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8,x)
```

```
[Out] a**8*x + 8*a**7*b*x**3/3 + 28*a**6*b**2*x**5/5 + 8*a**5*b**3*x**7 + 70*a**4
*b**4*x**9/9 + 56*a**3*b**5*x**11/11 + 28*a**2*b**6*x**13/13 + 8*a*b**7*x**
15/15 + b**8*x**17/17
```

Giac [A] time = 1.8359, size = 117, normalized size = 1.16

$$\frac{1}{17}b^8x^{17} + \frac{8}{15}ab^7x^{15} + \frac{28}{13}a^2b^6x^{13} + \frac{56}{11}a^3b^5x^{11} + \frac{70}{9}a^4b^4x^9 + 8a^5b^3x^7 + \frac{28}{5}a^6b^2x^5 + \frac{8}{3}a^7bx^3 + a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

3.114 $\int \frac{(a+bx^2)^8}{x^2}\,dx$

Optimal. Leaf size=100

$$\frac{28}{11}a^2b^6x^{11}+\frac{56}{9}a^3b^5x^9+10a^4b^4x^7+\frac{56}{5}a^5b^3x^5+\frac{28}{3}a^6b^2x^3+8a^7bx-\frac{a^8}{x}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Rubi [A] time = 0.0386043, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{11}a^2b^6x^{11}+\frac{56}{9}a^3b^5x^9+10a^4b^4x^7+\frac{56}{5}a^5b^3x^5+\frac{28}{3}a^6b^2x^3+8a^7bx-\frac{a^8}{x}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^2}\,dx &= \int \left(8a^7b+\frac{a^8}{x^2}+28a^6b^2x^2+56a^5b^3x^4+70a^4b^4x^6+56a^3b^5x^8+28a^2b^6x^{10}+8ab^7x^{12}+b^8x^{14}\right)dx\\ &= -\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.014578, size = 100, normalized size = 1.

$$\frac{28}{11}a^2b^6x^{11}+\frac{56}{9}a^3b^5x^9+10a^4b^4x^7+\frac{56}{5}a^5b^3x^5+\frac{28}{3}a^6b^2x^3+8a^7bx-\frac{a^8}{x}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Maple [A] time = 0.003, size = 89, normalized size = 0.9

$$-\frac{a^{8}}{x}+8\,a^{7}bx+\frac{28\,a^{6}b^{2}x^{3}}{3}+\frac{56\,a^{5}b^{3}x^{5}}{5}+10\,a^{4}b^{4}x^{7}+\frac{56\,a^{3}b^{5}x^{9}}{9}+\frac{28\,a^{2}b^{6}x^{11}}{11}+\frac{8\,ab^{7}x^{13}}{13}+\frac{b^{8}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^2,x)
```

```
[Out] -a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3*
b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

Maxima [A] time = 1.1504, size = 119, normalized size = 1.19

$$\frac{1}{15}\,b^{8}x^{15}+\frac{8}{13}\,ab^{7}x^{13}+\frac{28}{11}\,a^{2}b^{6}x^{11}+\frac{56}{9}\,a^{3}b^{5}x^{9}+10\,a^{4}b^{4}x^{7}+\frac{56}{5}\,a^{5}b^{3}x^{5}+\frac{28}{3}\,a^{6}b^{2}x^{3}+8\,a^{7}bx-\frac{a^{8}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

Fricas [A] time = 1.17631, size = 235, normalized size = 2.35

$$\frac{429\,b^{8}x^{16}+3960\,ab^{7}x^{14}+16380\,a^{2}b^{6}x^{12}+40040\,a^{3}b^{5}x^{10}+64350\,a^{4}b^{4}x^{8}+72072\,a^{5}b^{3}x^{6}+60060\,a^{6}b^{2}x^{4}+51480\,a^{7}bx^{2}}{6435\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="fricas")
```

```
[Out] 1/6435*(429*b^8*x^16 + 3960*a*b^7*x^14 + 16380*a^2*b^6*x^12 + 40040*a^3*b^5
*x^10 + 64350*a^4*b^4*x^8 + 72072*a^5*b^3*x^6 + 60060*a^6*b^2*x^4 + 51480*a
^7*b*x^2 - 6435*a^8)/x
```

Sympy [A] time = 0.3486, size = 99, normalized size = 0.99

$$-\frac{a^{8}}{x}+8a^{7}bx+\frac{28a^{6}b^{2}x^{3}}{3}+\frac{56a^{5}b^{3}x^{5}}{5}+10a^{4}b^{4}x^{7}+\frac{56a^{3}b^{5}x^{9}}{9}+\frac{28a^{2}b^{6}x^{11}}{11}+\frac{8ab^{7}x^{13}}{13}+\frac{b^{8}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**2,x)
```

```
[Out] -a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4*
b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/13
 + b**8*x**15/15
```

Giac [A] time = 2.81734, size = 119, normalized size = 1.19

$$\frac{1}{15}\,b^8x^{15}+\frac{8}{13}\,ab^7x^{13}+\frac{28}{11}\,a^2b^6x^{11}+\frac{56}{9}\,a^3b^5x^9+10\,a^4b^4x^7+\frac{56}{5}\,a^5b^3x^5+\frac{28}{3}\,a^6b^2x^3+8\,a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="giac")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

3.115 $\int \frac{(a+bx^2)^8}{x^4}\,dx$

Optimal. Leaf size=98

$$\frac{28}{9}a^2b^6x^9+8a^3b^5x^7+14a^4b^4x^5+\frac{56}{3}a^5b^3x^3+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

```
[Out] -a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Rubi [A] time = 0.0379941, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{9}a^2b^6x^9+8a^3b^5x^7+14a^4b^4x^5+\frac{56}{3}a^5b^3x^3+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^4,x]
```

```
[Out] -a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^4}\,dx &= \int \left(28a^6b^2+\frac{a^8}{x^4}+\frac{8a^7b}{x^2}+56a^5b^3x^2+70a^4b^4x^4+56a^3b^5x^6+28a^2b^6x^8+8ab^7x^{10}+b^8x^{12}\right)dx\\ &= -\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.0077405, size = 98, normalized size = 1.

$$\frac{28}{9}a^2b^6x^9+8a^3b^5x^7+14a^4b^4x^5+\frac{56}{3}a^5b^3x^3+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^4,x]
```

```
[Out] -a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Maple [A] time = 0.005, size = 89, normalized size = 0.9

$$-\frac{a^8}{3x^3}-8\frac{a^7b}{x}+28a^6b^2x+\frac{56a^5b^3x^3}{3}+14a^4b^4x^5+8a^3b^5x^7+\frac{28a^2b^6x^9}{9}+\frac{8ab^7x^{11}}{11}+\frac{b^8x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^4,x)
```

```
[Out] -1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b
^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

Maxima [A] time = 1.96558, size = 120, normalized size = 1.22

$$\frac{1}{13}b^8x^{13}+\frac{8}{11}ab^7x^{11}+\frac{28}{9}a^2b^6x^9+8a^3b^5x^7+14a^4b^4x^5+\frac{56}{3}a^5b^3x^3+28a^6b^2x-\frac{24a^7bx^2+a^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="maxima")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

Fricas [A] time = 1.25563, size = 232, normalized size = 2.37

$$\frac{99b^8x^{16}+936ab^7x^{14}+4004a^2b^6x^{12}+10296a^3b^5x^{10}+18018a^4b^4x^8+24024a^5b^3x^6+36036a^6b^2x^4-10296a^7bx^2}{1287x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="fricas")
```

```
[Out] 1/1287*(99*b^8*x^16 + 936*a*b^7*x^14 + 4004*a^2*b^6*x^12 + 10296*a^3*b^5*x^
10 + 18018*a^4*b^4*x^8 + 24024*a^5*b^3*x^6 + 36036*a^6*b^2*x^4 - 10296*a^7*
b*x^2 - 429*a^8)/x^3
```

Sympy [A] time = 0.385195, size = 99, normalized size = 1.01

$$28a^6b^2x+\frac{56a^5b^3x^3}{3}+14a^4b^4x^5+8a^3b^5x^7+\frac{28a^2b^6x^9}{9}+\frac{8ab^7x^{11}}{11}+\frac{b^8x^{13}}{13}-\frac{a^8+24a^7bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**4,x)
```

```
[Out] 28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**7
 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13 - (a**8 + 24*a**
7*b*x**2)/(3*x**3)
```

Giac [A] time = 1.91878, size = 120, normalized size = 1.22

$$\frac{1}{13}\,b^{8}x^{13} + \frac{8}{11}\,ab^{7}x^{11} + \frac{28}{9}\,a^{2}b^{6}x^{9} + 8\,a^{3}b^{5}x^{7} + 14\,a^{4}b^{4}x^{5} + \frac{56}{3}\,a^{5}b^{3}x^{3} + 28\,a^{6}b^{2}x - \frac{24\,a^{7}bx^{2} + a^{8}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="giac")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

3.116 $\int \frac{(a+bx^2)^8}{x^6}\,dx$

Optimal. Leaf size=100

$$4a^2b^6x^7+\frac{56}{5}a^3b^5x^5+\frac{70}{3}a^4b^4x^3+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

```
[Out] -a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Rubi [A] time = 0.0390975, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$4a^2b^6x^7+\frac{56}{5}a^3b^5x^5+\frac{70}{3}a^4b^4x^3+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^6,x]
```

```
[Out] -a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^6}\,dx &= \int \left(56a^5b^3+\frac{a^8}{x^6}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^2}+70a^4b^4x^2+56a^3b^5x^4+28a^2b^6x^6+8ab^7x^8+b^8x^{10}\right)dx\\ &= -\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.0093797, size = 100, normalized size = 1.

$$4a^2b^6x^7+\frac{56}{5}a^3b^5x^5+\frac{70}{3}a^4b^4x^3+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^6,x]
```

```
[Out] -a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Maple [A] time = 0.005, size = 89, normalized size = 0.9

$$-\frac{a^{8}}{5 x^{5}}-\frac{8 a^{7} b}{3 x^{3}}-28 \frac{a^{6} b^{2}}{x}+56 a^{5} b^{3} x+\frac{70 a^{4} b^{4} x^{3}}{3}+\frac{56 a^{3} b^{5} x^{5}}{5}+4 a^{2} b^{6} x^{7}+\frac{8 a b^{7} x^{9}}{9}+\frac{b^{8} x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^6,x)
```

```
[Out] -1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*
a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

Maxima [A] time = 2.64936, size = 123, normalized size = 1.23

$$\frac{1}{11} b^{8} x^{11}+\frac{8}{9} a b^{7} x^{9}+4 a^{2} b^{6} x^{7}+\frac{56}{5} a^{3} b^{5} x^{5}+\frac{70}{3} a^{4} b^{4} x^{3}+56 a^{5} b^{3} x-\frac{420 a^{6} b^{2} x^{4}+40 a^{7} b x^{2}+3 a^{8}}{15 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="maxima")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

Fricas [A] time = 1.17004, size = 227, normalized size = 2.27

$$\frac{45 b^{8} x^{16}+440 a b^{7} x^{14}+1980 a^{2} b^{6} x^{12}+5544 a^{3} b^{5} x^{10}+11550 a^{4} b^{4} x^{8}+27720 a^{5} b^{3} x^{6}-13860 a^{6} b^{2} x^{4}-1320 a^{7} b x^{2}-99 a^{8}}{495 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="fricas")
```

```
[Out] 1/495*(45*b^8*x^16 + 440*a*b^7*x^14 + 1980*a^2*b^6*x^12 + 5544*a^3*b^5*x^10
 + 11550*a^4*b^4*x^8 + 27720*a^5*b^3*x^6 - 13860*a^6*b^2*x^4 - 1320*a^7*b*x
^2 - 99*a^8)/x^5
```

Sympy [A] time = 0.451726, size = 100, normalized size = 1.

$$56a^{5}b^{3}x+\frac{70a^{4}b^{4}x^{3}}{3}+\frac{56a^{3}b^{5}x^{5}}{5}+4a^{2}b^{6}x^{7}+\frac{8ab^{7}x^{9}}{9}+\frac{b^{8}x^{11}}{11}-\frac{3a^{8}+40a^{7}bx^{2}+420a^{6}b^{2}x^{4}}{15x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**6,x)
```

```
[Out] 56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x*
*7 + 8*a*b**7*x**9/9 + b**8*x**11/11 - (3*a**8 + 40*a**7*b*x**2 + 420*a**6*
b**2*x**4)/(15*x**5)
```

Giac [A] time = 2.438, size = 123, normalized size = 1.23

$$\frac{1}{11}\,b^{8}x^{11}+\frac{8}{9}\,ab^{7}x^{9}+4\,a^{2}b^{6}x^{7}+\frac{56}{5}\,a^{3}b^{5}x^{5}+\frac{70}{3}\,a^{4}b^{4}x^{3}+56\,a^{5}b^{3}x-\frac{420\,a^{6}b^{2}x^{4}+40\,a^{7}bx^{2}+3\,a^{8}}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="giac")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

3.117 $\int \frac{(a+bx^2)^8}{x^8}\,dx$

Optimal. Leaf size=102

$$\frac{28}{5}a^2b^6x^5+\frac{56}{3}a^3b^5x^3-\frac{28a^6b^2}{3x^3}+70a^4b^4x-\frac{56a^5b^3}{x}-\frac{8a^7b}{5x^5}-\frac{a^8}{7x^7}+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

```
[Out] -a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Rubi [A] time = 0.036847, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{5}a^2b^6x^5+\frac{56}{3}a^3b^5x^3-\frac{28a^6b^2}{3x^3}+70a^4b^4x-\frac{56a^5b^3}{x}-\frac{8a^7b}{5x^5}-\frac{a^8}{7x^7}+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^8,x]
```

```
[Out] -a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^8}\,dx &= \int \left(70a^4b^4+\frac{a^8}{x^8}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^2}+56a^3b^5x^2+28a^2b^6x^4+8ab^7x^6+b^8x^8\right)dx\\ &= -\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0058527, size = 102, normalized size = 1.

$$\frac{28}{5}a^2b^6x^5+\frac{56}{3}a^3b^5x^3-\frac{28a^6b^2}{3x^3}+70a^4b^4x-\frac{56a^5b^3}{x}-\frac{8a^7b}{5x^5}-\frac{a^8}{7x^7}+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^8,x]
```

```
[Out] -a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Maple [A] time = 0.005, size = 89, normalized size = 0.9

$$-\frac{a^{8}}{7x^{7}}-\frac{8a^{7}b}{5x^{5}}-\frac{28a^{6}b^{2}}{3x^{3}}-56\frac{a^{5}b^{3}}{x}+70a^{4}b^{4}x+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{8ab^{7}x^{7}}{7}+\frac{b^{8}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^8,x)
```

```
[Out] -1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*
a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

Maxima [A] time = 1.42532, size = 123, normalized size = 1.21

$$\frac{1}{9}b^{8}x^{9}+\frac{8}{7}ab^{7}x^{7}+\frac{28}{5}a^{2}b^{6}x^{5}+\frac{56}{3}a^{3}b^{5}x^{3}+70a^{4}b^{4}x-\frac{5880a^{5}b^{3}x^{6}+980a^{6}b^{2}x^{4}+168a^{7}bx^{2}+15a^{8}}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

Fricas [A] time = 1.2508, size = 224, normalized size = 2.2

$$\frac{35b^{8}x^{16}+360ab^{7}x^{14}+1764a^{2}b^{6}x^{12}+5880a^{3}b^{5}x^{10}+22050a^{4}b^{4}x^{8}-17640a^{5}b^{3}x^{6}-2940a^{6}b^{2}x^{4}-504a^{7}bx^{2}-45}{315x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^8*x^16 + 360*a*b^7*x^14 + 1764*a^2*b^6*x^12 + 5880*a^3*b^5*x^10
 + 22050*a^4*b^4*x^8 - 17640*a^5*b^3*x^6 - 2940*a^6*b^2*x^4 - 504*a^7*b*x^2
 - 45*a^8)/x^7
```

Sympy [A] time = 0.511301, size = 100, normalized size = 0.98

$$70a^{4}b^{4}x+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{8ab^{7}x^{7}}{7}+\frac{b^{8}x^{9}}{9}-\frac{15a^{8}+168a^{7}bx^{2}+980a^{6}b^{2}x^{4}+5880a^{5}b^{3}x^{6}}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**8,x)
```

```
[Out] 70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7/
7 + b**8*x**9/9 - (15*a**8 + 168*a**7*b*x**2 + 980*a**6*b**2*x**4 + 5880*a*
*5*b**3*x**6)/(105*x**7)
```

Giac [A] time = 2.62742, size = 123, normalized size = 1.21

$$\frac{1}{9}b^8x^9 + \frac{8}{7}ab^7x^7 + \frac{28}{5}a^2b^6x^5 + \frac{56}{3}a^3b^5x^3 + 70a^4b^4x - \frac{5880a^5b^3x^6 + 980a^6b^2x^4 + 168a^7bx^2 + 15a^8}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="giac")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

3.118 $\int \frac{(a+bx^2)^8}{x^{10}}\,dx$

Optimal. Leaf size=102

$$-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}+\frac{28}{3}a^2b^6x^3-\frac{70a^4b^4}{x}+56a^3b^5x-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

```
[Out] -a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Rubi [A] time = 0.0390688, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}+\frac{28}{3}a^2b^6x^3-\frac{70a^4b^4}{x}+56a^3b^5x-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^10,x]
```

```
[Out] -a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{10}}\,dx &= \int \left(56a^3b^5+\frac{a^8}{x^{10}}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^2}+28a^2b^6x^2+8ab^7x^4+b^8x^6\right)dx\\ &= -\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0096779, size = 102, normalized size = 1.

$$-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}+\frac{28}{3}a^2b^6x^3-\frac{70a^4b^4}{x}+56a^3b^5x-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^10,x]
```

```
[Out] -a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Maple [A] time = 0.005, size = 89, normalized size = 0.9

$$-\frac{a^{8}}{9\,x^{9}}-\frac{8\,a^{7}b}{7\,x^{7}}-\frac{28\,a^{6}b^{2}}{5\,x^{5}}-\frac{56\,a^{5}b^{3}}{3\,x^{3}}-70\,\frac{a^{4}b^{4}}{x}+56\,a^{3}b^{5}x+\frac{28\,a^{2}b^{6}x^{3}}{3}+\frac{8\,ab^{7}x^{5}}{5}+\frac{b^{8}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^10,x)
```

```
[Out] -1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+5
6*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

Maxima [A] time = 2.33814, size = 123, normalized size = 1.21

$$\frac{1}{7}\,b^{8}x^{7}+\frac{8}{5}\,ab^{7}x^{5}+\frac{28}{3}\,a^{2}b^{6}x^{3}+56\,a^{3}b^{5}x-\frac{22050\,a^{4}b^{4}x^{8}+5880\,a^{5}b^{3}x^{6}+1764\,a^{6}b^{2}x^{4}+360\,a^{7}bx^{2}+35\,a^{8}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

Fricas [A] time = 1.19949, size = 224, normalized size = 2.2

$$\frac{45\,b^{8}x^{16}+504\,ab^{7}x^{14}+2940\,a^{2}b^{6}x^{12}+17640\,a^{3}b^{5}x^{10}-22050\,a^{4}b^{4}x^{8}-5880\,a^{5}b^{3}x^{6}-1764\,a^{6}b^{2}x^{4}-360\,a^{7}bx^{2}-35\,a^{8}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(45*b^8*x^16 + 504*a*b^7*x^14 + 2940*a^2*b^6*x^12 + 17640*a^3*b^5*x^1
0 - 22050*a^4*b^4*x^8 - 5880*a^5*b^3*x^6 - 1764*a^6*b^2*x^4 - 360*a^7*b*x^2
 - 35*a^8)/x^9
```

Sympy [A] time = 0.602902, size = 99, normalized size = 0.97

$$56a^{3}b^{5}x+\frac{28a^{2}b^{6}x^{3}}{3}+\frac{8ab^{7}x^{5}}{5}+\frac{b^{8}x^{7}}{7}-\frac{35a^{8}+360a^{7}bx^{2}+1764a^{6}b^{2}x^{4}+5880a^{5}b^{3}x^{6}+22050a^{4}b^{4}x^{8}}{315x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**10,x)
```

```
[Out] 56*a**3*b**5*x + 28*a**2*b**6*x**3/3 + 8*a*b**7*x**5/5 + b**8*x**7/7 - (35*
a**8 + 360*a**7*b*x**2 + 1764*a**6*b**2*x**4 + 5880*a**5*b**3*x**6 + 22050*
a**4*b**4*x**8)/(315*x**9)
```

Giac [A] time = 1.94556, size = 123, normalized size = 1.21

$$\frac{1}{7}\,b^{8}x^{7}+\frac{8}{5}\,ab^{7}x^{5}+\frac{28}{3}\,a^{2}b^{6}x^{3}+56\,a^{3}b^{5}x-\frac{22050\,a^{4}b^{4}x^{8}+5880\,a^{5}b^{3}x^{6}+1764\,a^{6}b^{2}x^{4}+360\,a^{7}bx^{2}+35\,a^{8}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="giac")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

3.119 $\int \frac{(a+bx^2)^8}{x^{12}}\,dx$

Optimal. Leaf size=100

$$-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

```
[Out] -a^8/(11*x^11) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Rubi [A] time = 0.0398323, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^12,x]
```

```
[Out] -a^8/(11*x^11) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{12}}\,dx &= \int \left(28a^2b^6+\frac{a^8}{x^{12}}+\frac{8a^7b}{x^{10}}+\frac{28a^6b^2}{x^8}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^2}+8ab^7x^2+b^8x^4\right)dx\\ &= -\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0103317, size = 100, normalized size = 1.

$$-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^12,x]
```

```
[Out] -a^8/(11*x^11) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Maple [A] time = 0.007, size = 89, normalized size = 0.9

$$-\frac{a^{8}}{11\,x^{11}}-\frac{8\,a^{7}b}{9\,x^{9}}-4\,\frac{a^{6}b^{2}}{x^{7}}-\frac{56\,a^{5}b^{3}}{5\,x^{5}}-\frac{70\,a^{4}b^{4}}{3\,x^{3}}-56\,\frac{a^{3}b^{5}}{x}+28\,a^{2}b^{6}x+\frac{8\,ab^{7}x^{3}}{3}+\frac{b^{8}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^12,x)
```

```
[Out] -1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^
3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

Maxima [A] time = 2.00433, size = 123, normalized size = 1.23

$$\frac{1}{5}\,b^{8}x^{5}+\frac{8}{3}\,ab^{7}x^{3}+28\,a^{2}b^{6}x-\frac{27720\,a^{3}b^{5}x^{10}+11550\,a^{4}b^{4}x^{8}+5544\,a^{5}b^{3}x^{6}+1980\,a^{6}b^{2}x^{4}+440\,a^{7}bx^{2}+45\,a^{8}}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="maxima")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

Fricas [A] time = 1.28241, size = 228, normalized size = 2.28

$$\frac{99\,b^{8}x^{16}+1320\,ab^{7}x^{14}+13860\,a^{2}b^{6}x^{12}-27720\,a^{3}b^{5}x^{10}-11550\,a^{4}b^{4}x^{8}-5544\,a^{5}b^{3}x^{6}-1980\,a^{6}b^{2}x^{4}-440\,a^{7}bx^{2}-4}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="fricas")
```

```
[Out] 1/495*(99*b^8*x^16 + 1320*a*b^7*x^14 + 13860*a^2*b^6*x^12 - 27720*a^3*b^5*x
^10 - 11550*a^4*b^4*x^8 - 5544*a^5*b^3*x^6 - 1980*a^6*b^2*x^4 - 440*a^7*b*x
^2 - 45*a^8)/x^11
```

Sympy [A] time = 0.759729, size = 97, normalized size = 0.97

$$28a^{2}b^{6}x+\frac{8ab^{7}x^{3}}{3}+\frac{b^{8}x^{5}}{5}-\frac{45a^{8}+440a^{7}bx^{2}+1980a^{6}b^{2}x^{4}+5544a^{5}b^{3}x^{6}+11550a^{4}b^{4}x^{8}+27720a^{3}b^{5}x^{10}}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**12,x)
```

```
[Out] 28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5 - (45*a**8 + 440*a**7*b*x**2
 + 1980*a**6*b**2*x**4 + 5544*a**5*b**3*x**6 + 11550*a**4*b**4*x**8 + 27720
*a**3*b**5*x**10)/(495*x**11)
```

<u>Giac</u> [A] time = 2.58107, size = 123, normalized size = 1.23

$$\frac{1}{5}\,b^{8}x^{5} + \frac{8}{3}\,ab^{7}x^{3} + 28\,a^{2}b^{6}x - \frac{27720\,a^{3}b^{5}x^{10} + 11550\,a^{4}b^{4}x^{8} + 5544\,a^{5}b^{3}x^{6} + 1980\,a^{6}b^{2}x^{4} + 440\,a^{7}bx^{2} + 45\,a^{8}}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="giac")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

3.120 $\int \frac{(a+bx^2)^8}{x^{14}} dx$

Optimal. Leaf size=98

$$-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}+8ab^7x+\frac{b^8x^3}{3}$$

```
[Out] -a^8/(13*x^13) - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Rubi [A] time = 0.0377404, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^14,x]
```

```
[Out] -a^8/(13*x^13) - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{14}} dx &= \int \left(8ab^7+\frac{a^8}{x^{14}}+\frac{8a^7b}{x^{12}}+\frac{28a^6b^2}{x^{10}}+\frac{56a^5b^3}{x^8}+\frac{70a^4b^4}{x^6}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^2}+b^8x^2\right) dx\\ &= -\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0097264, size = 98, normalized size = 1.

$$-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^14,x]
```

```
[Out] -a^8/(13*x^13) - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Maple [A] time = 0.008, size = 89, normalized size = 0.9

$$-\frac{a^{8}}{13\,x^{13}}-\frac{8\,a^{7}b}{11\,x^{11}}-\frac{28\,a^{6}b^{2}}{9\,x^{9}}-8\,\frac{a^{5}b^{3}}{x^{7}}-14\,\frac{a^{4}b^{4}}{x^{5}}-\frac{56\,a^{3}b^{5}}{3\,x^{3}}-28\,\frac{a^{2}b^{6}}{x}+8\,ab^{7}x+\frac{b^{8}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^14,x)
```

```
[Out] -1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^
5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

Maxima [A] time = 2.71249, size = 123, normalized size = 1.26

$$\frac{1}{3}\,b^{8}x^{3}+8\,ab^{7}x-\frac{36036\,a^{2}b^{6}x^{12}+24024\,a^{3}b^{5}x^{10}+18018\,a^{4}b^{4}x^{8}+10296\,a^{5}b^{3}x^{6}+4004\,a^{6}b^{2}x^{4}+936\,a^{7}bx^{2}+99\,a^{8}}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="maxima")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

Fricas [A] time = 1.12773, size = 234, normalized size = 2.39

$$\frac{429\,b^{8}x^{16}+10296\,ab^{7}x^{14}-36036\,a^{2}b^{6}x^{12}-24024\,a^{3}b^{5}x^{10}-18018\,a^{4}b^{4}x^{8}-10296\,a^{5}b^{3}x^{6}-4004\,a^{6}b^{2}x^{4}-936\,a^{7}bx^{2}-9}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="fricas")
```

```
[Out] 1/1287*(429*b^8*x^16 + 10296*a*b^7*x^14 - 36036*a^2*b^6*x^12 - 24024*a^3*b^
5*x^10 - 18018*a^4*b^4*x^8 - 10296*a^5*b^3*x^6 - 4004*a^6*b^2*x^4 - 936*a^7
*b*x^2 - 99*a^8)/x^13
```

Sympy [A] time = 0.888839, size = 95, normalized size = 0.97

$$8ab^{7}x+\frac{b^{8}x^{3}}{3}-\frac{99a^{8}+936a^{7}bx^{2}+4004a^{6}b^{2}x^{4}+10296a^{5}b^{3}x^{6}+18018a^{4}b^{4}x^{8}+24024a^{3}b^{5}x^{10}+36036a^{2}b^{6}x^{12}}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**14,x)
```

```
[Out] 8*a*b**7*x + b**8*x**3/3 - (99*a**8 + 936*a**7*b*x**2 + 4004*a**6*b**2*x**4
 + 10296*a**5*b**3*x**6 + 18018*a**4*b**4*x**8 + 24024*a**3*b**5*x**10 + 36
036*a**2*b**6*x**12)/(1287*x**13)
```

Giac [A] time = 2.13636, size = 123, normalized size = 1.26

$$\frac{1}{3}b^8x^3 + 8ab^7x - \frac{36036a^2b^6x^{12} + 24024a^3b^5x^{10} + 18018a^4b^4x^8 + 10296a^5b^3x^6 + 4004a^6b^2x^4 + 936a^7bx^2 + 99a^8}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="giac")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

3.121 $\int \frac{(a+bx^2)^8}{x^{16}} \, dx$

Optimal. Leaf size=99

$$-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a^7b}{13x^{13}}-\frac{a^8}{15x^{15}}-\frac{8ab^7}{x}+b^8x$$

```
[Out] -a^8/(15*x^15) - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Rubi [A] time = 0.0393084, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a^7b}{13x^{13}}-\frac{a^8}{15x^{15}}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^16,x]
```

```
[Out] -a^8/(15*x^15) - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{16}} \, dx &= \int \left(b^8+\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{12}}+\frac{56a^5b^3}{x^{10}}+\frac{70a^4b^4}{x^8}+\frac{56a^3b^5}{x^6}+\frac{28a^2b^6}{x^4}+\frac{8ab^7}{x^2}\right) dx\\ &= -\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x\end{aligned}$$

Mathematica [A] time = 0.0053595, size = 99, normalized size = 1.

$$-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a^7b}{13x^{13}}-\frac{a^8}{15x^{15}}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^16,x]
```

```
[Out] -a^8/(15*x^15) - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Maple [A] time = 0.007, size = 88, normalized size = 0.9

$$-\frac{a^8}{15\,x^{15}}-\frac{8\,a^7b}{13\,x^{13}}-\frac{28\,a^6b^2}{11\,x^{11}}-\frac{56\,a^5b^3}{9\,x^9}-10\,\frac{a^4b^4}{x^7}-\frac{56\,a^3b^5}{5\,x^5}-\frac{28\,a^2b^6}{3\,x^3}-8\,\frac{ab^7}{x}+b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^16,x)
```

```
[Out] -1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*b
^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

Maxima [A] time = 1.3335, size = 122, normalized size = 1.23

$$b^8x-\frac{51480\,ab^7x^{14}+60060\,a^2b^6x^{12}+72072\,a^3b^5x^{10}+64350\,a^4b^4x^8+40040\,a^5b^3x^6+16380\,a^6b^2x^4+3960\,a^7bx^2+}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="maxima")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

Fricas [A] time = 1.26084, size = 239, normalized size = 2.41

$$\frac{6435\,b^8x^{16}-51480\,ab^7x^{14}-60060\,a^2b^6x^{12}-72072\,a^3b^5x^{10}-64350\,a^4b^4x^8-40040\,a^5b^3x^6-16380\,a^6b^2x^4-3960\,a^7}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="fricas")
```

```
[Out] 1/6435*(6435*b^8*x^16 - 51480*a*b^7*x^14 - 60060*a^2*b^6*x^12 - 72072*a^3*b
^5*x^10 - 64350*a^4*b^4*x^8 - 40040*a^5*b^3*x^6 - 16380*a^6*b^2*x^4 - 3960*
a^7*b*x^2 - 429*a^8)/x^15
```

Sympy [A] time = 0.977237, size = 94, normalized size = 0.95

$$b^8x-\frac{429a^8+3960a^7bx^2+16380a^6b^2x^4+40040a^5b^3x^6+64350a^4b^4x^8+72072a^3b^5x^{10}+60060a^2b^6x^{12}+51480ab^7}{6435x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**16,x)
```

```
[Out] b**8*x - (429*a**8 + 3960*a**7*b*x**2 + 16380*a**6*b**2*x**4 + 40040*a**5*b
**3*x**6 + 64350*a**4*b**4*x**8 + 72072*a**3*b**5*x**10 + 60060*a**2*b**6*x
**12 + 51480*a*b**7*x**14)/(6435*x**15)
```

Giac [A] time = 2.15924, size = 122, normalized size = 1.23

$$b^8x - \frac{51480\,ab^7x^{14} + 60060\,a^2b^6x^{12} + 72072\,a^3b^5x^{10} + 64350\,a^4b^4x^8 + 40040\,a^5b^3x^6 + 16380\,a^6b^2x^4 + 3960\,a^7bx^2 + 429}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="giac")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

3.122 $\int \frac{(a+bx^2)^8}{x^{18}}\,dx$

Optimal. Leaf size=104

$$-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{17x^{17}}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

```
[Out] -a^8/(17*x^17) - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Rubi [A] time = 0.0393411, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{17x^{17}}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^18,x]
```

```
[Out] -a^8/(17*x^17) - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{18}}\,dx &= \int \left(\frac{a^8}{x^{18}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{10}}+\frac{56a^3b^5}{x^8}+\frac{28a^2b^6}{x^6}+\frac{8ab^7}{x^4}+\frac{b^8}{x^2}\right)dx\\ &= -\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}\end{aligned}$$

Mathematica [A] time = 0.0097595, size = 104, normalized size = 1.

$$-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{17x^{17}}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^18,x]
```

```
[Out] -a^8/(17*x^17) - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Maple [A] time = 0.006, size = 91, normalized size = 0.9

$$-\frac{a^8}{17\,x^{17}}-\frac{8\,a^7b}{15\,x^{15}}-\frac{28\,a^6b^2}{13\,x^{13}}-\frac{56\,a^5b^3}{11\,x^{11}}-\frac{70\,a^4b^4}{9\,x^9}-8\,\frac{a^3b^5}{x^7}-\frac{28\,a^2b^6}{5\,x^5}-\frac{8\,ab^7}{3\,x^3}-\frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^18,x)
```

```
[Out] -1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*a
^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

Maxima [A] time = 1.54452, size = 124, normalized size = 1.19

$$-\frac{109395\,b^8x^{16}+291720\,ab^7x^{14}+612612\,a^2b^6x^{12}+875160\,a^3b^5x^{10}+850850\,a^4b^4x^8+556920\,a^5b^3x^6+235620\,a^6b^2x^4+\cdots}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="maxima")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Fricas [A] time = 1.17504, size = 257, normalized size = 2.47

$$-\frac{109395\,b^8x^{16}+291720\,ab^7x^{14}+612612\,a^2b^6x^{12}+875160\,a^3b^5x^{10}+850850\,a^4b^4x^8+556920\,a^5b^3x^6+235620\,a^6b^2x^4+\cdots}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="fricas")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Sympy [A] time = 1.00391, size = 99, normalized size = 0.95

$$-\frac{6435a^8+58344a^7bx^2+235620a^6b^2x^4+556920a^5b^3x^6+850850a^4b^4x^8+875160a^3b^5x^{10}+612612a^2b^6x^{12}+291720a\cdots}{109395x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**18,x)
```

```
[Out] -(6435*a**8 + 58344*a**7*b*x**2 + 235620*a**6*b**2*x**4 + 556920*a**5*b**3*
x**6 + 850850*a**4*b**4*x**8 + 875160*a**3*b**5*x**10 + 612612*a**2*b**6*x*
*12 + 291720*a*b**7*x**14 + 109395*b**8*x**16)/(109395*x**17)
```

Giac [A] time = 2.39408, size = 124, normalized size = 1.19

$$-\frac{109395\,b^8x^{16}+291720\,ab^7x^{14}+612612\,a^2b^6x^{12}+875160\,a^3b^5x^{10}+850850\,a^4b^4x^8+556920\,a^5b^3x^6+235620\,a^6b^2}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="giac")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

3.123 $\int \frac{(a+bx^2)^8}{x^{20}}\,dx$

Optimal. Leaf size=106

$$-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a^7b}{17x^{17}}-\frac{a^8}{19x^{19}}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

```
[Out] -a^8/(19*x^19) - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Rubi [A] time = 0.0395873, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a^7b}{17x^{17}}-\frac{a^8}{19x^{19}}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^20,x]
```

```
[Out] -a^8/(19*x^19) - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{20}}\,dx &= \int \left(\frac{a^8}{x^{20}}+\frac{8a^7b}{x^{18}}+\frac{28a^6b^2}{x^{16}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^8}+\frac{8ab^7}{x^6}+\frac{b^8}{x^4}\right)dx\\ &= -\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0112797, size = 106, normalized size = 1.

$$-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a^7b}{17x^{17}}-\frac{a^8}{19x^{19}}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^20,x]
```

```
[Out] -a^8/(19*x^19) - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Maple [A] time = 0.007, size = 91, normalized size = 0.9

$$-\frac{a^{8}}{19\,x^{19}}-\frac{8\,a^{7}b}{17\,x^{17}}-\frac{28\,a^{6}b^{2}}{15\,x^{15}}-\frac{56\,a^{5}b^{3}}{13\,x^{13}}-\frac{70\,a^{4}b^{4}}{11\,x^{11}}-\frac{56\,a^{3}b^{5}}{9\,x^{9}}-4\,\frac{a^{2}b^{6}}{x^{7}}-\frac{8\,ab^{7}}{5\,x^{5}}-\frac{b^{8}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^20,x)
```

```
[Out] -1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11*
a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

Maxima [A] time = 2.64907, size = 124, normalized size = 1.17

$$-\frac{692835\,b^{8}x^{16}+3325608\,ab^{7}x^{14}+8314020\,a^{2}b^{6}x^{12}+12932920\,a^{3}b^{5}x^{10}+13226850\,a^{4}b^{4}x^{8}+8953560\,a^{5}b^{3}x^{6}+387}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="maxima")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Fricas [A] time = 1.17586, size = 273, normalized size = 2.58

$$-\frac{692835\,b^{8}x^{16}+3325608\,ab^{7}x^{14}+8314020\,a^{2}b^{6}x^{12}+12932920\,a^{3}b^{5}x^{10}+13226850\,a^{4}b^{4}x^{8}+8953560\,a^{5}b^{3}x^{6}+387}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="fricas")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Sympy [A] time = 1.05334, size = 99, normalized size = 0.93

$$-\frac{109395a^{8}+978120a^{7}bx^{2}+3879876a^{6}b^{2}x^{4}+8953560a^{5}b^{3}x^{6}+13226850a^{4}b^{4}x^{8}+12932920a^{3}b^{5}x^{10}+8314020a^{2}b}{2078505x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**20,x)
```

```
[Out] -(109395*a**8 + 978120*a**7*b*x**2 + 3879876*a**6*b**2*x**4 + 8953560*a**5*
b**3*x**6 + 13226850*a**4*b**4*x**8 + 12932920*a**3*b**5*x**10 + 8314020*a*
*2*b**6*x**12 + 3325608*a*b**7*x**14 + 692835*b**8*x**16)/(2078505*x**19)
```

Giac [A] time = 1.57089, size = 124, normalized size = 1.17

$$-\frac{692835\, b^8 x^{16} + 3325608\, a b^7 x^{14} + 8314020\, a^2 b^6 x^{12} + 12932920\, a^3 b^5 x^{10} + 13226850\, a^4 b^4 x^8 + 8953560\, a^5 b^3 x^6 + 38798}{2078505\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="giac")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

3.124 $\int \frac{x^{11}}{a+bx^2}\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^6}{6b^3}-\frac{a^3x^4}{4b^4}+\frac{a^4x^2}{2b^5}-\frac{a^5\log\left(a+bx^2\right)}{2b^6}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] time = 0.0558447, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{6b^3}-\frac{a^3x^4}{4b^4}+\frac{a^4x^2}{2b^5}-\frac{a^5\log\left(a+bx^2\right)}{2b^6}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}-\frac{a^5\log\left(a+bx^2\right)}{2b^6}
\end{aligned}$$

Mathematica [A] time = 0.005609, size = 79, normalized size = 1.

$$\frac{a^2x^6}{6b^3}-\frac{a^3x^4}{4b^4}+\frac{a^4x^2}{2b^5}-\frac{a^5\log\left(a+bx^2\right)}{2b^6}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Maple [A] time = 0.003, size = 68, normalized size = 0.9

$$\frac{a^4x^2}{2\,b^5}-\frac{a^3x^4}{4\,b^4}+\frac{a^2x^6}{6\,b^3}-\frac{ax^8}{8\,b^2}+\frac{x^{10}}{10\,b}-\frac{a^5\ln\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a),x)
```

```
[Out] 1/2*a^4*x^2/b^5-1/4*a^3*x^4/b^4+1/6*a^2*x^6/b^3-1/8*a*x^8/b^2+1/10*x^10/b-1
/2*a^5*ln(b*x^2+a)/b^6
```

Maxima [A] time = 1.34039, size = 92, normalized size = 1.16

$$-\frac{a^5\log\left(bx^2+a\right)}{2\,b^6}+\frac{12\,b^4x^{10}-15\,ab^3x^8+20\,a^2b^2x^6-30\,a^3bx^4+60\,a^4x^2}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^5*log(b*x^2 + a)/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a^2*b^
2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

Fricas [A] time = 1.24124, size = 153, normalized size = 1.94

$$\frac{12\,b^5x^{10}-15\,ab^4x^8+20\,a^2b^3x^6-30\,a^3b^2x^4+60\,a^4bx^2-60\,a^5\log\left(bx^2+a\right)}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*x^10 - 15*a*b^4*x^8 + 20*a^2*b^3*x^6 - 30*a^3*b^2*x^4 + 60*a^
4*b*x^2 - 60*a^5*log(b*x^2 + a))/b^6
```

Sympy [A] time = 0.319619, size = 68, normalized size = 0.86

$$-\frac{a^5\log\left(a+bx^2\right)}{2b^6}+\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a),x)
```

```
[Out] -a**5*log(a + b*x**2)/(2*b**6) + a**4*x**2/(2*b**5) - a**3*x**4/(4*b**4) +
a**2*x**6/(6*b**3) - a*x**8/(8*b**2) + x**10/(10*b)
```

Giac [A] time = 1.78218, size = 93, normalized size = 1.18

$$-\frac{a^5\log\left(\left|bx^2+a\right|\right)}{2\,b^6}+\frac{12\,b^4x^{10}-15\,ab^3x^8+20\,a^2b^2x^6-30\,a^3bx^4+60\,a^4x^2}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^5*log(abs(b*x^2 + a))/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a
^2*b^2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

3.125 $\int \frac{x^{10}}{a+bx^2} dx$

Optimal. Leaf size=81

$$\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rubi [A] time = 0.0350131, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{a+bx^2} dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^6}{b^2}+\frac{x^8}{b}-\frac{a^5}{b^5\left(a+bx^2\right)}\right) dx\\
&= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^5\int \frac{1}{a+bx^2} dx}{b^5}\\
&= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.0339796, size = 81, normalized size = 1.

$$\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Maple [A] time = 0.004, size = 71, normalized size = 0.9

$$\frac{x^9}{9\,b} - \frac{a x^7}{7\,b^2} + \frac{a^2 x^5}{5\,b^3} - \frac{a^3 x^3}{3\,b^4} + \frac{a^4 x}{b^5} - \frac{a^5}{b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a),x)
```

```
[Out] 1/9*x^9/b-1/7*a*x^7/b^2+1/5*a^2*x^5/b^3-1/3*a^3*x^3/b^4+a^4*x/b^5-a^5/b^5/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34757, size = 386, normalized size = 4.77

$$\left[\frac{70\,b^4 x^9 - 90\,ab^3 x^7 + 126\,a^2 b^2 x^5 - 210\,a^3 bx^3 + 315\,a^4 \sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 630\,a^4 x}{630\,b^5}, \frac{35\,b^4 x^9 - 45\,ab^3 x^7 + 63\,a^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*x^9 - 90*a*b^3*x^7 + 126*a^2*b^2*x^5 - 210*a^3*b*x^3 + 315*a
^4*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*a^4*x)/
b^5, 1/315*(35*b^4*x^9 - 45*a*b^3*x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 - 31
5*a^4*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315*a^4*x)/b^5]
```

Sympy [A] time = 0.341252, size = 119, normalized size = 1.47

$$\frac{a^4 x}{b^5} - \frac{a^3 x^3}{3b^4} + \frac{a^2 x^5}{5b^3} - \frac{a x^7}{7b^2} + \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} - \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} + \frac{x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a),x)
```

```
[Out] a**4*x/b**5 - a**3*x**3/(3*b**4) + a**2*x**5/(5*b**3) - a*x**7/(7*b**2) + s
qrt(-a**9/b**11)*log(x - b**5*sqrt(-a**9/b**11)/a**4)/2 - sqrt(-a**9/b**11)
*log(x + b**5*sqrt(-a**9/b**11)/a**4)/2 + x**9/(9*b)
```

Giac [A] time = 2.2614, size = 104, normalized size = 1.28

$$-\frac{a^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8x^9 - 45\,ab^7x^7 + 63\,a^2b^6x^5 - 105\,a^3b^5x^3 + 315\,a^4b^4x}{315\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*x^9 - 45*a*b^7*x
^7 + 63*a^2*b^6*x^5 - 105*a^3*b^5*x^3 + 315*a^4*b^4*x)/b^9
```

3.126 $\int \frac{x^9}{a+bx^2} dx$

Optimal. Leaf size=66

$$\frac{a^2x^4}{4b^3} - \frac{a^3x^2}{2b^4} + \frac{a^4\log(a+bx^2)}{2b^5} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

```
[Out] -(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.0443748, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^4}{4b^3} - \frac{a^3x^2}{2b^4} + \frac{a^4\log(a+bx^2)}{2b^5} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2),x]
```

```
[Out] -(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b} + \frac{a^4\log(a+bx^2)}{2b^5}
\end{aligned}$$

Mathematica [A] time = 0.0055564, size = 66, normalized size = 1.

$$\frac{a^2x^4}{4b^3} - \frac{a^3x^2}{2b^4} + \frac{a^4\log(a+bx^2)}{2b^5} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2),x]
```

```
[Out] -(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Maple [A] time = 0.004, size = 57, normalized size = 0.9

$$-\frac{a^3x^2}{2\,b^4}+\frac{a^2x^4}{4\,b^3}-\frac{ax^6}{6\,b^2}+\frac{x^8}{8\,b}+\frac{a^4\ln\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a),x)
```

```
[Out] -1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*ln(b*x^2+a
)/b^5
```

Maxima [A] time = 2.28757, size = 77, normalized size = 1.17

$$\frac{a^4\log\left(bx^2+a\right)}{2\,b^5}+\frac{3\,b^3x^8-4\,ab^2x^6+6\,a^2bx^4-12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^4*log(b*x^2 + a)/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 -
12*a^3*x^2)/b^4
```

Fricas [A] time = 1.28839, size = 123, normalized size = 1.86

$$\frac{3\,b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-12\,a^3bx^2+12\,a^4\log\left(bx^2+a\right)}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 12*a^3*b*x^2 + 12*a^4*log(b
*x^2 + a))/b^5
```

Sympy [A] time = 0.309818, size = 56, normalized size = 0.85

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a),x)
```

```
[Out] a**4*log(a + b*x**2)/(2*b**5) - a**3*x**2/(2*b**4) + a**2*x**4/(4*b**3) - a
*x**6/(6*b**2) + x**8/(8*b)
```

Giac [A] time = 2.39218, size = 78, normalized size = 1.18

$$\frac{a^4 \log \left(|bx^2 + a|\right)}{2\, b^5} + \frac{3\, b^3 x^8 - 4\, ab^2 x^6 + 6\, a^2 bx^4 - 12\, a^3 x^2}{24\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^4*log(abs(b*x^2 + a))/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x
^4 - 12*a^3*x^2)/b^4
```

3.127 $\int \frac{x^8}{a+bx^2}\,dx$

Optimal. Leaf size=68

$$\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.0295362, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a+bx^2}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^4\int \frac{1}{a+bx^2}\,dx}{b^4}\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0251603, size = 68, normalized size = 1.

$$\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.003, size = 60, normalized size = 0.9

$$\frac{x^7}{7\,b}-\frac{ax^5}{5\,b^2}+\frac{a^2x^3}{3\,b^3}-\frac{a^3x}{b^4}+\frac{a^4}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a),x)
```

```
[Out] 1/7*x^7/b-1/5*a*x^5/b^2+1/3*a^2*x^3/b^3-a^3*x/b^4+a^4/b^4/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32892, size = 336, normalized size = 4.94

$$\left[\frac{30\,b^3x^7-42\,ab^2x^5+70\,a^2bx^3+105\,a^3\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,a^3x}{210\,b^4},\ \frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3+105\,a^3\sqrt{\frac{a}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*x^7 - 42*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*sqrt(-a/b)*log((
b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*a^3*x)/b^4, 1/105*(15*b^3*
x^7 - 21*a*b^2*x^5 + 35*a^2*b*x^3 + 105*a^3*sqrt(a/b)*arctan(b*x*sqrt(a/b)/
a) - 105*a^3*x)/b^4]
```

Sympy [A] time = 0.329173, size = 107, normalized size = 1.57

$$-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}-\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a),x)
```

```
[Out] -a**3*x/b**4 + a**2*x**3/(3*b**3) - a*x**5/(5*b**2) - sqrt(-a**7/b**9)*log(
x - b**4*sqrt(-a**7/b**9)/a**3)/2 + sqrt(-a**7/b**9)*log(x + b**4*sqrt(-a**
7/b**9)/a**3)/2 + x**7/(7*b)
```

Giac [A] time = 2.50869, size = 88, normalized size = 1.29

$$\frac{a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6x^7 - 21\,ab^5x^5 + 35\,a^2b^4x^3 - 105\,a^3b^3x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*x^7 - 21*a*b^5*x^
5 + 35*a^2*b^4*x^3 - 105*a^3*b^3*x)/b^7
```

3.128 $\int \frac{x^7}{a+bx^2}\,dx$

Optimal. Leaf size=53

$$\frac{a^2x^2}{2b^3}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Rubi [A] time = 0.0350458, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^2}{2b^3}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.0053087, size = 53, normalized size = 1.

$$\frac{a^2x^2}{2b^3}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Maple [A] time = 0.003, size = 46, normalized size = 0.9

$$\frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b} - \frac{a^3\ln\left(bx^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a),x)
```

```
[Out] 1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*ln(b*x^2+a)/b^4
```

Maxima [A] time = 2.59227, size = 62, normalized size = 1.17

$$-\frac{a^3\log\left(bx^2+a\right)}{2b^4} + \frac{2b^2x^6 - 3abx^4 + 6a^2x^2}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*log(b*x^2 + a)/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3
```

Fricas [A] time = 1.2818, size = 99, normalized size = 1.87

$$\frac{2b^3x^6 - 3ab^2x^4 + 6a^2bx^2 - 6a^3\log\left(bx^2+a\right)}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^6 - 3*a*b^2*x^4 + 6*a^2*b*x^2 - 6*a^3*log(b*x^2 + a))/b^4
```

Sympy [A] time = 0.305953, size = 44, normalized size = 0.83

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a),x)
```

```
[Out] -a**3*log(a + b*x**2)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**
6/(6*b)
```

Giac [A] time = 2.38341, size = 63, normalized size = 1.19

$$-\frac{a^3 \log \left(|bx^2 + a|\right)}{2\,b^4} + \frac{2\,b^2x^6 - 3\,abx^4 + 6\,a^2x^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^3*log(abs(b*x^2 + a))/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)
/b^3
```

3.129 $\int \frac{x^6}{a+bx^2} dx$

Optimal. Leaf size=55

$$\frac{a^2x}{b^3} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.0239212, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x}{b^3} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{a+bx^2} dx &= \int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3\left(a+bx^2\right)}\right) dx \\
&= \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^3 \int \frac{1}{a+bx^2} dx}{b^3} \\
&= \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0222535, size = 55, normalized size = 1.

$$\frac{a^2x}{b^3} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

Maple [A] time = 0.003, size = 49, normalized size = 0.9

$$\frac{x^5}{5\,b}-\frac{ax^3}{3\,b^2}+\frac{a^2x}{b^3}-\frac{a^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a),x)
```

```
[Out] 1/5*x^5/b-1/3*a*x^3/b^2+a^2*x/b^3-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.17411, size = 278, normalized size = 5.05

$$\left[\frac{6\,b^2x^5-10\,abx^3+15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,a^2x}{30\,b^3},\frac{3\,b^2x^5-5\,abx^3-15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\,a^2x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*x^5 - 10*a*b*x^3 + 15*a^2*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-
a/b) - a)/(b*x^2 + a)) + 30*a^2*x)/b^3, 1/15*(3*b^2*x^5 - 5*a*b*x^3 - 15*a^
2*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*a^2*x)/b^3]
```

Sympy [A] time = 0.327934, size = 95, normalized size = 1.73

$$\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2}-\frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2}+\frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a),x)
```

```
[Out] a**2*x/b**3 - a*x**3/(3*b**2) + sqrt(-a**5/b**7)*log(x - b**3*sqrt(-a**5/b*
*7)/a**2)/2 - sqrt(-a**5/b**7)*log(x + b**3*sqrt(-a**5/b**7)/a**2)/2 + x**5
/(5*b)
```

Giac [A] time = 2.63796, size = 74, normalized size = 1.35

$$-\frac{a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4x^5 - 5\,ab^3x^3 + 15\,a^2b^2x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*x^5 - 5*a*b^3*x^3
+ 15*a^2*b^2*x)/b^5
```

3.130 $\int \frac{x^5}{a+bx^2} dx$

Optimal. Leaf size=40

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

```
[Out] -(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.0267045, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2),x]
```

```
[Out] -(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{ax^2}{2b^2} + \frac{x^4}{4b} + \frac{a^2 \log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0052148, size = 40, normalized size = 1.

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2),x]
```

```
[Out] -(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.002, size = 35, normalized size = 0.9

$$-\frac{ax^2}{2\,b^2}+\frac{x^4}{4\,b}+\frac{a^2\ln\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a),x)
```

```
[Out] -1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*ln(b*x^2+a)/b^3
```

Maxima [A] time = 1.88058, size = 46, normalized size = 1.15

$$\frac{a^2\log\left(bx^2+a\right)}{2\,b^3}+\frac{bx^4-2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Fricas [A] time = 1.23125, size = 73, normalized size = 1.82

$$\frac{b^2x^4-2\,abx^2+2\,a^2\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [A] time = 0.300961, size = 32, normalized size = 0.8

$$\frac{a^2\log\left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

Giac [A] time = 1.52663, size = 47, normalized size = 1.18

$$\frac{a^2 \log\left(\left|bx^2 + a\right|\right)}{2\,b^3} + \frac{bx^4 - 2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

3.131 $\int \frac{x^4}{a+bx^2} \, dx$

Optimal. Leaf size=42

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.0198166, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2),x]
```

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{a+bx^2} \, dx &= \int \left(-\frac{a}{b^2} + \frac{x^2}{b} + \frac{a^2}{b^2 \left(a+bx^2\right)}\right) dx \\ &= -\frac{ax}{b^2} + \frac{x^3}{3b} + \frac{a^2 \int \frac{1}{a+bx^2} \, dx}{b^2} \\ &= -\frac{ax}{b^2} + \frac{x^3}{3b} + \frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0191931, size = 42, normalized size = 1.

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2),x]
```

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A] time = 0.003, size = 38, normalized size = 0.9

$$\frac{x^3}{3b} - \frac{ax}{b^2} + \frac{a^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a),x)
```

```
[Out] 1/3*x^3/b-a*x/b^2+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29122, size = 217, normalized size = 5.17

$$\left[\frac{2\,bx^3 + 3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) - 6\,ax}{6\,b^2}, \frac{bx^3 + 3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 3\,ax}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

Sympy [B] time = 0.309992, size = 80, normalized size = 1.9

$$-\frac{ax}{b^2} - \frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x - \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x + \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

Giac [A] time = 1.44787, size = 54, normalized size = 1.29

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2x^3 - 3\,abx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3 - 3*a*b*x)/b^3
```

3.132 $\int \frac{x^3}{a+bx^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b} - \frac{a \log \left(a + bx^2\right)}{2b^2}$$

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0176019, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{x^2}{2b} - \frac{a \log \left(a + bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a + bx^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x}{a + bx} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a + bx)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{2b} - \frac{a \log \left(a + bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0042308, size = 27, normalized size = 1.

$$\frac{x^2}{2b} - \frac{a \log \left(a + bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.002, size = 24, normalized size = 0.9

$$\frac{x^2}{2\,b} - \frac{a\ln\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a),x)
```

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Maxima [A] time = 2.65654, size = 31, normalized size = 1.15

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.29075, size = 49, normalized size = 1.81

$$\frac{bx^2 - a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A] time = 0.280649, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Giac [A] time = 2.65943, size = 32, normalized size = 1.19

$$\frac{x^2}{2\,b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

3.133 $\int \frac{x^2}{a+bx^2} dx$

Optimal. Leaf size=31

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.0117742, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {321, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx^2}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.008222, size = 31, normalized size = 1.

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$\frac{x}{b}-\frac{a}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a),x)
```

```
[Out] x/b-1/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31892, size = 165, normalized size = 5.32

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [B] time = 0.286196, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

Giac [A] time = 2.13923, size = 35, normalized size = 1.13

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

3.134 $\int \frac{x}{a+bx^2} \, dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^2)}{2b}$$

```
[Out] Log[a + b*x^2]/(2*b)
```

Rubi [A] time = 0.0030103, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {260}

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a+bx^2} \, dx = \frac{\log(a+bx^2)}{2b}$$

Mathematica [A] time = 0.0021747, size = 15, normalized size = 1.

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a),x)
```

```
[Out] 1/2*ln(b*x^2+a)/b
```

Maxima [A] time = 1.96971, size = 18, normalized size = 1.2

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Fricas [A] time = 1.25512, size = 30, normalized size = 2.

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A] time = 0.101219, size = 10, normalized size = 0.67

$$\frac{\log\left(a + bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a),x)
```

```
[Out] log(a + b*x**2)/(2*b)
```

Giac [A] time = 1.73615, size = 19, normalized size = 1.27

$$\frac{\log\left(\left|bx^2 + a\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b
```

3.135 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0050293, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.0036061, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.001, size = 16, normalized size = 0.7

$$\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.20772, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] time = 0.119455, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A] time = 1.95056, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.136 $\int \frac{1}{x(a+bx^2)} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rubi [A] time = 0.0109357, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x(a+bx)} \, dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} \, dx, x, x^2\right)}{2a} - \frac{b \, \text{Subst}\left(\int \frac{1}{a+bx} \, dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0047294, size = 22, normalized size = 1.

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Maple [A] time = 0.003, size = 21, normalized size = 1.

$$\frac{\ln(x)}{a}-\frac{\ln\left(bx^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a),x)
```

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Maxima [A] time = 1.33694, size = 31, normalized size = 1.41

$$-\frac{\log\left(bx^2+a\right)}{2a}+\frac{\log\left(x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

Fricas [A] time = 1.34415, size = 49, normalized size = 2.23

$$-\frac{\log\left(bx^2+a\right)-2\log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A] time = 0.179144, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

Giac [A] time = 2.59884, size = 32, normalized size = 1.45

$$\frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

3.137 $\int \frac{1}{x^2(a+bx^2)} dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0121051, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)}\,dx &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.012191, size = 34, normalized size = 1.

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Maple [A] time = 0.004, size = 30, normalized size = 0.9

$$-\frac{1}{ax}-\frac{b}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/x-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27306, size = 173, normalized size = 5.09

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [B] time = 0.321033, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A] time = 1.72229, size = 39, normalized size = 1.15

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

3.138 $\int \frac{1}{x^3(a+bx^2)} \, dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.0222234, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0062615, size = 35, normalized size = 1.

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$-\frac{1}{2\,a x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b x^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a),x)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Maxima [A] time = 1.35467, size = 45, normalized size = 1.29

$$\frac{b\log\left(b x^2+a\right)}{2\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Fricas [A] time = 1.20812, size = 80, normalized size = 2.29

$$\frac{b x^2\log\left(b x^2+a\right)-2\,b x^2\log(x)-a}{2\,a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A] time = 0.398468, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Giac [A] time = 2.04053, size = 58, normalized size = 1.66

$$-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

3.139 $\int \frac{1}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=43

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0172627, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)}\,dx &= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0194677, size = 43, normalized size = 1.

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A] time = 0.006, size = 39, normalized size = 0.9

$$-\frac{1}{3\,a x^3}+\frac{b}{a^2 x}+\frac{b^2}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a),x)
```

```
[Out] -1/3/a/x^3+b/a^2/x+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30642, size = 234, normalized size = 5.44

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

Sympy [B] time = 0.3655, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Giac [A] time = 2.95686, size = 54, normalized size = 1.26

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

3.140 $\int \frac{1}{x^5(a+bx^2)}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rubi [A] time = 0.0282816, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.006423, size = 49, normalized size = 1.

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Maple [A] time = 0.007, size = 44, normalized size = 0.9

$$-\frac{1}{4\,a x^{4}}+\frac{b}{2\,a^{2} x^{2}}+\frac{b^{2} \ln \left(x \right)}{a^{3}}-\frac{b^{2} \ln \left(b \,x^{2}+a \right)}{2\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a),x)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Maxima [A] time = 2.11841, size = 63, normalized size = 1.29

$$-\frac{b^{2} \log \left(b x^{2}+a\right)}{2 \, a^{3}}+\frac{b^{2} \log \left(x^{2}\right)}{2 \, a^{3}}+\frac{2 \, b x^{2}-a}{4 \, a^{2} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + 1/2*b^2*log(x^2)/a^3 + 1/4*(2*b*x^2 - a)/(a^2
*x^4)
```

Fricas [A] time = 1.29664, size = 108, normalized size = 2.2

$$-\frac{2 \, b^{2} x^{4} \log \left(b x^{2}+a\right)-4 \, b^{2} x^{4} \log \left(x\right)-2 \, a b x^{2}+a^{2}}{4 \, a^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

Sympy [A] time = 0.452362, size = 42, normalized size = 0.86

$$\frac{-a+2bx^{2}}{4a^{2}x^{4}}+\frac{b^{2}\log{\left(x \right)}}{a^{3}}-\frac{b^{2}\log{\left(\frac{a}{b}+x^{2} \right)}}{2a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

Giac [A] time = 1.63094, size = 77, normalized size = 1.57

$$\frac{b^2 \log \left(x^2\right)}{2\,a^3} - \frac{b^2 \log \left(\left|bx^2 + a\right|\right)}{2\,a^3} - \frac{3\,b^2x^4 - 2\,abx^2 + a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

3.141 $\int \frac{1}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

```
[Out] -1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(7/2)
```

Rubi [A] time = 0.0245379, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(7/2)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(a+bx^2\right)}\,dx &= -\frac{1}{5ax^5}-\frac{b\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{5ax^5}+\frac{b}{3a^2x^3}+\frac{b^2\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^3}\\
&= -\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0228291, size = 58, normalized size = 1.

$$-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(7/2)
```

Maple [A] time = 0.007, size = 52, normalized size = 0.9

$$-\frac{1}{5\,ax^5}-\frac{b^2}{a^3x}+\frac{b}{3\,a^2x^3}-\frac{b^3}{a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a),x)
```

```
[Out] -1/5/a/x^5-b^2/a^3/x+1/3*b/a^2/x^3-b^3/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29944, size = 296, normalized size = 5.1

$$\left[\frac{15\,b^2x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-30\,b^2x^4+10\,abx^2-6\,a^2}{30\,a^3x^5},-\frac{15\,b^2x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+15\,b^2x^4-5\,abx^2+3\,a^2}{15\,a^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) - 30*b^2*x^4 + 10*a*b*x^2 - 6*a^2)/(a^3*x^5), -1/15*(15*b^2*x^5*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 15*b^2*x^4 - 5*a*b*x^2 + 3*a^2)/(a^3*x^5)]
```

Sympy [B] time = 0.433655, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{b^5}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3}+x\right)}{2}-\frac{3a^2-5abx^2+15b^2x^4}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**7)*log(-a**4*sqrt(-b**5/a**7)/b**3 + x)/2 - sqrt(-b**5/a**7)*
log(a**4*sqrt(-b**5/a**7)/b**3 + x)/2 - (3*a**2 - 5*a*b*x**2 + 15*b**2*x**4
)/(15*a**3*x**5)
```

Giac [A] time = 1.78794, size = 70, normalized size = 1.21

$$-\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}-\frac{15\,b^2x^4-5\,abx^2+3\,a^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

3.142 $\int \frac{1}{x^7(a+bx^2)}\,dx$

Optimal. Leaf size=63

$$-\frac{b^2}{2a^3x^2}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

```
[Out] -1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*Lo
g[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.0346818, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^2}{2a^3x^2}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*Lo
g[a + b*x^2])/(2*a^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.0068349, size = 63, normalized size = 1.

$$-\frac{b^2}{2a^3x^2}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*Lo
g[a + b*x^2])/(2*a^4)
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{1}{6\,a x^{6}}+\frac{b}{4\,a^{2} x^{4}}-\frac{b^{2}}{2\,a^{3} x^{2}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(b x^{2}+a\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a),x)
```

```
[Out] -1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)/
a^4
```

Maxima [A] time = 2.08265, size = 78, normalized size = 1.24

$$\frac{b^{3}\log\left(b x^{2}+a\right)}{2\,a^{4}}-\frac{b^{3}\log\left(x^{2}\right)}{2\,a^{4}}-\frac{6\,b^{2} x^{4}-3\,a b x^{2}+2\,a^{2}}{12\,a^{3} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/a^4 - 1/2*b^3*log(x^2)/a^4 - 1/12*(6*b^2*x^4 - 3*a*b
*x^2 + 2*a^2)/(a^3*x^6)
```

Fricas [A] time = 1.27312, size = 134, normalized size = 2.13

$$\frac{6\,b^{3} x^{6}\log\left(b x^{2}+a\right)-12\,b^{3} x^{6}\log\left(x\right)-6\,a b^{2} x^{4}+3\,a^{2} b x^{2}-2\,a^{3}}{12\,a^{4} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^3*x^6*log(b*x^2 + a) - 12*b^3*x^6*log(x) - 6*a*b^2*x^4 + 3*a^2*b*
x^2 - 2*a^3)/(a^4*x^6)
```

Sympy [A] time = 0.538223, size = 56, normalized size = 0.89

$$-\frac{2a^{2}-3abx^{2}+6b^{2}x^{4}}{12a^{3}x^{6}}-\frac{b^{3}\log\left(x\right)}{a^{4}}+\frac{b^{3}\log\left(\frac{a}{b}+x^{2}\right)}{2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a),x)
```

```
[Out] -(2*a**2 - 3*a*b*x**2 + 6*b**2*x**4)/(12*a**3*x**6) - b**3*log(x)/a**4 + b*
*3*log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 2.87401, size = 95, normalized size = 1.51

$$-\frac{b^3 \log\left(x^2\right)}{2\,a^4} + \frac{b^3 \log\left(\left|bx^2 + a\right|\right)}{2\,a^4} + \frac{11\,b^3x^6 - 6\,ab^2x^4 + 3\,a^2bx^2 - 2\,a^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(x^2)/a^4 + 1/2*b^3*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6
- 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)
```

3.143 $\int \frac{1}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=69

$$-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

```
[Out] -1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.0346157, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(a+bx^2\right)}\,dx &= -\frac{1}{7ax^7}-\frac{b\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}+\frac{b^2\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}-\frac{b^3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^3}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^4}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0246361, size = 69, normalized size = 1.

$$-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Maple [A] time = 0.005, size = 61, normalized size = 0.9

$$-\frac{1}{7\,ax^7}-\frac{b^2}{3\,a^3x^3}+\frac{b}{5\,a^2x^5}+\frac{b^3}{a^4x}+\frac{b^4}{a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a),x)
```

```
[Out] -1/7/a/x^7-1/3*b^2/a^3/x^3+1/5*b/a^2/x^5+b^3/a^4/x+b^4/a^4/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.17482, size = 352, normalized size = 5.1

$$\left[\frac{105\,b^3x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\,b^3x^6-70\,ab^2x^4+42\,a^2bx^2-30\,a^3}{210\,a^4x^7},\frac{105\,b^3x^7\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+105\,b^3x^6-35}{105\,a^4x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(105*b^3*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)) + 210*b^3*x^6 - 70*a*b^2*x^4 + 42*a^2*b*x^2 - 30*a^3)/(a^4*x^7), 1/105*
(105*b^3*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*b^3*x^6 - 35*a*b^2*x^4 + 2
1*a^2*b*x^2 - 15*a^3)/(a^4*x^7)]
```

Sympy [A] time = 0.53454, size = 112, normalized size = 1.62

$$-\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{-15a^3+21a^2bx^2-35ab^2x^4+105b^3x^6}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**7/a**9)*log(-a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + sqrt(-b**7/a**9)
*log(a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + (-15*a**3 + 21*a**2*b*x**2 - 35*a*
b**2*x**4 + 105*b**3*x**6)/(105*a**4*x**7)
```

Giac [A] time = 1.99494, size = 84, normalized size = 1.22

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}+\frac{105\,b^3x^6-35\,ab^2x^4+21\,a^2bx^2-15\,a^3}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

3.144 $\int \frac{1}{x^9\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=75

$$\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

```
[Out] -1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log
[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Rubi [A] time = 0.0408999, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log
[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{8ax^8}+\frac{b}{6a^2x^6}-\frac{b^2}{4a^3x^4}+\frac{b^3}{2a^4x^2}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}\end{aligned}$$

Mathematica [A] time = 0.0065807, size = 75, normalized size = 1.

$$\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log
[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Maple [A] time = 0.005, size = 66, normalized size = 0.9

$$-\frac{1}{8\,ax^{8}}+\frac{b}{6\,a^{2}x^{6}}-\frac{b^{2}}{4\,a^{3}x^{4}}+\frac{b^{3}}{2\,a^{4}x^{2}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx^{2}+a\right)}{2\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a),x)
```

```
[Out] -1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2*
b^4*ln(b*x^2+a)/a^5
```

Maxima [A] time = 2.47016, size = 93, normalized size = 1.24

$$-\frac{b^{4}\log\left(bx^{2}+a\right)}{2\,a^{5}}+\frac{b^{4}\log\left(x^{2}\right)}{2\,a^{5}}+\frac{12\,b^{3}x^{6}-6\,ab^{2}x^{4}+4\,a^{2}bx^{2}-3\,a^{3}}{24\,a^{4}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(b*x^2 + a)/a^5 + 1/2*b^4*log(x^2)/a^5 + 1/24*(12*b^3*x^6 - 6*a
*b^2*x^4 + 4*a^2*b*x^2 - 3*a^3)/(a^4*x^8)
```

Fricas [A] time = 1.25234, size = 159, normalized size = 2.12

$$-\frac{12\,b^{4}x^{8}\log\left(bx^{2}+a\right)-24\,b^{4}x^{8}\log\left(x\right)-12\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-4\,a^{3}bx^{2}+3\,a^{4}}{24\,a^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/24*(12*b^4*x^8*log(b*x^2 + a) - 24*b^4*x^8*log(x) - 12*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

Sympy [A] time = 0.683919, size = 68, normalized size = 0.91

$$\frac{-3a^{3}+4a^{2}bx^{2}-6ab^{2}x^{4}+12b^{3}x^{6}}{24a^{4}x^{8}}+\frac{b^{4}\log\left(x\right)}{a^{5}}-\frac{b^{4}\log\left(\frac{a}{b}+x^{2}\right)}{2a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x**2 - 6*a*b**2*x**4 + 12*b**3*x**6)/(24*a**4*x**8) + b
**4*log(x)/a**5 - b**4*log(a/b + x**2)/(2*a**5)
```

Giac [A] time = 2.74214, size = 109, normalized size = 1.45

$$\frac{b^4 \log\left(x^2\right)}{2\, a^5} - \frac{b^4 \log\left(\left|bx^2 + a\right|\right)}{2\, a^5} - \frac{25\, b^4 x^8 - 12\, ab^3 x^6 + 6\, a^2 b^2 x^4 - 4\, a^3 b x^2 + 3\, a^4}{24\, a^5 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2)/a^5 - 1/2*b^4*log(abs(b*x^2 + a))/a^5 - 1/24*(25*b^4*x^8 -
 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

3.145 $\int \frac{x^{13}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=94

$$\frac{a^2x^6}{2b^4}-\frac{a^3x^4}{b^5}+\frac{5a^4x^2}{2b^6}-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

```
[Out] (5*a^4*x^2)/(2*b^6) - (a^3*x^4)/b^5 + (a^2*x^6)/(2*b^4) - (a*x^8)/(4*b^3) +
 x^10/(10*b^2) - a^6/(2*b^7*(a + b*x^2)) - (3*a^5*Log[a + b*x^2])/b^7
```

Rubi [A] time = 0.0762538, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{2b^4}-\frac{a^3x^4}{b^5}+\frac{5a^4x^2}{2b^6}-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^2,x]
```

```
[Out] (5*a^4*x^2)/(2*b^6) - (a^3*x^4)/b^5 + (a^2*x^6)/(2*b^4) - (a*x^8)/(4*b^3) +
 x^10/(10*b^2) - a^6/(2*b^7*(a + b*x^2)) - (3*a^5*Log[a + b*x^2])/b^7
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13}}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.0299354, size = 83, normalized size = 0.88

$$\frac{10a^2b^3x^6-20a^3b^2x^4+50a^4bx^2-\frac{10a^6}{a+bx^2}-60a^5\log\left(a+bx^2\right)-5ab^4x^8+2b^5x^{10}}{20b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^2,x]
```

```
[Out] (50*a^4*b*x^2 - 20*a^3*b^2*x^4 + 10*a^2*b^3*x^6 - 5*a*b^4*x^8 + 2*b^5*x^10
- (10*a^6)/(a + b*x^2) - 60*a^5*Log[a + b*x^2])/(20*b^7)
```

Maple [A] time = 0.008, size = 85, normalized size = 0.9

$$\frac{5\,a^4x^2}{2\,b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2\,b^4}-\frac{ax^8}{4\,b^3}+\frac{x^{10}}{10\,b^2}-\frac{a^6}{2\,b^7\left(bx^2+a\right)}-3\,\frac{a^5\ln\left(bx^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^2,x)
```

```
[Out] 5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/2
*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

Maxima [A] time = 2.17364, size = 119, normalized size = 1.27

$$-\frac{a^6}{2\left(b^8x^2+ab^7\right)}-\frac{3\,a^5\log\left(bx^2+a\right)}{b^7}+\frac{2\,b^4x^{10}-5\,ab^3x^8+10\,a^2b^2x^6-20\,a^3bx^4+50\,a^4x^2}{20\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^6/(b^8*x^2 + a*b^7) - 3*a^5*log(b*x^2 + a)/b^7 + 1/20*(2*b^4*x^10 -
5*a*b^3*x^8 + 10*a^2*b^2*x^6 - 20*a^3*b*x^4 + 50*a^4*x^2)/b^6
```

Fricas [A] time = 1.27641, size = 221, normalized size = 2.35

$$\frac{2\,b^6x^{12}-3\,ab^5x^{10}+5\,a^2b^4x^8-10\,a^3b^3x^6+30\,a^4b^2x^4+50\,a^5bx^2-10\,a^6-60\left(a^5bx^2+a^6\right)\log\left(bx^2+a\right)}{20\left(b^8x^2+ab^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 - 3*a*b^5*x^10 + 5*a^2*b^4*x^8 - 10*a^3*b^3*x^6 + 30*a^4*b
^2*x^4 + 50*a^5*b*x^2 - 10*a^6 - 60*(a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^8*
x^2 + a*b^7)
```

Sympy [A] time = 0.439369, size = 88, normalized size = 0.94

$$-\frac{a^6}{2ab^7+2b^8x^2}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}+\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**2,x)
```

```
[Out] -a**6/(2*a*b**7 + 2*b**8*x**2) - 3*a**5*log(a + b*x**2)/b**7 + 5*a**4*x**2/
(2*b**6) - a**3*x**4/b**5 + a**2*x**6/(2*b**4) - a*x**8/(4*b**3) + x**10/(1
0*b**2)
```

Giac [A] time = 2.45078, size = 139, normalized size = 1.48

$$-\frac{3\,a^5\log\left(\left|bx^2+a\right|\right)}{b^7}+\frac{6\,a^5bx^2+5\,a^6}{2\left(bx^2+a\right)b^7}+\frac{2\,b^8x^{10}-5\,ab^7x^8+10\,a^2b^6x^6-20\,a^3b^5x^4+50\,a^4b^4x^2}{20\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3*a^5*log(abs(b*x^2 + a))/b^7 + 1/2*(6*a^5*b*x^2 + 5*a^6)/((b*x^2 + a)*b^7
) + 1/20*(2*b^8*x^10 - 5*a*b^7*x^8 + 10*a^2*b^6*x^6 - 20*a^3*b^5*x^4 + 50*a
^4*b^4*x^2)/b^10
```

3.146 $\int \frac{x^{12}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=105

$$\frac{11a^2x^5}{10b^4}-\frac{11a^3x^3}{6b^5}+\frac{11a^4x}{2b^6}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b(a+bx^2)}+\frac{11x^9}{18b^2}$$

```
[Out] (11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x
^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^11/(2*b*(a + b*x^2)) - (11*a^(9/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Rubi [A] time = 0.0455382, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{11a^2x^5}{10b^4}-\frac{11a^3x^3}{6b^5}+\frac{11a^4x}{2b^6}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b(a+bx^2)}+\frac{11x^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^2,x]
```

```
[Out] (11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x
^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^11/(2*b*(a + b*x^2)) - (11*a^(9/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^{11}}{2b\left(a+bx^2\right)} + \frac{11\int \frac{x^{10}}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^{11}}{2b\left(a+bx^2\right)} + \frac{11\int \left(\frac{a^4}{b^5} - \frac{a^3x^2}{b^4} + \frac{a^2x^4}{b^3} - \frac{ax^6}{b^2} + \frac{x^8}{b} - \frac{a^5}{b^5\left(a+bx^2\right)}\right)dx}{2b}$$

$$= \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} + \frac{11x^9}{18b^2} - \frac{x^{11}}{2b\left(a+bx^2\right)} - \frac{\left(11a^5\right)\int \frac{1}{a+bx^2}\,dx}{2b^6}$$

$$= \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} + \frac{11x^9}{18b^2} - \frac{x^{11}}{2b\left(a+bx^2\right)} - \frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

Mathematica [A] time = 0.0588059, size = 93, normalized size = 0.89

$$\frac{x\left(378a^2b^2x^4 - 840a^3bx^2 + \frac{315a^5}{a+bx^2} + 3150a^4 - 180ab^3x^6 + 70b^4x^8\right)}{630b^6} - \frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^2,x]
```

```
[Out] (x*(3150*a^4 - 840*a^3*b*x^2 + 378*a^2*b^2*x^4 - 180*a*b^3*x^6 + 70*b^4*x^8
 + (315*a^5)/(a + b*x^2)))/(630*b^6) - (11*a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(13/2))
```

Maple [A] time = 0.007, size = 90, normalized size = 0.9

$$\frac{x^9}{9\,b^2} - \frac{2\,ax^7}{7\,b^3} + \frac{3\,a^2x^5}{5\,b^4} - \frac{4\,a^3x^3}{3\,b^5} + 5\,\frac{a^4x}{b^6} + \frac{a^5x}{2\,b^6\left(bx^2+a\right)} - \frac{11\,a^5}{2\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^2,x)
```

```
[Out] 1/9*x^9/b^2-2/7*a*x^7/b^3+3/5*a^2*x^5/b^4-4/3*a^3*x^3/b^5+5*a^4*x/b^6+1/2/b
^6*a^5*x/(b*x^2+a)-11/2/b^6*a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27917, size = 528, normalized size = 5.03

$$\left[\frac{140\,b^5x^{11} - 220\,ab^4x^9 + 396\,a^2b^3x^7 - 924\,a^3b^2x^5 + 4620\,a^4bx^3 + 6930\,a^5x + 3465\left(a^4bx^2 + a^5\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{1260\left(b^7x^2 + ab^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*x^11 - 220*a*b^4*x^9 + 396*a^2*b^3*x^7 - 924*a^3*b^2*x^5 +
 4620*a^4*b*x^3 + 6930*a^5*x + 3465*(a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^2 + a*b^6), 1/630*(70*b^5*x^1
1 - 110*a*b^4*x^9 + 198*a^2*b^3*x^7 - 462*a^3*b^2*x^5 + 2310*a^4*b*x^3 + 34
65*a^5*x - 3465*(a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^2 + a*b^6)]
```

Sympy [A] time = 0.469092, size = 151, normalized size = 1.44

$$\frac{a^5x}{2ab^6 + 2b^7x^2} + \frac{5a^4x}{b^6} - \frac{4a^3x^3}{3b^5} + \frac{3a^2x^5}{5b^4} - \frac{2ax^7}{7b^3} + \frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4} - \frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4} + \frac{x^9}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**2,x)
```

```
[Out] a**5*x/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*x/b**6 - 4*a**3*x**3/(3*b**5) + 3*
a**2*x**5/(5*b**4) - 2*a*x**7/(7*b**3) + 11*sqrt(-a**9/b**13)*log(x - b**6*
sqrt(-a**9/b**13)/a**4)/4 - 11*sqrt(-a**9/b**13)*log(x + b**6*sqrt(-a**9/b*
*13)/a**4)/4 + x**9/(9*b**2)
```

Giac [A] time = 2.98895, size = 128, normalized size = 1.22

$$-\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} + \frac{a^5x}{2\left(bx^2 + a\right)b^6} + \frac{35\,b^{16}x^9 - 90\,ab^{15}x^7 + 189\,a^2b^{14}x^5 - 420\,a^3b^{13}x^3 + 1575\,a^4b^{12}x}{315\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/2*a^5*x/((b*x^2 + a)*b^
6) + 1/315*(35*b^16*x^9 - 90*a*b^15*x^7 + 189*a^2*b^14*x^5 - 420*a^3*b^13*x
^3 + 1575*a^4*b^12*x)/b^18
```

3.147 $\int \frac{x^{11}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=83

$$\frac{3a^2x^4}{4b^4} - \frac{2a^3x^2}{b^5} + \frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4\log(a+bx^2)}{2b^6} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] time = 0.0656377, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^4}{4b^4} - \frac{2a^3x^2}{b^5} + \frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4\log(a+bx^2)}{2b^6} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4a^3}{b^5} + \frac{3a^2x}{b^4} - \frac{2ax^2}{b^3} + \frac{x^3}{b^2} - \frac{a^5}{b^5(a+bx)^2} + \frac{5a^4}{b^5(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2} + \frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4\log(a+bx^2)}{2b^6}
\end{aligned}$$

Mathematica [A] time = 0.0232081, size = 72, normalized size = 0.87

$$\frac{18a^2b^2x^4 - 48a^3bx^2 + \frac{12a^5}{a+bx^2} + 60a^4\log(a+bx^2) - 8ab^3x^6 + 3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Maple [A] time = 0.009, size = 74, normalized size = 0.9

$$-2\,{\frac {{a}^{3}{x}^{2}}{{b}^{5}}}+{\frac {3\,{a}^{2}{x}^{4}}{4\,{b}^{4}}}-{\frac {a{x}^{6}}{3\,{b}^{3}}}+{\frac {{x}^{8}}{8\,{b}^{2}}}+{\frac {{a}^{5}}{2\,{b}^{6} \left(b{x}^{2}+a \right) }}+{\frac {5\,{a}^{4}\ln \left(b{x}^{2}+a \right) }{2\,{b}^{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^2,x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Maxima [A] time = 1.37893, size = 104, normalized size = 1.25

$$\frac {a^{5}}{2 \, {\left (b^{7} x^{2} + a b^{6}\right)}} + \frac {5 \, a^{4} \log \left (b x^{2} + a\right)}{2 \, b^{6}} + \frac {3 \, b^{3} x^{8} - 8 \, a b^{2} x^{6} + 18 \, a^{2} b x^{4} - 48 \, a^{3} x^{2}}{24 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Fricas [A] time = 1.24937, size = 198, normalized size = 2.39

$$\frac {3 \, b^{5} x^{10} - 5 \, a b^{4} x^{8} + 10 \, a^{2} b^{3} x^{6} - 30 \, a^{3} b^{2} x^{4} - 48 \, a^{4} b x^{2} + 12 \, a^{5} + 60 \, {\left (a^{4} b x^{2} + a^{5}\right)} \log \left (b x^{2} + a\right)}{24 \, {\left (b^{7} x^{2} + a b^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A] time = 0.42281, size = 80, normalized size = 0.96

$$\frac {a^{5}}{2 a b^{6} + 2 b^{7} x^{2}} + \frac {5 a^{4} \log {\left (a + b x^{2} \right)}}{2 b^{6}} - \frac {2 a^{3} x^{2}}{b^{5}} + \frac {3 a^{2} x^{4}}{4 b^{4}} - \frac {a x^{6}}{3 b^{3}} + \frac {x^{8}}{8 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**2,x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

Giac [A] time = 1.51511, size = 124, normalized size = 1.49

$$\frac{5\,a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\left(bx^2+a\right)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

3.148 $\int \frac{x^{10}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=92

$$\frac{3a^2x^3}{2b^4} - \frac{9a^3x}{2b^5} + \frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}} - \frac{9ax^5}{10b^3} - \frac{x^9}{2b\left(a+bx^2\right)} + \frac{9x^7}{14b^2}$$

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rubi [A] time = 0.0369984, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{3a^2x^3}{2b^4} - \frac{9a^3x}{2b^5} + \frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}} - \frac{9ax^5}{10b^3} - \frac{x^9}{2b\left(a+bx^2\right)} + \frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^2,x]
```


```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9\int \frac{x^8}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9\int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{\left(9a^4\right)\int \frac{1}{a+bx^2}\,dx}{2b^5}$$

$$= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Mathematica [A] time = 0.0541899, size = 82, normalized size = 0.89

$$\frac{x\left(70a^2bx^2-\frac{35a^4}{a+bx^2}-280a^3-28ab^2x^4+10b^3x^6\right)}{70b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^2,x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [A] time = 0.008, size = 78, normalized size = 0.9

$$\frac{x^7}{7\,b^2}-\frac{2\,ax^5}{5\,b^3}+\frac{a^2x^3}{b^4}-4\,\frac{a^3x}{b^5}-\frac{a^4x}{2\,b^5\left(bx^2+a\right)}+\frac{9\,a^4}{2\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^2,x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(b*x^2+a)+9
/2/b^5*a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28686, size = 459, normalized size = 4.99

$$\left[\frac{20\,b^4x^9-36\,ab^3x^7+84\,a^2b^2x^5-420\,a^3bx^3-630\,a^4x+315\,\left(a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\,\left(b^6x^2+ab^5\right)},\ \frac{10\,b^4x^9-18\,ab^3x^7+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

Sympy [A] time = 0.479109, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**2,x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

Giac [A] time = 3.23192, size = 113, normalized size = 1.23

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}-\frac{a^4x}{2\,\left(bx^2+a\right)b^5}+\frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

3.149 $\int \frac{x^9}{(a+bx^2)^2} dx$

Optimal. Leaf size=70

$$\frac{3a^2x^2}{2b^4} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rubi [A] time = 0.0544227, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^2}{2b^4} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^2,x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(a+bx^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^2} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.0231995, size = 60, normalized size = 0.86

$$\frac{9a^2bx^2 - \frac{3a^4}{a+bx^2} - 12a^3\log(a+bx^2) - 3ab^2x^4 + b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^2,x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

Maple [A] time = 0.009, size = 63, normalized size = 0.9

$$\frac{3\,a^2x^2}{2\,b^4}-\frac{ax^4}{2\,b^3}+\frac{x^6}{6\,b^2}-\frac{a^4}{2\,b^5\left(bx^2+a\right)}-2\,\frac{a^3\ln\left(bx^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^2,x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

Maxima [A] time = 2.62999, size = 88, normalized size = 1.26

$$-\frac{a^4}{2\left(b^6x^2+ab^5\right)}-\frac{2\,a^3\log\left(bx^2+a\right)}{b^5}+\frac{b^2x^6-3\,abx^4+9\,a^2x^2}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

Fricas [A] time = 1.18547, size = 166, normalized size = 2.37

$$\frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\left(a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] time = 0.403645, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**2,x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

Giac [A] time = 2.48879, size = 108, normalized size = 1.54

$$-\frac{2\,a^{3}\log\left(\left|bx^{2}+a\right|\right)}{b^{5}}+\frac{b^{4}x^{6}-3\,ab^{3}x^{4}+9\,a^{2}b^{2}x^{2}}{6\,b^{6}}+\frac{4\,a^{3}bx^{2}+3\,a^{4}}{2\left(bx^{2}+a\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

3.150 $\int \frac{x^8}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=79

$$\frac{7a^2x}{2b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b\left(a+bx^2\right)} + \frac{7x^5}{10b^2}$$

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] time = 0.0316641, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{7a^2x}{2b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b\left(a+bx^2\right)} + \frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^2,x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx = -\frac{x^7}{2b\left(a+bx^2\right)} + \frac{7\int \frac{x^6}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^7}{2b\left(a+bx^2\right)} + \frac{7\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3(a+bx^2)}\right)dx}{2b}$$

$$= \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} + \frac{7x^5}{10b^2} - \frac{x^7}{2b\left(a+bx^2\right)} - \frac{\left(7a^3\right)\int \frac{1}{a+bx^2}\,dx}{2b^4}$$

$$= \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} + \frac{7x^5}{10b^2} - \frac{x^7}{2b\left(a+bx^2\right)} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Mathematica [A] time = 0.0496657, size = 71, normalized size = 0.9

$$\frac{x\left(\frac{15a^3}{a+bx^2} + 90a^2 - 20abx^2 + 6b^2x^4\right)}{30b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^2,x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.008, size = 68, normalized size = 0.9

$$\frac{x^5}{5\,b^2} - \frac{2\,ax^3}{3\,b^3} + 3\,\frac{a^2x}{b^4} + \frac{a^3x}{2\,b^4\left(bx^2+a\right)} - \frac{7\,a^3}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^2,x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2/b^4*a^3/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32196, size = 409, normalized size = 5.18

$$\left[\frac{12\,b^{3}x^{7}-28\,ab^{2}x^{5}+140\,a^{2}bx^{3}+210\,a^{3}x+105\,\left(a^{2}bx^{2}+a^{3}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{60\,\left(b^{5}x^{2}+ab^{4}\right)},\frac{6\,b^{3}x^{7}-14\,ab^{2}x^{5}+70\,a^{2}bx^{3}+10}{30}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

Sympy [A] time = 0.436557, size = 124, normalized size = 1.57

$$\frac{a^{3}x}{2ab^{4}+2b^{5}x^{2}}+\frac{3a^{2}x}{b^{4}}-\frac{2ax^{3}}{3b^{3}}+\frac{7\sqrt{-\frac{a^{5}}{b^{9}}}\log\left(x-\frac{b^{4}\sqrt{-\frac{a^{5}}{b^{9}}}}{a^{2}}\right)}{4}-\frac{7\sqrt{-\frac{a^{5}}{b^{9}}}\log\left(x+\frac{b^{4}\sqrt{-\frac{a^{5}}{b^{9}}}}{a^{2}}\right)}{4}+\frac{x^{5}}{5b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**2,x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

Giac [A] time = 2.56612, size = 99, normalized size = 1.25

$$-\frac{7\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^{4}}+\frac{a^{3}x}{2\,\left(bx^{2}+a\right)b^{4}}+\frac{3\,b^{8}x^{5}-10\,ab^{7}x^{3}+45\,a^{2}b^{6}x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

3.151 $\int \frac{x^7}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rubi [A] time = 0.0422548, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^2,x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0202011, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^2,x]
```

```
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

Maple [A] time = 0.008, size = 52, normalized size = 0.9

$$-\frac{ax^2}{b^3}+\frac{x^4}{4\,b^2}+\frac{a^3}{2\,b^4\left(bx^2+a\right)}+\frac{3\,a^2\ln\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^2,x)
```

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Maxima [A] time = 1.88142, size = 73, normalized size = 1.28

$$\frac{a^3}{2\left(b^5x^2+ab^4\right)}+\frac{3\,a^2\log\left(bx^2+a\right)}{2\,b^4}+\frac{bx^4-4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

Fricas [A] time = 1.20495, size = 143, normalized size = 2.51

$$\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\left(a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 0.391561, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4+2b^5x^2}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**2,x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

Giac [A] time = 2.57938, size = 90, normalized size = 1.58

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{b^2x^4-4\,abx^2}{4\,b^4}-\frac{3\,a^2bx^2+2\,a^3}{2\left(bx^2+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

3.152 $\int \frac{x^6}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.0269408, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^2,x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx = -\frac{x^5}{2b\left(a+bx^2\right)} + \frac{5\int \frac{x^4}{a+bx^2}\,dx}{2b}$$
$$= -\frac{x^5}{2b\left(a+bx^2\right)} + \frac{5\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{2b}$$
$$= -\frac{5ax}{2b^3} + \frac{5x^3}{6b^2} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{\left(5a^2\right)\int \frac{1}{a+bx^2}\,dx}{2b^3}$$
$$= -\frac{5ax}{2b^3} + \frac{5x^3}{6b^2} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.0435879, size = 60, normalized size = 0.91

$$\frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3} + \frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^2,x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.008, size = 57, normalized size = 0.9

$$\frac{x^3}{3\,b^2} - 2\,\frac{ax}{b^3} - \frac{a^2x}{2\,b^3\left(bx^2+a\right)} + \frac{5\,a^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^2,x)
```

```
[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2/b^3*a^2*x/(b*x^2+a)+5/2/b^3*a^2/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28089, size = 348, normalized size = 5.27

$$\left[\frac{4\,b^2x^5 - 20\,abx^3 - 30\,a^2x + 15\left(abx^2 + a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\left(b^4x^2 + ab^3\right)}, \frac{2\,b^2x^5 - 10\,abx^3 - 15\,a^2x + 15\left(abx^2 + a^2\right)\sqrt{\frac{a}{b}}\,\text{arcta}}{6\left(b^4x^2 + ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 0.418157, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3 + 2b^4x^2} - \frac{2ax}{b^3} - \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x - \frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x + \frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**2,x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

Giac [A] time = 2.57837, size = 82, normalized size = 1.24

$$\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} - \frac{a^2x}{2\left(bx^2 + a\right)b^3} + \frac{b^4x^3 - 6\,ab^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

3.153 $\int \frac{x^5}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rubi [A] time = 0.031901, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^2,x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0224456, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)+bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^2,x]
```

```
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.009, size = 41, normalized size = 0.9

$$\frac{x^2}{2\,b^2} - \frac{a^2}{2\,b^3\left(bx^2+a\right)} - \frac{a\ln\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Maxima [A] time = 1.41121, size = 58, normalized size = 1.32

$$-\frac{a^2}{2\left(b^4x^2+ab^3\right)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Fricas [A] time = 1.22029, size = 113, normalized size = 2.57

$$\frac{b^2x^4+abx^2-a^2-2\left(abx^2+a^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

Sympy [A] time = 0.368169, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3+2b^4x^2} - \frac{a\log\left(a+bx^2\right)}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**2,x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

Giac [A] time = 3.29296, size = 66, normalized size = 1.5

$$\frac{x^2}{2\,b^2} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\left(bx^2 + a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

3.154 $\int \frac{x^4}{(a+bx^2)^2} dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rubi [A] time = 0.016912, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^2,x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = -\frac{x^3}{2b(a+bx^2)} + \frac{3\int \frac{x^2}{a+bx^2}\,dx}{2b}$$
$$= \frac{3x}{2b^2} - \frac{x^3}{2b(a+bx^2)} - \frac{(3a)\int \frac{1}{a+bx^2}\,dx}{2b^2}$$
$$= \frac{3x}{2b^2} - \frac{x^3}{2b(a+bx^2)} - \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.0314233, size = 51, normalized size = 0.93

$$\frac{ax}{2b^2(a+bx^2)} - \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^2,x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

Maple [A] time = 0.007, size = 43, normalized size = 0.8

$$\frac{x}{b^2} + \frac{ax}{2\,b^2(bx^2+a)} - \frac{3\,a}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2,x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33109, size = 285, normalized size = 5.18

$$\left[\frac{4\,bx^3 + 3\left(bx^2+a\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,ax}{4\left(b^3x^2+ab^2\right)}, \frac{2\,bx^3 - 3\left(bx^2+a\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3\,ax}{2\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

Sympy [A] time = 0.387008, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2 + 2b^3x^2} + \frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2,x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

Giac [A] time = 2.84691, size = 57, normalized size = 1.04

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{ax}{2\left(bx^2 + a\right)b^2} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

3.155 $\int \frac{x^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rubi [A] time = 0.0237993, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0076065, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^2,x]
```

```
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.008, size = 30, normalized size = 0.9

$$\frac{a}{2\,b^2\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2,x)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.95675, size = 43, normalized size = 1.3

$$\frac{a}{2\left(b^3x^2+ab^2\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.29161, size = 76, normalized size = 2.3

$$\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)+a}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.331859, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2+2b^3x^2}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2,x)
```

```
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

Giac [A] time = 2.69281, size = 65, normalized size = 1.97

$$-\frac{\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
```

3.156 $\int \frac{x^2}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.0124618, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^2,x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^2} \, dx &= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2} \, dx}{2b}\\ &= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.019776, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^2,x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$-\frac{x}{2\,b\left(bx^2+a\right)}+\frac{1}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21179, size = 263, normalized size = 5.84

$$\left[-\frac{2\,abx+\left(bx^2+a\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\left(ab^3x^2+a^2b^2\right)},-\frac{abx-\left(bx^2+a\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] time = 0.341995, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2,x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

Giac [A] time = 3.15065, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b} - \frac{x}{2\left(bx^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

3.157 $\int \frac{x}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

```
[Out] -1/(2*b*(a + b*x^2))
```

Rubi [A] time = 0.0029703, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^2,x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2} \, dx = -\frac{1}{2b\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0018684, size = 16, normalized size = 1.

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^2,x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Maple [A] time = 0.001, size = 15, normalized size = 0.9

$$-\frac{1}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

Maxima [A] time = 2.07251, size = 19, normalized size = 1.19

$$-\frac{1}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

Fricas [A] time = 1.2853, size = 30, normalized size = 1.88

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Sympy [A] time = 0.296921, size = 15, normalized size = 0.94

$$-\frac{1}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2,x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

Giac [A] time = 3.56301, size = 19, normalized size = 1.19

$$-\frac{1}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

3.158 $\int \frac{1}{(a+bx^2)^2} dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0095894, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2} dx &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2} dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0235936, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$\frac{x}{2a(bx^2+a)}+\frac{1}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32964, size = 261, normalized size = 5.8

$$\left[\frac{2abx-(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{4(a^2b^2x^2+a^3b)},\frac{abx+(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2(a^2b^2x^2+a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] time = 0.357141, size = 78, normalized size = 1.73

$$\frac{x}{2a^2+2abx^2}-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Giac [A] time = 3.35825, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2\left(bx^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

3.159 $\int \frac{1}{x\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rubi [A] time = 0.0267591, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx,x,x^2\right)\\
&= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0121456, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2),x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.01, size = 35, normalized size = 0.9

$$\frac{1}{2\,a\left(bx^{2}+a\right)}+\frac{\ln\left(x\right)}{a^{2}}-\frac{\ln\left(bx^{2}+a\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Maxima [A] time = 1.96636, size = 50, normalized size = 1.32

$$\frac{1}{2\left(abx^{2}+a^{2}\right)}-\frac{\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{\log\left(x^{2}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A] time = 1.27075, size = 108, normalized size = 2.84

$$-\frac{\left(bx^{2}+a\right)\log\left(bx^{2}+a\right)-2\left(bx^{2}+a\right)\log\left(x\right)-a}{2\left(a^{2}bx^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

Sympy [A] time = 0.425711, size = 34, normalized size = 0.89

$$\frac{1}{2a^{2}+2abx^{2}}+\frac{\log\left(x\right)}{a^{2}}-\frac{\log\left(\frac{a}{b}+x^{2}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Giac [A] time = 1.7849, size = 63, normalized size = 1.66

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{bx^2 + 2\,a}{2\,\left(bx^2 + a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

3.160 $\int \frac{1}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rubi [A] time = 0.0168707, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax\left(a+bx^2\right)} + \frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.0348756, size = 54, normalized size = 0.95

$$-\frac{bx}{2a^2\left(a+bx^2\right)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

Maple [A] time = 0.008, size = 46, normalized size = 0.8

$$-\frac{1}{a^2x} - \frac{bx}{2\,a^2\left(bx^2+a\right)} - \frac{3\,b}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-1/2*b/a^2*x/(b*x^2+a)-3/2*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27124, size = 288, normalized size = 5.05

$$\left[-\frac{6\,bx^2 - 3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) + 4\,a}{4\left(a^2bx^3+a^3x\right)}, -\frac{3\,bx^2 + 3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] time = 0.444057, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{2a+3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A] time = 2.56312, size = 63, normalized size = 1.11

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{3\,bx^2+2\,a}{2\,\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

3.161 $\int \frac{1}{x^3(a+bx^2)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rubi [A] time = 0.0361045, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx, x, x^2\right)\\ &= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0378983, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.011, size = 46, normalized size = 0.9

$$-\frac{1}{2\,a^2x^2}-\frac{b}{2\,a^2\left(bx^2+a\right)}-2\,\frac{b\ln\left(x\right)}{a^3}+\frac{b\ln\left(bx^2+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Maxima [A] time = 1.29151, size = 70, normalized size = 1.43

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

Fricas [A] time = 1.2973, size = 157, normalized size = 3.2

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 0.530411, size = 49, normalized size = 1.

$$-\frac{a+2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2,x)
```

```
[Out] -(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

Giac [A] time = 2.63517, size = 69, normalized size = 1.41

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\,\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

3.162 $\int \frac{1}{x^4(a+bx^2)^2} dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.0257021, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3\left(a+bx^2\right)} - \frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.0389508, size = 67, normalized size = 0.99

$$\frac{b^2x}{2a^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/(3*a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.01, size = 59, normalized size = 0.9

$$-\frac{1}{3\,a^2x^3} + 2\,\frac{b}{a^3x} + \frac{b^2x}{2\,a^3\left(bx^2+a\right)} + \frac{5\,b^2}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32605, size = 359, normalized size = 5.28

$$\left[\frac{30\,b^2x^4 + 20\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4\,a^2}{12\left(a^3bx^5 + a^4x^3\right)}, \frac{15\,b^2x^4 + 10\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{\frac{b}{a}}\,\mathrm{arctan}}{6\left(a^3bx^5 + a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A] time = 0.588076, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A] time = 1.89991, size = 80, normalized size = 1.18

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2x}{2\left(bx^2 + a\right)a^3} + \frac{6\,bx^2 - a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

3.163 $\int \frac{1}{x^5\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{b^2}{2a^3\left(a+bx^2\right)}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.0439952, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^2}{2a^3\left(a+bx^2\right)}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.0488009, size = 57, normalized size = 0.86

$$\frac{a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)-6b^2\log\left(a+bx^2\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

Maple [A] time = 0.012, size = 61, normalized size = 0.9

$$-\frac{1}{4\,a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2\,a^3\left(bx^2+a\right)}+3\,\frac{b^2\ln\left(x\right)}{a^4}-\frac{3\,b^2\ln\left(bx^2+a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

Maxima [A] time = 1.14977, size = 95, normalized size = 1.44

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\left(a^3bx^6+a^4x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Fricas [A] time = 1.24134, size = 184, normalized size = 2.79

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log\left(x\right)}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A] time = 0.689847, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log\left(x\right)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 2.41803, size = 116, normalized size = 1.76

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4} - \frac{3\,b^2\log\left(\left|bx^2 + a\right|\right)}{2\,a^4} + \frac{3\,b^3x^2 + 4\,ab^2}{2\left(bx^2 + a\right)a^4} - \frac{9\,b^2x^4 - 4\,abx^2 + a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

3.164 $\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi [A] time = 0.0329955, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^5\left(a+bx^2\right)} + \frac{7\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{7}{10a^2x^5} + \frac{1}{2ax^5\left(a+bx^2\right)} - \frac{(7b)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a^2}$$

$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} + \frac{1}{2ax^5\left(a+bx^2\right)} + \frac{\left(7b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^3}$$

$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} + \frac{1}{2ax^5\left(a+bx^2\right)} - \frac{\left(7b^3\right)\int \frac{1}{a+bx^2}\,dx}{2a^4}$$

$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} + \frac{1}{2ax^5\left(a+bx^2\right)} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Mathematica [A] time = 0.0436866, size = 80, normalized size = 0.99

$$-\frac{b^3x}{2a^4\left(a+bx^2\right)} - \frac{3b^2}{a^4x} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}} + \frac{2b}{3a^3x^3} - \frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/(5*a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a +
b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A] time = 0.011, size = 70, normalized size = 0.9

$$-\frac{1}{5\,a^2x^5} - 3\,\frac{b^2}{a^4x} + \frac{2\,b}{3\,a^3x^3} - \frac{b^3x}{2\,a^4\left(bx^2+a\right)} - \frac{7\,b^3}{2\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3-1/2*b^3/a^4*x/(b*x^2+a)-7/2*b^3/a^4/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27195, size = 423, normalized size = 5.22

$$\left[-\frac{210\,b^{3}x^{6}+140\,ab^{2}x^{4}-28\,a^{2}bx^{2}+12\,a^{3}-105\,\left(b^{3}x^{7}+ab^{2}x^{5}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{60\,\left(a^{4}bx^{7}+a^{5}x^{5}\right)},-\frac{105\,b^{3}x^{6}+70\,ab^{2}x^{4}-14\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

Sympy [A] time = 0.676042, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^{5}}{a^{9}}}\log\left(-\frac{a^{5}\sqrt{-\frac{b^{5}}{a^{9}}}}{b^{3}}+x\right)}{4}-\frac{7\sqrt{-\frac{b^{5}}{a^{9}}}\log\left(\frac{a^{5}\sqrt{-\frac{b^{5}}{a^{9}}}}{b^{3}}+x\right)}{4}-\frac{6a^{3}-14a^{2}bx^{2}+70ab^{2}x^{4}+105b^{3}x^{6}}{30a^{5}x^{5}+30a^{4}bx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2,x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - (6*a**3 - 14*a**2*b*x**2 + 70*a
*b**2*x**4 + 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Giac [A] time = 2.14657, size = 95, normalized size = 1.17

$$-\frac{7\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^{4}}-\frac{b^{3}x}{2\,\left(bx^{2}+a\right)a^{4}}-\frac{45\,b^{2}x^{4}-10\,abx^{2}+3\,a^{2}}{15\,a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

3.165 $\int \frac{1}{x^7(a+bx^2)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3}{2a^4(a+bx^2)}-\frac{3b^2}{2a^4x^2}+\frac{2b^3\log(a+bx^2)}{a^5}-\frac{4b^3\log(x)}{a^5}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

```
[Out] -1/(6*a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*x^
2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5
```

Rubi [A] time = 0.0539037, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^3}{2a^4(a+bx^2)}-\frac{3b^2}{2a^4x^2}+\frac{2b^3\log(a+bx^2)}{a^5}-\frac{4b^3\log(x)}{a^5}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -1/(6*a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*x^
2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7(a+bx^2)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{b^3}{2a^4(a+bx^2)}-\frac{4b^3\log(x)}{a^5}+\frac{2b^3\log(a+bx^2)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.0536143, size = 68, normalized size = 0.85

$$\frac{a\left(-\frac{a^2}{x^6}-\frac{3b^3}{a+bx^2}+\frac{3ab}{x^4}-\frac{9b^2}{x^2}\right)+12b^3\log(a+bx^2)-24b^3\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a^2/x^6) + (3*a*b)/x^4 - (9*b^2)/x^2 - (3*b^3)/(a + b*x^2)) - 24*b^3*
Log[x] + 12*b^3*Log[a + b*x^2])/(6*a^5)
```

Maple [A] time = 0.012, size = 73, normalized size = 0.9

$$-\frac{1}{6\,a^{2}x^{6}}+\frac{b}{2\,a^{3}x^{4}}-\frac{3\,b^{2}}{2\,a^{4}x^{2}}-\frac{b^{3}}{2\,a^{4}\left(bx^{2}+a\right)}-4\,\frac{b^{3}\ln\left(x\right)}{a^{5}}+2\,\frac{b^{3}\ln\left(bx^{2}+a\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2,x)
```

```
[Out] -1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*b^2/a^4/x^2-1/2*b^3/a^4/(b*x^2+a)-4*b^3*ln(x
)/a^5+2*b^3*ln(b*x^2+a)/a^5
```

Maxima [A] time = 2.20893, size = 107, normalized size = 1.34

$$-\frac{12\,b^{3}x^{6}+6\,ab^{2}x^{4}-2\,a^{2}bx^{2}+a^{3}}{6\left(a^{4}bx^{8}+a^{5}x^{6}\right)}+\frac{2\,b^{3}\log\left(bx^{2}+a\right)}{a^{5}}-\frac{2\,b^{3}\log\left(x^{2}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 6*a*b^2*x^4 - 2*a^2*b*x^2 + a^3)/(a^4*b*x^8 + a^5*x^6) +
 2*b^3*log(b*x^2 + a)/a^5 - 2*b^3*log(x^2)/a^5
```

Fricas [A] time = 1.36778, size = 209, normalized size = 2.61

$$-\frac{12\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-2\,a^{3}bx^{2}+a^{4}-12\left(b^{4}x^{8}+ab^{3}x^{6}\right)\log\left(bx^{2}+a\right)+24\left(b^{4}x^{8}+ab^{3}x^{6}\right)\log\left(x\right)}{6\left(a^{5}bx^{8}+a^{6}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4 - 12*(b^4*x^8 + a*b^
3*x^6)*log(b*x^2 + a) + 24*(b^4*x^8 + a*b^3*x^6)*log(x))/(a^5*b*x^8 + a^6*x
^6)
```

Sympy [A] time = 0.833802, size = 78, normalized size = 0.98

$$-\frac{a^{3}-2a^{2}bx^{2}+6ab^{2}x^{4}+12b^{3}x^{6}}{6a^{5}x^{6}+6a^{4}bx^{8}}-\frac{4b^{3}\log\left(x\right)}{a^{5}}+\frac{2b^{3}\log\left(\frac{a}{b}+x^{2}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2,x)
```

```
[Out] -(a**3 - 2*a**2*b*x**2 + 6*a*b**2*x**4 + 12*b**3*x**6)/(6*a**5*x**6 + 6*a**
4*b*x**8) - 4*b**3*log(x)/a**5 + 2*b**3*log(a/b + x**2)/a**5
```

Giac [A] time = 2.76559, size = 134, normalized size = 1.68

$$-\frac{2\,b^3\log\left(x^2\right)}{a^5}+\frac{2\,b^3\log\left(\left|bx^2+a\right|\right)}{a^5}-\frac{4\,b^4x^2+5\,ab^3}{2\left(bx^2+a\right)a^5}+\frac{22\,b^3x^6-9\,ab^2x^4+3\,a^2bx^2-a^3}{6\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*b^3*log(x^2)/a^5 + 2*b^3*log(abs(b*x^2 + a))/a^5 - 1/2*(4*b^4*x^2 + 5*a*
b^3)/((b*x^2 + a)*a^5) + 1/6*(22*b^3*x^6 - 9*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)
/(a^5*x^6)
```

3.166 $\int \frac{1}{x^8(a+bx^2)^2} dx$

Optimal. Leaf size=94

$$-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

```
[Out] -9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5
*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(11/2))
```

Rubi [A] time = 0.0431358, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5
*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(11/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{9\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{9}{14a^2x^7} + \frac{1}{2ax^7\left(a+bx^2\right)} - \frac{(9b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{2a^2}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} + \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{\left(9b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a^3}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{1}{2ax^7\left(a+bx^2\right)} - \frac{\left(9b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^4}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{9b^3}{2a^5x} + \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{\left(9b^4\right)\int \frac{1}{a+bx^2}\,dx}{2a^5}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{9b^3}{2a^5x} + \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

Mathematica [A] time = 0.0532909, size = 91, normalized size = 0.97

$$\frac{b^4x}{2a^5\left(a+bx^2\right)} - \frac{b^2}{a^4x^3} + \frac{4b^3}{a^5x} + \frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}} + \frac{2b}{5a^3x^5} - \frac{1}{7a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/(7*a^2*x^7) + (2*b)/(5*a^3*x^5) - b^2/(a^4*x^3) + (4*b^3)/(a^5*x) + (b^4
*x)/(2*a^5*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/
2))
```

Maple [A] time = 0.01, size = 81, normalized size = 0.9

$$-\frac{1}{7\,a^2x^7} + 4\,\frac{b^3}{a^5x} - \frac{b^2}{a^4x^3} + \frac{2\,b}{5\,a^3x^5} + \frac{b^4x}{2\,a^5\left(bx^2+a\right)} + \frac{9\,b^4}{2\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/7/a^2/x^7+4*b^3/a^5/x-b^2/a^4/x^3+2/5*b/a^3/x^5+1/2*b^4/a^5*x/(b*x^2+a)+
9/2*b^4/a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.1774, size = 470, normalized size = 5.

$$\left[\frac{630\,b^4x^8 + 420\,ab^3x^6 - 84\,a^2b^2x^4 + 36\,a^3bx^2 - 20\,a^4 + 315\,\left(b^4x^9 + ab^3x^7\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{140\,\left(a^5bx^9 + a^6x^7\right)}, \frac{315\,b^4x^8 + 210\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(630*b^4*x^8 + 420*a*b^3*x^6 - 84*a^2*b^2*x^4 + 36*a^3*b*x^2 - 20*a^
4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^5*b*x^9 + a^6*x^7), 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 4
2*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(b/a)
*arctan(x*sqrt(b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

Sympy [A] time = 1.00635, size = 138, normalized size = 1.47

$$-\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4} + x\right)}{4} + \frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4} + x\right)}{4} + \frac{-10a^4 + 18a^3bx^2 - 42a^2b^2x^4 + 210ab^3x^6 + 315b^4x^8}{70a^6x^7 + 70a^5bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**2,x)
```

```
[Out] -9*sqrt(-b**7/a**11)*log(-a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + 9*sqrt(-b**7
/a**11)*log(a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + (-10*a**4 + 18*a**3*b*x**2
 - 42*a**2*b**2*x**4 + 210*a*b**3*x**6 + 315*b**4*x**8)/(70*a**6*x**7 + 70*
a**5*b*x**9)
```

Giac [A] time = 2.32981, size = 109, normalized size = 1.16

$$\frac{9\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5} + \frac{b^4x}{2\,\left(bx^2 + a\right)a^5} + \frac{140\,b^3x^6 - 35\,ab^2x^4 + 14\,a^2bx^2 - 5\,a^3}{35\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/2*b^4*x/((b*x^2 + a)*a^5)
 + 1/35*(140*b^3*x^6 - 35*a*b^2*x^4 + 14*a^2*b*x^2 - 5*a^3)/(a^5*x^7)
```

3.167 $\int \frac{1}{x^9(a+bx^2)^2}\,dx$

Optimal. Leaf size=93

$$\frac{b^4}{2a^5(a+bx^2)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}-\frac{5b^4\log(a+bx^2)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

```
[Out] -1/(8*a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2) +
b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*a^
6)
```

Rubi [A] time = 0.0652697, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^4}{2a^5(a+bx^2)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}-\frac{5b^4\log(a+bx^2)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] -1/(8*a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2) +
b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*a^
6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9(a+bx^2)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^5(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{1}{8a^2x^8}+\frac{b}{3a^3x^6}-\frac{3b^2}{4a^4x^4}+\frac{2b^3}{a^5x^2}+\frac{b^4}{2a^5(a+bx^2)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx^2)}{2a^6}
\end{aligned}$$

Mathematica [A] time = 0.0711963, size = 79, normalized size = 0.85

$$\frac{a\left(\frac{8a^2b}{x^6}-\frac{3a^3}{x^8}-\frac{18ab^2}{x^4}+12b^3\left(\frac{b}{a+bx^2}+\frac{4}{x^2}\right)\right)-60b^4\log(a+bx^2)+120b^4\log(x)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] (a*((-3*a^3)/x^8 + (8*a^2*b)/x^6 - (18*a*b^2)/x^4 + 12*b^3*(4/x^2 + b/(a +
b*x^2))) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/(24*a^6)
```

Maple [A] time = 0.013, size = 84, normalized size = 0.9

$$-\frac{1}{8\,a^2x^8}+\frac{b}{3\,a^3x^6}-\frac{3\,b^2}{4\,a^4x^4}+2\,\frac{b^3}{a^5x^2}+\frac{b^4}{2\,a^5\left(bx^2+a\right)}+5\,\frac{b^4\ln\left(x\right)}{a^6}-\frac{5\,b^4\ln\left(bx^2+a\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^2,x)
```

```
[Out] -1/8/a^2/x^8+1/3*b/a^3/x^6-3/4*b^2/a^4/x^4+2*b^3/a^5/x^2+1/2*b^4/a^5/(b*x^2
+a)+5*b^4*ln(x)/a^6-5/2*b^4*ln(b*x^2+a)/a^6
```

Maxima [A] time = 2.10876, size = 124, normalized size = 1.33

$$\frac{60\,b^4x^8+30\,ab^3x^6-10\,a^2b^2x^4+5\,a^3bx^2-3\,a^4}{24\left(a^5bx^{10}+a^6x^8\right)}-\frac{5\,b^4\log\left(bx^2+a\right)}{2\,a^6}+\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(60*b^4*x^8 + 30*a*b^3*x^6 - 10*a^2*b^2*x^4 + 5*a^3*b*x^2 - 3*a^4)/(a^
5*b*x^10 + a^6*x^8) - 5/2*b^4*log(b*x^2 + a)/a^6 + 5/2*b^4*log(x^2)/a^6
```

Fricas [A] time = 1.14426, size = 242, normalized size = 2.6

$$\frac{60\,ab^4x^8+30\,a^2b^3x^6-10\,a^3b^2x^4+5\,a^4bx^2-3\,a^5-60\left(b^5x^{10}+ab^4x^8\right)\log\left(bx^2+a\right)+120\left(b^5x^{10}+ab^4x^8\right)\log\left(x\right)}{24\left(a^6bx^{10}+a^7x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(60*a*b^4*x^8 + 30*a^2*b^3*x^6 - 10*a^3*b^2*x^4 + 5*a^4*b*x^2 - 3*a^5
- 60*(b^5*x^10 + a*b^4*x^8)*log(b*x^2 + a) + 120*(b^5*x^10 + a*b^4*x^8)*log
(x))/(a^6*b*x^10 + a^7*x^8)
```

Sympy [A] time = 1.29343, size = 94, normalized size = 1.01

$$\frac{-3a^4+5a^3bx^2-10a^2b^2x^4+30ab^3x^6+60b^4x^8}{24a^6x^8+24a^5bx^{10}}+\frac{5b^4\log\left(x\right)}{a^6}-\frac{5b^4\log\left(\frac{a}{b}+x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x**2 - 10*a**2*b**2*x**4 + 30*a*b**3*x**6 + 60*b**4*x**
8)/(24*a**6*x**8 + 24*a**5*b*x**10) + 5*b**4*log(x)/a**6 - 5*b**4*log(a/b +
 x**2)/(2*a**6)
```

Giac [A] time = 2.67453, size = 149, normalized size = 1.6

$$\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}-\frac{5\,b^4\log\left(\left|bx^2+a\right|\right)}{2\,a^6}+\frac{5\,b^5x^2+6\,ab^4}{2\left(bx^2+a\right)a^6}-\frac{125\,b^4x^8-48\,ab^3x^6+18\,a^2b^2x^4-8\,a^3bx^2+3\,a^4}{24\,a^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^4*log(x^2)/a^6 - 5/2*b^4*log(abs(b*x^2 + a))/a^6 + 1/2*(5*b^5*x^2 + 6
*a*b^4)/((b*x^2 + a)*a^6) - 1/24*(125*b^4*x^8 - 48*a*b^3*x^6 + 18*a^2*b^2*x
^4 - 8*a^3*b*x^2 + 3*a^4)/(a^6*x^8)
```

3.168 $\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=114

$$\frac{a^2x^6}{b^5}-\frac{5a^3x^4}{2b^6}+\frac{15a^4x^2}{2b^7}-\frac{7a^6}{2b^8\left(a+bx^2\right)}+\frac{a^7}{4b^8\left(a+bx^2\right)^2}-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^8}{8b^4}+\frac{x^{10}}{10b^3}$$

```
[Out] (15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b
^4) + x^10/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2
)) - (21*a^5*Log[a + b*x^2])/(2*b^8)
```

Rubi [A] time = 0.103378, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{b^5}-\frac{5a^3x^4}{2b^6}+\frac{15a^4x^2}{2b^7}-\frac{7a^6}{2b^8\left(a+bx^2\right)}+\frac{a^7}{4b^8\left(a+bx^2\right)^2}-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^8}{8b^4}+\frac{x^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^3,x]
```

```
[Out] (15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b
^4) + x^10/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2
)) - (21*a^5*Log[a + b*x^2])/(2*b^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^7}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right)dx,\right.\\
&= \frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}+\frac{x^{10}}{10b^3}+\frac{a^7}{4b^8\left(a+bx^2\right)^2}-\frac{7a^6}{2b^8\left(a+bx^2\right)}-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}
\end{aligned}$$

Mathematica [A] time = 0.0294884, size = 97, normalized size = 0.85

$$\frac{40a^2b^3x^6-100a^3b^2x^4+300a^4bx^2-\frac{140a^6}{a+bx^2}+\frac{10a^7}{(a+bx^2)^2}-420a^5\log\left(a+bx^2\right)-15ab^4x^8+4b^5x^{10}}{40b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^3,x]
```

```
[Out] (300*a^4*b*x^2 - 100*a^3*b^2*x^4 + 40*a^2*b^3*x^6 - 15*a*b^4*x^8 + 4*b^5*x^
10 + (10*a^7)/(a + b*x^2)^2 - (140*a^6)/(a + b*x^2) - 420*a^5*Log[a + b*x^2
])/(40*b^8)
```

Maple [A] time = 0.01, size = 101, normalized size = 0.9

$$\frac{15\,a^4x^2}{2\,b^7}-\frac{5\,a^3x^4}{2\,b^6}+\frac{a^2x^6}{b^5}-\frac{3\,ax^8}{8\,b^4}+\frac{x^{10}}{10\,b^3}+\frac{a^7}{4\,b^8\left(bx^2+a\right)^2}-\frac{7\,a^6}{2\,b^8\left(bx^2+a\right)}-\frac{21\,a^5\ln\left(bx^2+a\right)}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^3,x)
```

```
[Out] 15/2*a^4*x^2/b^7-5/2*a^3*x^4/b^6+a^2*x^6/b^5-3/8*a*x^8/b^4+1/10*x^10/b^3+1/
4*a^7/b^8/(b*x^2+a)^2-7/2*a^6/b^8/(b*x^2+a)-21/2*a^5*ln(b*x^2+a)/b^8
```

Maxima [A] time = 1.66549, size = 150, normalized size = 1.32

$$-\frac{14\,a^6bx^2+13\,a^7}{4\left(b^{10}x^4+2\,ab^9x^2+a^2b^8\right)}-\frac{21\,a^5\log\left(bx^2+a\right)}{2\,b^8}+\frac{4\,b^4x^{10}-15\,ab^3x^8+40\,a^2b^2x^6-100\,a^3bx^4+300\,a^4x^2}{40\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(14*a^6*b*x^2 + 13*a^7)/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8) - 21/2*a^5*
log(b*x^2 + a)/b^8 + 1/40*(4*b^4*x^10 - 15*a*b^3*x^8 + 40*a^2*b^2*x^6 - 100
*a^3*b*x^4 + 300*a^4*x^2)/b^7
```

Fricas [A] time = 1.23095, size = 298, normalized size = 2.61

$$\frac{4\,b^7x^{14}-7\,ab^6x^{12}+14\,a^2b^5x^{10}-35\,a^3b^4x^8+140\,a^4b^3x^6+500\,a^5b^2x^4+160\,a^6bx^2-130\,a^7-420\left(a^5b^2x^4+2\,a^6bx^2+a^7\right.}{40\left(b^{10}x^4+2\,ab^9x^2+a^2b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^7*x^14 - 7*a*b^6*x^12 + 14*a^2*b^5*x^10 - 35*a^3*b^4*x^8 + 140*a^
4*b^3*x^6 + 500*a^5*b^2*x^4 + 160*a^6*b*x^2 - 130*a^7 - 420*(a^5*b^2*x^4 +
```

```
2*a^6*b*x^2 + a^7)*log(b*x^2 + a))/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8)
```

Sympy [A] time = 0.668941, size = 117, normalized size = 1.03

$$-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}+\frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}-\frac{13a^7+14a^6bx^2}{4a^2b^8+8ab^9x^2+4b^{10}x^4}+\frac{x^{10}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x**2)/(2*b**8) + 15*a**4*x**2/(2*b**7) - 5*a**3*x**4/(2*
b**6) + a**2*x**6/b**5 - 3*a*x**8/(8*b**4) - (13*a**7 + 14*a**6*b*x**2)/(4*
a**2*b**8 + 8*a*b**9*x**2 + 4*b**10*x**4) + x**10/(10*b**3)
```

Giac [A] time = 2.40065, size = 154, normalized size = 1.35

$$-\frac{21\,a^5\log\left(\left|bx^2+a\right|\right)}{2\,b^8}+\frac{63\,a^5b^2x^4+112\,a^6bx^2+50\,a^7}{4\left(bx^2+a\right)^2b^8}+\frac{4\,b^{12}x^{10}-15\,ab^{11}x^8+40\,a^2b^{10}x^6-100\,a^3b^9x^4+300\,a^4b^8x^2}{40\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -21/2*a^5*log(abs(b*x^2 + a))/b^8 + 1/4*(63*a^5*b^2*x^4 + 112*a^6*b*x^2 + 5
0*a^7)/((b*x^2 + a)^2*b^8) + 1/40*(4*b^12*x^10 - 15*a*b^11*x^8 + 40*a^2*b^1
0*x^6 - 100*a^3*b^9*x^4 + 300*a^4*b^8*x^2)/b^15
```

3.169 $\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=100

$$\frac{3a^2x^4}{2b^5}-\frac{5a^3x^2}{b^6}+\frac{3a^5}{b^7\left(a+bx^2\right)}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

```
[Out] (-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^
6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2
])/(2*b^7)
```

Rubi [A] time = 0.0821957, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^4}{2b^5}-\frac{5a^3x^2}{b^6}+\frac{3a^5}{b^7\left(a+bx^2\right)}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^
6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2
])/(2*b^7)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^6}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{3a^5}{b^7\left(a+bx^2\right)}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}
\end{aligned}$$

Mathematica [A] time = 0.0272045, size = 85, normalized size = 0.85

$$\frac{12a^2b^2x^4-40a^3bx^2+\frac{24a^5}{a+bx^2}-\frac{2a^6}{(a+bx^2)^2}+60a^4\log\left(a+bx^2\right)-4ab^3x^6+b^4x^8}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-40*a^3*b*x^2 + 12*a^2*b^2*x^4 - 4*a*b^3*x^6 + b^4*x^8 - (2*a^6)/(a + b*x^
2)^2 + (24*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(8*b^7)
```

Maple [A] time = 0.009, size = 91, normalized size = 0.9

$$-5\frac{a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}-\frac{a^6}{4b^7\left(bx^2+a\right)^2}+3\frac{a^5}{b^7\left(bx^2+a\right)}+\frac{15a^4\ln\left(bx^2+a\right)}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^3,x)
```

```
[Out] -5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^2
+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*ln(b*x^2+a)/b^7
```

Maxima [A] time = 2.31735, size = 134, normalized size = 1.34

$$\frac{12a^5bx^2+11a^6}{4\left(b^9x^4+2ab^8x^2+a^2b^7\right)}+\frac{15a^4\log\left(bx^2+a\right)}{2b^7}+\frac{b^3x^8-4ab^2x^6+12a^2bx^4-40a^3x^2}{8b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*a^5*b*x^2 + 11*a^6)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 15/2*a^4*lo
g(b*x^2 + a)/b^7 + 1/8*(b^3*x^8 - 4*a*b^2*x^6 + 12*a^2*b*x^4 - 40*a^3*x^2)/
b^6
```

Fricas [A] time = 1.20368, size = 261, normalized size = 2.61

$$\frac{b^6x^{12}-2ab^5x^{10}+5a^2b^4x^8-20a^3b^3x^6-68a^4b^2x^4-16a^5bx^2+22a^6+60\left(a^4b^2x^4+2a^5bx^2+a^6\right)\log\left(bx^2+a\right)}{8\left(b^9x^4+2ab^8x^2+a^2b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 - 2*a*b^5*x^10 + 5*a^2*b^4*x^8 - 20*a^3*b^3*x^6 - 68*a^4*b^2*
x^4 - 16*a^5*b*x^2 + 22*a^6 + 60*(a^4*b^2*x^4 + 2*a^5*b*x^2 + a^6)*log(b*x^
```

```
2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

Sympy [A] time = 0.597757, size = 104, normalized size = 1.04

$$\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{11a^6+12a^5bx^2}{4a^2b^7+8ab^8x^2+4b^9x^4}+\frac{x^8}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*x**4/(2*b**5)
- a*x**6/(2*b**4) + (11*a**6 + 12*a**5*b*x**2)/(4*a**2*b**7 + 8*a*b**8*x**2
 + 4*b**9*x**4) + x**8/(8*b**3)
```

Giac [A] time = 1.81621, size = 138, normalized size = 1.38

$$\frac{15\,a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^7}-\frac{45\,a^4b^2x^4+78\,a^5bx^2+34\,a^6}{4\left(bx^2+a\right)^2b^7}+\frac{b^9x^8-4\,ab^8x^6+12\,a^2b^7x^4-40\,a^3b^6x^2}{8\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*a^4*log(abs(b*x^2 + a))/b^7 - 1/4*(45*a^4*b^2*x^4 + 78*a^5*b*x^2 + 34*
a^6)/((b*x^2 + a)^2*b^7) + 1/8*(b^9*x^8 - 4*a*b^8*x^6 + 12*a^2*b^7*x^4 - 40
*a^3*b^6*x^2)/b^12
```

3.170 $\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=87

$$\frac{3a^2x^2}{b^5}-\frac{5a^4}{2b^6\left(a+bx^2\right)}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}$$

```
[Out] (3*a^2*x^2)/b^5 - (3*a*x^4)/(4*b^4) + x^6/(6*b^3) + a^5/(4*b^6*(a + b*x^2)^
2) - (5*a^4)/(2*b^6*(a + b*x^2)) - (5*a^3*Log[a + b*x^2])/b^6
```

Rubi [A] time = 0.0715145, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^2}{b^5}-\frac{5a^4}{2b^6\left(a+bx^2\right)}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*x^2)/b^5 - (3*a*x^4)/(4*b^4) + x^6/(6*b^3) + a^5/(4*b^6*(a + b*x^2)^
2) - (5*a^4)/(2*b^6*(a + b*x^2)) - (5*a^3*Log[a + b*x^2])/b^6
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^5}{(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^4}{2b^6\left(a+bx^2\right)}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0232649, size = 75, normalized size = 0.86

$$\frac{36a^2bx^2-\frac{30a^4}{a+bx^2}+\frac{3a^5}{\left(a+bx^2\right)^2}-60a^3\log\left(a+bx^2\right)-9ab^2x^4+2b^3x^6}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^3,x]
```

```
[Out] (36*a^2*b*x^2 - 9*a*b^2*x^4 + 2*b^3*x^6 + (3*a^5)/(a + b*x^2)^2 - (30*a^4)/
(a + b*x^2) - 60*a^3*Log[a + b*x^2])/(12*b^6)
```

Maple [A] time = 0.01, size = 80, normalized size = 0.9

$$3\,{\frac{{a}^{2}{x}^{2}}{{b}^{5}}}-{\frac{3\,a{x}^{4}}{4\,{b}^{4}}}+{\frac{{x}^{6}}{6\,{b}^{3}}}+{\frac{{a}^{5}}{4\,{b}^{6}\left(b{x}^{2}+a\right)^{2}}}-{\frac{5\,{a}^{4}}{2\,{b}^{6}\left(b{x}^{2}+a\right)}}-5\,{\frac{{a}^{3}\ln\left(b{x}^{2}+a\right)}{{b}^{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^3,x)
```

```
[Out] 3*a^2*x^2/b^5-3/4*a*x^4/b^4+1/6*x^6/b^3+1/4*a^5/b^6/(b*x^2+a)^2-5/2*a^4/b^6
/(b*x^2+a)-5*a^3*ln(b*x^2+a)/b^6
```

Maxima [A] time = 1.25385, size = 120, normalized size = 1.38

$$-\frac{10\,a^{4}bx^{2}+9\,a^{5}}{4\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}-\frac{5\,a^{3}\log\left(bx^{2}+a\right)}{b^{6}}+\frac{2\,b^{2}x^{6}-9\,abx^{4}+36\,a^{2}x^{2}}{12\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(10*a^4*b*x^2 + 9*a^5)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) - 5*a^3*log(b
*x^2 + a)/b^6 + 1/12*(2*b^2*x^6 - 9*a*b*x^4 + 36*a^2*x^2)/b^5
```

Fricas [A] time = 1.17652, size = 240, normalized size = 2.76

$$\frac{2\,b^{5}x^{10}-5\,ab^{4}x^{8}+20\,a^{2}b^{3}x^{6}+63\,a^{3}b^{2}x^{4}+6\,a^{4}bx^{2}-27\,a^{5}-60\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)\log\left(bx^{2}+a\right)}{12\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 - 5*a*b^4*x^8 + 20*a^2*b^3*x^6 + 63*a^3*b^2*x^4 + 6*a^4*b*
x^2 - 27*a^5 - 60*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^8*x^
4 + 2*a*b^7*x^2 + a^2*b^6)
```

Sympy [A] time = 0.579237, size = 90, normalized size = 1.03

$$-\frac{5a^{3}\log\left(a+bx^{2}\right)}{b^{6}}+\frac{3a^{2}x^{2}}{b^{5}}-\frac{3ax^{4}}{4b^{4}}-\frac{9a^{5}+10a^{4}bx^{2}}{4a^{2}b^{6}+8ab^{7}x^{2}+4b^{8}x^{4}}+\frac{x^{6}}{6b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**3,x)
```

```
[Out] -5*a**3*log(a + b*x**2)/b**6 + 3*a**2*x**2/b**5 - 3*a*x**4/(4*b**4) - (9*a*
*5 + 10*a**4*b*x**2)/(4*a**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) + x**6/(6*
b**3)
```

Giac [A] time = 1.80929, size = 124, normalized size = 1.43

$$-\frac{5 a^3 \log\left(\left|bx^2 + a\right|\right)}{b^6} + \frac{30 a^3 b^2 x^4 + 50 a^4 b x^2 + 21 a^5}{4 \left(bx^2 + a\right)^2 b^6} + \frac{2 b^6 x^6 - 9 a b^5 x^4 + 36 a^2 b^4 x^2}{12 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*a^3*log(abs(b*x^2 + a))/b^6 + 1/4*(30*a^3*b^2*x^4 + 50*a^4*b*x^2 + 21*a^
5)/((b*x^2 + a)^2*b^6) + 1/12*(2*b^6*x^6 - 9*a*b^5*x^4 + 36*a^2*b^4*x^2)/b^
9
```

3.171 $\int \frac{x^9}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=74

$$-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

```
[Out] (-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5
*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5
```

Rubi [A] time = 0.0581404, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5
*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^3} dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{(a+bx)^3} dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.0229814, size = 63, normalized size = 0.85

$$\frac{-\frac{a^4}{\left(a+bx^2\right)^2}+\frac{8a^3}{a+bx^2}+12a^2\log\left(a+bx^2\right)-6abx^2+b^2x^4}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-6*a*b*x^2 + b^2*x^4 - a^4/(a + b*x^2)^2 + (8*a^3)/(a + b*x^2) + 12*a^2*Lo
g[a + b*x^2])/(4*b^5)
```

Maple [A] time = 0.01, size = 69, normalized size = 0.9

$$-\frac{3\,ax^{2}}{2\,b^{4}}+\frac{x^{4}}{4\,b^{3}}-\frac{a^{4}}{4\,b^{5}\left(bx^{2}+a\right)^{2}}+2\,\frac{a^{3}}{b^{5}\left(bx^{2}+a\right)}+3\,\frac{a^{2}\ln\left(bx^{2}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^3,x)
```

```
[Out] -3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a^
2*ln(b*x^2+a)/b^5
```

Maxima [A] time = 2.06585, size = 104, normalized size = 1.41

$$\frac{8\,a^{3}bx^{2}+7\,a^{4}}{4\left(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5}\right)}+\frac{3\,a^{2}\log\left(bx^{2}+a\right)}{b^{5}}+\frac{bx^{4}-6\,ax^{2}}{4\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*x^2 + 7*a^4)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) + 3*a^2*log(b*x
^2 + a)/b^5 + 1/4*(b*x^4 - 6*a*x^2)/b^4
```

Fricas [A] time = 1.13331, size = 211, normalized size = 2.85

$$\frac{b^{4}x^{8}-4\,ab^{3}x^{6}-11\,a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+7\,a^{4}+12\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)\log\left(bx^{2}+a\right)}{4\left(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 - 4*a*b^3*x^6 - 11*a^2*b^2*x^4 + 2*a^3*b*x^2 + 7*a^4 + 12*(a^2
*b^2*x^4 + 2*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*
b^5)
```

Sympy [A] time = 0.561601, size = 78, normalized size = 1.05

$$\frac{3a^{2}\log\left(a+bx^{2}\right)}{b^{5}}-\frac{3ax^{2}}{2b^{4}}+\frac{7a^{4}+8a^{3}bx^{2}}{4a^{2}b^{5}+8ab^{6}x^{2}+4b^{7}x^{4}}+\frac{x^{4}}{4b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**3,x)
```

```
[Out] 3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + (7*a**4 + 8*a**3*b*x**2)/
(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) + x**4/(4*b**3)
```

Giac [A] time = 2.63261, size = 108, normalized size = 1.46

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{b^5}+\frac{b^3x^4-6\,ab^2x^2}{4\,b^6}-\frac{18\,a^2b^2x^4+28\,a^3bx^2+11\,a^4}{4\,\left(bx^2+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x^2 + a))/b^5 + 1/4*(b^3*x^4 - 6*a*b^2*x^2)/b^6 - 1/4*(18*a
^2*b^2*x^4 + 28*a^3*b*x^2 + 11*a^4)/((b*x^2 + a)^2*b^5)
```

3.172 $\int \frac{x^7}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{a^3}{4b^4\left(a+bx^2\right)^2}-\frac{3a^2}{2b^4\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{2b^4}+\frac{x^2}{2b^3}$$

```
[Out] x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*
a*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0458384, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{4b^4\left(a+bx^2\right)^2}-\frac{3a^2}{2b^4\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{2b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^3,x]
```

```
[Out] x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*
a*Log[a + b*x^2])/(2*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2b^3}+\frac{a^3}{4b^4\left(a+bx^2\right)^2}-\frac{3a^2}{2b^4\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0544749, size = 48, normalized size = 0.74

$$-\frac{\frac{a^2\left(5a+6bx^2\right)}{\left(a+bx^2\right)^2}+6a\log\left(a+bx^2\right)-2bx^2}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^3,x]
```

```
[Out] -(-2*b*x^2 + (a^2*(5*a + 6*b*x^2))/(a + b*x^2)^2 + 6*a*Log[a + b*x^2])/(4*b
^4)
```

Maple [A] time = 0.009, size = 58, normalized size = 0.9

$$\frac{x^2}{2\,b^3}+\frac{a^3}{4\,b^4\left(bx^2+a\right)^2}-\frac{3\,a^2}{2\,b^4\left(bx^2+a\right)}-\frac{3\,a\ln\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^3,x)
```

```
[Out] 1/2*x^2/b^3+1/4*a^3/b^4/(b*x^2+a)^2-3/2*a^2/b^4/(b*x^2+a)-3/2*a*ln(b*x^2+a)
/b^4
```

Maxima [A] time = 2.33507, size = 89, normalized size = 1.37

$$-\frac{6\,a^2bx^2+5\,a^3}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}+\frac{x^2}{2\,b^3}-\frac{3\,a\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.19138, size = 186, normalized size = 2.86

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [A] time = 0.538994, size = 66, normalized size = 1.02

$$-\frac{3a\log\left(a+bx^2\right)}{2b^4}-\frac{5a^3+6a^2bx^2}{4a^2b^4+8ab^5x^2+4b^6x^4}+\frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/(2*b**4) - (5*a**3 + 6*a**2*b*x**2)/(4*a**2*b**4 + 8*a
*b**5*x**2 + 4*b**6*x**4) + x**2/(2*b**3)
```

Giac [A] time = 1.71611, size = 84, normalized size = 1.29

$$\frac{x^2}{2\,b^3} - \frac{3\,a\log\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{9\,ab^2x^4 + 12\,a^2bx^2 + 4\,a^3}{4\,\left(bx^2 + a\right)^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^3 - 3/2*a*log(abs(b*x^2 + a))/b^4 + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x
^2 + 4*a^3)/((b*x^2 + a)^2*b^4)
```

3.173 $\int \frac{x^5}{(a+bx^2)^3} dx$

Optimal. Leaf size=49

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] -a^2/(4*b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + Log[a + b*x^2]/(2*b^3)
```

Rubi [A] time = 0.0377695, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^3,x]
```

```
[Out] -a^2/(4*b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + Log[a + b*x^2]/(2*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0183525, size = 39, normalized size = 0.8

$$\frac{\frac{a\left(3a+4bx^2\right)}{\left(a+bx^2\right)^2}+2\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x^2))/(a + b*x^2)^2 + 2*Log[a + b*x^2])/(4*b^3)
```

Maple [A] time = 0.009, size = 46, normalized size = 0.9

$$-\frac{a^2}{4\,b^3\left(bx^2+a\right)^2}+\frac{a}{b^3\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3,x)
```

```
[Out] -1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

Maxima [A] time = 2.11304, size = 74, normalized size = 1.51

$$\frac{4\,abx^2+3\,a^2}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3
```

Fricas [A] time = 1.15257, size = 143, normalized size = 2.92

$$\frac{4\,abx^2+3\,a^2+2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [A] time = 0.468551, size = 53, normalized size = 1.08

$$\frac{3a^2+4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x**2)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + log(a +
 b*x**2)/(2*b**3)
```

Giac [A] time = 2.02583, size = 57, normalized size = 1.16

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^3}-\frac{3\,bx^4+2\,ax^2}{4\,\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^3 - 1/4*(3*b*x^4 + 2*a*x^2)/((b*x^2 + a)^2*b^2)
```

3.174 $\int \frac{x^3}{(a+bx^2)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4a(a+bx^2)^2}$$

[Out] x^4/(4*a*(a + b*x^2)^2)

Rubi [A] time = 0.0038325, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{x^4}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^2)^3,x]

[Out] x^4/(4*a*(a + b*x^2)^2)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^3} dx = \frac{x^4}{4a(a+bx^2)^2}$$

Mathematica [A] time = 0.0076782, size = 24, normalized size = 1.26

$$-\frac{a+2bx^2}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*x^2)^3,x]

[Out] -(a + 2*b*x^2)/(4*b^2*(a + b*x^2)^2)

Maple [A] time = 0.007, size = 31, normalized size = 1.6

$$\frac{a}{4\,b^2(bx^2+a)^2} - \frac{1}{2\,b^2(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^3,x)
```

```
[Out] 1/4/b^2*a/(b*x^2+a)^2-1/2/b^2/(b*x^2+a)
```

Maxima [B] time = 2.15848, size = 49, normalized size = 2.58

$$-\frac{2\,bx^2+a}{4\,\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [B] time = 1.24151, size = 73, normalized size = 3.84

$$-\frac{2\,bx^2+a}{4\,\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] time = 0.41778, size = 36, normalized size = 1.89

$$-\frac{a+2bx^2}{4a^2b^2+8ab^3x^2+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**3,x)
```

```
[Out] -(a + 2*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

Giac [A] time = 1.36925, size = 30, normalized size = 1.58

$$-\frac{2\,bx^2+a}{4\,\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2)
```

3.175 $\int \frac{x}{(a+bx^2)^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{4b(a+bx^2)^2}$$

```
[Out] -1/(4*b*(a + b*x^2)^2)
```

Rubi [A] time = 0.0028312, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^3,x]
```

```
[Out] -1/(4*b*(a + b*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^3} dx = -\frac{1}{4b(a+bx^2)^2}$$

Mathematica [A] time = 0.0021633, size = 16, normalized size = 1.

$$-\frac{1}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^3,x]
```

```
[Out] -1/(4*b*(a + b*x^2)^2)
```

Maple [A] time = 0.001, size = 15, normalized size = 0.9

$$-\frac{1}{4b(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^3,x)
```

```
[Out] -1/4/b/(b*x^2+a)^2
```

Maxima [A] time = 2.11486, size = 19, normalized size = 1.19

$$-\frac{1}{4\left(bx^2+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

Fricas [A] time = 1.1138, size = 51, normalized size = 3.19

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [A] time = 0.420224, size = 27, normalized size = 1.69

$$-\frac{1}{4a^2b+8ab^2x^2+4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**3,x)
```

```
[Out] -1/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)
```

Giac [A] time = 2.00536, size = 19, normalized size = 1.19

$$-\frac{1}{4\left(bx^2+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

3.176 $\int \frac{1}{x(a+bx^2)^3} dx$

Optimal. Leaf size=54

$$\frac{1}{2a^2(a+bx^2)} - \frac{\log(a+bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{4a(a+bx^2)^2}$$

```
[Out] 1/(4*a*(a + b*x^2)^2) + 1/(2*a^2*(a + b*x^2)) + Log[x]/a^3 - Log[a + b*x^2]
/(2*a^3)
```

Rubi [A] time = 0.0380604, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2a^2(a+bx^2)} - \frac{\log(a+bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a + b*x^2)^2) + 1/(2*a^2*(a + b*x^2)) + Log[x]/a^3 - Log[a + b*x^2]
/(2*a^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{1}{4a(a+bx^2)^2} + \frac{1}{2a^2(a+bx^2)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx^2)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.0343808, size = 43, normalized size = 0.8

$$\frac{\frac{a(3a+2bx^2)}{(a+bx^2)^2} - 2\log(a+bx^2) + 4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x^2))/(a + b*x^2)^2 + 4*Log[x] - 2*Log[a + b*x^2])/(4*a^3)
```

Maple [A] time = 0.011, size = 49, normalized size = 0.9

$$\frac{1}{4\,a\left(bx^2+a\right)^2}+\frac{1}{2\,a^2\left(bx^2+a\right)}+\frac{\ln\left(x\right)}{a^3}-\frac{\ln\left(bx^2+a\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^3,x)
```

```
[Out] 1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

Maxima [A] time = 2.5015, size = 81, normalized size = 1.5

$$\frac{2\,bx^2+3\,a}{4\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/
a^3 + 1/2*log(x^2)/a^3
```

Fricas [A] time = 1.22897, size = 196, normalized size = 3.63

$$\frac{2\,abx^2+3\,a^2-2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+2\,abx^2+a^2\right)\log\left(x\right)}{4\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

Sympy [A] time = 0.632291, size = 56, normalized size = 1.04

$$\frac{3a+2bx^2}{4a^4+8a^3bx^2+4a^2b^2x^4}+\frac{\log\left(x\right)}{a^3}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**3,x)
```

```
[Out] (3*a + 2*b*x**2)/(4*a**4 + 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**3
- log(a/b + x**2)/(2*a**3)
```

Giac [A] time = 1.93902, size = 80, normalized size = 1.48

$$\frac{\log\left(x^2\right)}{2\,a^3}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3}+\frac{3\,b^2x^4+8\,abx^2+6\,a^2}{4\left(bx^2+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 + a))/a^3 + 1/4*(3*b^2*x^4 + 8*a*b*x^2
 + 6*a^2)/((b*x^2 + a)^2*a^3)
```

3.177 $\int \frac{1}{x^3(a+bx^2)^3} dx$

Optimal. Leaf size=67

$$-\frac{b}{a^3(a+bx^2)}-\frac{b}{4a^2(a+bx^2)^2}+\frac{3b\log(a+bx^2)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

```
[Out] -1/(2*a^3*x^2) - b/(4*a^2*(a + b*x^2)^2) - b/(a^3*(a + b*x^2)) - (3*b*Log[x
])/a^4 + (3*b*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.0465404, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b}{a^3(a+bx^2)}-\frac{b}{4a^2(a+bx^2)^2}+\frac{3b\log(a+bx^2)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/(2*a^3*x^2) - b/(4*a^2*(a + b*x^2)^2) - b/(a^3*(a + b*x^2)) - (3*b*Log[x
])/a^4 + (3*b*Log[a + b*x^2])/(2*a^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{1}{2a^3x^2}-\frac{b}{4a^2(a+bx^2)^2}-\frac{b}{a^3(a+bx^2)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx^2)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.0535047, size = 59, normalized size = 0.88

$$-\frac{\frac{a(2a^2+9abx^2+6b^2x^4)}{x^2(a+bx^2)^2}-6b\log(a+bx^2)+12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -((a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4))/(x^2*(a + b*x^2)^2) + 12*b*Log[x] - 6
*b*Log[a + b*x^2])/(4*a^4)
```

Maple [A] time = 0.012, size = 62, normalized size = 0.9

$$-\frac{1}{2\,a^3x^2}-\frac{b}{4\,a^2\left(bx^2+a\right)^2}-\frac{b}{a^3\left(bx^2+a\right)}-3\,\frac{b\ln(x)}{a^4}+\frac{3\,b\ln\left(bx^2+a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2/a^3/x^2-1/4*b/a^2/(b*x^2+a)^2-b/a^3/(b*x^2+a)-3*b*ln(x)/a^4+3/2*b*ln(b
*x^2+a)/a^4
```

Maxima [A] time = 1.95167, size = 104, normalized size = 1.55

$$-\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{4\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{2\,a^4}-\frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
+ 3/2*b*log(b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4
```

Fricas [A] time = 1.24778, size = 247, normalized size = 3.69

$$-\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3-6\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\log\left(bx^2+a\right)+12\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A] time = 0.781277, size = 78, normalized size = 1.16

$$-\frac{2a^2+9abx^2+6b^2x^4}{4a^5x^2+8a^4bx^4+4a^3b^2x^6}-\frac{3b\log(x)}{a^4}+\frac{3b\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**3,x)
```

```
[Out] -(2*a**2 + 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) - 3*b*log(x)/a**4 + 3*b*log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 2.91433, size = 111, normalized size = 1.66

$$-\frac{3\,b\log\left(x^2\right)}{2\,a^4}+\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{2\,a^4}-\frac{9\,b^3x^4+22\,ab^2x^2+14\,a^2b}{4\left(bx^2+a\right)^2a^4}+\frac{3\,bx^2-a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*b*log(x^2)/a^4 + 3/2*b*log(abs(b*x^2 + a))/a^4 - 1/4*(9*b^3*x^4 + 22*a
*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4) + 1/2*(3*b*x^2 - a)/(a^4*x^2)
```

3.178 $\int \frac{1}{x^5(a+bx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{3b^2}{2a^4(a+bx^2)}+\frac{b^2}{4a^3(a+bx^2)^2}-\frac{3b^2\log(a+bx^2)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b}{2a^4x^2}-\frac{1}{4a^3x^4}$$

```
[Out] -1/(4*a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a + b*x^2)^2) + (3*b^2)/(2
*a^4*(a + b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a + b*x^2])/a^5
```

Rubi [A] time = 0.0600754, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{3b^2}{2a^4(a+bx^2)}+\frac{b^2}{4a^3(a+bx^2)^2}-\frac{3b^2\log(a+bx^2)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b}{2a^4x^2}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -1/(4*a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a + b*x^2)^2) + (3*b^2)/(2
*a^4*(a + b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a + b*x^2])/a^5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(a+bx^2)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4a^3x^4}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3(a+bx^2)^2}+\frac{3b^2}{2a^4(a+bx^2)}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log(a+bx^2)}{a^5}
\end{aligned}$$

Mathematica [A] time = 0.0470941, size = 74, normalized size = 0.86

$$\frac{\frac{a(4a^2bx^2-a^3+18ab^2x^4+12b^3x^6)}{x^4(a+bx^2)^2}-12b^2\log(a+bx^2)+24b^2\log(x)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x^2 + 18*a*b^2*x^4 + 12*b^3*x^6))/(x^4*(a + b*x^2)^2) +
 24*b^2*Log[x] - 12*b^2*Log[a + b*x^2])/(4*a^5)
```

Maple [A] time = 0.013, size = 79, normalized size = 0.9

$$-\frac{1}{4\,a^3x^4}+\frac{3\,b}{2\,a^4x^2}+\frac{b^2}{4\,a^3\left(bx^2+a\right)^2}+\frac{3\,b^2}{2\,a^4\left(bx^2+a\right)}+6\,\frac{b^2\ln(x)}{a^5}-3\,\frac{b^2\ln\left(bx^2+a\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^3,x)
```

```
[Out] -1/4/a^3/x^4+3/2*b/a^4/x^2+1/4*b^2/a^3/(b*x^2+a)^2+3/2*b^2/a^4/(b*x^2+a)+6*
b^2*ln(x)/a^5-3*b^2*ln(b*x^2+a)/a^5
```

Maxima [A] time = 2.15005, size = 124, normalized size = 1.44

$$\frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{a^5}+\frac{3\,b^2\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/(a^4*b^2*x^8 + 2*a^5*b*
x^6 + a^6*x^4) - 3*b^2*log(b*x^2 + a)/a^5 + 3*b^2*log(x^2)/a^5
```

Fricas [A] time = 1.27323, size = 274, normalized size = 3.19

$$\frac{12\,ab^3x^6+18\,a^2b^2x^4+4\,a^3bx^2-a^4-12\left(b^4x^8+2\,ab^3x^6+a^2b^2x^4\right)\log\left(bx^2+a\right)+24\left(b^4x^8+2\,ab^3x^6+a^2b^2x^4\right)\log(x)}{4\left(a^5b^2x^8+2\,a^6bx^6+a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^3*x^6 + 18*a^2*b^2*x^4 + 4*a^3*b*x^2 - a^4 - 12*(b^4*x^8 + 2*a*
b^3*x^6 + a^2*b^2*x^4)*log(b*x^2 + a) + 24*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2
*x^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

Sympy [A] time = 1.00023, size = 90, normalized size = 1.05

$$\frac{-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6}{4a^6x^4+8a^5bx^6+4a^4b^2x^8}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x**2 + 18*a*b**2*x**4 + 12*b**3*x**6)/(4*a**6*x**4 + 8*a*
*5*b*x**6 + 4*a**4*b**2*x**8) + 6*b**2*log(x)/a**5 - 3*b**2*log(a/b + x**2)
/a**5
```

Giac [A] time = 2.20303, size = 108, normalized size = 1.26

$$\frac{3\,b^2\log\left(x^2\right)}{a^5}-\frac{3\,b^2\log\left(\left|bx^2+a\right|\right)}{a^5}+\frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\left(bx^4+ax^2\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*b^2*log(x^2)/a^5 - 3*b^2*log(abs(b*x^2 + a))/a^5 + 1/4*(12*b^3*x^6 + 18*a
*b^2*x^4 + 4*a^2*b*x^2 - a^3)/((b*x^4 + a*x^2)^2*a^4)
```

3.179 $\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=95

$$-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}-\frac{3b^2}{a^5x^2}+\frac{5b^3\log\left(a+bx^2\right)}{a^6}-\frac{10b^3\log(x)}{a^6}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

```
[Out] -1/(6*a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a + b*
x^2)^2) - (2*b^3)/(a^5*(a + b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a +
b*x^2])/a^6
```

Rubi [A] time = 0.067537, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}-\frac{3b^2}{a^5x^2}+\frac{5b^3\log\left(a+bx^2\right)}{a^6}-\frac{10b^3\log(x)}{a^6}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/(6*a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a + b*
x^2)^2) - (2*b^3)/(a^5*(a + b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a +
b*x^2])/a^6
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log\left(a+bx^2\right)}{a^6}
\end{aligned}$$

Mathematica [A] time = 0.0668509, size = 85, normalized size = 0.89

$$-\frac{\frac{a\left(20a^2b^2x^4-5a^3bx^2+2a^4+90ab^3x^6+60b^4x^8\right)}{x^6\left(a+bx^2\right)^2}-60b^3\log\left(a+bx^2\right)+120b^3\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -((a*(2*a^4 - 5*a^3*b*x^2 + 20*a^2*b^2*x^4 + 90*a*b^3*x^6 + 60*b^4*x^8))/(x
^6*(a + b*x^2)^2) + 120*b^3*Log[x] - 60*b^3*Log[a + b*x^2])/(12*a^6)
```

Maple [A] time = 0.012, size = 90, normalized size = 1.

$$-\frac{1}{6\,a^3x^6}+\frac{3\,b}{4\,a^4x^4}-3\,\frac{b^2}{a^5x^2}-\frac{b^3}{4\,a^4\left(bx^2+a\right)^2}-2\,\frac{b^3}{a^5\left(bx^2+a\right)}-10\,\frac{b^3\ln(x)}{a^6}+5\,\frac{b^3\ln\left(bx^2+a\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^3,x)
```

```
[Out] -1/6/a^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-1/4*b^3/a^4/(b*x^2+a)^2-2*b^3/a^5/
(b*x^2+a)-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

Maxima [A] time = 1.80771, size = 139, normalized size = 1.46

$$-\frac{60\,b^4x^8+90\,ab^3x^6+20\,a^2b^2x^4-5\,a^3bx^2+2\,a^4}{12\left(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6\right)}+\frac{5\,b^3\log\left(bx^2+a\right)}{a^6}-\frac{5\,b^3\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*b^4*x^8 + 90*a*b^3*x^6 + 20*a^2*b^2*x^4 - 5*a^3*b*x^2 + 2*a^4)/(a
^5*b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) + 5*b^3*log(b*x^2 + a)/a^6 - 5*b^3*log
(x^2)/a^6
```

Fricas [A] time = 1.26026, size = 308, normalized size = 3.24

$$-\frac{60\,ab^4x^8+90\,a^2b^3x^6+20\,a^3b^2x^4-5\,a^4bx^2+2\,a^5-60\left(b^5x^{10}+2\,ab^4x^8+a^2b^3x^6\right)\log\left(bx^2+a\right)+120\left(b^5x^{10}+2\,ab^4\right.}{12\left(a^6b^2x^{10}+2\,a^7bx^8+a^8x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*a*b^4*x^8 + 90*a^2*b^3*x^6 + 20*a^3*b^2*x^4 - 5*a^4*b*x^2 + 2*a^5
 - 60*(b^5*x^10 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(b*x^2 + a) + 120*(b^5*x^10
 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(x))/(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6
```

)

Sympy [A] time = 1.71902, size = 104, normalized size = 1.09

$$-\frac{2a^4-5a^3bx^2+20a^2b^2x^4+90ab^3x^6+60b^4x^8}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**3,x)
```

```
[Out] -(2*a**4 - 5*a**3*b*x**2 + 20*a**2*b**2*x**4 + 90*a*b**3*x**6 + 60*b**4*x**
8)/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) - 10*b**3*log(x)/a*
*6 + 5*b**3*log(a/b + x**2)/a**6
```

Giac [A] time = 2.10982, size = 149, normalized size = 1.57

$$-\frac{5\,b^3\log\left(x^2\right)}{a^6}+\frac{5\,b^3\log\left(\left|bx^2+a\right|\right)}{a^6}-\frac{30\,b^5x^4+68\,ab^4x^2+39\,a^2b^3}{4\left(bx^2+a\right)^2a^6}+\frac{110\,b^3x^6-36\,ab^2x^4+9\,a^2bx^2-2\,a^3}{12\,a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*b^3*log(x^2)/a^6 + 5*b^3*log(abs(b*x^2 + a))/a^6 - 1/4*(30*b^5*x^4 + 68*
a*b^4*x^2 + 39*a^2*b^3)/((b*x^2 + a)^2*a^6) + 1/12*(110*b^3*x^6 - 36*a*b^2*
x^4 + 9*a^2*b*x^2 - 2*a^3)/(a^6*x^6)
```

3.180 $\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=112

$$\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^3}{a^6x^2}-\frac{3b^2}{2a^5x^4}-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

```
[Out] -1/(8*a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2) +
b^4/(4*a^5*(a + b*x^2)^2) + (5*b^4)/(2*a^6*(a + b*x^2)) + (15*b^4*Log[x])/a
^7 - (15*b^4*Log[a + b*x^2])/(2*a^7)
```

Rubi [A] time = 0.0798041, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^3}{a^6x^2}-\frac{3b^2}{2a^5x^4}-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] -1/(8*a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2) +
b^4/(4*a^5*(a + b*x^2)^2) + (5*b^4)/(2*a^6*(a + b*x^2)) + (15*b^4*Log[x])/a
^7 - (15*b^4*Log[a + b*x^2])/(2*a^7)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx)}\right)d\right.\\ &= -\frac{1}{8a^3x^8}+\frac{b}{2a^4x^6}-\frac{3b^2}{2a^5x^4}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(\cdot}{2a^7}\end{aligned}$$

Mathematica [A] time = 0.0576169, size = 96, normalized size = 0.86

$$\frac{\frac{a\left(20a^2b^3x^6-5a^3b^2x^4+2a^4bx^2-a^5+90ab^4x^8+60b^5x^{10}\right)}{x^8\left(a+bx^2\right)^2}-60b^4\log\left(a+bx^2\right)+120b^4\log(x)}{8a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x^2 - 5*a^3*b^2*x^4 + 20*a^2*b^3*x^6 + 90*a*b^4*x^8 + 6
0*b^5*x^10))/(x^8*(a + b*x^2)^2) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/
(8*a^7)
```

Maple [A] time = 0.014, size = 101, normalized size = 0.9

$$-\frac{1}{8\,a^3x^8}+\frac{b}{2\,a^4x^6}-\frac{3\,b^2}{2\,a^5x^4}+5\,\frac{b^3}{a^6x^2}+\frac{b^4}{4\,a^5\left(bx^2+a\right)^2}+\frac{5\,b^4}{2\,a^6\left(bx^2+a\right)}+15\,\frac{b^4\ln(x)}{a^7}-\frac{15\,b^4\ln\left(bx^2+a\right)}{2\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^3,x)
```

```
[Out] -1/8/a^3/x^8+1/2*b/a^4/x^6-3/2*b^2/a^5/x^4+5*b^3/a^6/x^2+1/4*b^4/a^5/(b*x^2
+a)^2+5/2*b^4/a^6/(b*x^2+a)+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

Maxima [A] time = 2.71982, size = 154, normalized size = 1.38

$$\frac{60\,b^5x^{10}+90\,ab^4x^8+20\,a^2b^3x^6-5\,a^3b^2x^4+2\,a^4bx^2-a^5}{8\left(a^6b^2x^{12}+2\,a^7bx^{10}+a^8x^8\right)}-\frac{15\,b^4\log\left(bx^2+a\right)}{2\,a^7}+\frac{15\,b^4\log\left(x^2\right)}{2\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(60*b^5*x^10 + 90*a*b^4*x^8 + 20*a^2*b^3*x^6 - 5*a^3*b^2*x^4 + 2*a^4*b*
x^2 - a^5)/(a^6*b^2*x^12 + 2*a^7*b*x^10 + a^8*x^8) - 15/2*b^4*log(b*x^2 + a
)/a^7 + 15/2*b^4*log(x^2)/a^7
```

Fricas [A] time = 1.24437, size = 329, normalized size = 2.94

$$\frac{60\,ab^5x^{10}+90\,a^2b^4x^8+20\,a^3b^3x^6-5\,a^4b^2x^4+2\,a^5bx^2-a^6-60\left(b^6x^{12}+2\,ab^5x^{10}+a^2b^4x^8\right)\log\left(bx^2+a\right)+120\left(b^6x^{12}\right.}{8\left(a^7b^2x^{12}+2\,a^8bx^{10}+a^9x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(60*a*b^5*x^10 + 90*a^2*b^4*x^8 + 20*a^3*b^3*x^6 - 5*a^4*b^2*x^4 + 2*a^
5*b*x^2 - a^6 - 60*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(b*x^2 + a) +
```

```
120*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(x))/(a^7*b^2*x^12 + 2*a^8*
b*x^10 + a^9*x^8)
```

Sympy [A] time = 3.1706, size = 116, normalized size = 1.04

$$\frac{-a^5 + 2a^4bx^2 - 5a^3b^2x^4 + 20a^2b^3x^6 + 90ab^4x^8 + 60b^5x^{10}}{8a^8x^8 + 16a^7bx^{10} + 8a^6b^2x^{12}} + \frac{15b^4 \log(x)}{a^7} - \frac{15b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x**2 - 5*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 90*a*b**4*x
**8 + 60*b**5*x**10)/(8*a**8*x**8 + 16*a**7*b*x**10 + 8*a**6*b**2*x**12) +
15*b**4*log(x)/a**7 - 15*b**4*log(a/b + x**2)/(2*a**7)
```

Giac [A] time = 1.60655, size = 161, normalized size = 1.44

$$\frac{15\,b^4 \log\left(x^2\right)}{2\,a^7} - \frac{15\,b^4 \log\left(\left|bx^2 + a\right|\right)}{2\,a^7} + \frac{45\,b^6x^4 + 100\,ab^5x^2 + 56\,a^2b^4}{4\left(bx^2 + a\right)^2 a^7} - \frac{125\,b^4x^8 - 40\,ab^3x^6 + 12\,a^2b^2x^4 - 4\,a^3bx^2 + a}{8\,a^7x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*b^4*log(x^2)/a^7 - 15/2*b^4*log(abs(b*x^2 + a))/a^7 + 1/4*(45*b^6*x^4
+ 100*a*b^5*x^2 + 56*a^2*b^4)/((b*x^2 + a)^2*a^7) - 1/8*(125*b^4*x^8 - 40*a
*b^3*x^6 + 12*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)/(a^7*x^8)
```

3.181 $\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{33a^2x^3}{8b^5}-\frac{99a^3x}{8b^6}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{11x^9}{8b^2\left(a+bx^2\right)}-\frac{99ax^5}{40b^4}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{99x^7}{56b^3}$$

```
[Out] (-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)
/(56*b^3) - x^11/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a
^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rubi [A] time = 0.0484199, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{33a^2x^3}{8b^5}-\frac{99a^3x}{8b^6}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{11x^9}{8b^2\left(a+bx^2\right)}-\frac{99ax^5}{40b^4}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{99x^7}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^3,x]
```

```
[Out] (-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)
/(56*b^3) - x^11/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a
^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{11}}{4b\left(a+bx^2\right)^2} + \frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^{11}}{4b\left(a+bx^2\right)^2} - \frac{11x^9}{8b^2\left(a+bx^2\right)} + \frac{99\int \frac{x^8}{a+bx^2}\,dx}{8b^2}$$

$$= -\frac{x^{11}}{4b\left(a+bx^2\right)^2} - \frac{11x^9}{8b^2\left(a+bx^2\right)} + \frac{99\int \left(-\frac{a^3}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^4}{b^2} + \frac{x^6}{b} + \frac{a^4}{b^4\left(a+bx^2\right)}\right)dx}{8b^2}$$

$$= -\frac{99a^3x}{8b^6} + \frac{33a^2x^3}{8b^5} - \frac{99ax^5}{40b^4} + \frac{99x^7}{56b^3} - \frac{x^{11}}{4b\left(a+bx^2\right)^2} - \frac{11x^9}{8b^2\left(a+bx^2\right)} + \frac{\left(99a^4\right)\int \frac{1}{a+bx^2}\,dx}{8b^6}$$

$$= -\frac{99a^3x}{8b^6} + \frac{33a^2x^3}{8b^5} - \frac{99ax^5}{40b^4} + \frac{99x^7}{56b^3} - \frac{x^{11}}{4b\left(a+bx^2\right)^2} - \frac{11x^9}{8b^2\left(a+bx^2\right)} + \frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

Mathematica [A] time = 0.0588633, size = 99, normalized size = 0.89

$$\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}} - \frac{-264a^2b^3x^7 + 1848a^3b^2x^5 + 5775a^4bx^3 + 3465a^5x + 88ab^4x^9 - 40b^5x^{11}}{280b^6\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^3,x]
```

```
[Out] -(3465*a^5*x + 5775*a^4*b*x^3 + 1848*a^3*b^2*x^5 - 264*a^2*b^3*x^7 + 88*a*b
^4*x^9 - 40*b^5*x^11)/(280*b^6*(a + b*x^2)^2) + (99*a^(7/2)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*b^(13/2))
```

Maple [A] time = 0.01, size = 99, normalized size = 0.9

$$\frac{x^7}{7\,b^3} - \frac{3\,ax^5}{5\,b^4} + 2\,\frac{a^2x^3}{b^5} - 10\,\frac{a^3x}{b^6} - \frac{21\,a^4x^3}{8\,b^5\left(bx^2+a\right)^2} - \frac{19\,a^5x}{8\,b^6\left(bx^2+a\right)^2} + \frac{99\,a^4}{8\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^3,x)
```

```
[Out] 1/7*x^7/b^3-3/5*a*x^5/b^4+2*a^2*x^3/b^5-10*a^3*x/b^6-21/8/b^5*a^4/(b*x^2+a)
^2*x^3-19/8/b^6*a^5/(b*x^2+a)^2*x+99/8/b^6*a^4/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26598, size = 614, normalized size = 5.53

$$\left[\frac{80\, b^{5} x^{11} - 176\, a b^{4} x^{9} + 528\, a^{2} b^{3} x^{7} - 3696\, a^{3} b^{2} x^{5} - 11550\, a^{4} b x^{3} - 6930\, a^{5} x + 3465\, \left(a^{3} b^{2} x^{4} + 2\, a^{4} b x^{2} + a^{5}\right) \sqrt{-\frac{a}{b}} \log\left(\frac{b x^{2}}{}\right.}{560\, \left(b^{8} x^{4} + 2\, a b^{7} x^{2} + a^{2} b^{6}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*b^5*x^11 - 176*a*b^4*x^9 + 528*a^2*b^3*x^7 - 3696*a^3*b^2*x^5 -
11550*a^4*b*x^3 - 6930*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^4 + 2*a*b^7*x
^2 + a^2*b^6), 1/280*(40*b^5*x^11 - 88*a*b^4*x^9 + 264*a^2*b^3*x^7 - 1848*a
^3*b^2*x^5 - 5775*a^4*b*x^3 - 3465*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2
+ a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)
]
```

Sympy [A] time = 0.619371, size = 160, normalized size = 1.44

$$-\frac{10a^{3}x}{b^{6}} + \frac{2a^{2}x^{3}}{b^{5}} - \frac{3ax^{5}}{5b^{4}} - \frac{99\sqrt{-\frac{a^{7}}{b^{13}}}\log\left(x - \frac{b^{6}\sqrt{-\frac{a^{7}}{b^{13}}}}{a^{3}}\right)}{16} + \frac{99\sqrt{-\frac{a^{7}}{b^{13}}}\log\left(x + \frac{b^{6}\sqrt{-\frac{a^{7}}{b^{13}}}}{a^{3}}\right)}{16} - \frac{19a^{5}x + 21a^{4}bx^{3}}{8a^{2}b^{6} + 16ab^{7}x^{2} + 8b^{8}x^{4}} + \frac{x^{7}}{7b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**3,x)
```

```
[Out] -10*a**3*x/b**6 + 2*a**2*x**3/b**5 - 3*a*x**5/(5*b**4) - 99*sqrt(-a**7/b**1
3)*log(x - b**6*sqrt(-a**7/b**13)/a**3)/16 + 99*sqrt(-a**7/b**13)*log(x + b
**6*sqrt(-a**7/b**13)/a**3)/16 - (19*a**5*x + 21*a**4*b*x**3)/(8*a**2*b**6
+ 16*a*b**7*x**2 + 8*b**8*x**4) + x**7/(7*b**3)
```

Giac [A] time = 2.76754, size = 130, normalized size = 1.17

$$\frac{99\, a^{4} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{8\, \sqrt{a b} b^{6}} - \frac{21\, a^{4} b x^{3} + 19\, a^{5} x}{8\, {\left(b x^{2} + a\right)}^{2} b^{6}} + \frac{5\, b^{18} x^{7} - 21\, a b^{17} x^{5} + 70\, a^{2} b^{16} x^{3} - 350\, a^{3} b^{15} x}{35\, b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*a^4*b*x^3 + 19*a^5
*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*b^18*x^7 - 21*a*b^17*x^5 + 70*a^2*b^16*x^
3 - 350*a^3*b^15*x)/b^21
```

3.182 $\int \frac{x^{10}}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=98

$$\frac{63a^2x}{8b^5}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{21ax^3}{8b^4}-\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{63x^5}{40b^3}$$

```
[Out] (63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a +
 b*x^2)^2) - (9*x^7)/(8*b^2*(a + b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*b^(11/2))
```

Rubi [A] time = 0.0407871, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{63a^2x}{8b^5}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{21ax^3}{8b^4}-\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{63x^5}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^3,x]
```

```
[Out] (63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a +
 b*x^2)^2) - (9*x^7)/(8*b^2*(a + b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*b^(11/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^9}{4b\left(a+bx^2\right)^2} + \frac{9\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} + \frac{63\int \frac{x^6}{a+bx^2}\,dx}{8b^2}$$

$$= -\frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} + \frac{63\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3\left(a+bx^2\right)}\right)dx}{8b^2}$$

$$= \frac{63a^2x}{8b^5} - \frac{21ax^3}{8b^4} + \frac{63x^5}{40b^3} - \frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} - \frac{\left(63a^3\right)\int \frac{1}{a+bx^2}\,dx}{8b^5}$$

$$= \frac{63a^2x}{8b^5} - \frac{21ax^3}{8b^4} + \frac{63x^5}{40b^3} - \frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} - \frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Mathematica [A] time = 0.0472025, size = 88, normalized size = 0.9

$$\frac{168a^2b^2x^5 + 525a^3bx^3 + 315a^4x - 24ab^3x^7 + 8b^4x^9}{40b^5\left(a+bx^2\right)^2} - \frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^3,x]
```

```
[Out] (315*a^4*x + 525*a^3*b*x^3 + 168*a^2*b^2*x^5 - 24*a*b^3*x^7 + 8*b^4*x^9)/(4
0*b^5*(a + b*x^2)^2) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2)
)
```

Maple [A] time = 0.008, size = 88, normalized size = 0.9

$$\frac{x^5}{5\,b^3} - \frac{ax^3}{b^4} + 6\,\frac{a^2x}{b^5} + \frac{17\,a^3x^3}{8\,b^4\left(bx^2+a\right)^2} + \frac{15\,a^4x}{8\,b^5\left(bx^2+a\right)^2} - \frac{63\,a^3}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^3,x)
```

```
[Out] 1/5*x^5/b^3-a*x^3/b^4+6*a^2*x/b^5+17/8/b^4*a^3/(b*x^2+a)^2*x^3+15/8/b^5*a^4
/(b*x^2+a)^2*x-63/8/b^5*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2842, size = 547, normalized size = 5.58

$$\left[\frac{16\,b^4x^9 - 48\,ab^3x^7 + 336\,a^2b^2x^5 + 1050\,a^3bx^3 + 630\,a^4x + 315\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2+a}\right)}{80\left(b^7x^4 + 2\,ab^6x^2 + a^2b^5\right)}, \frac{8\,b^4\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/80*(16*b^4*x^9 - 48*a*b^3*x^7 + 336*a^2*b^2*x^5 + 1050*a^3*b*x^3 + 630*a
^4*x + 315*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 - 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/40*(8*b^
4*x^9 - 24*a*b^3*x^7 + 168*a^2*b^2*x^5 + 525*a^3*b*x^3 + 315*a^4*x - 315*(a
^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x^4
 + 2*a*b^6*x^2 + a^2*b^5)]
```

Sympy [A] time = 0.610995, size = 144, normalized size = 1.47

$$\frac{6a^2x}{b^5} - \frac{ax^3}{b^4} + \frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16} - \frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16} + \frac{15a^4x + 17a^3bx^3}{8a^2b^5 + 16ab^6x^2 + 8b^7x^4} + \frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**3,x)
```

```
[Out] 6*a**2*x/b**5 - a*x**3/b**4 + 63*sqrt(-a**5/b**11)*log(x - b**5*sqrt(-a**5/
b**11)/a**2)/16 - 63*sqrt(-a**5/b**11)*log(x + b**5*sqrt(-a**5/b**11)/a**2)
/16 + (15*a**4*x + 17*a**3*b*x**3)/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x
**4) + x**5/(5*b**3)
```

Giac [A] time = 2.73309, size = 113, normalized size = 1.15

$$-\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{17\,a^3bx^3 + 15\,a^4x}{8\left(bx^2 + a\right)^2b^5} + \frac{b^{12}x^5 - 5\,ab^{11}x^3 + 30\,a^2b^{10}x}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*a^3*b*x^3 + 15*a^
4*x)/((b*x^2 + a)^2*b^5) + 1/5*(b^12*x^5 - 5*a*b^11*x^3 + 30*a^2*b^10*x)/b^
15
```

3.183 $\int \frac{x^8}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=85

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{7x^5}{8b^2\left(a+bx^2\right)}-\frac{35ax}{8b^4}-\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{35x^3}{24b^3}$$

```
[Out] (-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(
8*b^2*(a + b*x^2)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rubi [A] time = 0.0351929, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{7x^5}{8b^2\left(a+bx^2\right)}-\frac{35ax}{8b^4}-\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{35x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^3,x]
```

```
[Out] (-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(
8*b^2*(a + b*x^2)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{7\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35\int \frac{x^4}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{8b^2}\\
&= -\frac{35ax}{8b^4}+\frac{35x^3}{24b^3}-\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{\left(35a^2\right)\int \frac{1}{a+bx^2}\,dx}{8b^4}\\
&= -\frac{35ax}{8b^4}+\frac{35x^3}{24b^3}-\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0447759, size = 77, normalized size = 0.91

$$
\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{175a^2bx^3+105a^3x+56ab^2x^5-8b^3x^7}{24b^4\left(a+bx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^3,x]
```

```
[Out] -(105*a^3*x + 175*a^2*b*x^3 + 56*a*b^2*x^5 - 8*b^3*x^7)/(24*b^4*(a + b*x^2)
^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Maple [A] time = 0.01, size = 77, normalized size = 0.9

$$
\frac{x^3}{3\,b^3}-3\,\frac{ax}{b^4}-\frac{13\,a^2x^3}{8\,b^3\left(bx^2+a\right)^2}-\frac{11\,a^3x}{8\,b^4\left(bx^2+a\right)^2}+\frac{35\,a^2}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^3,x)
```

```
[Out] 1/3*x^3/b^3-3*a*x/b^4-13/8/b^3*a^2/(b*x^2+a)^2*x^3-11/8/b^4*a^3/(b*x^2+a)^2
*x+35/8/b^4*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29079, size = 493, normalized size = 5.8

$$\left[\frac{16\,b^3x^7 - 112\,ab^2x^5 - 350\,a^2bx^3 - 210\,a^3x + 105\left(ab^2x^4 + 2\,a^2bx^2 + a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{48\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)}, \frac{8\,b^3x^7 - 56\,ab^2x^5 - 17}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^3*x^7 - 112*a*b^2*x^5 - 350*a^2*b*x^3 - 210*a^3*x + 105*(a*b^2*
x^4 + 2*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*(8*b^3*x^7 - 56*a*b^2*x^5
 - 175*a^2*b*x^3 - 105*a^3*x + 105*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [A] time = 0.579039, size = 131, normalized size = 1.54

$$-\frac{3ax}{b^4} - \frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x - \frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16} + \frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x + \frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16} - \frac{11a^3x + 13a^2bx^3}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4} + \frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**3,x)
```

```
[Out] -3*a*x/b**4 - 35*sqrt(-a**3/b**9)*log(x - b**4*sqrt(-a**3/b**9)/a)/16 + 35*
sqrt(-a**3/b**9)*log(x + b**4*sqrt(-a**3/b**9)/a)/16 - (11*a**3*x + 13*a**2
*b*x**3)/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4) + x**3/(3*b**3)
```

Giac [A] time = 1.53496, size = 99, normalized size = 1.16

$$\frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} - \frac{13\,a^2bx^3 + 11\,a^3x}{8\left(bx^2 + a\right)^2b^4} + \frac{b^6x^3 - 9\,ab^5x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*a^2*b*x^3 + 11*a^3
*x)/((b*x^2 + a)^2*b^4) + 1/3*(b^6*x^3 - 9*a*b^5*x)/b^9
```

3.184 $\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}-\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{15x}{8b^3}$$

```
[Out] (15*x)/(8*b^3) - x^5/(4*b*(a + b*x^2)^2) - (5*x^3)/(8*b^2*(a + b*x^2)) - (1
5*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Rubi [A] time = 0.0252003, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}-\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{15x}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*x)/(8*b^3) - x^5/(4*b*(a + b*x^2)^2) - (5*x^3)/(8*b^2*(a + b*x^2)) - (1
5*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^3}\,dx = -\frac{x^5}{4b(a+bx^2)^2} + \frac{5\int \frac{x^4}{(a+bx^2)^2}\,dx}{4b}$$
$$= -\frac{x^5}{4b(a+bx^2)^2} - \frac{5x^3}{8b^2(a+bx^2)} + \frac{15\int \frac{x^2}{a+bx^2}\,dx}{8b^2}$$
$$= \frac{15x}{8b^3} - \frac{x^5}{4b(a+bx^2)^2} - \frac{5x^3}{8b^2(a+bx^2)} - \frac{(15a)\int \frac{1}{a+bx^2}\,dx}{8b^3}$$
$$= \frac{15x}{8b^3} - \frac{x^5}{4b(a+bx^2)^2} - \frac{5x^3}{8b^2(a+bx^2)} - \frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.0459202, size = 66, normalized size = 0.89

$$\frac{15a^2x + 25abx^3 + 8b^2x^5}{8b^3(a+bx^2)^2} - \frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*a^2*x + 25*a*b*x^3 + 8*b^2*x^5)/(8*b^3*(a + b*x^2)^2) - (15*Sqrt[a]*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Maple [A] time = 0.008, size = 63, normalized size = 0.9

$$\frac{x}{b^3} + \frac{9\,ax^3}{8\,b^2(bx^2+a)^2} + \frac{7\,a^2x}{8\,b^3(bx^2+a)^2} - \frac{15\,a}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^3,x)
```

```
[Out] x/b^3+9/8/b^2*a/(b*x^2+a)^2*x^3+7/8/b^3*a^2/(b*x^2+a)^2*x-15/8/b^3*a/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32053, size = 425, normalized size = 5.74

$$\left[\frac{16\,b^{2}x^{5}+50\,abx^{3}+30\,a^{2}x+15\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{16\,\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)},\ \frac{8\,b^{2}x^{5}+25\,abx^{3}+15\,a^{2}x-15\,\left(b^{2}x^{4}+\right.}{8\,\left(b^{5}x^{4}+2\,ab^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4 +
2*a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^4 + 2*a*b^4*x^2
+ a^2*b^3)]
```

Sympy [A] time = 0.545711, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{a}{b^{7}}}\log\left(-b^{3}\sqrt{-\frac{a}{b^{7}}}+x\right)}{16}-\frac{15\sqrt{-\frac{a}{b^{7}}}\log\left(b^{3}\sqrt{-\frac{a}{b^{7}}}+x\right)}{16}+\frac{7a^{2}x+9abx^{3}}{8a^{2}b^{3}+16ab^{4}x^{2}+8b^{5}x^{4}}+\frac{x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-a/b**7)*log(-b**3*sqrt(-a/b**7) + x)/16 - 15*sqrt(-a/b**7)*log(b**
3*sqrt(-a/b**7) + x)/16 + (7*a**2*x + 9*a*b*x**3)/(8*a**2*b**3 + 16*a*b**4*
x**2 + 8*b**5*x**4) + x/b**3
```

Giac [A] time = 3.03909, size = 73, normalized size = 0.99

$$-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^{3}}+\frac{x}{b^{3}}+\frac{9\,abx^{3}+7\,a^{2}x}{8\,\left(bx^{2}+a\right)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3 + 1/8*(9*a*b*x^3 + 7*
a^2*x)/((b*x^2 + a)^2*b^3)
```

3.185 $\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}}-\frac{x^3}{4b\left(a+bx^2\right)^2}$$

```
[Out] -x^3/(4*b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0197249, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}}-\frac{x^3}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^3,x]
```

```
[Out] -x^3/(4*b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^3}{4b\left(a+bx^2\right)^2}+\frac{3\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx}{4b}\\ &= -\frac{x^3}{4b\left(a+bx^2\right)^2}-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\int \frac{1}{a+bx^2}\,dx}{8b^2}\\ &= -\frac{x^3}{4b\left(a+bx^2\right)^2}-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0384839, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}}-\frac{3ax+5bx^3}{8b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^3,x]
```

```
[Out] -(3*a*x + 5*b*x^3)/(8*b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.007, size = 47, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^2}\left(-\frac{5x^3}{8b}-\frac{3ax}{8b^2}\right)+\frac{3}{8b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^3,x)
```

```
[Out] (-5/8*x^3/b-3/8*a*x/b^2)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28876, size = 404, normalized size = 6.31

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},-\frac{5\,ab^2x^3+3\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}}{8\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [A] time = 0.48288, size = 109, normalized size = 1.7

$$-\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}-\frac{3ax+5bx^3}{8a^2b^2+16ab^3x^2+8b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a*b**5))*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*b
**5))*log(a*b**2*sqrt(-1/(a*b**5)) + x)/16 - (3*a*x + 5*b*x**3)/(8*a**2*b**
2 + 16*a*b**3*x**2 + 8*b**4*x**4)
```

Giac [A] time = 1.36908, size = 61, normalized size = 0.95

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2}-\frac{5\,bx^3+3\,ax}{8\,\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/8*(5*b*x^3 + 3*a*x)/((b*x^2 +
 a)^2*b^2)
```

3.186 $\int \frac{x^2}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

```
[Out] -x/(4*b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0183032, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^3,x]
```

```
[Out] -x/(4*b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^3}\,dx = -\frac{x}{4b(a+bx^2)^2} + \frac{\int \frac{1}{(a+bx^2)^2}\,dx}{4b}$$
$$= -\frac{x}{4b(a+bx^2)^2} + \frac{x}{8ab(a+bx^2)} + \frac{\int \frac{1}{a+bx^2}\,dx}{8ab}$$
$$= -\frac{x}{4b(a+bx^2)^2} + \frac{x}{8ab(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.0271433, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\sqrt{b}x(bx^2-a)}{(a+bx^2)^2} + \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-a + b*x^2))/(a + b*x^2)^2 + ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.007, size = 49, normalized size = 0.8

$$\frac{1}{(bx^2+a)^2}\left(\frac{x^3}{8a} - \frac{x}{8b}\right) + \frac{1}{8ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^3,x)
```

```
[Out] (1/8/a*x^3-1/8*x/b)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25451, size = 394, normalized size = 6.06

$$\left[\frac{2ab^2x^3 - 2a^2bx - (b^2x^4 + 2abx^2 + a^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{16(a^2b^4x^4 + 2a^3b^3x^2 + a^4b^2)}, \frac{ab^2x^3 - a^2bx + (b^2x^4 + 2abx^2 + a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right.}{8(a^2b^4x^4 + 2a^3b^3x^2 + a^4b^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [B] time = 0.493554, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16}+\frac{-ax+bx^3}{8a^3b+16a^2b^2x^2+8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**3))*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(a
**3*b**3))*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + (-a*x + b*x**3)/(8*a**
3*b + 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

Giac [A] time = 1.75238, size = 68, normalized size = 1.05

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}ab}+\frac{bx^3-ax}{8\left(bx^2+a\right)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(b*x^3 - a*x)/((b*x^2 + a)^
2*a*b)
```

3.187 $\int \frac{1}{(a+bx^2)^3} dx$

Optimal. Leaf size=62

$$\frac{3x}{8a^2(a+bx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{x}{4a(a+bx^2)^2}$$

```
[Out] x/(4*a*(a + b*x^2)^2) + (3*x)/(8*a^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0163593, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{3x}{8a^2(a+bx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{x}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + b*x^2)^2) + (3*x)/(8*a^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^3} dx &= \frac{x}{4a(a+bx^2)^2} + \frac{3\int \frac{1}{(a+bx^2)^2} dx}{4a} \\
&= \frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3\int \frac{1}{a+bx^2} dx}{8a^2} \\
&= \frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0345738, size = 55, normalized size = 0.89

$$\frac{5ax+3bx^3}{8a^2\left(a+bx^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3),x]
```

```
[Out] (5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.003, size = 51, normalized size = 0.8

$$\frac{x}{4\,a\left(bx^2+a\right)^2}+\frac{3\,x}{8\,a^2\left(bx^2+a\right)}+\frac{3}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24934, size = 401, normalized size = 6.47

$$\left[\frac{6\,ab^2x^3+10\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)},\frac{3\,ab^2x^3+5\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\,\mathrm{ar}}{8\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] time = 0.508068, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{5ax+3bx^3}{8a^4+16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**5*
b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16 + (5*a*x + 3*b*x**3)/(8*a**4 + 16*a*
*3*b*x**2 + 8*a**2*b**2*x**4)
```

Giac [A] time = 2.86886, size = 61, normalized size = 0.98

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2}+\frac{3\,bx^3+5\,ax}{8\left(bx^2+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)/((b*x^2 +
 a)^2*a^2)
```

3.188 $\int \frac{1}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=76

$$\frac{5}{8a^2x(a+bx^2)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax(a+bx^2)^2}$$

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*Sqr
t[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] time = 0.0258144, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{5}{8a^2x(a+bx^2)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*Sqr
t[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$
$$= \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5}{8a^2x\left(a+bx^2\right)} + \frac{15\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$
$$= -\frac{15}{8a^3x} + \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5}{8a^2x\left(a+bx^2\right)} - \frac{(15b)\int \frac{1}{a+bx^2}\,dx}{8a^3}$$
$$= -\frac{15}{8a^3x} + \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5}{8a^2x\left(a+bx^2\right)} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.0387667, size = 68, normalized size = 0.89

$$-\frac{8a^2+25abx^2+15b^2x^4}{8a^3x\left(a+bx^2\right)^2} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)/(8*a^3*x*(a + b*x^2)^2) - (15*Sqrt[b]*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Maple [A] time = 0.01, size = 66, normalized size = 0.9

$$-\frac{1}{a^3x} - \frac{7\,b^2x^3}{8\,a^3\left(bx^2+a\right)^2} - \frac{9\,bx}{8\,a^2\left(bx^2+a\right)^2} - \frac{15\,b}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-7/8/a^3*b^2/(b*x^2+a)^2*x^3-9/8/a^2*b/(b*x^2+a)^2*x-15/8/a^3*b/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32791, size = 428, normalized size = 5.63

$$\left[-\frac{30\,b^{2}x^{4}+50\,abx^{2}-15\,\left(b^{2}x^{5}+2\,abx^{3}+a^{2}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+16\,a^{2}}{16\,\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)},\,-\frac{15\,b^{2}x^{4}+25\,abx^{2}+15\,\left(b^{2}x^{5}+2\,ab\right.}{8\,\left(a^{3}b^{2}x^{5}+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

Sympy [A] time = 0.636164, size = 114, normalized size = 1.5

$$\frac{15\sqrt{-\frac{b}{a^{7}}}\log\left(-\frac{a^{4}\sqrt{-\frac{b}{a^{7}}}}{b}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{a^{7}}}\log\left(\frac{a^{4}\sqrt{-\frac{b}{a^{7}}}}{b}+x\right)}{16}-\frac{8a^{2}+25abx^{2}+15b^{2}x^{4}}{8a^{5}x+16a^{4}bx^{3}+8a^{3}b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-b/a**7)*log(-a**4*sqrt(-b/a**7)/b + x)/16 - 15*sqrt(-b/a**7)*log(a
**4*sqrt(-b/a**7)/b + x)/16 - (8*a**2 + 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5
*x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Giac [A] time = 2.10063, size = 77, normalized size = 1.01

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^{3}}-\frac{7\,b^{2}x^{3}+9\,abx}{8\,\left(bx^{2}+a\right)^{2}a^{3}}-\frac{1}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/8*(7*b^2*x^3 + 9*a*b*x)/(
(b*x^2 + a)^2*a^3) - 1/(a^3*x)
```

3.189 $\int \frac{1}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=87

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

```
[Out] -35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a + b*x^2)^2) + 7/(8*a^2*
x^3*(a + b*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] time = 0.0336019, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a + b*x^2)^2) + 7/(8*a^2*
x^3*(a + b*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$
$$= \frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^2}$$
$$= -\frac{35}{24a^3x^3}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}-\frac{(35b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^3}$$
$$= -\frac{35}{24a^3x^3}+\frac{35b}{8a^4x}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{\left(35b^2\right)\int \frac{1}{a+bx^2}\,dx}{8a^4}$$
$$= -\frac{35}{24a^3x^3}+\frac{35b}{8a^4x}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Mathematica [A] time = 0.0408913, size = 79, normalized size = 0.91

$$\frac{56a^2bx^2-8a^3+175ab^2x^4+105b^3x^6}{24a^4x^3\left(a+bx^2\right)^2}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4*x^3*(a + b*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Maple [A] time = 0.012, size = 79, normalized size = 0.9

$$-\frac{1}{3\,a^3x^3}+3\,\frac{b}{a^4x}+\frac{11\,b^3x^3}{8\,a^4\left(bx^2+a\right)^2}+\frac{13\,b^2x}{8\,a^3\left(bx^2+a\right)^2}+\frac{35\,b^2}{8\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+3*b/a^4/x+11/8/a^4*b^3/(b*x^2+a)^2*x^3+13/8/a^3*b^2/(b*x^2+a)^
2*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24974, size = 504, normalized size = 5.79

$$\left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)},\ \frac{105\,b^3x^6+175\,ab^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [A] time = 0.852244, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**3,x)
```

```
[Out] -35*sqrt(-b**3/a**9)*log(-a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + 35*sqrt(-b**
3/a**9)*log(a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + (-8*a**3 + 56*a**2*b*x**2
+ 175*a*b**2*x**4 + 105*b**3*x**6)/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4
*b**2*x**7)
```

Giac [A] time = 2.12686, size = 96, normalized size = 1.1

$$\frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}+\frac{11\,b^3x^3+13\,ab^2x}{8\left(bx^2+a\right)^2a^4}+\frac{9\,bx^2-a}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/8*(11*b^3*x^3 + 13*a*b^2
*x)/((b*x^2 + a)^2*a^4) + 1/3*(9*b*x^2 - a)/(a^4*x^3)
```

3.190 $\int \frac{1}{x^6(a+bx^2)^3} dx$

Optimal. Leaf size=100

$$-\frac{63b^2}{8a^5x}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{21b}{8a^4x^3}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{63}{40a^3x^5}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

```
[Out] -63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a
+ b*x^2)^2) + 9/(8*a^2*x^5*(a + b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2))
```

Rubi [A] time = 0.041861, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{63b^2}{8a^5x}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{21b}{8a^4x^3}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{63}{40a^3x^5}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a
+ b*x^2)^2) + 9/(8*a^2*x^5*(a + b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx}{4a}$$
$$= \frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9}{8a^2x^5\left(a+bx^2\right)}+\frac{63\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{8a^2}$$
$$= -\frac{63}{40a^3x^5}+\frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{(63b)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^3}$$
$$= -\frac{63}{40a^3x^5}+\frac{21b}{8a^4x^3}+\frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9}{8a^2x^5\left(a+bx^2\right)}+\frac{\left(63b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^4}$$
$$= -\frac{63}{40a^3x^5}+\frac{21b}{8a^4x^3}-\frac{63b^2}{8a^5x}+\frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{\left(63b^3\right)\int \frac{1}{a+bx^2}\,dx}{8a^5}$$
$$= -\frac{63}{40a^3x^5}+\frac{21b}{8a^4x^3}-\frac{63b^2}{8a^5x}+\frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Mathematica [A] time = 0.050852, size = 90, normalized size = 0.9

$$-\frac{168a^2b^2x^4-24a^3bx^2+8a^4+525ab^3x^6+315b^4x^8}{40a^5x^5\left(a+bx^2\right)^2}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -(8*a^4 - 24*a^3*b*x^2 + 168*a^2*b^2*x^4 + 525*a*b^3*x^6 + 315*b^4*x^8)/(40
*a^5*x^5*(a + b*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11
/2))
```

Maple [A] time = 0.012, size = 89, normalized size = 0.9

$$-\frac{1}{5\,a^3x^5}-6\,\frac{b^2}{a^5x}+\frac{b}{a^4x^3}-\frac{15\,b^4x^3}{8\,a^5\left(bx^2+a\right)^2}-\frac{17\,b^3x}{8\,a^4\left(bx^2+a\right)^2}-\frac{63\,b^3}{8\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5/a^3/x^5-6*b^2/a^5/x+b/a^4/x^3-15/8/a^5*b^4/(b*x^2+a)^2*x^3-17/8/a^4*b^
3/(b*x^2+a)^2*x-63/8/a^5*b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25866, size = 560, normalized size = 5.6

$$\left[-\frac{630\,b^4x^8+1050\,ab^3x^6+336\,a^2b^2x^4-48\,a^3bx^2+16\,a^4-315\left(b^4x^9+2\,ab^3x^7+a^2b^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{80\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*b^4*x^8 + 1050*a*b^3*x^6 + 336*a^2*b^2*x^4 - 48*a^3*b*x^2 + 16*
a^4 - 315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), -1/
40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4 +
315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

Sympy [A] time = 1.21165, size = 150, normalized size = 1.5

$$\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16}-\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16}-\frac{8a^4-24a^3bx^2+168a^2b^2x^4+525ab^3x^6+315b^4x^8}{40a^7x^5+80a^6bx^7+40a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**3,x)
```

```
[Out] 63*sqrt(-b**5/a**11)*log(-a**6*sqrt(-b**5/a**11)/b**3 + x)/16 - 63*sqrt(-b*
*5/a**11)*log(a**6*sqrt(-b**5/a**11)/b**3 + x)/16 - (8*a**4 - 24*a**3*b*x**
2 + 168*a**2*b**2*x**4 + 525*a*b**3*x**6 + 315*b**4*x**8)/(40*a**7*x**5 + 8
0*a**6*b*x**7 + 40*a**5*b**2*x**9)
```

Giac [A] time = 2.51769, size = 108, normalized size = 1.08

$$-\frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^5}-\frac{15\,b^4x^3+17\,ab^3x}{8\left(bx^2+a\right)^2a^5}-\frac{30\,b^2x^4-5\,abx^2+a^2}{5\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/8*(15*b^4*x^3 + 17*a*b^
3*x)/((b*x^2 + a)^2*a^5) - 1/5*(30*b^2*x^4 - 5*a*b*x^2 + a^2)/(a^5*x^5)
```

3.191 $\int \frac{1}{x^8(a+bx^2)^3}\,dx$

Optimal. Leaf size=113

$$-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b}{40a^4x^5}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{99}{56a^3x^7}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

```
[Out] -99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8
*a^6*x) + 1/(4*a*x^7*(a + b*x^2)^2) + 11/(8*a^2*x^7*(a + b*x^2)) + (99*b^(7
/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rubi [A] time = 0.0558603, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b}{40a^4x^5}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{99}{56a^3x^7}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8
*a^6*x) + 1/(4*a*x^7*(a + b*x^2)^2) + 11/(8*a^2*x^7*(a + b*x^2)) + (99*b^(7
/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{99\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{99}{56a^3x^7} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} - \frac{(99b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{8a^3}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{\left(99b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^4}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} - \frac{\left(99b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^5}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{99b^3}{8a^6x} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{\left(99b^4\right)\int \frac{1}{a+bx^2}\,dx}{8a^6}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{99b^3}{8a^6x} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

Mathematica [A] time = 0.0560113, size = 101, normalized size = 0.89

$$\frac{1848a^2b^3x^6 - 264a^3b^2x^4 + 88a^4bx^2 - 40a^5 + 5775ab^4x^8 + 3465b^5x^{10}}{280a^6x^7\left(a+bx^2\right)^2} + \frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] (-40*a^5 + 88*a^4*b*x^2 - 264*a^3*b^2*x^4 + 1848*a^2*b^3*x^6 + 5775*a*b^4*x
^8 + 3465*b^5*x^10)/(280*a^6*x^7*(a + b*x^2)^2) + (99*b^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Maple [A] time = 0.011, size = 101, normalized size = 0.9

$$-\frac{1}{7a^3x^7} + 10\frac{b^3}{a^6x} - 2\frac{b^2}{a^5x^3} + \frac{3b}{5a^4x^5} + \frac{19b^5x^3}{8a^6\left(bx^2+a\right)^2} + \frac{21b^4x}{8a^5\left(bx^2+a\right)^2} + \frac{99b^4}{8a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^3,x)
```

```
[Out] -1/7/a^3/x^7+10*b^3/a^6/x-2*b^2/a^5/x^3+3/5*b/a^4/x^5+19/8/a^6*b^5/(b*x^2+a
)^2*x^3+21/8/a^5*b^4/(b*x^2+a)^2*x+99/8/a^6*b^4/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.278, size = 630, normalized size = 5.58

$$\left[\frac{6930\,b^5x^{10} + 11550\,ab^4x^8 + 3696\,a^2b^3x^6 - 528\,a^3b^2x^4 + 176\,a^4bx^2 - 80\,a^5 + 3465\left(b^5x^{11} + 2\,ab^4x^9 + a^2b^3x^7\right)\sqrt{-\frac{b}{a}}\log\left(\right.}{560\left(a^6b^2x^{11} + 2\,a^7bx^9 + a^8x^7\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(6930*b^5*x^10 + 11550*a*b^4*x^8 + 3696*a^2*b^3*x^6 - 528*a^3*b^2*x^
4 + 176*a^4*b*x^2 - 80*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x^9 + a^2*b^3*x^7)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2
*a^7*b*x^9 + a^8*x^7), 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3
*x^6 - 264*a^3*b^2*x^4 + 88*a^4*b*x^2 - 40*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x
^9 + a^2*b^3*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^
9 + a^8*x^7)]
```

Sympy [A] time = 2.24989, size = 162, normalized size = 1.43

$$-\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4} + x\right)}{16} + \frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4} + x\right)}{16} + \frac{-40a^5 + 88a^4bx^2 - 264a^3b^2x^4 + 1848a^2b^3x^6 + 5775ab^4x}{280a^8x^7 + 560a^7bx^9 + 280a^6b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**3,x)
```

```
[Out] -99*sqrt(-b**7/a**13)*log(-a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + 99*sqrt(-b
**7/a**13)*log(a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + (-40*a**5 + 88*a**4*b*
x**2 - 264*a**3*b**2*x**4 + 1848*a**2*b**3*x**6 + 5775*a*b**4*x**8 + 3465*b
**5*x**10)/(280*a**8*x**7 + 560*a**7*b*x**9 + 280*a**6*b**2*x**11)
```

Giac [A] time = 2.98601, size = 126, normalized size = 1.12

$$\frac{99\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6} + \frac{19\,b^5x^3 + 21\,ab^4x}{8\left(bx^2 + a\right)^2a^6} + \frac{350\,b^3x^6 - 70\,ab^2x^4 + 21\,a^2bx^2 - 5\,a^3}{35\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*x^3 + 21*a*b^4
*x)/((b*x^2 + a)^2*a^6) + 1/35*(350*b^3*x^6 - 70*a*b^2*x^4 + 21*a^2*b*x^2 -
 5*a^3)/(a^6*x^7)
```

3.192 $\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=216

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{154a^6}{b^{13}\left(a+b\right.}$$

```
[Out] (55*a^2*x^2)/(2*b^12) - (5*a*x^4)/(2*b^11) + x^6/(6*b^10) - a^12/(18*b^13*(
a + b*x^2)^9) + (3*a^11)/(4*b^13*(a + b*x^2)^8) - (33*a^10)/(7*b^13*(a + b*
x^2)^7) + (55*a^9)/(3*b^13*(a + b*x^2)^6) - (99*a^8)/(2*b^13*(a + b*x^2)^5)
 + (99*a^7)/(b^13*(a + b*x^2)^4) - (154*a^6)/(b^13*(a + b*x^2)^3) + (198*a^
5)/(b^13*(a + b*x^2)^2) - (495*a^4)/(2*b^13*(a + b*x^2)) - (110*a^3*Log[a +
 b*x^2])/b^13
```

Rubi [A] time = 0.255003, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{154a^6}{b^{13}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^25/(a + b*x^2)^10,x]
```

```
[Out] (55*a^2*x^2)/(2*b^12) - (5*a*x^4)/(2*b^11) + x^6/(6*b^10) - a^12/(18*b^13*(
a + b*x^2)^9) + (3*a^11)/(4*b^13*(a + b*x^2)^8) - (33*a^10)/(7*b^13*(a + b*
x^2)^7) + (55*a^9)/(3*b^13*(a + b*x^2)^6) - (99*a^8)/(2*b^13*(a + b*x^2)^5)
 + (99*a^7)/(b^13*(a + b*x^2)^4) - (154*a^6)/(b^13*(a + b*x^2)^3) + (198*a^
5)/(b^13*(a + b*x^2)^2) - (495*a^4)/(2*b^13*(a + b*x^2)) - (110*a^3*Log[a +
 b*x^2])/b^13
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^{12}}{(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{55a^2}{b^{12}} - \frac{10ax}{b^{11}} + \frac{x^2}{b^{10}} + \frac{a^{12}}{b^{12}(a+bx)^{10}} - \frac{12a^{11}}{b^{12}(a+bx)^9} + \frac{66a^{10}}{b^{12}(a+bx)^8} - \frac{220a^9}{b^{12}(a+bx)^7}\right.\right.$$

$$= \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} - \frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9} + \frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8} - \frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7} + \frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}$$

Mathematica [A] time = 0.0448646, size = 169, normalized size = 0.78

$$-\frac{-2772a^2b^{10}x^{20} - 43218a^3b^9x^{18} - 139482a^4b^8x^{16} - 58968a^5b^7x^{14} + 638568a^6b^6x^{12} + 1831032a^7b^5x^{10} + 2529576a^8b}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^25/(a + b*x^2)^10,x]
```

```
[Out] -(35201*a^12 + 289089*a^11*b*x^2 + 1031616*a^10*b^2*x^4 + 2074464*a^9*b^3*x
^6 + 2529576*a^8*b^4*x^8 + 1831032*a^7*b^5*x^10 + 638568*a^6*b^6*x^12 - 589
68*a^5*b^7*x^14 - 139482*a^4*b^8*x^16 - 43218*a^3*b^9*x^18 - 2772*a^2*b^10*
x^20 + 252*a*b^11*x^22 - 42*b^12*x^24 + 27720*a^3*(a + b*x^2)^9*Log[a + b*x
^2])/(252*b^13*(a + b*x^2)^9)
```

Maple [A] time = 0.019, size = 199, normalized size = 0.9

$$\frac{55\,a^2x^2}{2\,b^{12}} - \frac{5\,ax^4}{2\,b^{11}} + \frac{x^6}{6\,b^{10}} - \frac{a^{12}}{18\,b^{13}\left(bx^2+a\right)^9} + \frac{3\,a^{11}}{4\,b^{13}\left(bx^2+a\right)^8} - \frac{33\,a^{10}}{7\,b^{13}\left(bx^2+a\right)^7} + \frac{55\,a^9}{3\,b^{13}\left(bx^2+a\right)^6} - \frac{99\,a^8}{2\,b^{13}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(b*x^2+a)^10,x)
```

```
[Out] 55/2*a^2*x^2/b^12-5/2*a*x^4/b^11+1/6*x^6/b^10-1/18*a^12/b^13/(b*x^2+a)^9+3/
4*a^11/b^13/(b*x^2+a)^8-33/7*a^10/b^13/(b*x^2+a)^7+55/3*a^9/b^13/(b*x^2+a)^
6-99/2*a^8/b^13/(b*x^2+a)^5+99*a^7/b^13/(b*x^2+a)^4-154*a^6/b^13/(b*x^2+a)^
3+198*a^5/b^13/(b*x^2+a)^2-495/2*a^4/b^13/(b*x^2+a)-110*a^3*ln(b*x^2+a)/b^1
3
```

Maxima [A] time = 2.87715, size = 327, normalized size = 1.51

$$-\frac{62370\,a^4b^8x^{16} + 449064\,a^5b^7x^{14} + 1435896\,a^6b^6x^{12} + 2652804\,a^7b^5x^{10} + 3089394\,a^8b^4x^8 + 2318316\,a^9b^3x^6 + 1093}{252\left(b^{22}x^{18} + 9\,ab^{21}x^{16} + 36\,a^2b^{20}x^{14} + 84\,a^3b^{19}x^{12} + 126\,a^4b^{18}x^{10} + 126\,a^5b^{17}x^8 + 84\,a^6b^{16}x^6 + 36\,a^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(62370*a^4*b^8*x^16 + 449064*a^5*b^7*x^14 + 1435896*a^6*b^6*x^12 + 2
652804*a^7*b^5*x^10 + 3089394*a^8*b^4*x^8 + 2318316*a^9*b^3*x^6 + 1093356*a
```

```
^10*b^2*x^4 + 296019*a^11*b*x^2 + 35201*a^12)/(b^22*x^18 + 9*a*b^21*x^16 +
36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*a^4*b^18*x^10 + 126*a^5*b^17*x^8
+ 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^8*b^14*x^2 + a^9*b^13) - 110*a^3*
log(b*x^2 + a)/b^13 + 1/6*(b^2*x^6 - 15*a*b*x^4 + 165*a^2*x^2)/b^12
```

Fricas [A] time = 1.27799, size = 842, normalized size = 3.9

$$\frac{42\,b^{12}x^{24} - 252\,ab^{11}x^{22} + 2772\,a^2b^{10}x^{20} + 43218\,a^3b^9x^{18} + 139482\,a^4b^8x^{16} + 58968\,a^5b^7x^{14} - 638568\,a^6b^6x^{12} - 1831032}{252\left(b^{22}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(42*b^12*x^24 - 252*a*b^11*x^22 + 2772*a^2*b^10*x^20 + 43218*a^3*b^9*
x^18 + 139482*a^4*b^8*x^16 + 58968*a^5*b^7*x^14 - 638568*a^6*b^6*x^12 - 183
1032*a^7*b^5*x^10 - 2529576*a^8*b^4*x^8 - 2074464*a^9*b^3*x^6 - 1031616*a^1
0*b^2*x^4 - 289089*a^11*b*x^2 - 35201*a^12 - 27720*(a^3*b^9*x^18 + 9*a^4*b^
8*x^16 + 36*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 126*a^7*b^5*x^10 + 126*a^8*b^4
*x^8 + 84*a^9*b^3*x^6 + 36*a^10*b^2*x^4 + 9*a^11*b*x^2 + a^12)*log(b*x^2 +
a))/(b^22*x^18 + 9*a*b^21*x^16 + 36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*
a^4*b^18*x^10 + 126*a^5*b^17*x^8 + 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^
8*b^14*x^2 + a^9*b^13)
```

Sympy [A] time = 8.6922, size = 258, normalized size = 1.19

$$-\frac{110a^3\log\left(a + bx^2\right)}{b^{13}} + \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} - \frac{35201a^{12} + 296019a^{11}bx^2 + 1093356a^{10}b^2x^4 + 2318316a^9b^3x^6 + 3089394a^8b^4}{252a^9b^{13} + 2268a^8b^{14}x^2 + 9072a^7b^{15}x^4 + 21168a^6b^{16}x^6 + 31752a^5b^{17}x^8 + 31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**25/(b*x**2+a)**10,x)
```

```
[Out] -110*a**3*log(a + b*x**2)/b**13 + 55*a**2*x**2/(2*b**12) - 5*a*x**4/(2*b**1
1) - (35201*a**12 + 296019*a**11*b*x**2 + 1093356*a**10*b**2*x**4 + 2318316
*a**9*b**3*x**6 + 3089394*a**8*b**4*x**8 + 2652804*a**7*b**5*x**10 + 143589
6*a**6*b**6*x**12 + 449064*a**5*b**7*x**14 + 62370*a**4*b**8*x**16)/(252*a*
*9*b**13 + 2268*a**8*b**14*x**2 + 9072*a**7*b**15*x**4 + 21168*a**6*b**16*x
**6 + 31752*a**5*b**17*x**8 + 31752*a**4*b**18*x**10 + 21168*a**3*b**19*x**
12 + 9072*a**2*b**20*x**14 + 2268*a*b**21*x**16 + 252*b**22*x**18) + x**6/(
6*b**10)
```

Giac [A] time = 2.45032, size = 227, normalized size = 1.05

$$-\frac{110\,a^3\log\left(\left|bx^2 + a\right|\right)}{b^{13}} + \frac{78419\,a^3b^9x^{18} + 643401\,a^4b^8x^{16} + 2374020\,a^5b^7x^{14} + 5151300\,a^6b^6x^{12} + 7227990\,a^7b^5x^{10} + 67}{252\left(bx^2 + a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*a^3*log(abs(b*x^2 + a))/b^13 + 1/252*(78419*a^3*b^9*x^18 + 643401*a^4*
b^8*x^16 + 2374020*a^5*b^7*x^14 + 5151300*a^6*b^6*x^12 + 7227990*a^7*b^5*x^
10 + 6791400*a^8*b^4*x^8 + 4268880*a^9*b^3*x^6 + 1729728*a^10*b^2*x^4 + 409
752*a^11*b*x^2 + 43218*a^12)/((b*x^2 + a)^9*b^13) + 1/6*(b^20*x^6 - 15*a*b^
19*x^4 + 165*a^2*b^18*x^2)/b^30
```

3.193 $\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=205

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{77a}{b^{12}\left(a+\right.}$$

```
[Out] (-5*a*x^2)/b^11 + x^4/(4*b^10) + a^11/(18*b^12*(a + b*x^2)^9) - (11*a^10)/(
16*b^12*(a + b*x^2)^8) + (55*a^9)/(14*b^12*(a + b*x^2)^7) - (55*a^8)/(4*b^1
2*(a + b*x^2)^6) + (33*a^7)/(b^12*(a + b*x^2)^5) - (231*a^6)/(4*b^12*(a + b
*x^2)^4) + (77*a^5)/(b^12*(a + b*x^2)^3) - (165*a^4)/(2*b^12*(a + b*x^2)^2)
 + (165*a^3)/(2*b^12*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^12)
```

Rubi [A] time = 0.21444, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{77a}{b^{12}\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/(a + b*x^2)^10,x]
```

```
[Out] (-5*a*x^2)/b^11 + x^4/(4*b^10) + a^11/(18*b^12*(a + b*x^2)^9) - (11*a^10)/(
16*b^12*(a + b*x^2)^8) + (55*a^9)/(14*b^12*(a + b*x^2)^7) - (55*a^8)/(4*b^1
2*(a + b*x^2)^6) + (33*a^7)/(b^12*(a + b*x^2)^5) - (231*a^6)/(4*b^12*(a + b
*x^2)^4) + (77*a^5)/(b^12*(a + b*x^2)^3) - (165*a^4)/(2*b^12*(a + b*x^2)^2)
 + (165*a^3)/(2*b^12*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^12)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{x^{11}}{(a+bx)^{10}}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\frac{330a^7}{b^{11}(a+b\right.\right.$$

$$=-\frac{5ax^2}{b^{11}}+\frac{x^4}{4b^{10}}+\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{3}{b^{12}\left(a\right.}$$

Mathematica [A] time = 0.0282518, size = 158, normalized size = 0.77

$$\frac{-36288a^2b^9x^{18}-77112a^3b^8x^{16}+190512a^4b^7x^{14}+1220688a^5b^6x^{12}+2704212a^6b^5x^{10}+3402756a^7b^4x^8+2656584a}{1008b^{12}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/(a + b*x^2)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x^2 + 1281096*a^9*b^2*x^4 + 2656584*a^8*b^3*x^6
 + 3402756*a^7*b^4*x^8 + 2704212*a^6*b^5*x^10 + 1220688*a^5*b^6*x^12 + 1905
12*a^4*b^7*x^14 - 77112*a^3*b^8*x^16 - 36288*a^2*b^9*x^18 - 2772*a*b^10*x^2
0 + 252*b^11*x^22 + 27720*a^2*(a + b*x^2)^9*Log[a + b*x^2])/(1008*b^12*(a +
 b*x^2)^9)
```

Maple [A] time = 0.018, size = 188, normalized size = 0.9

$$-5\,{\frac{a{x}^{2}}{{b}^{11}}}+{\frac{{x}^{4}}{4\,{b}^{10}}}+{\frac{{a}^{11}}{18\,{b}^{12} \left(b{x}^{2}+a \right) ^{9}}}-{\frac{11\,{a}^{10}}{16\,{b}^{12} \left(b{x}^{2}+a \right) ^{8}}}+{\frac{55\,{a}^{9}}{14\,{b}^{12} \left(b{x}^{2}+a \right) ^{7}}}-{\frac{55\,{a}^{8}}{4\,{b}^{12} \left(b{x}^{2}+a \right) ^{6}}}+33\,{\frac{{a}^{7}}{{b}^{12} \left(b{x}^{2}+a \right) ^{5}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^2+a)^10,x)
```

```
[Out] -5*a*x^2/b^11+1/4*x^4/b^10+1/18*a^11/b^12/(b*x^2+a)^9-11/16*a^10/b^12/(b*x^
2+a)^8+55/14*a^9/b^12/(b*x^2+a)^7-55/4*a^8/b^12/(b*x^2+a)^6+33*a^7/b^12/(b*
x^2+a)^5-231/4*a^6/b^12/(b*x^2+a)^4+77*a^5/b^12/(b*x^2+a)^3-165/2*a^4/b^12/
(b*x^2+a)^2+165/2*a^3/b^12/(b*x^2+a)+55/2*a^2*ln(b*x^2+a)/b^12
```

Maxima [A] time = 2.69169, size = 312, normalized size = 1.52

$$\frac{83160\,a^{3}b^{8}x^{16}+582120\,a^{4}b^{7}x^{14}+1823976\,a^{5}b^{6}x^{12}+3318084\,a^{6}b^{5}x^{10}+3817044\,a^{7}b^{4}x^{8}+2835756\,a^{8}b^{3}x^{6}+132620}{1008\,\left(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^{2}b^{19}x^{14}+84\,a^{3}b^{18}x^{12}+126\,a^{4}b^{17}x^{10}+126\,a^{5}b^{16}x^{8}+84\,a^{6}b^{15}x^{6}+36\,a^{7}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(83160*a^3*b^8*x^16 + 582120*a^4*b^7*x^14 + 1823976*a^5*b^6*x^12 + 3
318084*a^6*b^5*x^10 + 3817044*a^7*b^4*x^8 + 2835756*a^8*b^3*x^6 + 1326204*a
^9*b^2*x^4 + 356499*a^10*b*x^2 + 42131*a^11)/(b^21*x^18 + 9*a*b^20*x^16 + 3
6*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 +
 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12) + 55/2*a^2*
log(b*x^2 + a)/b^12 + 1/4*(b*x^4 - 20*a*x^2)/b^11
```

Fricas [A] time = 1.22048, size = 817, normalized size = 3.99

$$\frac{252\,b^{11}x^{22}-2772\,ab^{10}x^{20}-36288\,a^{2}b^{9}x^{18}-77112\,a^{3}b^{8}x^{16}+190512\,a^{4}b^{7}x^{14}+1220688\,a^{5}b^{6}x^{12}+2704212\,a^{6}b^{5}x^{10}+}{1008\,\left(b^{21}x^{1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(252*b^11*x^22 - 2772*a*b^10*x^20 - 36288*a^2*b^9*x^18 - 77112*a^3*b
^8*x^16 + 190512*a^4*b^7*x^14 + 1220688*a^5*b^6*x^12 + 2704212*a^6*b^5*x^10
 + 3402756*a^7*b^4*x^8 + 2656584*a^8*b^3*x^6 + 1281096*a^9*b^2*x^4 + 351459
*a^10*b*x^2 + 42131*a^11 + 27720*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^
7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*
x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*log(b*x^2 + a))/(b^21*x^18 + 9*
a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126
*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^
12)
```

Sympy [A] time = 8.4607, size = 245, normalized size = 1.2

$$\frac{55a^2\log\left(a+bx^2\right)}{2b^{12}}-\frac{5ax^2}{b^{11}}+\frac{42131a^{11}+356499a^{10}bx^2+1326204a^9b^2x^4+2835756a^8b^3x^6+3817044a^7b^4x^8+3318}{1008a^9b^{12}+9072a^8b^{13}x^2+36288a^7b^{14}x^4+84672a^6b^{15}x^6+127008a^5b^{16}x^8+127008a^4b^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x**2)/(2*b**12) - 5*a*x**2/b**11 + (42131*a**11 + 356499*
a**10*b*x**2 + 1326204*a**9*b**2*x**4 + 2835756*a**8*b**3*x**6 + 3817044*a*
*7*b**4*x**8 + 3318084*a**6*b**5*x**10 + 1823976*a**5*b**6*x**12 + 582120*a
**4*b**7*x**14 + 83160*a**3*b**8*x**16)/(1008*a**9*b**12 + 9072*a**8*b**13*
x**2 + 36288*a**7*b**14*x**4 + 84672*a**6*b**15*x**6 + 127008*a**5*b**16*x*
*8 + 127008*a**4*b**17*x**10 + 84672*a**3*b**18*x**12 + 36288*a**2*b**19*x*
*14 + 9072*a*b**20*x**16 + 1008*b**21*x**18) + x**4/(4*b**10)
```

Giac [A] time = 3.20379, size = 212, normalized size = 1.03

$$\frac{55\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^{12}}+\frac{b^{10}x^4-20\,ab^9x^2}{4\,b^{20}}-\frac{78419\,a^2b^9x^{18}+622611\,a^3b^8x^{16}+2240964\,a^4b^7x^{14}+4763220\,a^5b^6x^{12}+656}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*a^2*log(abs(b*x^2 + a))/b^12 + 1/4*(b^10*x^4 - 20*a*b^9*x^2)/b^20 - 1/
1008*(78419*a^2*b^9*x^18 + 622611*a^3*b^8*x^16 + 2240964*a^4*b^7*x^14 + 476
3220*a^5*b^6*x^12 + 6562710*a^6*b^5*x^10 + 6063750*a^7*b^4*x^8 + 3751440*a^
8*b^3*x^6 + 1496880*a^9*b^2*x^4 + 349272*a^10*b*x^2 + 36288*a^11)/((b*x^2 +
 a)^9*b^12)
```

3.194 $\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=188

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{3}{b^{11}\left(a\right.}$$

```
[Out] x^2/(2*b^10) - a^10/(18*b^11*(a + b*x^2)^9) + (5*a^9)/(8*b^11*(a + b*x^2)^8
) - (45*a^8)/(14*b^11*(a + b*x^2)^7) + (10*a^7)/(b^11*(a + b*x^2)^6) - (21*
a^6)/(b^11*(a + b*x^2)^5) + (63*a^5)/(2*b^11*(a + b*x^2)^4) - (35*a^4)/(b^1
1*(a + b*x^2)^3) + (30*a^3)/(b^11*(a + b*x^2)^2) - (45*a^2)/(2*b^11*(a + b*
x^2)) - (5*a*Log[a + b*x^2])/b^11
```

Rubi [A] time = 0.1894, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{3}{b^{11}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^21/(a + b*x^2)^10,x]
```

```
[Out] x^2/(2*b^10) - a^10/(18*b^11*(a + b*x^2)^9) + (5*a^9)/(8*b^11*(a + b*x^2)^8
) - (45*a^8)/(14*b^11*(a + b*x^2)^7) + (10*a^7)/(b^11*(a + b*x^2)^6) - (21*
a^6)/(b^11*(a + b*x^2)^5) + (63*a^5)/(2*b^11*(a + b*x^2)^4) - (35*a^4)/(b^1
1*(a + b*x^2)^3) + (30*a^3)/(b^11*(a + b*x^2)^2) - (45*a^2)/(2*b^11*(a + b*
x^2)) - (5*a*Log[a + b*x^2])/b^11
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^{10}}{(a+bx)^{10}}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\right.\right.$$

$$=\frac{x^2}{2b^{10}}-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.0336789, size = 145, normalized size = 0.77

$$-\frac{2268a^2b^8x^{16}+54432a^3b^7x^{14}+197568a^4b^6x^{12}+375732a^5b^5x^{10}+439236a^6b^4x^8+328104a^7b^3x^6+153576a^8b^2x^4+4}{504b^{11}\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^21/(a + b*x^2)^10,x]
```

```
[Out] -(4861*a^10 + 41229*a^9*b*x^2 + 153576*a^8*b^2*x^4 + 328104*a^7*b^3*x^6 + 4
39236*a^6*b^4*x^8 + 375732*a^5*b^5*x^10 + 197568*a^4*b^6*x^12 + 54432*a^3*b
^7*x^14 + 2268*a^2*b^8*x^16 - 2268*a*b^9*x^18 - 252*b^10*x^20 + 2520*a*(a +
 b*x^2)^9*Log[a + b*x^2])/(504*b^11*(a + b*x^2)^9)
```

Maple [A] time = 0.017, size = 177, normalized size = 0.9

$$\frac{x^2}{2\,b^{10}}-\frac{a^{10}}{18\,b^{11}\left(bx^2+a\right)^9}+\frac{5\,a^9}{8\,b^{11}\left(bx^2+a\right)^8}-\frac{45\,a^8}{14\,b^{11}\left(bx^2+a\right)^7}+10\,\frac{a^7}{b^{11}\left(bx^2+a\right)^6}-21\,\frac{a^6}{b^{11}\left(bx^2+a\right)^5}+\frac{63\,a^5}{2\,b^{11}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(b*x^2+a)^10,x)
```

```
[Out] 1/2*x^2/b^10-1/18*a^10/b^11/(b*x^2+a)^9+5/8*a^9/b^11/(b*x^2+a)^8-45/14*a^8/
b^11/(b*x^2+a)^7+10*a^7/b^11/(b*x^2+a)^6-21*a^6/b^11/(b*x^2+a)^5+63/2*a^5/b
^11/(b*x^2+a)^4-35*a^4/b^11/(b*x^2+a)^3+30*a^3/b^11/(b*x^2+a)^2-45/2*a^2/b^
11/(b*x^2+a)-5*a*ln(b*x^2+a)/b^11
```

Maxima [A] time = 2.56957, size = 297, normalized size = 1.58

$$-\frac{11340\,a^2b^8x^{16}+75600\,a^3b^7x^{14}+229320\,a^4b^6x^{12}+407484\,a^5b^5x^{10}+460404\,a^6b^4x^8+337176\,a^7b^3x^6+155844\,a^8b^2x^4}{504\left(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x^8+84\,a^6b^{14}x^6+36\,a^7b^{13}x^4+9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(11340*a^2*b^8*x^16 + 75600*a^3*b^7*x^14 + 229320*a^4*b^6*x^12 + 407
484*a^5*b^5*x^10 + 460404*a^6*b^4*x^8 + 337176*a^7*b^3*x^6 + 155844*a^8*b^2
*x^4 + 41481*a^9*b*x^2 + 4861*a^10)/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^1
8*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b
^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11) + 1/2*x^2/b^10 - 5*a
*log(b*x^2 + a)/b^11
```

Fricas [A] time = 1.28917, size = 767, normalized size = 4.08

$$\frac{252\,b^{10}x^{20}+2268\,ab^9x^{18}-2268\,a^2b^8x^{16}-54432\,a^3b^7x^{14}-197568\,a^4b^6x^{12}-375732\,a^5b^5x^{10}-439236\,a^6b^4x^8-328104}{504\left(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^10*x^20 + 2268*a*b^9*x^18 - 2268*a^2*b^8*x^16 - 54432*a^3*b^7*
x^14 - 197568*a^4*b^6*x^12 - 375732*a^5*b^5*x^10 - 439236*a^6*b^4*x^8 - 328
104*a^7*b^3*x^6 - 153576*a^8*b^2*x^4 - 41229*a^9*b*x^2 - 4861*a^10 - 2520*(
a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b
^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 +
 a^10)*log(b*x^2 + a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a
^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*
a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11)
```

Sympy [A] time = 8.48979, size = 231, normalized size = 1.23

$$-\frac{5a\log\left(a+bx^2\right)}{b^{11}}-\frac{4861a^{10}+41481a^9bx^2+155844a^8b^2x^4+337176a^7b^3x^6+460404a^6b^4x^8+407484a^5b^5x^{10}}{504a^9b^{11}+4536a^8b^{12}x^2+18144a^7b^{13}x^4+42336a^6b^{14}x^6+63504a^5b^{15}x^8+63504a^4b^{16}x^{10}+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**21/(b*x**2+a)**10,x)
```

```
[Out] -5*a*log(a + b*x**2)/b**11 - (4861*a**10 + 41481*a**9*b*x**2 + 155844*a**8*
b**2*x**4 + 337176*a**7*b**3*x**6 + 460404*a**6*b**4*x**8 + 407484*a**5*b**
5*x**10 + 229320*a**4*b**6*x**12 + 75600*a**3*b**7*x**14 + 11340*a**2*b**8*
x**16)/(504*a**9*b**11 + 4536*a**8*b**12*x**2 + 18144*a**7*b**13*x**4 + 423
36*a**6*b**14*x**6 + 63504*a**5*b**15*x**8 + 63504*a**4*b**16*x**10 + 42336
*a**3*b**17*x**12 + 18144*a**2*b**18*x**14 + 4536*a*b**19*x**16 + 504*b**20
*x**18) + x**2/(2*b**10)
```

Giac [A] time = 2.47873, size = 188, normalized size = 1.

$$\frac{x^2}{2\,b^{10}}-\frac{5\,a\log\left(\left|bx^2+a\right|\right)}{b^{11}}+\frac{7129\,ab^9x^{18}+52821\,a^2b^8x^{16}+181044\,a^3b^7x^{14}+369516\,a^4b^6x^{12}+490770\,a^5b^5x^{10}+43}{504\left(bx^2+a\right)^9b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^10 - 5*a*log(abs(b*x^2 + a))/b^11 + 1/504*(7129*a*b^9*x^18 + 5282
1*a^2*b^8*x^16 + 181044*a^3*b^7*x^14 + 369516*a^4*b^6*x^12 + 490770*a^5*b^5
*x^10 + 437850*a^6*b^4*x^8 + 261660*a^7*b^3*x^6 + 100800*a^8*b^2*x^4 + 2268
0*a^9*b*x^2 + 2268*a^10)/((b*x^2 + a)^9*b^11)
```

3.195 $\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=179

$$\frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{4b^{10}\left(a+bx^2\right)^4}+\frac{14a^3}{b^{10}\left(a+b\right.}$$

```
[Out] a^9/(18*b^10*(a + b*x^2)^9) - (9*a^8)/(16*b^10*(a + b*x^2)^8) + (18*a^7)/(7
*b^10*(a + b*x^2)^7) - (7*a^6)/(b^10*(a + b*x^2)^6) + (63*a^5)/(5*b^10*(a +
 b*x^2)^5) - (63*a^4)/(4*b^10*(a + b*x^2)^4) + (14*a^3)/(b^10*(a + b*x^2)^3
) - (9*a^2)/(b^10*(a + b*x^2)^2) + (9*a)/(2*b^10*(a + b*x^2)) + Log[a + b*x
^2]/(2*b^10)
```

Rubi [A] time = 0.169356, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{4b^{10}\left(a+bx^2\right)^4}+\frac{14a^3}{b^{10}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^2)^10,x]
```

```
[Out] a^9/(18*b^10*(a + b*x^2)^9) - (9*a^8)/(16*b^10*(a + b*x^2)^8) + (18*a^7)/(7
*b^10*(a + b*x^2)^7) - (7*a^6)/(b^10*(a + b*x^2)^6) + (63*a^5)/(5*b^10*(a +
 b*x^2)^5) - (63*a^4)/(4*b^10*(a + b*x^2)^4) + (14*a^3)/(b^10*(a + b*x^2)^3
) - (9*a^2)/(b^10*(a + b*x^2)^2) + (9*a)/(2*b^10*(a + b*x^2)) + Log[a + b*x
^2]/(2*b^10)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^9}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a^4}{b^9(a+bx)^5}\right.\right.\\ &= \frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63}{4b^{10}\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.0317584, size = 116, normalized size = 0.65

$$\frac{\frac{a\left(388080a^2b^6x^{12}+661500a^3b^5x^{10}+725004a^4b^4x^8+518616a^5b^3x^6+235224a^6b^2x^4+61641a^7bx^2+7129a^8+136080ab^7x^{14}+22680b^8x^{16}\right)}{\left(a+bx^2\right)^9}+2520\log\left(a+\right.}{5040b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^2)^10,x]
```

```
[Out] ((a*(7129*a^8 + 61641*a^7*b*x^2 + 235224*a^6*b^2*x^4 + 518616*a^5*b^3*x^6 +
 725004*a^4*b^4*x^8 + 661500*a^3*b^5*x^10 + 388080*a^2*b^6*x^12 + 136080*a*
b^7*x^14 + 22680*b^8*x^16))/(a + b*x^2)^9 + 2520*Log[a + b*x^2])/(5040*b^10
)
```

Maple [A] time = 0.011, size = 166, normalized size = 0.9

$$\frac{a^9}{18\,b^{10}\left(bx^2+a\right)^9}-\frac{9\,a^8}{16\,b^{10}\left(bx^2+a\right)^8}+\frac{18\,a^7}{7\,b^{10}\left(bx^2+a\right)^7}-7\frac{a^6}{b^{10}\left(bx^2+a\right)^6}+\frac{63\,a^5}{5\,b^{10}\left(bx^2+a\right)^5}-\frac{63\,a^4}{4\,b^{10}\left(bx^2+a\right)^4}+14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a)
^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+a)
^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1/2*
ln(b*x^2+a)/b^10
```

Maxima [A] time = 2.32338, size = 282, normalized size = 1.58

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+235224\,a^7b^2}{5040\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 6615
00*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*
x^4 + 61641*a^8*b*x^2 + 7129*a^9)/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*
x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^1
3*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10) + 1/2*log(b*x^2 + a)/b
^10
```

Fricas [A] time = 1.27218, size = 714, normalized size = 3.99

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+235224\,a^7b^2}{5040\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 6615
00*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*
x^4 + 61641*a^8*b*x^2 + 7129*a^9 + 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b
^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3
*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a))/(b^19*x^18 + 9*a
*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*
a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^1
0)
```

Sympy [A] time = 7.94604, size = 219, normalized size = 1.22

$$\frac{7129a^9 + 61641a^8bx^2 + 235224a^7b^2x^4 + 518616a^6b^3x^6 + 725004a^5b^4x^8 + 661500a^4b^5x^{10} + 388080a^3b^6x^{12}}{5040a^9b^{10} + 45360a^8b^{11}x^2 + 181440a^7b^{12}x^4 + 423360a^6b^{13}x^6 + 635040a^5b^{14}x^8 + 635040a^4b^{15}x^{10} + 423360a^3b^{16}x^{12} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x**2 + 235224*a**7*b**2*x**4 + 518616*a**6*b**3*x
**6 + 725004*a**5*b**4*x**8 + 661500*a**4*b**5*x**10 + 388080*a**3*b**6*x**
12 + 136080*a**2*b**7*x**14 + 22680*a*b**8*x**16)/(5040*a**9*b**10 + 45360*
a**8*b**11*x**2 + 181440*a**7*b**12*x**4 + 423360*a**6*b**13*x**6 + 635040*
a**5*b**14*x**8 + 635040*a**4*b**15*x**10 + 423360*a**3*b**16*x**12 + 18144
0*a**2*b**17*x**14 + 45360*a*b**18*x**16 + 5040*b**19*x**18) + log(a + b*x*
*2)/(2*b**10)
```

Giac [A] time = 1.31202, size = 161, normalized size = 0.9

$$\frac{\log\left(\left|bx^2 + a\right|\right)}{2\,b^{10}} - \frac{7129\,b^8x^{18} + 41481\,ab^7x^{16} + 120564\,a^2b^6x^{14} + 210756\,a^3b^5x^{12} + 236754\,a^4b^4x^{10} + 173250\,a^5b^3x^8 + 80}{5040\left(bx^2 + a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^10 - 1/5040*(7129*b^8*x^18 + 41481*a*b^7*x^16 + 1
20564*a^2*b^6*x^14 + 210756*a^3*b^5*x^12 + 236754*a^4*b^4*x^10 + 173250*a^5
*b^3*x^8 + 80220*a^6*b^2*x^6 + 21420*a^7*b*x^4 + 2520*a^8*x^2)/((b*x^2 + a)
^9*b^9)
```

3.196 $\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

```
[Out] x^18/(18*a*(a + b*x^2)^9)
```

Rubi [A] time = 0.003412, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/(a + b*x^2)^10,x]
```

```
[Out] x^18/(18*a*(a + b*x^2)^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = \frac{x^{18}}{18a(a+bx^2)^9}$$

Mathematica [B] time = 0.0193256, size = 101, normalized size = 5.32

$$-\frac{84a^2b^6x^{12}+126a^3b^5x^{10}+126a^4b^4x^8+84a^5b^3x^6+36a^6b^2x^4+9a^7bx^2+a^8+36ab^7x^{14}+9b^8x^{16}}{18b^9(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/(a + b*x^2)^10,x]
```

```
[Out] -(a^8 + 9*a^7*b*x^2 + 36*a^6*b^2*x^4 + 84*a^5*b^3*x^6 + 126*a^4*b^4*x^8 + 1
26*a^3*b^5*x^10 + 84*a^2*b^6*x^12 + 36*a*b^7*x^14 + 9*b^8*x^16)/(18*b^9*(a
+ b*x^2)^9)
```

Maple [B] time = 0.01, size = 150, normalized size = 7.9

$$-\frac{a^8}{18\,b^9\left(bx^2+a\right)^9}+7\,\frac{a^3}{b^9\left(bx^2+a\right)^4}-\frac{14\,a^2}{3\,b^9\left(bx^2+a\right)^3}+\frac{14\,a^5}{3\,b^9\left(bx^2+a\right)^6}+\frac{a^7}{2\,b^9\left(bx^2+a\right)^8}+2\,\frac{a}{b^9\left(bx^2+a\right)^2}-7\,\frac{a^4}{b^9\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^8/b^9/(b*x^2+a)^9+7*a^3/b^9/(b*x^2+a)^4-14/3*a^2/b^9/(b*x^2+a)^3+14
/3*a^5/b^9/(b*x^2+a)^6+1/2*a^7/b^9/(b*x^2+a)^8+2/b^9*a/(b*x^2+a)^2-7*a^4/b^
9/(b*x^2+a)^5-1/2/b^9/(b*x^2+a)-2*a^6/b^9/(b*x^2+a)^7
```

Maxima [B] time = 2.01939, size = 257, normalized size = 13.53

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\left(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+9\,a^8b^{10}x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

Fricas [B] time = 1.20712, size = 423, normalized size = 22.26

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\left(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+9\,a^8b^{10}x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

Sympy [B] time = 7.60824, size = 202, normalized size = 10.63

$$-\frac{a^8+9a^7bx^2+36a^6b^2x^4+84a^5b^3x^6+126a^4b^4x^8+126a^3b^5x^{10}+84a^2b^6x^{12}+36ab^7x^{14}+9b^8x^{16}}{18a^9b^9+162a^8b^{10}x^2+648a^7b^{11}x^4+1512a^6b^{12}x^6+2268a^5b^{13}x^8+2268a^4b^{14}x^{10}+1512a^3b^{15}x^{12}+648a^2b^{16}x^{14}+162a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**2+a)**10,x)
```

```
[Out] -(a**8 + 9*a**7*b*x**2 + 36*a**6*b**2*x**4 + 84*a**5*b**3*x**6 + 126*a**4*b
**4*x**8 + 126*a**3*b**5*x**10 + 84*a**2*b**6*x**12 + 36*a*b**7*x**14 + 9*b
**8*x**16)/(18*a**9*b**9 + 162*a**8*b**10*x**2 + 648*a**7*b**11*x**4 + 1512
*a**6*b**12*x**6 + 2268*a**5*b**13*x**8 + 2268*a**4*b**14*x**10 + 1512*a**3
*b**15*x**12 + 648*a**2*b**16*x**14 + 162*a*b**17*x**16 + 18*b**18*x**18)
```

Giac [B] time = 2.71458, size = 134, normalized size = 7.05

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,\left(bx^2+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/((b*x^
2 + a)^9*b^9)
```

3.197 $\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=39

$$\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}+\frac{x^{16}}{18a\left(a+bx^2\right)^9}$$

```
[Out] x^16/(18*a*(a + b*x^2)^9) + x^16/(144*a^2*(a + b*x^2)^8)
```

Rubi [A] time = 0.0173839, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}+\frac{x^{16}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^10,x]
```

```
[Out] x^16/(18*a*(a + b*x^2)^9) + x^16/(144*a^2*(a + b*x^2)^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{x^{16}}{18a\left(a+bx^2\right)^9} + \frac{\text{Subst}\left(\int \frac{x^7}{(a+bx)^9}\,dx, x, x^2\right)}{18a}$$
$$= \frac{x^{16}}{18a\left(a+bx^2\right)^9} + \frac{x^{16}}{144a^2\left(a+bx^2\right)^8}$$

Mathematica [B] time = 0.0151744, size = 90, normalized size = 2.31

$$-\frac{126a^2b^5x^{10}+126a^3b^4x^8+84a^4b^3x^6+36a^5b^2x^4+9a^6bx^2+a^7+84ab^6x^{12}+36b^7x^{14}}{144b^8\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^10,x]
```

```
[Out] -(a^7 + 9*a^6*b*x^2 + 36*a^5*b^2*x^4 + 84*a^4*b^3*x^6 + 126*a^3*b^4*x^8 + 1
26*a^2*b^5*x^10 + 84*a*b^6*x^12 + 36*b^7*x^14)/(144*b^8*(a + b*x^2)^9)
```

Maple [B] time = 0.008, size = 133, normalized size = 3.4

$$\frac{a^7}{18\,b^8\left(bx^2+a\right)^9}+\frac{7\,a}{6\,b^8\left(bx^2+a\right)^3}-\frac{7\,a^6}{16\,b^8\left(bx^2+a\right)^8}-\frac{1}{4\,b^8\left(bx^2+a\right)^2}-\frac{21\,a^2}{8\,b^8\left(bx^2+a\right)^4}+\frac{7\,a^3}{2\,b^8\left(bx^2+a\right)^5}-\frac{35}{12\,b^8\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^10,x)
```

```
[Out] 1/18/b^8*a^7/(b*x^2+a)^9+7/6/b^8*a/(b*x^2+a)^3-7/16*a^6/b^8/(b*x^2+a)^8-1/4
/b^8/(b*x^2+a)^2-21/8*a^2/b^8/(b*x^2+a)^4+7/2*a^3/b^8/(b*x^2+a)^5-35/12*a^4
/b^8/(b*x^2+a)^6+3/2*a^5/b^8/(b*x^2+a)^7
```

Maxima [B] time = 2.72399, size = 242, normalized size = 6.21

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4+9\,a^8b^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*
x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^
12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

Fricas [B] time = 1.22697, size = 400, normalized size = 10.26

$$-\frac{36\,b^{7}x^{14}+84\,ab^{6}x^{12}+126\,a^{2}b^{5}x^{10}+126\,a^{3}b^{4}x^{8}+84\,a^{4}b^{3}x^{6}+36\,a^{5}b^{2}x^{4}+9\,a^{6}bx^{2}+a^{7}}{144\left(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^{2}b^{15}x^{14}+84\,a^{3}b^{14}x^{12}+126\,a^{4}b^{13}x^{10}+126\,a^{5}b^{12}x^{8}+84\,a^{6}b^{11}x^{6}+36\,a^{7}b^{10}x^{4}+9\,a^{8}b^{9}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*
x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^
12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

Sympy [B] time = 7.67083, size = 190, normalized size = 4.87

$$-\frac{a^{7}+9a^{6}bx^{2}+36a^{5}b^{2}x^{4}+84a^{4}b^{3}x^{6}+126a^{3}b^{4}x^{8}+126a^{2}b^{5}x^{10}+84ab^{6}x^{12}+36b^{7}x^{14}}{144a^{9}b^{8}+1296a^{8}b^{9}x^{2}+5184a^{7}b^{10}x^{4}+12096a^{6}b^{11}x^{6}+18144a^{5}b^{12}x^{8}+18144a^{4}b^{13}x^{10}+12096a^{3}b^{14}x^{12}+5184a^{2}b^{15}x^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**10,x)
```

```
[Out] -(a**7 + 9*a**6*b*x**2 + 36*a**5*b**2*x**4 + 84*a**4*b**3*x**6 + 126*a**3*b
**4*x**8 + 126*a**2*b**5*x**10 + 84*a*b**6*x**12 + 36*b**7*x**14)/(144*a**9
*b**8 + 1296*a**8*b**9*x**2 + 5184*a**7*b**10*x**4 + 12096*a**6*b**11*x**6
+ 18144*a**5*b**12*x**8 + 18144*a**4*b**13*x**10 + 12096*a**3*b**14*x**12 +
 5184*a**2*b**15*x**14 + 1296*a*b**16*x**16 + 144*b**17*x**18)
```

Giac [B] time = 2.31156, size = 119, normalized size = 3.05

$$-\frac{36\,b^{7}x^{14}+84\,ab^{6}x^{12}+126\,a^{2}b^{5}x^{10}+126\,a^{3}b^{4}x^{8}+84\,a^{4}b^{3}x^{6}+36\,a^{5}b^{2}x^{4}+9\,a^{6}bx^{2}+a^{7}}{144\left(bx^{2}+a\right)^{9}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/((b*x^2 + a)^9*b^8)
```

3.198 $\int \frac{x^{13}}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=58

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

```
[Out] x^14/(18*a*(a + b*x^2)^9) + x^14/(72*a^2*(a + b*x^2)^8) + x^14/(504*a^3*(a
+ b*x^2)^7)
```

Rubi [A] time = 0.0269605, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^10,x]
```

```
[Out] x^14/(18*a*(a + b*x^2)^9) + x^14/(72*a^2*(a + b*x^2)^8) + x^14/(504*a^3*(a
+ b*x^2)^7)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9} + \frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^9}\,dx, x, x^2\right)}{9a}$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9} + \frac{x^{14}}{72a^2\left(a+bx^2\right)^8} + \frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^8}\,dx, x, x^2\right)}{72a^2}$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9} + \frac{x^{14}}{72a^2\left(a+bx^2\right)^8} + \frac{x^{14}}{504a^3\left(a+bx^2\right)^7}$$

Mathematica [A] time = 0.017492, size = 79, normalized size = 1.36

$$-\frac{126a^2b^4x^8 + 84a^3b^3x^6 + 36a^4b^2x^4 + 9a^5bx^2 + a^6 + 126ab^5x^{10} + 84b^6x^{12}}{504b^7\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^10,x]
```

```
[Out] -(a^6 + 9*a^5*b*x^2 + 36*a^4*b^2*x^4 + 84*a^3*b^3*x^6 + 126*a^2*b^4*x^8 + 1
26*a*b^5*x^10 + 84*b^6*x^12)/(504*b^7*(a + b*x^2)^9)
```

Maple [B] time = 0.01, size = 116, normalized size = 2.

$$-\frac{a^6}{18\,b^7\left(bx^2+a\right)^9} + \frac{5\,a^3}{3\,b^7\left(bx^2+a\right)^6} + \frac{3\,a^5}{8\,b^7\left(bx^2+a\right)^8} - \frac{1}{6\,b^7\left(bx^2+a\right)^3} - \frac{3\,a^2}{2\,b^7\left(bx^2+a\right)^5} + \frac{3\,a}{4\,b^7\left(bx^2+a\right)^4} - \frac{15\,a^4}{14\,b^7\left(bx^2 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^6/b^7/(b*x^2+a)^9+5/3*a^3/b^7/(b*x^2+a)^6+3/8*a^5/b^7/(b*x^2+a)^8-1
/6/b^7/(b*x^2+a)^3-3/2*a^2/b^7/(b*x^2+a)^5+3/4/b^7*a/(b*x^2+a)^4-15/14*a^4/
b^7/(b*x^2+a)^7
```

Maxima [B] time = 2.76735, size = 227, normalized size = 3.91

$$-\frac{84\,b^6x^{12} + 126\,ab^5x^{10} + 126\,a^2b^4x^8 + 84\,a^3b^3x^6 + 36\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{504\left(b^{16}x^{18} + 9\,ab^{15}x^{16} + 36\,a^2b^{14}x^{14} + 84\,a^3b^{13}x^{12} + 126\,a^4b^{12}x^{10} + 126\,a^5b^{11}x^8 + 84\,a^6b^{10}x^6 + 36\,a^7b^9x^4 + 9\,a^8b^8x^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
```

```
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

Fricas [B] time = 1.16262, size = 374, normalized size = 6.45

$$-\frac{84\,b^{6}x^{12} + 126\,ab^{5}x^{10} + 126\,a^{2}b^{4}x^{8} + 84\,a^{3}b^{3}x^{6} + 36\,a^{4}b^{2}x^{4} + 9\,a^{5}bx^{2} + a^{6}}{504\,\left(b^{16}x^{18} + 9\,ab^{15}x^{16} + 36\,a^{2}b^{14}x^{14} + 84\,a^{3}b^{13}x^{12} + 126\,a^{4}b^{12}x^{10} + 126\,a^{5}b^{11}x^{8} + 84\,a^{6}b^{10}x^{6} + 36\,a^{7}b^{9}x^{4} + 9\,a^{8}b^{8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

Sympy [B] time = 7.59069, size = 178, normalized size = 3.07

$$-\frac{a^{6} + 9a^{5}bx^{2} + 36a^{4}b^{2}x^{4} + 84a^{3}b^{3}x^{6} + 126a^{2}b^{4}x^{8} + 126ab^{5}x^{10} + 84b^{6}x^{12}}{504a^{9}b^{7} + 4536a^{8}b^{8}x^{2} + 18144a^{7}b^{9}x^{4} + 42336a^{6}b^{10}x^{6} + 63504a^{5}b^{11}x^{8} + 63504a^{4}b^{12}x^{10} + 42336a^{3}b^{13}x^{12} + 18144a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**10,x)
```

```
[Out] -(a**6 + 9*a**5*b*x**2 + 36*a**4*b**2*x**4 + 84*a**3*b**3*x**6 + 126*a**2*b
**4*x**8 + 126*a*b**5*x**10 + 84*b**6*x**12)/(504*a**9*b**7 + 4536*a**8*b**
8*x**2 + 18144*a**7*b**9*x**4 + 42336*a**6*b**10*x**6 + 63504*a**5*b**11*x*
*8 + 63504*a**4*b**12*x**10 + 42336*a**3*b**13*x**12 + 18144*a**2*b**14*x**
14 + 4536*a*b**15*x**16 + 504*b**16*x**18)
```

Giac [A] time = 1.84436, size = 104, normalized size = 1.79

$$-\frac{84\,b^{6}x^{12} + 126\,ab^{5}x^{10} + 126\,a^{2}b^{4}x^{8} + 84\,a^{3}b^{3}x^{6} + 36\,a^{4}b^{2}x^{4} + 9\,a^{5}bx^{2} + a^{6}}{504\,{\left(bx^{2} + a\right)}^{9}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/((b*x^2 + a)^9*b^7)
```

3.199 $\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=77

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

```
[Out] x^12/(18*a*(a + b*x^2)^9) + x^12/(48*a^2*(a + b*x^2)^8) + x^12/(168*a^3*(a
+ b*x^2)^7) + x^12/(1008*a^4*(a + b*x^2)^6)
```

Rubi [A] time = 0.0389887, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^10,x]
```

```
[Out] x^12/(18*a*(a + b*x^2)^9) + x^12/(48*a^2*(a + b*x^2)^8) + x^12/(168*a^3*(a
+ b*x^2)^7) + x^12/(1008*a^4*(a + b*x^2)^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^5}{(a+bx)^{10}}\,dx,x,x^2\right)\\
&= \frac{x^{12}}{18a\left(a+bx^2\right)^9}+\frac{\operatorname{Subst}\left(\int \frac{x^5}{(a+bx)^9}\,dx,x,x^2\right)}{6a}\\
&= \frac{x^{12}}{18a\left(a+bx^2\right)^9}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{\operatorname{Subst}\left(\int \frac{x^5}{(a+bx)^8}\,dx,x,x^2\right)}{24a^2}\\
&= \frac{x^{12}}{18a\left(a+bx^2\right)^9}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{\operatorname{Subst}\left(\int \frac{x^5}{(a+bx)^7}\,dx,x,x^2\right)}{168a^3}\\
&= \frac{x^{12}}{18a\left(a+bx^2\right)^9}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}
\end{aligned}
$$

Mathematica [A] time = 0.021162, size = 68, normalized size = 0.88

$$
-\frac{84a^2b^3x^6+36a^3b^2x^4+9a^4bx^2+a^5+126ab^4x^8+126b^5x^{10}}{1008b^6\left(a+bx^2\right)^9}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^10,x]
```

```
[Out] -(a^5 + 9*a^4*b*x^2 + 36*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 126*a*b^4*x^8 + 126
*b^5*x^10)/(1008*b^6*(a + b*x^2)^9)
```

Maple [A] time = 0.009, size = 99, normalized size = 1.3

$$
-\frac{5\,a^4}{16\,b^6\left(bx^2+a\right)^8}-\frac{5\,a^2}{6\,b^6\left(bx^2+a\right)^6}+\frac{a^5}{18\,b^6\left(bx^2+a\right)^9}+\frac{a}{2\,b^6\left(bx^2+a\right)^5}-\frac{1}{8\,b^6\left(bx^2+a\right)^4}+\frac{5\,a^3}{7\,b^6\left(bx^2+a\right)^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^10,x)
```

```
[Out] -5/16*a^4/b^6/(b*x^2+a)^8-5/6*a^2/b^6/(b*x^2+a)^6+1/18/b^6*a^5/(b*x^2+a)^9+
1/2/b^6*a/(b*x^2+a)^5-1/8/b^6/(b*x^2+a)^4+5/7*a^3/b^6/(b*x^2+a)^7
```

Maxima [B] time = 2.36448, size = 212, normalized size = 2.75

$$
-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\left(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b^8x^4+9\,a^8b^7\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

Fricas [B] time = 1.24011, size = 350, normalized size = 4.55

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\left(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b^8x^4+9\,a^8b^7x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

Sympy [B] time = 7.2542, size = 167, normalized size = 2.17

$$-\frac{a^5+9a^4bx^2+36a^3b^2x^4+84a^2b^3x^6+126ab^4x^8+126b^5x^{10}}{1008a^9b^6+9072a^8b^7x^2+36288a^7b^8x^4+84672a^6b^9x^6+127008a^5b^{10}x^8+127008a^4b^{11}x^{10}+84672a^3b^{12}x^{12}+36288a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**10,x)
```

```
[Out] -(a**5 + 9*a**4*b*x**2 + 36*a**3*b**2*x**4 + 84*a**2*b**3*x**6 + 126*a*b**4
*x**8 + 126*b**5*x**10)/(1008*a**9*b**6 + 9072*a**8*b**7*x**2 + 36288*a**7*
b**8*x**4 + 84672*a**6*b**9*x**6 + 127008*a**5*b**10*x**8 + 127008*a**4*b**
11*x**10 + 84672*a**3*b**12*x**12 + 36288*a**2*b**13*x**14 + 9072*a*b**14*x
**16 + 1008*b**15*x**18)
```

Giac [A] time = 2.57667, size = 89, normalized size = 1.16

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,{\left(bx^2+a\right)}^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/((b*x^2 + a)^9*b^6)
```

3.200 $\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

```
[Out] -a^4/(18*b^5*(a + b*x^2)^9) + a^3/(4*b^5*(a + b*x^2)^8) - (3*a^2)/(7*b^5*(a
 + b*x^2)^7) + a/(3*b^5*(a + b*x^2)^6) - 1/(10*b^5*(a + b*x^2)^5)
```

Rubi [A] time = 0.0676645, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^10,x]
```

```
[Out] -a^4/(18*b^5*(a + b*x^2)^9) + a^3/(4*b^5*(a + b*x^2)^8) - (3*a^2)/(7*b^5*(a
 + b*x^2)^7) + a/(3*b^5*(a + b*x^2)^6) - 1/(10*b^5*(a + b*x^2)^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{10}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right)dx,x,x^2\right)\\
&= -\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}
\end{aligned}$$

Mathematica [A] time = 0.0150951, size = 57, normalized size = 0.63

$$-\frac{36a^2b^2x^4+9a^3bx^2+a^4+84ab^3x^6+126b^4x^8}{1260b^5\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^10,x]
```

```
[Out] -(a^4 + 9*a^3*b*x^2 + 36*a^2*b^2*x^4 + 84*a*b^3*x^6 + 126*b^4*x^8)/(1260*b^
5*(a + b*x^2)^9)
```

Maple [A] time = 0.008, size = 82, normalized size = 0.9

$$-\frac{a^4}{18\,b^5\left(bx^2+a\right)^9}+\frac{a^3}{4\,b^5\left(bx^2+a\right)^8}-\frac{3\,a^2}{7\,b^5\left(bx^2+a\right)^7}+\frac{a}{3\,b^5\left(bx^2+a\right)^6}-\frac{1}{10\,b^5\left(bx^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1
/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

Maxima [A] time = 2.25224, size = 197, normalized size = 2.16

$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7x^4+9\,a^8b^6x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

Fricas [A] time = 1.21416, size = 323, normalized size = 3.55

$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7x^4+9\,a^8b^6x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

Sympy [A] time = 7.24991, size = 155, normalized size = 1.7

$$-\frac{a^4 + 9a^3bx^2 + 36a^2b^2x^4 + 84ab^3x^6 + 126b^4x^8}{1260a^9b^5 + 11340a^8b^6x^2 + 45360a^7b^7x^4 + 105840a^6b^8x^6 + 158760a^5b^9x^8 + 158760a^4b^{10}x^{10} + 105840a^3b^{11}x^{12} + 45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**10,x)
```

```
[Out] -(a**4 + 9*a**3*b*x**2 + 36*a**2*b**2*x**4 + 84*a*b**3*x**6 + 126*b**4*x**8
)/(1260*a**9*b**5 + 11340*a**8*b**6*x**2 + 45360*a**7*b**7*x**4 + 105840*a*
*6*b**8*x**6 + 158760*a**5*b**9*x**8 + 158760*a**4*b**10*x**10 + 105840*a**
3*b**11*x**12 + 45360*a**2*b**12*x**14 + 11340*a*b**13*x**16 + 1260*b**14*x
**18)
```

Giac [A] time = 1.84398, size = 74, normalized size = 0.81

$$-\frac{126\,b^4x^8 + 84\,ab^3x^6 + 36\,a^2b^2x^4 + 9\,a^3bx^2 + a^4}{1260\,\left(bx^2 + a\right)^9 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
(b*x^2 + a)^9*b^5)
```

3.201 $\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=72

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

```
[Out] a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4
*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)
```

Rubi [A] time = 0.0530002, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^10,x]
```

```
[Out] a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4
*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right)dx,x,x^2\right)\\ &= \frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}\end{aligned}$$

Mathematica [A] time = 0.0105129, size = 46, normalized size = 0.64

$$-\frac{9a^2bx^2+a^3+36ab^2x^4+84b^3x^6}{1008b^4\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^10,x]
```

```
[Out] -(a^3 + 9*a^2*b*x^2 + 36*a*b^2*x^4 + 84*b^3*x^6)/(1008*b^4*(a + b*x^2)^9)
```

Maple [A] time = 0.008, size = 65, normalized size = 0.9

$$\frac{a^3}{18\,b^4\left(bx^2+a\right)^9}-\frac{3\,a^2}{16\,b^4\left(bx^2+a\right)^8}+\frac{3\,a}{14\,b^4\left(bx^2+a\right)^7}-\frac{1}{12\,b^4\left(bx^2+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1/
12/b^4/(b*x^2+a)^6
```

Maxima [B] time = 2.81793, size = 182, normalized size = 2.53

$$-\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\left(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^2b^{11}x^{14}+84\,a^3b^{10}x^{12}+126\,a^4b^9x^{10}+126\,a^5b^8x^8+84\,a^6b^7x^6+36\,a^7b^6x^4+9\,a^8b^5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

Fricas [B] time = 1.19305, size = 297, normalized size = 4.12

$$-\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\left(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^2b^{11}x^{14}+84\,a^3b^{10}x^{12}+126\,a^4b^9x^{10}+126\,a^5b^8x^8+84\,a^6b^7x^6+36\,a^7b^6x^4+9\,a^8b^5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

Sympy [B] time = 7.12123, size = 143, normalized size = 1.99

$$-\frac{a^3+9a^2bx^2+36ab^2x^4+84b^3x^6}{1008a^9b^4+9072a^8b^5x^2+36288a^7b^6x^4+84672a^6b^7x^6+127008a^5b^8x^8+127008a^4b^9x^{10}+84672a^3b^{10}x^{12}+36288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**10,x)
```

```
[Out] -(a**3 + 9*a**2*b*x**2 + 36*a*b**2*x**4 + 84*b**3*x**6)/(1008*a**9*b**4 + 9
072*a**8*b**5*x**2 + 36288*a**7*b**6*x**4 + 84672*a**6*b**7*x**6 + 127008*a
**5*b**8*x**8 + 127008*a**4*b**9*x**10 + 84672*a**3*b**10*x**12 + 36288*a**
2*b**11*x**14 + 9072*a*b**12*x**16 + 1008*b**13*x**18)
```

Giac [A] time = 2.53198, size = 59, normalized size = 0.82

$$-\frac{84\,b^3x^6 + 36\,ab^2x^4 + 9\,a^2bx^2 + a^3}{1008\,(bx^2 + a)^9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/((b*x^2 + a)^9*b^4)
```

3.202 $\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

```
[Out] -a^2/(18*b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*x^
2)^7)
```

Rubi [A] time = 0.0388855, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^10,x]
```

```
[Out] -a^2/(18*b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*x^
2)^7)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}\end{aligned}$$

Mathematica [A] time = 0.0126977, size = 35, normalized size = 0.66

$$-\frac{a^2+9abx^2+36b^2x^4}{504b^3\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^10,x]
```

```
[Out] -(a^2 + 9*a*b*x^2 + 36*b^2*x^4)/(504*b^3*(a + b*x^2)^9)
```

Maple [A] time = 0.007, size = 48, normalized size = 0.9

$$-\frac{a^2}{18\,b^3\left(bx^2+a\right)^9}+\frac{a}{8\,b^3\left(bx^2+a\right)^8}-\frac{1}{14\,b^3\left(bx^2+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

Maxima [B] time = 2.34149, size = 167, normalized size = 3.15

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\left(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4+9\,a^8b^4x^2+a^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

Fricas [B] time = 1.18631, size = 271, normalized size = 5.11

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\left(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4+9\,a^8b^4x^2+a^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

Sympy [B] time = 7.05524, size = 131, normalized size = 2.47

$$-\frac{a^2+9abx^2+36b^2x^4}{504a^9b^3+4536a^8b^4x^2+18144a^7b^5x^4+42336a^6b^6x^6+63504a^5b^7x^8+63504a^4b^8x^{10}+42336a^3b^9x^{12}+18144a^2b^{10}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**10,x)
```

```
[Out] -(a**2 + 9*a*b*x**2 + 36*b**2*x**4)/(504*a**9*b**3 + 4536*a**8*b**4*x**2 +
18144*a**7*b**5*x**4 + 42336*a**6*b**6*x**6 + 63504*a**5*b**7*x**8 + 63504*
a**4*b**8*x**10 + 42336*a**3*b**9*x**12 + 18144*a**2*b**10*x**14 + 4536*a*b
**11*x**16 + 504*b**12*x**18)
```

Giac [A] time = 2.32654, size = 45, normalized size = 0.85

$$-\frac{36\,b^2x^4 + 9\,abx^2 + a^2}{504\,\left(bx^2 + a\right)^9 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/((b*x^2 + a)^9*b^3)
```

3.203 $\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=34

$$\frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}$$

```
[Out] a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)
```

Rubi [A] time = 0.0251604, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^10,x]
```

```
[Out] a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{10}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{10}}+\frac{1}{b(a+bx)^9}\right)dx, x, x^2\right)\\
&= \frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}
\end{aligned}$$

Mathematica [A] time = 0.0079257, size = 24, normalized size = 0.71

$$-\frac{a+9bx^2}{144b^2\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^10,x]
```

```
[Out] -(a + 9*b*x^2)/(144*b^2*(a + b*x^2)^9)
```

Maple [A] time = 0.007, size = 31, normalized size = 0.9

$$\frac{a}{18\,b^2\left(bx^2+a\right)^9}-\frac{1}{16\,b^2\left(bx^2+a\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8
```

Maxima [B] time = 2.60832, size = 153, normalized size = 4.5

$$-\frac{9\,bx^2+a}{144\left(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4+9\,a^8b^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

Fricas [B] time = 1.19908, size = 247, normalized size = 7.26

$$-\frac{9\,bx^2+a}{144\left(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4+9\,a^8b^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

Sympy [B] time = 6.96263, size = 119, normalized size = 3.5

$$-\frac{a+9bx^2}{144a^9b^2+1296a^8b^3x^2+5184a^7b^4x^4+12096a^6b^5x^6+18144a^5b^6x^8+18144a^4b^7x^{10}+12096a^3b^8x^{12}+5184a^2b^9x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**10,x)
```

```
[Out] -(a + 9*b*x**2)/(144*a**9*b**2 + 1296*a**8*b**3*x**2 + 5184*a**7*b**4*x**4
+ 12096*a**6*b**5*x**6 + 18144*a**5*b**6*x**8 + 18144*a**4*b**7*x**10 + 120
96*a**3*b**8*x**12 + 5184*a**2*b**9*x**14 + 1296*a*b**10*x**16 + 144*b**11*
x**18)
```

Giac [A] time = 1.69398, size = 30, normalized size = 0.88

$$-\frac{9\,bx^2+a}{144\,\left(bx^2+a\right)^9 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(9*b*x^2 + a)/((b*x^2 + a)^9*b^2)
```

3.204 $\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{18b\left(a+bx^2\right)^9}$$

```
[Out] -1/(18*b*(a + b*x^2)^9)
```

Rubi [A] time = 0.0027757, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^10,x]

[Out] -1/(18*b*(a + b*x^2)^9)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx = -\frac{1}{18b\left(a+bx^2\right)^9}$$

Mathematica [A] time = 0.0023917, size = 16, normalized size = 1.

$$-\frac{1}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^10,x]

[Out] -1/(18*b*(a + b*x^2)^9)
```

Maple [A] time = 0., size = 15, normalized size = 0.9

$$-\frac{1}{18\,b\left(bx^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^10,x)
```

```
[Out] -1/18/b/(b*x^2+a)^9
```

Maxima [A] time = 2.24934, size = 19, normalized size = 1.19

$$-\frac{1}{18\left(bx^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

Fricas [B] time = 1.23636, size = 223, normalized size = 13.94

$$-\frac{1}{18\left(b^{10}x^{18}+9\,ab^9x^{16}+36\,a^2b^8x^{14}+84\,a^3b^7x^{12}+126\,a^4b^6x^{10}+126\,a^5b^5x^8+84\,a^6b^4x^6+36\,a^7b^3x^4+9\,a^8b^2x^2+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18/(b^10*x^18 + 9*a*b^9*x^16 + 36*a^2*b^8*x^14 + 84*a^3*b^7*x^12 + 126*a
^4*b^6*x^10 + 126*a^5*b^5*x^8 + 84*a^6*b^4*x^6 + 36*a^7*b^3*x^4 + 9*a^8*b^2
*x^2 + a^9*b)
```

Sympy [B] time = 7.01308, size = 110, normalized size = 6.88

$$-\frac{1}{18a^9b+162a^8b^2x^2+648a^7b^3x^4+1512a^6b^4x^6+2268a^5b^5x^8+2268a^4b^6x^{10}+1512a^3b^7x^{12}+648a^2b^8x^{14}+162ab^9x^{16}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**10,x)
```

```
[Out] -1/(18*a**9*b + 162*a**8*b**2*x**2 + 648*a**7*b**3*x**4 + 1512*a**6*b**4*x*
*6 + 2268*a**5*b**5*x**8 + 2268*a**4*b**6*x**10 + 1512*a**3*b**7*x**12 + 64
8*a**2*b**8*x**14 + 162*a*b**9*x**16 + 18*b**10*x**18)
```

Giac [A] time = 2.06648, size = 19, normalized size = 1.19

$$-\frac{1}{18\left(bx^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

3.205 $\int \frac{1}{x(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=166

$$\frac{1}{2a^9(a+bx^2)}+\frac{1}{4a^8(a+bx^2)^2}+\frac{1}{6a^7(a+bx^2)^3}+\frac{1}{8a^6(a+bx^2)^4}+\frac{1}{10a^5(a+bx^2)^5}+\frac{1}{12a^4(a+bx^2)^6}+\frac{1}{14a^3(a+bx^2)}$$

```
[Out] 1/(18*a*(a + b*x^2)^9) + 1/(16*a^2*(a + b*x^2)^8) + 1/(14*a^3*(a + b*x^2)^7
) + 1/(12*a^4*(a + b*x^2)^6) + 1/(10*a^5*(a + b*x^2)^5) + 1/(8*a^6*(a + b*x
^2)^4) + 1/(6*a^7*(a + b*x^2)^3) + 1/(4*a^8*(a + b*x^2)^2) + 1/(2*a^9*(a +
b*x^2)) + Log[x]/a^10 - Log[a + b*x^2]/(2*a^10)
```

Rubi [A] time = 0.128463, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2a^9(a+bx^2)}+\frac{1}{4a^8(a+bx^2)^2}+\frac{1}{6a^7(a+bx^2)^3}+\frac{1}{8a^6(a+bx^2)^4}+\frac{1}{10a^5(a+bx^2)^5}+\frac{1}{12a^4(a+bx^2)^6}+\frac{1}{14a^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^10),x]
```

```
[Out] 1/(18*a*(a + b*x^2)^9) + 1/(16*a^2*(a + b*x^2)^8) + 1/(14*a^3*(a + b*x^2)^7
) + 1/(12*a^4*(a + b*x^2)^6) + 1/(10*a^5*(a + b*x^2)^5) + 1/(8*a^6*(a + b*x
^2)^4) + 1/(6*a^7*(a + b*x^2)^3) + 1/(4*a^8*(a + b*x^2)^2) + 1/(2*a^9*(a +
b*x^2)) + Log[x]/a^10 - Log[a + b*x^2]/(2*a^10)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x(a+bx^2)^{10}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{10}}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\,\text{Subst}\left(\int\left(\frac{1}{a^{10}x}-\frac{b}{a(a+bx)^{10}}-\frac{b}{a^2(a+bx)^9}-\frac{b}{a^3(a+bx)^8}-\frac{b}{a^4(a+bx)^7}-\frac{b}{a^5(a+bx)^6}-\frac{}{a^6(a}\right.\right.$$

$$=\frac{1}{18a(a+bx^2)^9}+\frac{1}{16a^2(a+bx^2)^8}+\frac{1}{14a^3(a+bx^2)^7}+\frac{1}{12a^4(a+bx^2)^6}+\frac{1}{10a^5(a+bx^2)^5}+\frac{}{8a^6(a}$$

Mathematica [A] time = 0.0816384, size = 120, normalized size = 0.72

$$\frac{\frac{a\left(80220a^2b^6x^{12}+173250a^3b^5x^{10}+236754a^4b^4x^8+210756a^5b^3x^6+120564a^6b^2x^4+41481a^7bx^2+7129a^8+21420ab^7x^{14}+2520b^8x^{16}\right)}{\left(a+bx^2\right)^9}-2520\log\left(a+bx^2\right.}{5040a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^10),x]
```

```
[Out] ((a*(7129*a^8 + 41481*a^7*b*x^2 + 120564*a^6*b^2*x^4 + 210756*a^5*b^3*x^6 +
 236754*a^4*b^4*x^8 + 173250*a^3*b^5*x^10 + 80220*a^2*b^6*x^12 + 21420*a*b^
7*x^14 + 2520*b^8*x^16))/(a + b*x^2)^9 + 5040*Log[x] - 2520*Log[a + b*x^2])
/(5040*a^10)
```

Maple [A] time = 0.018, size = 147, normalized size = 0.9

$$\frac{1}{18\,a\left(bx^2+a\right)^9}+\frac{1}{16\,a^2\left(bx^2+a\right)^8}+\frac{1}{14\,a^3\left(bx^2+a\right)^7}+\frac{1}{12\,a^4\left(bx^2+a\right)^6}+\frac{1}{10\,a^5\left(bx^2+a\right)^5}+\frac{1}{8\,a^6\left(bx^2+a\right)^4}+\frac{}{6\,a^7\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^10,x)
```

```
[Out] 1/18/a/(b*x^2+a)^9+1/16/a^2/(b*x^2+a)^8+1/14/a^3/(b*x^2+a)^7+1/12/a^4/(b*x^
2+a)^6+1/10/a^5/(b*x^2+a)^5+1/8/a^6/(b*x^2+a)^4+1/6/a^7/(b*x^2+a)^3+1/4/a^8
/(b*x^2+a)^2+1/2/a^9/(b*x^2+a)+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

Maxima [A] time = 1.35835, size = 289, normalized size = 1.74

$$\frac{2520\,b^8x^{16}+21420\,ab^7x^{14}+80220\,a^2b^6x^{12}+173250\,a^3b^5x^{10}+236754\,a^4b^4x^8+210756\,a^5b^3x^6+120564\,a^6b^2x^4+4}{5040\left(a^9b^9x^{18}+9\,a^{10}b^8x^{16}+36\,a^{11}b^7x^{14}+84\,a^{12}b^6x^{12}+126\,a^{13}b^5x^{10}+126\,a^{14}b^4x^8+84\,a^{15}b^3x^6+36\,a^{16}b^2x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(2520*b^8*x^16 + 21420*a*b^7*x^14 + 80220*a^2*b^6*x^12 + 173250*a^3*
b^5*x^10 + 236754*a^4*b^4*x^8 + 210756*a^5*b^3*x^6 + 120564*a^6*b^2*x^4 + 4
1481*a^7*b*x^2 + 7129*a^8)/(a^9*b^9*x^18 + 9*a^10*b^8*x^16 + 36*a^11*b^7*x^
14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a^14*b^4*x^8 + 84*a^15*b^3*
x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) - 1/2*log(b*x^2 + a)/a^10 + 1/
2*log(x^2)/a^10
```

Fricas [B] time = 1.34803, size = 936, normalized size = 5.64

$$\frac{2520\,ab^8x^{16}+21420\,a^2b^7x^{14}+80220\,a^3b^6x^{12}+173250\,a^4b^5x^{10}+236754\,a^5b^4x^8+210756\,a^6b^3x^6+120564\,a^7b^2x^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(2520*a*b^8*x^16 + 21420*a^2*b^7*x^14 + 80220*a^3*b^6*x^12 + 173250*
a^4*b^5*x^10 + 236754*a^5*b^4*x^8 + 210756*a^6*b^3*x^6 + 120564*a^7*b^2*x^4
 + 41481*a^8*b*x^2 + 7129*a^9 - 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*
x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^
6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a) + 5040*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^
5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(x))/(a
^10*b^9*x^18 + 9*a^11*b^8*x^16 + 36*a^12*b^7*x^14 + 84*a^13*b^6*x^12 + 126*
a^14*b^5*x^10 + 126*a^15*b^4*x^8 + 84*a^16*b^3*x^6 + 36*a^17*b^2*x^4 + 9*a^
18*b*x^2 + a^19)
```

Sympy [A] time = 77.2405, size = 223, normalized size = 1.34

$$\frac{7129a^8 + 41481a^7bx^2 + 120564a^6b^2x^4 + 210756a^5b^3x^6 + 236754a^4b^4x^8 + 173250a^3b^5x^{10} + 80220a^2b^6x^{12} \cdots}{5040a^{18} + 45360a^{17}bx^2 + 181440a^{16}b^2x^4 + 423360a^{15}b^3x^6 + 635040a^{14}b^4x^8 + 635040a^{13}b^5x^{10} + 423360a^{12}b^6x^{12} + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x**2 + 120564*a**6*b**2*x**4 + 210756*a**5*b**3*x
**6 + 236754*a**4*b**4*x**8 + 173250*a**3*b**5*x**10 + 80220*a**2*b**6*x**1
2 + 21420*a*b**7*x**14 + 2520*b**8*x**16)/(5040*a**18 + 45360*a**17*b*x**2
+ 181440*a**16*b**2*x**4 + 423360*a**15*b**3*x**6 + 635040*a**14*b**4*x**8
+ 635040*a**13*b**5*x**10 + 423360*a**12*b**6*x**12 + 181440*a**11*b**7*x**
14 + 45360*a**10*b**8*x**16 + 5040*a**9*b**9*x**18) + log(x)/a**10 - log(a/
b + x**2)/(2*a**10)
```

Giac [A] time = 2.27445, size = 184, normalized size = 1.11

$$\frac{\log\left(x^2\right)}{2\,a^{10}} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^{10}} + \frac{7129\,b^9x^{18} + 66681\,ab^8x^{16} + 278064\,a^2b^7x^{14} + 679056\,a^3b^6x^{12} + 1071504\,a^4b^5x^{10} + 113500 \cdots}{5040\left(bx^2 + a\right)^9 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^10 - 1/2*log(abs(b*x^2 + a))/a^10 + 1/5040*(7129*b^9*x^18 +
66681*a*b^8*x^16 + 278064*a^2*b^7*x^14 + 679056*a^3*b^6*x^12 + 1071504*a^4*
b^5*x^10 + 1135008*a^5*b^4*x^8 + 809592*a^6*b^3*x^6 + 377208*a^7*b^2*x^4 +
105642*a^8*b*x^2 + 14258*a^9)/((b*x^2 + a)^9*a^10)
```

3.206 $\int \frac{1}{x^3(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=184

$$-\frac{9b}{2a^{10}\left(a+bx^2\right)}-\frac{2b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{6a^8\left(a+bx^2\right)^3}-\frac{3b}{4a^7\left(a+bx^2\right)^4}-\frac{b}{2a^6\left(a+bx^2\right)^5}-\frac{b}{3a^5\left(a+bx^2\right)^6}-\frac{3b}{14a^4\left(a+bx^2\right.}$$

```
[Out] -1/(2*a^10*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3*b
)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)^5
) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^9*
(a + b*x^2)^2) - (9*b)/(2*a^10*(a + b*x^2)) - (10*b*Log[x])/a^11 + (5*b*Log
[a + b*x^2])/a^11
```

Rubi [A] time = 0.185709, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{9b}{2a^{10}\left(a+bx^2\right)}-\frac{2b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{6a^8\left(a+bx^2\right)^3}-\frac{3b}{4a^7\left(a+bx^2\right)^4}-\frac{b}{2a^6\left(a+bx^2\right)^5}-\frac{b}{3a^5\left(a+bx^2\right)^6}-\frac{3b}{14a^4\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^10),x]
```


```
[Out] -1/(2*a^10*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3*b
)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)^5
) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^9*
(a + b*x^2)^2) - (9*b)/(2*a^10*(a + b*x^2)) - (10*b*Log[x])/a^11 + (5*b*Log
[a + b*x^2])/a^11
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{10}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\frac{5}{a^6(a}\right.\right.\\ &= -\frac{1}{2a^{10}x^2}-\frac{b}{18a^2\left(a+bx^2\right)^9}-\frac{b}{8a^3\left(a+bx^2\right)^8}-\frac{3b}{14a^4\left(a+bx^2\right)^7}-\frac{b}{3a^5\left(a+bx^2\right)^6}-\frac{b}{2a^6\left(a+bx\right.}\end{aligned}$$

Mathematica [A] time = 0.120819, size = 136, normalized size = 0.74

$$-\frac{\frac{a\left(80220a^2b^7x^{14}+173250a^3b^6x^{12}+236754a^4b^5x^{10}+210756a^5b^4x^8+120564a^6b^3x^6+41481a^7b^2x^4+7129a^8bx^2+252a^9+21420ab^8x^{16}+2520b^9x^{18}\right)}{x^2\left(a+bx^2\right)^9}-2520b\log}{504a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -((a*(252*a^9 + 7129*a^8*b*x^2 + 41481*a^7*b^2*x^4 + 120564*a^6*b^3*x^6 + 2
10756*a^5*b^4*x^8 + 236754*a^4*b^5*x^10 + 173250*a^3*b^6*x^12 + 80220*a^2*b
^7*x^14 + 21420*a*b^8*x^16 + 2520*b^9*x^18))/(x^2*(a + b*x^2)^9) + 5040*b*L
og[x] - 2520*b*Log[a + b*x^2])/(504*a^11)
```

Maple [A] time = 0.019, size = 167, normalized size = 0.9

$$-\frac{1}{2\,a^{10}x^2}-\frac{b}{18\,a^2\left(bx^2+a\right)^9}-\frac{b}{8\,a^3\left(bx^2+a\right)^8}-\frac{3\,b}{14\,a^4\left(bx^2+a\right)^7}-\frac{b}{3\,a^5\left(bx^2+a\right)^6}-\frac{b}{2\,a^6\left(bx^2+a\right)^5}-\frac{3\,b}{4\,a^7\left(bx^2+a\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^10,x)
```

```
[Out] -1/2/a^10/x^2-1/18*b/a^2/(b*x^2+a)^9-1/8*b/a^3/(b*x^2+a)^8-3/14*b/a^4/(b*x^
2+a)^7-1/3*b/a^5/(b*x^2+a)^6-1/2*b/a^6/(b*x^2+a)^5-3/4*b/a^7/(b*x^2+a)^4-7/
6*b/a^8/(b*x^2+a)^3-2*b/a^9/(b*x^2+a)^2-9/2*b/a^10/(b*x^2+a)-10*b*ln(x)/a^1
1+5*b*ln(b*x^2+a)/a^11
```

Maxima [A] time = 1.26732, size = 312, normalized size = 1.7

$$-\frac{2520\,b^9x^{18}+21420\,ab^8x^{16}+80220\,a^2b^7x^{14}+173250\,a^3b^6x^{12}+236754\,a^4b^5x^{10}+210756\,a^5b^4x^8+120564\,a^6b^3x^6+41}{504\left(a^{10}b^9x^{20}+9\,a^{11}b^8x^{18}+36\,a^{12}b^7x^{16}+84\,a^{13}b^6x^{14}+126\,a^{14}b^5x^{12}+126\,a^{15}b^4x^{10}+84\,a^{16}b^3x^8+36\,a^{17}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(2520*b^9*x^18 + 21420*a*b^8*x^16 + 80220*a^2*b^7*x^14 + 173250*a^3*
b^6*x^12 + 236754*a^4*b^5*x^10 + 210756*a^5*b^4*x^8 + 120564*a^6*b^3*x^6 +
41481*a^7*b^2*x^4 + 7129*a^8*b*x^2 + 252*a^9)/(a^10*b^9*x^20 + 9*a^11*b^8*x
^18 + 36*a^12*b^7*x^16 + 84*a^13*b^6*x^14 + 126*a^14*b^5*x^12 + 126*a^15*b^
4*x^10 + 84*a^16*b^3*x^8 + 36*a^17*b^2*x^6 + 9*a^18*b*x^4 + a^19*x^2) + 5*b
*log(b*x^2 + a)/a^11 - 5*b*log(x^2)/a^11
```

Fricas [B] time = 1.38937, size = 996, normalized size = 5.41

$$-\frac{2520\,ab^9x^{18}+21420\,a^2b^8x^{16}+80220\,a^3b^7x^{14}+173250\,a^4b^6x^{12}+236754\,a^5b^5x^{10}+210756\,a^6b^4x^8+120564\,a^7b^3x^6+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(2520*a*b^9*x^18 + 21420*a^2*b^8*x^16 + 80220*a^3*b^7*x^14 + 173250*
a^4*b^6*x^12 + 236754*a^5*b^5*x^10 + 210756*a^6*b^4*x^8 + 120564*a^7*b^3*x^
6 + 41481*a^8*b^2*x^4 + 7129*a^9*b*x^2 + 252*a^10 - 2520*(b^10*x^20 + 9*a*b
^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x^14 + 126*a^4*b^6*x^12 + 126*a^5*b^
5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(b
*x^2 + a) + 5040*(b^10*x^20 + 9*a*b^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x
^14 + 126*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6
 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(x))/(a^11*b^9*x^20 + 9*a^12*b^8*x^18 + 36
*a^13*b^7*x^16 + 84*a^14*b^6*x^14 + 126*a^15*b^5*x^12 + 126*a^16*b^4*x^10 +
 84*a^17*b^3*x^8 + 36*a^18*b^2*x^6 + 9*a^19*b*x^4 + a^20*x^2)
```

Sympy [A] time = 168.877, size = 243, normalized size = 1.32

$$-\frac{252a^9+7129a^8bx^2+41481a^7b^2x^4+120564a^6b^3x^6+210756a^5b^4x^8+236754a^4b^5x^{10}+173250a^3b^6x^{12}+8022\cdots}{504a^{19}x^2+4536a^{18}bx^4+18144a^{17}b^2x^6+42336a^{16}b^3x^8+63504a^{15}b^4x^{10}+63504a^{14}b^5x^{12}+42336a^{13}b^6x^{14}+1814\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**10,x)
```

```
[Out] -(252*a**9 + 7129*a**8*b*x**2 + 41481*a**7*b**2*x**4 + 120564*a**6*b**3*x**
6 + 210756*a**5*b**4*x**8 + 236754*a**4*b**5*x**10 + 173250*a**3*b**6*x**12
 + 80220*a**2*b**7*x**14 + 21420*a*b**8*x**16 + 2520*b**9*x**18)/(504*a**19
*x**2 + 4536*a**18*b*x**4 + 18144*a**17*b**2*x**6 + 42336*a**16*b**3*x**8 +
 63504*a**15*b**4*x**10 + 63504*a**14*b**5*x**12 + 42336*a**13*b**6*x**14 +
 18144*a**12*b**7*x**16 + 4536*a**11*b**8*x**18 + 504*a**10*b**9*x**20) - 1
0*b*log(x)/a**11 + 5*b*log(a/b + x**2)/a**11
```

Giac [A] time = 1.94949, size = 215, normalized size = 1.17

$$-\frac{5\,b\log\left(x^2\right)}{a^{11}}+\frac{5\,b\log\left(\left|bx^2+a\right|\right)}{a^{11}}+\frac{10\,bx^2-a}{2\,a^{11}x^2}-\frac{7129\,b^{10}x^{18}+66429\,ab^9x^{16}+275796\,a^2b^8x^{14}+669984\,a^3b^7x^{12}+1\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -5*b*log(x^2)/a^11 + 5*b*log(abs(b*x^2 + a))/a^11 + 1/2*(10*b*x^2 - a)/(a^1
1*x^2) - 1/504*(7129*b^10*x^18 + 66429*a*b^9*x^16 + 275796*a^2*b^8*x^14 + 6
69984*a^3*b^7*x^12 + 1050336*a^4*b^6*x^10 + 1103256*a^5*b^5*x^8 + 777840*a^
6*b^4*x^6 + 356040*a^7*b^3*x^4 + 96570*a^8*b^2*x^2 + 11990*a^9*b)/((b*x^2 +
 a)^9*a^11)
```

3.207 $\int \frac{1}{x^5 \left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=217

$$\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+bx^2\right)^4}+\frac{3b^2}{2a^7\left(a+bx^2\right)^5}+\frac{5b^2}{6a^6\left(a+bx^2\right)^6}+\frac{3b^2}{7a^5\left(a+bx^2\right)^7}$$

```
[Out] -1/(4*a^10*x^4) + (5*b)/(a^11*x^2) + b^2/(18*a^3*(a + b*x^2)^9) + (3*b^2)/(
16*a^4*(a + b*x^2)^8) + (3*b^2)/(7*a^5*(a + b*x^2)^7) + (5*b^2)/(6*a^6*(a +
 b*x^2)^6) + (3*b^2)/(2*a^7*(a + b*x^2)^5) + (21*b^2)/(8*a^8*(a + b*x^2)^4)
 + (14*b^2)/(3*a^9*(a + b*x^2)^3) + (9*b^2)/(a^10*(a + b*x^2)^2) + (45*b^2)
/(2*a^11*(a + b*x^2)) + (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x^2])/(2*a
^12)
```

Rubi [A] time = 0.220003, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+bx^2\right)^4}+\frac{3b^2}{2a^7\left(a+bx^2\right)^5}+\frac{5b^2}{6a^6\left(a+bx^2\right)^6}+\frac{3b^2}{7a^5\left(a+bx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] -1/(4*a^10*x^4) + (5*b)/(a^11*x^2) + b^2/(18*a^3*(a + b*x^2)^9) + (3*b^2)/(
16*a^4*(a + b*x^2)^8) + (3*b^2)/(7*a^5*(a + b*x^2)^7) + (5*b^2)/(6*a^6*(a +
 b*x^2)^6) + (3*b^2)/(2*a^7*(a + b*x^2)^5) + (21*b^2)/(8*a^8*(a + b*x^2)^4)
 + (14*b^2)/(3*a^9*(a + b*x^2)^3) + (9*b^2)/(a^10*(a + b*x^2)^2) + (45*b^2)
/(2*a^11*(a + b*x^2)) + (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x^2])/(2*a
^12)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^5(a+bx^2)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^{10}x^3} - \frac{10b}{a^{11}x^2} + \frac{55b^2}{a^{12}x} - \frac{b^3}{a^3(a+bx)^{10}} - \frac{3b^3}{a^4(a+bx)^9} - \frac{6b^3}{a^5(a+bx)^8} - \frac{10b^3}{a^6(a+bx)^7}\right.\right.$$

$$= -\frac{1}{4a^{10}x^4} + \frac{5b}{a^{11}x^2} + \frac{b^2}{18a^3(a+bx^2)^9} + \frac{3b^2}{16a^4(a+bx^2)^8} + \frac{3b^2}{7a^5(a+bx^2)^7} + \frac{5b^2}{6a^6(a+bx^2)^6} + \ldots$$

Mathematica [A] time = 0.0930556, size = 151, normalized size = 0.7

$$\frac{\frac{a(882420a^2b^8x^{16}+1905750a^3b^7x^{14}+2604294a^4b^6x^{12}+2318316a^5b^5x^{10}+1326204a^6b^4x^8+456291a^7b^3x^6+78419a^8b^2x^4+2772a^9bx^2-252a^{10}+235620ab^9x^{18}+\ldots}{x^4(a+bx^2)^9}}{1008a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x^2 + 78419*a^8*b^2*x^4 + 456291*a^7*b^3*x^6 +
1326204*a^6*b^4*x^8 + 2318316*a^5*b^5*x^10 + 2604294*a^4*b^6*x^12 + 1905750
*a^3*b^7*x^14 + 882420*a^2*b^8*x^16 + 235620*a*b^9*x^18 + 27720*b^10*x^20))
/(x^4*(a + b*x^2)^9) + 55440*b^2*Log[x] - 27720*b^2*Log[a + b*x^2])/(1008*a
^12)
```

Maple [A] time = 0.021, size = 198, normalized size = 0.9

$$-\frac{1}{4\,a^{10}x^4} + 5\,\frac{b}{a^{11}x^2} + \frac{b^2}{18\,a^3(bx^2+a)^9} + \frac{3\,b^2}{16\,a^4(bx^2+a)^8} + \frac{3\,b^2}{7\,a^5(bx^2+a)^7} + \frac{5\,b^2}{6\,a^6(bx^2+a)^6} + \frac{3\,b^2}{2\,a^7(bx^2+a)^5} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^10,x)
```

```
[Out] -1/4/a^10/x^4+5*b/a^11/x^2+1/18*b^2/a^3/(b*x^2+a)^9+3/16*b^2/a^4/(b*x^2+a)^
8+3/7*b^2/a^5/(b*x^2+a)^7+5/6*b^2/a^6/(b*x^2+a)^6+3/2*b^2/a^7/(b*x^2+a)^5+2
1/8*b^2/a^8/(b*x^2+a)^4+14/3*b^2/a^9/(b*x^2+a)^3+9*b^2/a^10/(b*x^2+a)^2+45/
2*b^2/a^11/(b*x^2+a)+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

Maxima [A] time = 3.03586, size = 332, normalized size = 1.53

$$\frac{27720\,b^{10}x^{20} + 235620\,ab^9x^{18} + 882420\,a^2b^8x^{16} + 1905750\,a^3b^7x^{14} + 2604294\,a^4b^6x^{12} + 2318316\,a^5b^5x^{10} + 1326204\ldots}{1008\left(a^{11}b^9x^{22} + 9\,a^{12}b^8x^{20} + 36\,a^{13}b^7x^{18} + 84\,a^{14}b^6x^{16} + 126\,a^{15}b^5x^{14} + 126\,a^{16}b^4x^{12} + 84\,a\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(27720*b^10*x^20 + 235620*a*b^9*x^18 + 882420*a^2*b^8*x^16 + 1905750
*a^3*b^7*x^14 + 2604294*a^4*b^6*x^12 + 2318316*a^5*b^5*x^10 + 1326204*a^6*b
```

```
^4*x^8 + 456291*a^7*b^3*x^6 + 78419*a^8*b^2*x^4 + 2772*a^9*b*x^2 - 252*a^10
)/(a^11*b^9*x^22 + 9*a^12*b^8*x^20 + 36*a^13*b^7*x^18 + 84*a^14*b^6*x^16 +
126*a^15*b^5*x^14 + 126*a^16*b^4*x^12 + 84*a^17*b^3*x^10 + 36*a^18*b^2*x^8
+ 9*a^19*b*x^6 + a^20*x^4) - 55/2*b^2*log(b*x^2 + a)/a^12 + 55/2*b^2*log(x^
2)/a^12
```

Fricas [B] time = 1.32716, size = 1053, normalized size = 4.85

$$\frac{27720\,ab^{10}x^{20} + 235620\,a^2b^9x^{18} + 882420\,a^3b^8x^{16} + 1905750\,a^4b^7x^{14} + 2604294\,a^5b^6x^{12} + 2318316\,a^6b^5x^{10} + 1326204\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(27720*a*b^10*x^20 + 235620*a^2*b^9*x^18 + 882420*a^3*b^8*x^16 + 190
5750*a^4*b^7*x^14 + 2604294*a^5*b^6*x^12 + 2318316*a^6*b^5*x^10 + 1326204*a
^7*b^4*x^8 + 456291*a^8*b^3*x^6 + 78419*a^9*b^2*x^4 + 2772*a^10*b*x^2 - 252
*a^11 - 27720*(b^11*x^22 + 9*a*b^10*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^1
6 + 126*a^4*b^7*x^14 + 126*a^5*b^6*x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8
+ 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(b*x^2 + a) + 55440*(b^11*x^22 + 9*a*b^10
*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^16 + 126*a^4*b^7*x^14 + 126*a^5*b^6*
x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8 + 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(
x))/(a^12*b^9*x^22 + 9*a^13*b^8*x^20 + 36*a^14*b^7*x^18 + 84*a^15*b^6*x^16
+ 126*a^16*b^5*x^14 + 126*a^17*b^4*x^12 + 84*a^18*b^3*x^10 + 36*a^19*b^2*x^
8 + 9*a^20*b*x^6 + a^21*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.69591, size = 235, normalized size = 1.08

$$\frac{55\,b^2\log\left(x^2\right)}{2\,a^{12}} - \frac{55\,b^2\log\left(\left|bx^2 + a\right|\right)}{2\,a^{12}} - \frac{165\,b^2x^4 - 20\,abx^2 + a^2}{4\,a^{12}x^4} + \frac{78419\,b^{11}x^{18} + 728451\,ab^{10}x^{16} + 3013596\,a^2b^9x^{14} + 7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*b^2*log(x^2)/a^12 - 55/2*b^2*log(abs(b*x^2 + a))/a^12 - 1/4*(165*b^2*x
^4 - 20*a*b*x^2 + a^2)/(a^12*x^4) + 1/1008*(78419*b^11*x^18 + 728451*a*b^10
*x^16 + 3013596*a^2*b^9*x^14 + 7290444*a^3*b^8*x^12 + 11372256*a^4*b^7*x^10
 + 11871216*a^5*b^6*x^8 + 8302224*a^6*b^5*x^6 + 3757680*a^7*b^4*x^4 + 10017
90*a^8*b^3*x^2 + 120550*a^9*b^2)/((b*x^2 + a)^9*a^12)
```

3.208 $\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=226

$$-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{7b^3}{a^9\left(a+bx^2\right)^4}-\frac{7b^3}{2a^8\left(a+bx^2\right)^5}-\frac{5b^3}{3a^7\left(a+bx^2\right)^6}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}$$

```
[Out] -1/(6*a^10*x^6) + (5*b)/(2*a^11*x^4) - (55*b^2)/(2*a^12*x^2) - b^3/(18*a^4*
(a + b*x^2)^9) - b^3/(4*a^5*(a + b*x^2)^8) - (5*b^3)/(7*a^6*(a + b*x^2)^7)
- (5*b^3)/(3*a^7*(a + b*x^2)^6) - (7*b^3)/(2*a^8*(a + b*x^2)^5) - (7*b^3)/(
a^9*(a + b*x^2)^4) - (14*b^3)/(a^10*(a + b*x^2)^3) - (30*b^3)/(a^11*(a + b*
x^2)^2) - (165*b^3)/(2*a^12*(a + b*x^2)) - (220*b^3*Log[x])/a^13 + (110*b^3
*Log[a + b*x^2])/a^13
```

Rubi [A] time = 0.233352, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{7b^3}{a^9\left(a+bx^2\right)^4}-\frac{7b^3}{2a^8\left(a+bx^2\right)^5}-\frac{5b^3}{3a^7\left(a+bx^2\right)^6}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/(6*a^10*x^6) + (5*b)/(2*a^11*x^4) - (55*b^2)/(2*a^12*x^2) - b^3/(18*a^4*
(a + b*x^2)^9) - b^3/(4*a^5*(a + b*x^2)^8) - (5*b^3)/(7*a^6*(a + b*x^2)^7)
- (5*b^3)/(3*a^7*(a + b*x^2)^6) - (7*b^3)/(2*a^8*(a + b*x^2)^5) - (7*b^3)/(
a^9*(a + b*x^2)^4) - (14*b^3)/(a^10*(a + b*x^2)^3) - (30*b^3)/(a^11*(a + b*
x^2)^2) - (165*b^3)/(2*a^12*(a + b*x^2)) - (220*b^3*Log[x])/a^13 + (110*b^3
*Log[a + b*x^2])/a^13
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^{10}x^4} - \frac{10b}{a^{11}x^3} + \frac{55b^2}{a^{12}x^2} - \frac{220b^3}{a^{13}x} + \frac{b^4}{a^4(a+bx)^{10}} + \frac{4b^4}{a^5(a+bx)^9} + \frac{10b^4}{a^6(a+bx)^8} + \frac{2}{a^7(a}\right.\right.$$
$$= -\frac{1}{6a^{10}x^6} + \frac{5b}{2a^{11}x^4} - \frac{55b^2}{2a^{12}x^2} - \frac{b^3}{18a^4\left(a+bx^2\right)^9} - \frac{b^3}{4a^5\left(a+bx^2\right)^8} - \frac{5b^3}{7a^6\left(a+bx^2\right)^7} - \frac{5b^3}{3a^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.114229, size = 162, normalized size = 0.72

$$-\frac{\frac{a\left(882420a^2b^9x^{18}+1905750a^3b^8x^{16}+2604294a^4b^7x^{14}+2318316a^5b^6x^{12}+1326204a^6b^5x^{10}+456291a^7b^4x^8+78419a^8b^3x^6+2772a^9b^2x^4-252a^{10}bx^2+42a^{11}+23562\right.}{x^6\left(a+bx^2\right)^9}}{252a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -((a*(42*a^11 - 252*a^10*b*x^2 + 2772*a^9*b^2*x^4 + 78419*a^8*b^3*x^6 + 456
291*a^7*b^4*x^8 + 1326204*a^6*b^5*x^10 + 2318316*a^5*b^6*x^12 + 2604294*a^4
*b^7*x^14 + 1905750*a^3*b^8*x^16 + 882420*a^2*b^9*x^18 + 235620*a*b^10*x^20
 + 27720*b^11*x^22))/(x^6*(a + b*x^2)^9) + 55440*b^3*Log[x] - 27720*b^3*Log
[a + b*x^2])/(252*a^13)
```

Maple [A] time = 0.019, size = 209, normalized size = 0.9

$$-\frac{1}{6\,a^{10}x^6} + \frac{5\,b}{2\,a^{11}x^4} - \frac{55\,b^2}{2\,a^{12}x^2} - \frac{b^3}{18\,a^4\left(bx^2+a\right)^9} - \frac{b^3}{4\,a^5\left(bx^2+a\right)^8} - \frac{5\,b^3}{7\,a^6\left(bx^2+a\right)^7} - \frac{5\,b^3}{3\,a^7\left(bx^2+a\right)^6} - \frac{7\,b^3}{2\,a^8\left(bx^2+a\right)^5} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^10,x)
```

```
[Out] -1/6/a^10/x^6+5/2*b/a^11/x^4-55/2*b^2/a^12/x^2-1/18*b^3/a^4/(b*x^2+a)^9-1/4
*b^3/a^5/(b*x^2+a)^8-5/7*b^3/a^6/(b*x^2+a)^7-5/3*b^3/a^7/(b*x^2+a)^6-7/2*b^
3/a^8/(b*x^2+a)^5-7*b^3/a^9/(b*x^2+a)^4-14*b^3/a^10/(b*x^2+a)^3-30*b^3/a^11
/(b*x^2+a)^2-165/2*b^3/a^12/(b*x^2+a)-220*b^3*ln(x)/a^13+110*b^3*ln(b*x^2+a
)/a^13
```

Maxima [A] time = 2.58007, size = 347, normalized size = 1.54

$$-\frac{27720\,b^{11}x^{22} + 235620\,ab^{10}x^{20} + 882420\,a^2b^9x^{18} + 1905750\,a^3b^8x^{16} + 2604294\,a^4b^7x^{14} + 2318316\,a^5b^6x^{12} + 1326204\,}{252\left(a^{12}b^9x^{24} + 9\,a^{13}b^8x^{22} + 36\,a^{14}b^7x^{20} + 84\,a^{15}b^6x^{18} + 126\,a^{16}b^5x^{16} + 126\,a^{17}b^4x^{14} +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(27720*b^11*x^22 + 235620*a*b^10*x^20 + 882420*a^2*b^9*x^18 + 190575
0*a^3*b^8*x^16 + 2604294*a^4*b^7*x^14 + 2318316*a^5*b^6*x^12 + 1326204*a^6*
```

```
b^5*x^10 + 456291*a^7*b^4*x^8 + 78419*a^8*b^3*x^6 + 2772*a^9*b^2*x^4 - 252*
a^10*b*x^2 + 42*a^11)/(a^12*b^9*x^24 + 9*a^13*b^8*x^22 + 36*a^14*b^7*x^20 +
 84*a^15*b^6*x^18 + 126*a^16*b^5*x^16 + 126*a^17*b^4*x^14 + 84*a^18*b^3*x^1
2 + 36*a^19*b^2*x^10 + 9*a^20*b*x^8 + a^21*x^6) + 110*b^3*log(b*x^2 + a)/a^
13 - 110*b^3*log(x^2)/a^13
```

Fricas [B] time = 1.39205, size = 1087, normalized size = 4.81

$$-\frac{27720\,ab^{11}x^{22}+235620\,a^2b^{10}x^{20}+882420\,a^3b^9x^{18}+1905750\,a^4b^8x^{16}+2604294\,a^5b^7x^{14}+2318316\,a^6b^6x^{12}+132}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(27720*a*b^11*x^22 + 235620*a^2*b^10*x^20 + 882420*a^3*b^9*x^18 + 19
05750*a^4*b^8*x^16 + 2604294*a^5*b^7*x^14 + 2318316*a^6*b^6*x^12 + 1326204*
a^7*b^5*x^10 + 456291*a^8*b^4*x^8 + 78419*a^9*b^3*x^6 + 2772*a^10*b^2*x^4 -
 252*a^11*b*x^2 + 42*a^12 - 27720*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*
x^20 + 84*a^3*b^9*x^18 + 126*a^4*b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x
^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4*x^8 + a^9*b^3*x^6)*log(b*x^2 + a) + 55440
*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*x^20 + 84*a^3*b^9*x^18 + 126*a^4*
b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4
*x^8 + a^9*b^3*x^6)*log(x))/(a^13*b^9*x^24 + 9*a^14*b^8*x^22 + 36*a^15*b^7*
x^20 + 84*a^16*b^6*x^18 + 126*a^17*b^5*x^16 + 126*a^18*b^4*x^14 + 84*a^19*b
^3*x^12 + 36*a^20*b^2*x^10 + 9*a^21*b*x^8 + a^22*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.02488, size = 252, normalized size = 1.12

$$-\frac{110\,b^3\log\left(x^2\right)}{a^{13}}+\frac{110\,b^3\log\left(\left|bx^2+a\right|\right)}{a^{13}}+\frac{1210\,b^3x^6-165\,ab^2x^4+15\,a^2bx^2-a^3}{6\,a^{13}x^6}-\frac{78419\,b^{12}x^{18}+726561\,ab^{11}x^{16}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*b^3*log(x^2)/a^13 + 110*b^3*log(abs(b*x^2 + a))/a^13 + 1/6*(1210*b^3*x
^6 - 165*a*b^2*x^4 + 15*a^2*b*x^2 - a^3)/(a^13*x^6) - 1/252*(78419*b^12*x^1
8 + 726561*a*b^11*x^16 + 2996964*a^2*b^10*x^14 + 7225764*a^3*b^9*x^12 + 112
26726*a^4*b^8*x^10 + 11663316*a^5*b^7*x^8 + 8108184*a^6*b^6*x^6 + 3641256*a
^7*b^5*x^4 + 960210*a^8*b^4*x^2 + 113620*a^9*b^3)/((b*x^2 + a)^9*a^13)
```

3.209 $\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=231

$$\frac{7436429a^2x}{65536b^{12}} - \frac{7436429a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{25/2}} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

```
[Out] (7436429*a^2*x)/(65536*b^12) - (7436429*a*x^3)/(196608*b^11) + (7436429*x^5
)/(327680*b^10) - x^23/(18*b*(a + b*x^2)^9) - (23*x^21)/(288*b^2*(a + b*x^2
)^8) - (23*x^19)/(192*b^3*(a + b*x^2)^7) - (437*x^17)/(2304*b^4*(a + b*x^2)
^6) - (7429*x^15)/(23040*b^5*(a + b*x^2)^5) - (7429*x^13)/(12288*b^6*(a + b
*x^2)^4) - (96577*x^11)/(73728*b^7*(a + b*x^2)^3) - (1062347*x^9)/(294912*b
^8*(a + b*x^2)^2) - (1062347*x^7)/(65536*b^9*(a + b*x^2)) - (7436429*a^(5/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(25/2))
```

Rubi [A] time = 0.167187, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{7436429a^2x}{65536b^{12}} - \frac{7436429a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{25/2}} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^24/(a + b*x^2)^10,x]
```

```
[Out] (7436429*a^2*x)/(65536*b^12) - (7436429*a*x^3)/(196608*b^11) + (7436429*x^5
)/(327680*b^10) - x^23/(18*b*(a + b*x^2)^9) - (23*x^21)/(288*b^2*(a + b*x^2
)^8) - (23*x^19)/(192*b^3*(a + b*x^2)^7) - (437*x^17)/(2304*b^4*(a + b*x^2)
^6) - (7429*x^15)/(23040*b^5*(a + b*x^2)^5) - (7429*x^13)/(12288*b^6*(a + b
*x^2)^4) - (96577*x^11)/(73728*b^7*(a + b*x^2)^3) - (1062347*x^9)/(294912*b
^8*(a + b*x^2)^2) - (1062347*x^7)/(65536*b^9*(a + b*x^2)) - (7436429*a^(5/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(25/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{24}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{23}}{18b\left(a+bx^2\right)^9} + \frac{23\int \frac{x^{22}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} + \frac{161\int \frac{x^{20}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} + \frac{437\int \frac{x^{18}}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} + \frac{7429\int \frac{x^{16}}{\left(a+bx^2\right)^6}\,d}{2304b^4}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^{15}}{23040b^5\left(a+bx^2\right.}$$

$$= \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} + \frac{7436429x^5}{327680b^{10}} - \frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right.}$$

$$= \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} + \frac{7436429x^5}{327680b^{10}} - \frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0887179, size = 166, normalized size = 0.72

$$\frac{\frac{\sqrt{b}x\left(94961664a^2b^9x^{18}+1469632311a^3b^8x^{16}+7323998514a^4b^7x^{14}+19562592546a^5b^6x^{12}+32314857354a^6b^5x^{10}+34810986496a^7b^4x^8+24648575094a^8b^3x^6+\right.}{\left(a+bx^2\right)^9}}{2949120b^{25/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(334639305*a^11 + 2900207310*a^10*b*x^2 + 11110024926*a^9*b^2*x
^4 + 24648575094*a^8*b^3*x^6 + 34810986496*a^7*b^4*x^8 + 32314857354*a^6*b^
5*x^10 + 19562592546*a^5*b^6*x^12 + 7323998514*a^4*b^7*x^14 + 1469632311*a^
3*b^8*x^16 + 94961664*a^2*b^9*x^18 - 4521984*a*b^10*x^20 + 589824*b^11*x^22
))/(a + b*x^2)^9 - 334639305*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2949120*
b^(25/2))
```

Maple [A] time = 0.018, size = 228, normalized size = 1.

$$\frac{x^5}{5\,b^{10}}-\frac{10\,a x^3}{3\,b^{11}}+55\,\frac{a^2 x}{b^{12}}+\frac{3831949\,a^{11} x}{65536\,b^{12}\left(b x^2+a\right)^9}+\frac{48340777\,a^{10} x^3}{98304\,b^{11}\left(b x^2+a\right)^9}+\frac{297702839\,a^9 x^5}{163840\,b^{10}\left(b x^2+a\right)^9}+\frac{631790371\,a^8 x^7}{163840\,b^9\left(b x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(b*x^2+a)^10,x)
```

```
[Out] 1/5*x^5/b^10-10/3*a*x^3/b^11+55*a^2*x/b^12+3831949/65536/b^12*a^11/(b*x^2+a
)^9*x+48340777/98304/b^11*a^10/(b*x^2+a)^9*x^3+297702839/163840/b^10*a^9/(b
*x^2+a)^9*x^5+631790371/163840/b^9*a^8/(b*x^2+a)^9*x^7+463199/90/b^8*a^7/(b
*x^2+a)^9*x^9+725918941/163840/b^7*a^6/(b*x^2+a)^9*x^11+394553929/163840/b^
6*a^5/(b*x^2+a)^9*x^13+74539223/98304/b^5*a^4/(b*x^2+a)^9*x^15+6981491/6553
6/b^4*a^3/(b*x^2+a)^9*x^17-7436429/65536/b^12*a^3/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2238, size = 1877, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/5898240*(1179648*b^11*x^23 - 9043968*a*b^10*x^21 + 189923328*a^2*b^9*x^1
9 + 2939264622*a^3*b^8*x^17 + 14647997028*a^4*b^7*x^15 + 39125185092*a^5*b^
6*x^13 + 64629714708*a^6*b^5*x^11 + 69621972992*a^7*b^4*x^9 + 49297150188*a
^8*b^3*x^7 + 22220049852*a^9*b^2*x^5 + 5800414620*a^10*b*x^3 + 669278610*a^
11*x + 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84*a^5*
b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a^9*b^2
*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^
12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x
```

```
^4 + 9*a^8*b^13*x^2 + a^9*b^12), 1/2949120*(589824*b^11*x^23 - 4521984*a*b^
10*x^21 + 94961664*a^2*b^9*x^19 + 1469632311*a^3*b^8*x^17 + 7323998514*a^4*
b^7*x^15 + 19562592546*a^5*b^6*x^13 + 32314857354*a^6*b^5*x^11 + 3481098649
6*a^7*b^4*x^9 + 24648575094*a^8*b^3*x^7 + 11110024926*a^9*b^2*x^5 + 2900207
310*a^10*b*x^3 + 334639305*a^11*x - 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^1
6 + 36*a^4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8
+ 84*a^8*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(a/b)*arctan(b
*x*sqrt(a/b)/a))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^1
8*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^
14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12)]
```

Sympy [A] time = 8.65098, size = 314, normalized size = 1.36

$$\frac{55a^2x}{b^{12}} - \frac{10ax^3}{3b^{11}} + \frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x - \frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072} - \frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x + \frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072} + \frac{172437705a^{11}x + 14\ldots}{2949120a^9b^{12} + 265420\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*x/b**12 - 10*a*x**3/(3*b**11) + 7436429*sqrt(-a**5/b**25)*log(x - b
**12*sqrt(-a**5/b**25)/a**2)/131072 - 7436429*sqrt(-a**5/b**25)*log(x + b**
12*sqrt(-a**5/b**25)/a**2)/131072 + (172437705*a**11*x + 1450223310*a**10*b
*x**3 + 5358651102*a**9*b**2*x**5 + 11372226678*a**8*b**3*x**7 + 1517810483
2*a**7*b**4*x**9 + 13066540938*a**6*b**5*x**11 + 7101970722*a**5*b**6*x**13
 + 2236176690*a**4*b**7*x**15 + 314167095*a**3*b**8*x**17)/(2949120*a**9*b*
*12 + 26542080*a**8*b**13*x**2 + 106168320*a**7*b**14*x**4 + 247726080*a**6
*b**15*x**6 + 371589120*a**5*b**16*x**8 + 371589120*a**4*b**17*x**10 + 2477
26080*a**3*b**18*x**12 + 106168320*a**2*b**19*x**14 + 26542080*a*b**20*x**1
6 + 2949120*b**21*x**18) + x**5/(5*b**10)
```

Giac [A] time = 2.65837, size = 219, normalized size = 0.95

$$-\frac{7436429\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{12}} + \frac{314167095\,a^3b^8x^{17} + 2236176690\,a^4b^7x^{15} + 7101970722\,a^5b^6x^{13} + 13066540938\,a^6b^5x\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^12) + 1/2949120*(3141
67095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^5*b^6*x^13 + 13
066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 11372226678*a^8*b^3*x^7
+ 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705*a^11*x)/((b*x^
2 + a)^9*b^12) + 1/15*(3*b^40*x^5 - 50*a*b^39*x^3 + 825*a^2*b^38*x)/b^50
```

3.210 $\int \frac{x^{22}}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=218

$$\frac{1616615a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{23/2}} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5} - \frac{4199x^{1}}{12288b^6\left(a+\right.}$$

```
[Out] (-1616615*a*x)/(65536*b^11) + (1616615*x^3)/(196608*b^10) - x^21/(18*b*(a +
 b*x^2)^9) - (7*x^19)/(96*b^2*(a + b*x^2)^8) - (19*x^17)/(192*b^3*(a + b*x^
2)^7) - (323*x^15)/(2304*b^4*(a + b*x^2)^6) - (323*x^13)/(1536*b^5*(a + b*x
^2)^5) - (4199*x^11)/(12288*b^6*(a + b*x^2)^4) - (46189*x^9)/(73728*b^7*(a
+ b*x^2)^3) - (46189*x^7)/(32768*b^8*(a + b*x^2)^2) - (323323*x^5)/(65536*b
^9*(a + b*x^2)) + (1616615*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(2
3/2))
```

Rubi [A] time = 0.140587, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{1616615a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{23/2}} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5} - \frac{4199x^{1}}{12288b^6\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^22/(a + b*x^2)^10,x]
```


```
[Out] (-1616615*a*x)/(65536*b^11) + (1616615*x^3)/(196608*b^10) - x^21/(18*b*(a +
 b*x^2)^9) - (7*x^19)/(96*b^2*(a + b*x^2)^8) - (19*x^17)/(192*b^3*(a + b*x^
2)^7) - (323*x^15)/(2304*b^4*(a + b*x^2)^6) - (323*x^13)/(1536*b^5*(a + b*x
^2)^5) - (4199*x^11)/(12288*b^6*(a + b*x^2)^4) - (46189*x^9)/(73728*b^7*(a
+ b*x^2)^3) - (46189*x^7)/(32768*b^8*(a + b*x^2)^2) - (323323*x^5)/(65536*b
^9*(a + b*x^2)) + (1616615*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(2
3/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{21}}{18b\left(a+bx^2\right)^9} + \frac{7\int \frac{x^{20}}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} + \frac{133\int \frac{x^{18}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} + \frac{323\int \frac{x^{16}}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} + \frac{1615\int \frac{x^{14}}{\left(a+bx^2\right)^6}\,dx}{768b^4}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{1616615ax}{65536b^{11}} + \frac{1616615x^3}{196608b^{10}} - \frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{1}}{2304b^4\left(a+\right.}$$

$$= -\frac{1616615ax}{65536b^{11}} + \frac{1616615x^3}{196608b^{10}} - \frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{1}}{2304b^4\left(a+\right.}$$

Mathematica [A] time = 0.0764073, size = 155, normalized size = 0.71

$$\frac{\frac{\sqrt{b}x\left(-63897057a^2b^8x^{16}-318434718a^3b^7x^{14}-850547502a^4b^6x^{12}-1404993798a^5b^5x^{10}-1513521152a^6b^4x^8-1071677178a^7b^3x^6-483044562a^8b^2x^4-1260959\right.}{\left(a+bx^2\right)^9}}{589824b^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^22/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(-14549535*a^10 - 126095970*a^9*b*x^2 - 483044562*a^8*b^2*x^4 -
 1071677178*a^7*b^3*x^6 - 1513521152*a^6*b^4*x^8 - 1404993798*a^5*b^5*x^10
- 850547502*a^4*b^6*x^12 - 318434718*a^3*b^7*x^14 - 63897057*a^2*b^8*x^16 -
 4128768*a*b^9*x^18 + 196608*b^10*x^20))/(a + b*x^2)^9 + 14549535*a^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*b^(23/2))
```

Maple [A] time = 0.017, size = 217, normalized size = 1.

$$\frac{x^3}{3\,b^{10}}-10\,\frac{ax}{b^{11}}-\frac{961255\,a^{10}x}{65536\,b^{11}\left(bx^2+a\right)^9}-\frac{12201403\,a^9x^3}{98304\,b^{10}\left(bx^2+a\right)^9}-\frac{15137633\,a^8x^5}{32768\,b^9\left(bx^2+a\right)^9}-\frac{32405717\,a^7x^7}{32768\,b^8\left(bx^2+a\right)^9}-\frac{24013\,a^6}{18\,b^7\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(b*x^2+a)^10,x)
```

```
[Out] 1/3*x^3/b^10-10*a*x/b^11-961255/65536/b^11*a^10/(b*x^2+a)^9*x-12201403/9830
4/b^10*a^9/(b*x^2+a)^9*x^3-15137633/32768/b^9*a^8/(b*x^2+a)^9*x^5-32405717/
32768/b^8*a^7/(b*x^2+a)^9*x^7-24013/18/b^7*a^6/(b*x^2+a)^9*x^9-38143787/327
68/b^6*a^5/(b*x^2+a)^9*x^11-21103775/32768/b^5*a^4/(b*x^2+a)^9*x^13-2043552
5/98304/b^4*a^3/(b*x^2+a)^9*x^15-1987865/65536/b^3*a^2/(b*x^2+a)^9*x^17+161
6615/65536/b^11*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22736, size = 1760, normalized size = 8.07

$$\left[\frac{393216\,b^{10}x^{21}-8257536\,ab^9x^{19}-127794114\,a^2b^8x^{17}-636869436\,a^3b^7x^{15}-1701095004\,a^4b^6x^{13}-2809987596\,a^5b^5x^{11}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(393216*b^10*x^21 - 8257536*a*b^9*x^19 - 127794114*a^2*b^8*x^17
- 636869436*a^3*b^7*x^15 - 1701095004*a^4*b^6*x^13 - 2809987596*a^5*b^5*x^1
1 - 3027042304*a^6*b^4*x^9 - 2143354356*a^7*b^3*x^7 - 966089124*a^8*b^2*x^5
 - 252191940*a^9*b*x^3 - 29099070*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8
*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*
x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(-a/b)*log(
(b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^20*x^18 + 9*a*b^19*x^16 + 3
6*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 +
```

```
 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11), 1/589824*(
196608*b^10*x^21 - 4128768*a*b^9*x^19 - 63897057*a^2*b^8*x^17 - 318434718*a
^3*b^7*x^15 - 850547502*a^4*b^6*x^13 - 1404993798*a^5*b^5*x^11 - 1513521152
*a^6*b^4*x^9 - 1071677178*a^7*b^3*x^7 - 483044562*a^8*b^2*x^5 - 126095970*a
^9*b*x^3 - 14549535*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3
*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b
^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(a/b)*arctan(b*x*sqrt(a/b
)/a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 12
6*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*
a^8*b^12*x^2 + a^9*b^11)]
```

Sympy [A] time = 8.3777, size = 298, normalized size = 1.37

$$-\frac{10ax}{b^{11}} - \frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\log\left(x - \frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072} + \frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\log\left(x + \frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072} - \frac{8651295a^{10}x + 73208418a^9}{589824a^9b^{11} + 5308416a^8b^{12}x^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**22/(b*x**2+a)**10,x)
```

```
[Out] -10*a*x/b**11 - 1616615*sqrt(-a**3/b**23)*log(x - b**11*sqrt(-a**3/b**23)/a
)/131072 + 1616615*sqrt(-a**3/b**23)*log(x + b**11*sqrt(-a**3/b**23)/a)/131
072 - (8651295*a**10*x + 73208418*a**9*b*x**3 + 272477394*a**8*b**2*x**5 +
583302906*a**7*b**3*x**7 + 786857984*a**6*b**4*x**9 + 686588166*a**5*b**5*x
**11 + 379867950*a**4*b**6*x**13 + 122613150*a**3*b**7*x**15 + 17890785*a**
2*b**8*x**17)/(589824*a**9*b**11 + 5308416*a**8*b**12*x**2 + 21233664*a**7*
b**13*x**4 + 49545216*a**6*b**14*x**6 + 74317824*a**5*b**15*x**8 + 74317824
*a**4*b**16*x**10 + 49545216*a**3*b**17*x**12 + 21233664*a**2*b**18*x**14 +
 5308416*a*b**19*x**16 + 589824*b**20*x**18) + x**3/(3*b**10)
```

Giac [A] time = 2.71424, size = 203, normalized size = 0.93

$$\frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{11}} - \frac{17890785\,a^2b^8x^{17} + 122613150\,a^3b^7x^{15} + 379867950\,a^4b^6x^{13} + 686588166\,a^5b^5x^{11} + 786}{589824\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) - 1/589824*(178907
85*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b^6*x^13 + 6865881
66*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^3*x^7 + 272477394
*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/((b*x^2 + a)^9*b^11) +
1/3*(b^20*x^3 - 30*a*b^19*x)/b^30
```

3.211 $\int \frac{x^{20}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=207

$$-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+bx^2\right)^5}-\frac{46189x^9}{258048b^6\left(a+bx^2\right)^4}-\frac{4618}{172032b^7}$$

```
[Out] (230945*x)/(65536*b^10) - x^19/(18*b*(a + b*x^2)^9) - (19*x^17)/(288*b^2*(a
 + b*x^2)^8) - (323*x^15)/(4032*b^3*(a + b*x^2)^7) - (1615*x^13)/(16128*b^4
*(a + b*x^2)^6) - (4199*x^11)/(32256*b^5*(a + b*x^2)^5) - (46189*x^9)/(2580
48*b^6*(a + b*x^2)^4) - (46189*x^7)/(172032*b^7*(a + b*x^2)^3) - (46189*x^5
)/(98304*b^8*(a + b*x^2)^2) - (230945*x^3)/(196608*b^9*(a + b*x^2)) - (2309
45*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(21/2))
```

Rubi [A] time = 0.124125, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+bx^2\right)^5}-\frac{46189x^9}{258048b^6\left(a+bx^2\right)^4}-\frac{4618}{172032b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^20/(a + b*x^2)^10,x]
```


```
[Out] (230945*x)/(65536*b^10) - x^19/(18*b*(a + b*x^2)^9) - (19*x^17)/(288*b^2*(a
 + b*x^2)^8) - (323*x^15)/(4032*b^3*(a + b*x^2)^7) - (1615*x^13)/(16128*b^4
*(a + b*x^2)^6) - (4199*x^11)/(32256*b^5*(a + b*x^2)^5) - (46189*x^9)/(2580
48*b^6*(a + b*x^2)^4) - (46189*x^7)/(172032*b^7*(a + b*x^2)^3) - (46189*x^5
)/(98304*b^8*(a + b*x^2)^2) - (230945*x^3)/(196608*b^9*(a + b*x^2)) - (2309
45*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(21/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{20}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{19}}{18b\left(a+bx^2\right)^9}+\frac{19\int \frac{x^{18}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}+\frac{323\int \frac{x^{16}}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}+\frac{1615\int \frac{x^{14}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}+\frac{20995\int \frac{x^{12}}{\left(a+bx^2\right.}}{16128b^4}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+b\right.}$$

$$= \frac{230945x}{65536b^{10}}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{}{322}$$

$$= \frac{230945x}{65536b^{10}}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{}{322}$$

Mathematica [A] time = 0.071281, size = 144, normalized size = 0.7

$$\frac{\frac{\sqrt{b}x\left(318434718a^2b^7x^{14}+850547502a^3b^6x^{12}+1404993798a^4b^5x^{10}+1513521152a^5b^4x^8+1071677178a^6b^3x^6+483044562a^7b^2x^4+126095970a^8bx^2+14549535a\right.}{\left(a+bx^2\right)^9}}{4128768b^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(14549535*a^9 + 126095970*a^8*b*x^2 + 483044562*a^7*b^2*x^4 + 1
071677178*a^6*b^3*x^6 + 1513521152*a^5*b^4*x^8 + 1404993798*a^4*b^5*x^10 +
850547502*a^3*b^6*x^12 + 318434718*a^2*b^7*x^14 + 63897057*a*b^8*x^16 + 412
8768*b^9*x^18))/(a + b*x^2)^9 - 14549535*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
```

```
])/(4128768*b^(21/2))
```

Maple [A] time = 0.018, size = 203, normalized size = 1.

$$\frac{x}{b^{10}}+\frac{165409\,a^{9}x}{65536\,b^{10}\left(bx^{2}+a\right)^{9}}+\frac{2117549\,a^{8}x^{3}}{98304\,b^{9}\left(bx^{2}+a\right)^{9}}+\frac{2654039\,a^{7}x^{5}}{32768\,b^{8}\left(bx^{2}+a\right)^{9}}+\frac{40270037\,a^{6}x^{7}}{229376\,b^{7}\left(bx^{2}+a\right)^{9}}+\frac{30313\,a^{5}x^{9}}{126\,b^{6}\left(bx^{2}+a\right)^{9}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(b*x^2+a)^10,x)
```

```
[Out] x/b^10+165409/65536/b^10*a^9/(b*x^2+a)^9*x+2117549/98304/b^9*a^8/(b*x^2+a)^
9*x^3+2654039/32768/b^8*a^7/(b*x^2+a)^9*x^5+40270037/229376/b^7*a^6/(b*x^2+
a)^9*x^7+30313/126/b^6*a^5/(b*x^2+a)^9*x^9+49153835/229376/b^5*a^4/(b*x^2+a
)^9*x^11+3997865/32768/b^4*a^3/(b*x^2+a)^9*x^13+4042835/98304/b^3*a^2/(b*x^
2+a)^9*x^15+424415/65536/b^2*a/(b*x^2+a)^9*x^17-230945/65536/b^10*a/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28223, size = 1683, normalized size = 8.13

$$\left[\frac{8257536\,b^{9}x^{19}+127794114\,ab^{8}x^{17}+636869436\,a^{2}b^{7}x^{15}+1701095004\,a^{3}b^{6}x^{13}+2809987596\,a^{4}b^{5}x^{11}+3027042304\,a}{8257536}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(8257536*b^9*x^19 + 127794114*a*b^8*x^17 + 636869436*a^2*b^7*x^1
5 + 1701095004*a^3*b^6*x^13 + 2809987596*a^4*b^5*x^11 + 3027042304*a^5*b^4*
x^9 + 2143354356*a^6*b^3*x^7 + 966089124*a^7*b^2*x^5 + 252191940*a^8*b*x^3
+ 29099070*a^9*x + 14549535*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84
*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^
7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16
*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^1
2*x^4 + 9*a^8*b^11*x^2 + a^9*b^10), 1/4128768*(4128768*b^9*x^19 + 63897057*
a*b^8*x^17 + 318434718*a^2*b^7*x^15 + 850547502*a^3*b^6*x^13 + 1404993798*a
^4*b^5*x^11 + 1513521152*a^5*b^4*x^9 + 1071677178*a^6*b^3*x^7 + 483044562*a
^7*b^2*x^5 + 126095970*a^8*b*x^3 + 14549535*a^9*x - 14549535*(b^9*x^18 + 9*
```

```
a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5
*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*
a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36
*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10)]
```

Sympy [A] time = 8.2095, size = 274, normalized size = 1.32

$$\frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(-b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}-\frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}+\frac{10420767a^9x+88937058a^8\cdots}{4128768a^9b^{10}+37158912a^8b^{11}x^2+1486\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-a/b**21)*log(-b**10*sqrt(-a/b**21) + x)/131072 - 230945*sqrt(-
a/b**21)*log(b**10*sqrt(-a/b**21) + x)/131072 + (10420767*a**9*x + 88937058
*a**8*b*x**3 + 334408914*a**7*b**2*x**5 + 724860666*a**6*b**3*x**7 + 993296
384*a**5*b**4*x**9 + 884769030*a**4*b**5*x**11 + 503730990*a**3*b**6*x**13
+ 169799070*a**2*b**7*x**15 + 26738145*a*b**8*x**17)/(4128768*a**9*b**10 +
37158912*a**8*b**11*x**2 + 148635648*a**7*b**12*x**4 + 346816512*a**6*b**13
*x**6 + 520224768*a**5*b**14*x**8 + 520224768*a**4*b**15*x**10 + 346816512*
a**3*b**16*x**12 + 148635648*a**2*b**17*x**14 + 37158912*a*b**18*x**16 + 41
28768*b**19*x**18) + x/b**10
```

Giac [A] time = 2.53835, size = 177, normalized size = 0.86

$$-\frac{230945\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{10}}+\frac{x}{b^{10}}+\frac{26738145\,ab^8x^{17}+169799070\,a^2b^7x^{15}+503730990\,a^3b^6x^{13}+884769030\,a^4b^5x^{11}\cdots}{4128\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10 + 1/4128768
*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6*x^13 + 8
84769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*x^7 + 334
408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/((b*x^2 + a)^9*b^
10)
```

3.212 $\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=197

$$-\frac{17x^{15}}{288b^2(a+bx^2)^8}-\frac{85x^{13}}{1344b^3(a+bx^2)^7}-\frac{1105x^{11}}{16128b^4(a+bx^2)^6}-\frac{2431x^9}{32256b^5(a+bx^2)^5}-\frac{2431x^7}{28672b^6(a+bx^2)^4}-\frac{2431x}{24576b^7(a}$$

```
[Out] -x^17/(18*b*(a + b*x^2)^9) - (17*x^15)/(288*b^2*(a + b*x^2)^8) - (85*x^13)/
(1344*b^3*(a + b*x^2)^7) - (1105*x^11)/(16128*b^4*(a + b*x^2)^6) - (2431*x^
9)/(32256*b^5*(a + b*x^2)^5) - (2431*x^7)/(28672*b^6*(a + b*x^2)^4) - (2431
*x^5)/(24576*b^7*(a + b*x^2)^3) - (12155*x^3)/(98304*b^8*(a + b*x^2)^2) - (
12155*x)/(65536*b^9*(a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(655
36*Sqrt[a]*b^(19/2))
```

Rubi [A] time = 0.113243, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$-\frac{17x^{15}}{288b^2(a+bx^2)^8}-\frac{85x^{13}}{1344b^3(a+bx^2)^7}-\frac{1105x^{11}}{16128b^4(a+bx^2)^6}-\frac{2431x^9}{32256b^5(a+bx^2)^5}-\frac{2431x^7}{28672b^6(a+bx^2)^4}-\frac{2431x}{24576b^7(a}$$

Antiderivative was successfully verified.

```
[In] Int[x^18/(a + b*x^2)^10,x]
```


```
[Out] -x^17/(18*b*(a + b*x^2)^9) - (17*x^15)/(288*b^2*(a + b*x^2)^8) - (85*x^13)/
(1344*b^3*(a + b*x^2)^7) - (1105*x^11)/(16128*b^4*(a + b*x^2)^6) - (2431*x^
9)/(32256*b^5*(a + b*x^2)^5) - (2431*x^7)/(28672*b^6*(a + b*x^2)^4) - (2431
*x^5)/(24576*b^7*(a + b*x^2)^3) - (12155*x^3)/(98304*b^8*(a + b*x^2)^2) - (
12155*x)/(65536*b^9*(a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(655
36*Sqrt[a]*b^(19/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{18}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{17}}{18b\left(a+bx^2\right)^9} + \frac{17\int \frac{x^{16}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} + \frac{85\int \frac{x^{14}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} + \frac{1105\int \frac{x^{12}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} + \frac{12155\int \frac{x^{10}}{\left(a+bx^2\right)}}{16128b^4}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{2431x^9}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{2431x^9}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{2431x^9}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{2431x^9}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{2431x^9}{32256b^5\left(a+b\right.}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{2431x^9}{32256b^5\left(a+b\right.}$$

Mathematica [A] time = 0.073698, size = 134, normalized size = 0.68

$$\frac{\frac{765765\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{b}x\left(44765658a^2b^6x^{12}+73947042a^3b^5x^{10}+79659008a^4b^4x^8+56404062a^5b^3x^6+25423398a^6b^2x^4+6636630a^7bx^2+765765a^8+167597\right.}{\left(a+bx^2\right)^9}}{4128768b^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^18/(a + b*x^2)^10,x]
```

```
[Out] (-((Sqrt[b]*x*(765765*a^8 + 6636630*a^7*b*x^2 + 25423398*a^6*b^2*x^4 + 5640
4062*a^5*b^3*x^6 + 79659008*a^4*b^4*x^8 + 73947042*a^3*b^5*x^10 + 44765658*
a^2*b^6*x^12 + 16759722*a*b^7*x^14 + 3363003*b^8*x^16))/(a + b*x^2)^9) + (7
65765*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(4128768*b^(19/2))
```

Maple [A] time = 0.014, size = 124, normalized size = 0.6

$$\frac{1}{\left(bx^{2}+a\right)^{9}}\left(-\frac{12155\,a^{8}x}{65536\,b^{9}}-\frac{158015\,a^{7}x^{3}}{98304\,b^{8}}-\frac{201773\,a^{6}x^{5}}{32768\,b^{7}}-\frac{3133559\,a^{5}x^{7}}{229376\,b^{6}}-\frac{2431\,a^{4}x^{9}}{126\,b^{5}}-\frac{4108169\,a^{3}x^{11}}{229376\,b^{4}}-\frac{355283\,a^{2}x^{13}}{32768\,b^{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(b*x^2+a)^10,x)
```

```
[Out] (-12155/65536*a^8/b^9*x-158015/98304*a^7/b^8*x^3-201773/32768*a^6/b^7*x^5-3
133559/229376*a^5/b^6*x^7-2431/126*a^4/b^5*x^9-4108169/229376*a^3/b^4*x^11-
355283/32768*a^2/b^3*x^13-399041/98304/b^2*a*x^15-53381/65536/b*x^17)/(b*x^
2+a)^9+12155/65536/b^9/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53849, size = 1615, normalized size = 8.2

$$\left[-\frac{6726006\,ab^{9}x^{17}+33519444\,a^{2}b^{8}x^{15}+89531316\,a^{3}b^{7}x^{13}+147894084\,a^{4}b^{6}x^{11}+159318016\,a^{5}b^{5}x^{9}+112808124\,a^{6}b^{4}x}{8257536\left(ab^{19}x^{18}+9\,a^{2}b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(6726006*a*b^9*x^17 + 33519444*a^2*b^8*x^15 + 89531316*a^3*b^7*
x^13 + 147894084*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 112808124*a^6*b^4*x
^7 + 50846796*a^7*b^3*x^5 + 13273260*a^8*b^2*x^3 + 1531530*a^9*b*x + 765765
*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5
*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a
^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^19*x^18
+ 9*a^2*b^18*x^16 + 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10
 + 126*a^6*b^14*x^8 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 +
a^10*b^10), -1/4128768*(3363003*a*b^9*x^17 + 16759722*a^2*b^8*x^15 + 447656
58*a^3*b^7*x^13 + 73947042*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 56404062*a
^6*b^4*x^7 + 25423398*a^7*b^3*x^5 + 6636630*a^8*b^2*x^3 + 765765*a^9*b*x -
765765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a
^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x
^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^19*x^18 + 9*a^2*b^18*x^16 +
 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10 + 126*a^6*b^14*x^8
 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 + a^10*b^10)]
```

Sympy [A] time = 7.77787, size = 275, normalized size = 1.4

$$-\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(-ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}-\frac{765765a^8x+6636\ldots}{4128768a^9b^9+37158912a^8b^{10}x^2+148\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**18/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a*b**19))*log(-a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + 1215
5*sqrt(-1/(a*b**19))*log(a*b**9*sqrt(-1/(a*b**19)) + x)/131072 - (765765*a*
*8*x + 6636630*a**7*b*x**3 + 25423398*a**6*b**2*x**5 + 56404062*a**5*b**3*x
**7 + 79659008*a**4*b**4*x**9 + 73947042*a**3*b**5*x**11 + 44765658*a**2*b*
*6*x**13 + 16759722*a*b**7*x**15 + 3363003*b**8*x**17)/(4128768*a**9*b**9 +
 37158912*a**8*b**10*x**2 + 148635648*a**7*b**11*x**4 + 346816512*a**6*b**1
2*x**6 + 520224768*a**5*b**13*x**8 + 520224768*a**4*b**14*x**10 + 346816512
*a**3*b**15*x**12 + 148635648*a**2*b**16*x**14 + 37158912*a*b**17*x**16 + 4
128768*b**18*x**18)
```

Giac [A] time = 2.5509, size = 165, normalized size = 0.84

$$\frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}b^9}-\frac{3363003\,b^8x^{17}+16759722\,ab^7x^{15}+44765658\,a^2b^6x^{13}+73947042\,a^3b^5x^{11}+79659008\,a^4b^4x\ldots}{4128768\left(bx^2+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9) - 1/4128768*(3363003*b^8*
x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13 + 73947042*a^3*b^5*x^11
+ 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 25423398*a^6*b^2*x^5 + 6636
630*a^7*b*x^3 + 765765*a^8*x)/((b*x^2 + a)^9*b^9)
```

3.213 $\int \frac{x^{16}}{(a+bx^2)^{10}} dx$

Optimal. Leaf size=198

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}-\frac{5x^{13}}{96b^2(a+bx^2)^8}-\frac{65x^{11}}{1344b^3(a+bx^2)^7}-\frac{715x^9}{16128b^4(a+bx^2)^6}-\frac{143x^7}{3584b^5(a+bx^2)^5}-\frac{143x^5}{4096b^6(a+bx^2)^4}$$

```
[Out] -x^15/(18*b*(a + b*x^2)^9) - (5*x^13)/(96*b^2*(a + b*x^2)^8) - (65*x^11)/(1
344*b^3*(a + b*x^2)^7) - (715*x^9)/(16128*b^4*(a + b*x^2)^6) - (143*x^7)/(3
584*b^5*(a + b*x^2)^5) - (143*x^5)/(4096*b^6*(a + b*x^2)^4) - (715*x^3)/(24
576*b^7*(a + b*x^2)^3) - (715*x)/(32768*b^8*(a + b*x^2)^2) + (715*x)/(65536
*a*b^8*(a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^(1
7/2))
```

Rubi [A] time = 0.122302, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}-\frac{5x^{13}}{96b^2(a+bx^2)^8}-\frac{65x^{11}}{1344b^3(a+bx^2)^7}-\frac{715x^9}{16128b^4(a+bx^2)^6}-\frac{143x^7}{3584b^5(a+bx^2)^5}-\frac{143x^5}{4096b^6(a+bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a + b*x^2)^10,x]
```



```
[Out] -x^15/(18*b*(a + b*x^2)^9) - (5*x^13)/(96*b^2*(a + b*x^2)^8) - (65*x^11)/(1
344*b^3*(a + b*x^2)^7) - (715*x^9)/(16128*b^4*(a + b*x^2)^6) - (143*x^7)/(3
584*b^5*(a + b*x^2)^5) - (143*x^5)/(4096*b^6*(a + b*x^2)^4) - (715*x^3)/(24
576*b^7*(a + b*x^2)^3) - (715*x)/(32768*b^8*(a + b*x^2)^2) + (715*x)/(65536
*a*b^8*(a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^(1
7/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{16}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{15}}{18b\left(a+bx^2\right)^9} + \frac{5\int \frac{x^{14}}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} + \frac{65\int \frac{x^{12}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} + \frac{715\int \frac{x^{10}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} + \frac{715\int \frac{x^8}{\left(a+bx^2\right)^6}\,dx}{1792b^4}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x^7}{3584b^5\left(a+bx^2 \cdots\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x^7}{3584b^5\left(a+bx^2 \cdots\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x^7}{3584b^5\left(a+bx^2 \cdots\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x^7}{3584b^5\left(a+bx^2 \cdots\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x^7}{3584b^5\left(a+bx^2 \cdots\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x^7}{3584b^5\left(a+bx^2 \cdots\right.}$$

Mathematica [A] time = 0.0685814, size = 138, normalized size = 0.7

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-2633274a^2b^6x^{12}-4349826a^3b^5x^{10}-4685824a^4b^4x^8-3317886a^5b^3x^6-1495494a^6b^2x^4-390390a^7bx^2-45045a^8-985866ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+450\cdots}{4128768a^{3/2}b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 - 985866*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(3/2)*b^(17/2))
```

Maple [A] time = 0.013, size = 124, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{715\,a^7x}{65536\,b^8}-\frac{9295\,a^6x^3}{98304\,b^7}-\frac{11869\,a^5x^5}{32768\,b^6}-\frac{184327\,a^4x^7}{229376\,b^5}-\frac{143\,a^3x^9}{126\,b^4}-\frac{241657\,a^2x^{11}}{229376\,b^3}-\frac{20899\,ax^{13}}{32768\,b^2}-\frac{23473\,x^{15}}{98304\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b*x^2+a)^10,x)
```

```
[Out] (-715/65536*a^7/b^8*x-9295/98304*a^6/b^7*x^3-11869/32768*a^5/b^6*x^5-184327
/229376*a^4/b^5*x^7-143/126*a^3/b^4*x^9-241657/229376*a^2/b^3*x^11-20899/32
768/b^2*a*x^13-23473/98304/b*x^15+715/65536/a*x^17)/(b*x^2+a)^9+715/65536/a
/b^8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32484, size = 1580, normalized size = 7.98

$$\left[\frac{90090\,ab^9x^{17}-1971732\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4x^7-2990988\,a}{8257536\left(a^2b^{18}x^{18}+9\,a^3b^{17}x^{16}+36\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 - 1971732*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^18*x^18 + 9*a^3*b^17*x^
16 + 36*a^4*b^16*x^14 + 84*a^5*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13
*x^8 + 84*a^8*b^12*x^6 + 36*a^9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9), 1/4
128768*(45045*a*b^9*x^17 - 985866*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 434
9826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^7
*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a))/(a^2*b^18*x^18 + 9*a^3*b^17*x^16 + 36*a^4*b^16*x^14 + 84*a^5
*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13*x^8 + 84*a^8*b^12*x^6 + 36*a^
9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9)]
```

Sympy [A] time = 7.65989, size = 289, normalized size = 1.46

$$-\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(-a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{-45045a^8x}{4128768a^{10}b^8+37158912a^9b^9x^2+14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**3*b**17))*log(-a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/131072
 + 715*sqrt(-1/(a**3*b**17))*log(a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 - 985866*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**10*b
**8 + 37158912*a**9*b**9*x**2 + 148635648*a**8*b**10*x**4 + 346816512*a**7*
b**11*x**6 + 520224768*a**6*b**12*x**8 + 520224768*a**5*b**13*x**10 + 34681
6512*a**4*b**14*x**12 + 148635648*a**3*b**15*x**14 + 37158912*a**2*b**16*x*
*16 + 4128768*a*b**17*x**18)
```

Giac [A] time = 2.9788, size = 173, normalized size = 0.87

$$\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}ab^8}+\frac{45045\,b^8x^{17}-985866\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-331788}{4128768\left(bx^2+a\right)^9ab^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8) + 1/4128768*(45045*b^8*x^
17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4685
824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a*b^8)
```

3.214 $\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=199

$$\frac{143x}{65536a^2b^7\left(a+bx^2\right)}+\frac{143\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}-\frac{13x^{11}}{288b^2\left(a+bx^2\right)^8}-\frac{143x^9}{4032b^3\left(a+bx^2\right)^7}-\frac{143x^7}{5376b^4\left(a+bx^2\right)^6}-\frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

```
[Out] -x^13/(18*b*(a + b*x^2)^9) - (13*x^11)/(288*b^2*(a + b*x^2)^8) - (143*x^9)/
(4032*b^3*(a + b*x^2)^7) - (143*x^7)/(5376*b^4*(a + b*x^2)^6) - (143*x^5)/(
7680*b^5*(a + b*x^2)^5) - (143*x^3)/(12288*b^6*(a + b*x^2)^4) - (143*x)/(24
576*b^7*(a + b*x^2)^3) + (143*x)/(98304*a*b^7*(a + b*x^2)^2) + (143*x)/(655
36*a^2*b^7*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(5/2)*
b^(15/2))
```

Rubi [A] time = 0.122993, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{143x}{65536a^2b^7\left(a+bx^2\right)}+\frac{143\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}-\frac{13x^{11}}{288b^2\left(a+bx^2\right)^8}-\frac{143x^9}{4032b^3\left(a+bx^2\right)^7}-\frac{143x^7}{5376b^4\left(a+bx^2\right)^6}-\frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^2)^10,x]
```

```
[Out] -x^13/(18*b*(a + b*x^2)^9) - (13*x^11)/(288*b^2*(a + b*x^2)^8) - (143*x^9)/
(4032*b^3*(a + b*x^2)^7) - (143*x^7)/(5376*b^4*(a + b*x^2)^6) - (143*x^5)/(
7680*b^5*(a + b*x^2)^5) - (143*x^3)/(12288*b^6*(a + b*x^2)^4) - (143*x)/(24
576*b^7*(a + b*x^2)^3) + (143*x)/(98304*a*b^7*(a + b*x^2)^2) + (143*x)/(655
36*a^2*b^7*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(5/2)*
b^(15/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{13}}{18b\left(a+bx^2\right)^9} + \frac{13\int \frac{x^{12}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} + \frac{143\int \frac{x^{10}}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} + \frac{143\int \frac{x^8}{\left(a+bx^2\right)^7}\,dx}{448b^3}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} + \frac{143\int \frac{x^6}{\left(a+bx^2\right)^6}\,d}{768b^4}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x^5}{7680b^5\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0594352, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-2633274a^2b^6x^{12}-4349826a^3b^5x^{10}-4685824a^4b^4x^8-3317886a^5b^3x^6-1495494a^6b^2x^4-390390a^7bx^2-45045a^8+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+450}{20643840a^{5/2}b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(5/2)*b^(15/2))
```

Maple [A] time = 0.014, size = 122, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{143\,a^6x}{65536\,b^7}-\frac{1859\,a^5x^3}{98304\,b^6}-\frac{11869\,a^4x^5}{163840\,b^5}-\frac{184327\,a^3x^7}{1146880\,b^4}-\frac{143\,a^2x^9}{630\,b^3}-\frac{241657\,ax^{11}}{1146880\,b^2}-\frac{20899\,x^{13}}{163840\,b}+\frac{1859\,x^{15}}{98304\,a}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2+a)^10,x)
```

```
[Out] (-143/65536*a^6/b^7*x-1859/98304*a^5/b^6*x^3-11869/163840*a^4/b^5*x^5-18432
7/1146880*a^3/b^4*x^7-143/630*a^2/b^3*x^9-241657/1146880/b^2*a*x^11-20899/1
63840/b*x^13+1859/98304/a*x^15+143/65536*b/a^2*x^17)/(b*x^2+a)^9+143/65536/
a^2/b^7/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29004, size = 1581, normalized size = 7.94

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4x^7-2990988\,a}{41287680\left(a^3b^{17}x^{18}+9\,a^4b^{16}x^{16}+36\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^17*x^18 + 9*a^4*b^16*x^
16 + 36*a^5*b^15*x^14 + 84*a^6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12
*x^8 + 84*a^9*b^11*x^6 + 36*a^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 43
49826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^3*b^17*x^18 + 9*a^4*b^16*x^16 + 36*a^5*b^15*x^14 + 84*a^
6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12*x^8 + 84*a^9*b^11*x^6 + 36*a
^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8)]
```

Sympy [A] time = 7.47003, size = 291, normalized size = 1.46

$$-\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(-a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{-450}{20643840a^{11}b^7+185794560a^{10}b^8x^2 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**5*b**15))*log(-a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/131072
 + 143*sqrt(-1/(a**5*b**15))*log(a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**11*
b**7 + 185794560*a**10*b**8*x**2 + 743178240*a**9*b**9*x**4 + 1734082560*a*
*8*b**10*x**6 + 2601123840*a**7*b**11*x**8 + 2601123840*a**6*b**12*x**10 +
1734082560*a**5*b**13*x**12 + 743178240*a**4*b**14*x**14 + 185794560*a**3*b
**15*x**16 + 20643840*a**2*b**16*x**18)
```

Giac [A] time = 2.59407, size = 173, normalized size = 0.87

$$\frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^2b^7}+\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-331788}{20643840\left(bx^2+a\right)^9a^2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4
685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^2*b^7)
```

3.215 $\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=200

$$\frac{55x}{65536a^3b^6(a+bx^2)}+\frac{55x}{98304a^2b^6(a+bx^2)^2}+\frac{55\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}-\frac{11x^9}{288b^2(a+bx^2)^8}-\frac{11x^7}{448b^3(a+bx^2)^7}-\frac{11x^5}{768b^4(a+bx^2}$$

```
[Out] -x^11/(18*b*(a + b*x^2)^9) - (11*x^9)/(288*b^2*(a + b*x^2)^8) - (11*x^7)/(4
48*b^3*(a + b*x^2)^7) - (11*x^5)/(768*b^4*(a + b*x^2)^6) - (11*x^3)/(1536*b
^5*(a + b*x^2)^5) - (11*x)/(4096*b^6*(a + b*x^2)^4) + (11*x)/(24576*a*b^6*(
a + b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a + b*x^2)^2) + (55*x)/(65536*a^3*b^
6*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2))
```

Rubi [A] time = 0.120337, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{55x}{65536a^3b^6(a+bx^2)}+\frac{55x}{98304a^2b^6(a+bx^2)^2}+\frac{55\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}-\frac{11x^9}{288b^2(a+bx^2)^8}-\frac{11x^7}{448b^3(a+bx^2)^7}-\frac{11x^5}{768b^4(a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^10,x]
```

```
[Out] -x^11/(18*b*(a + b*x^2)^9) - (11*x^9)/(288*b^2*(a + b*x^2)^8) - (11*x^7)/(4
48*b^3*(a + b*x^2)^7) - (11*x^5)/(768*b^4*(a + b*x^2)^6) - (11*x^3)/(1536*b
^5*(a + b*x^2)^5) - (11*x)/(4096*b^6*(a + b*x^2)^4) + (11*x)/(24576*a*b^6*(
a + b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a + b*x^2)^2) + (55*x)/(65536*a^3*b^
6*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{11}}{18b\left(a+bx^2\right)^9} + \frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} + \frac{11\int \frac{x^8}{\left(a+bx^2\right)^8}\,dx}{32b^2}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} + \frac{11\int \frac{x^6}{\left(a+bx^2\right)^7}\,dx}{64b^3}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} + \frac{55\int \frac{x^4}{\left(a+bx^2\right)^6}\,dx}{768b^4}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.0690834, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(115038a^2b^6x^{12}-334602a^3b^5x^{10}-360448a^4b^4x^8-255222a^5b^3x^6-115038a^6b^2x^4-30030a^7bx^2-3465a^8+30030ab^7x^{14}+3465b^8x^{16}\right)}{\left(a+bx^2\right)^9}+3465\tan^{-1}\left(\right.}{4128768a^{7/2}b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 - 255
222*a^5*b^3*x^6 - 360448*a^4*b^4*x^8 - 334602*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(7/2)*b^(13/2))
```

Maple [A] time = 0.013, size = 122, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{55\,a^5x}{65536\,b^6}-\frac{715\,a^4x^3}{98304\,b^5}-\frac{913\,a^3x^5}{32768\,b^4}-\frac{14179\,a^2x^7}{229376\,b^3}-\frac{11\,ax^9}{126\,b^2}-\frac{18589\,x^{11}}{229376\,b}+\frac{913\,x^{13}}{32768\,a}+\frac{715\,bx^{15}}{98304\,a^2}+\frac{55\,b^2x^{17}}{65536\,a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^10,x)
```

```
[Out] (-55/65536*a^5/b^6*x-715/98304*a^4/b^5*x^3-913/32768*a^3/b^4*x^5-14179/2293
76*a^2/b^3*x^7-11/126/b^2*a*x^9-18589/229376/b*x^11+913/32768/a*x^13+715/98
304*b/a^2*x^15+55/65536*b^2/a^3*x^17)/(b*x^2+a)^9+55/65536/a^3/b^6/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32055, size = 1551, normalized size = 7.76

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}-669204\,a^4b^6x^{11}-720896\,a^5b^5x^9-510444\,a^6b^4x^7-230076\,a^7b^3x^5-6}{8257536\,\left(a^4b^{16}x^{18}+9\,a^5b^{15}x^{16}+36\,a^6b^{14}x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 - 66
9204*a^4*b^6*x^11 - 720896*a^5*b^5*x^9 - 510444*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6
*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*b^12*x^10 + 126*a^9*b^11*x^8 + 84*a
^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*b^8*x^2 + a^13*b^7), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 - 334602*a^4*b^6*x^
11 - 360448*a^5*b^5*x^9 - 255222*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b
^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*
b^12*x^10 + 126*a^9*b^11*x^8 + 84*a^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*
b^8*x^2 + a^13*b^7)]
```

Sympy [A] time = 7.41752, size = 291, normalized size = 1.46

$$-\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(-a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{-3465 \cdots}{4128768a^{12}b^6+37158912a^{11}b^7x^2+148 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**7*b**13))*log(-a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ 55*sqrt(-1/(a**7*b**13))*log(a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 - 255222*a**5*b
**3*x**7 - 360448*a**4*b**4*x**9 - 334602*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**12*b**6 + 37158
912*a**11*b**7*x**2 + 148635648*a**10*b**8*x**4 + 346816512*a**9*b**9*x**6
+ 520224768*a**8*b**10*x**8 + 520224768*a**7*b**11*x**10 + 346816512*a**6*b
**12*x**12 + 148635648*a**5*b**13*x**14 + 37158912*a**4*b**14*x**16 + 41287
68*a**3*b**15*x**18)
```

Giac [A] time = 1.89854, size = 173, normalized size = 0.86

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^3b^6}+\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}-334602\,a^3b^5x^{11}-360448\,a^4b^4x^9-255222\,a^5b^3x^7 \cdots}{4128768\left(bx^2+a\right)^9a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*a^3*b^5*x^11 - 360448*
a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^3*b^6)
```

3.216 $\int \frac{x^{10}}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=201

$$\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+b\right.}$$

```
[Out] -x^9/(18*b*(a + b*x^2)^9) - x^7/(32*b^2*(a + b*x^2)^8) - x^5/(64*b^3*(a + b
*x^2)^7) - (5*x^3)/(768*b^4*(a + b*x^2)^6) - x/(512*b^5*(a + b*x^2)^5) + x/
(4096*a*b^5*(a + b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a + b*x^2)^3) + (35*x)/(
98304*a^3*b^5*(a + b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a + b*x^2)) + (35*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))
```

Rubi [A] time = 0.119051, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^10,x]
```

```
[Out] -x^9/(18*b*(a + b*x^2)^9) - x^7/(32*b^2*(a + b*x^2)^8) - x^5/(64*b^3*(a + b
*x^2)^7) - (5*x^3)/(768*b^4*(a + b*x^2)^6) - x/(512*b^5*(a + b*x^2)^5) + x/
(4096*a*b^5*(a + b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a + b*x^2)^3) + (35*x)/(
98304*a^3*b^5*(a + b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a + b*x^2)) + (35*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^9}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{x^8}{\left(a+bx^2\right)^9}\,dx}{2b}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} + \frac{7\int \frac{x^6}{\left(a+bx^2\right)^8}\,dx}{32b^2}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} + \frac{5\int \frac{x^4}{\left(a+bx^2\right)^7}\,dx}{64b^3}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} + \frac{5\int \frac{x^2}{\left(a+bx^2\right)^6}\,dx}{256b^4}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5} + \cdots$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5} + \cdots$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5} + \cdots$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5} + \cdots$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5} + \cdots$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5} + \cdots$$

Mathematica [A] time = 0.0692791, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(10458a^2b^6x^{12}+23202a^3b^5x^{10}-32768a^4b^4x^8-23202a^5b^3x^6-10458a^6b^2x^4-2730a^7bx^2-315a^8+2730ab^7x^{14}+315b^8x^{16}\right)}{\left(a+bx^2\right)^9}+315\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{589824a^{9/2}b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 - 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(9/2)*b^(11/2))
```

Maple [A] time = 0.013, size = 122, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{35\,a^4x}{65536\,b^5}-\frac{455\,a^3x^3}{98304\,b^4}-\frac{581\,a^2x^5}{32768\,b^3}-\frac{1289\,ax^7}{32768\,b^2}-\frac{x^9}{18\,b}+\frac{1289\,x^{11}}{32768\,a}+\frac{581\,bx^{13}}{32768\,a^2}+\frac{455\,b^2x^{15}}{98304\,a^3}+\frac{35\,b^3x^{17}}{65536\,a^4}\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^10,x)
```

```
[Out] (-35/65536*a^4*x/b^5-455/98304*a^3*x^3/b^4-581/32768*a^2*x^5/b^3-1289/32768
*a*x^7/b^2-1/18*x^9/b+1289/32768/a*x^11+581/32768*b/a^2*x^13+455/98304*b^2/
a^3*x^15+35/65536*b^3/a^4*x^17)/(b*x^2+a)^9+35/65536/a^4/b^5/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30969, size = 1523, normalized size = 7.58

$$\left[\frac{630\,ab^9x^{17} + 5460\,a^2b^8x^{15} + 20916\,a^3b^7x^{13} + 46404\,a^4b^6x^{11} - 65536\,a^5b^5x^9 - 46404\,a^6b^4x^7 - 20916\,a^7b^3x^5 - 5460\,a^8b^2}{1179648\left(a^5b^{15}x^{18} + 9\,a^6b^{14}x^{16} + 36\,a^7b^{13}x^{14} + 84\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 - 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^5*b^15*x^18 + 9*a^6*b^14*x^16 + 36*a^7*b^13*x^14
 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*b^10*x^8 + 84*a^11*b^9*x
^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 - 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^15*x^18 + 9*a^6*b^14
*x^16 + 36*a^7*b^13*x^14 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*
b^10*x^8 + 84*a^11*b^9*x^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6)]
```

Sympy [A] time = 7.45544, size = 291, normalized size = 1.45

$$-\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(-a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{-315a^8x -}{589824a^{13}b^5 + 5308416a^{12}b^6x^2 + 2123366}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**9*b**11))*log(-a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ 35*sqrt(-1/(a**9*b**11))*log(a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 - 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**13*b**5 + 5308416*a**12*b
**6*x**2 + 21233664*a**11*b**7*x**4 + 49545216*a**10*b**8*x**6 + 74317824*a
**9*b**9*x**8 + 74317824*a**8*b**10*x**10 + 49545216*a**7*b**11*x**12 + 212
33664*a**6*b**12*x**14 + 5308416*a**5*b**13*x**16 + 589824*a**4*b**14*x**18
)
```

<u>Giac</u> [A] time = 1.84067, size = 173, normalized size = 0.86

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^4 b^5} + \frac{315\, b^8 x^{17} + 2730\, ab^7 x^{15} + 10458\, a^2 b^6 x^{13} + 23202\, a^3 b^5 x^{11} - 32768\, a^4 b^4 x^9 - 23202\, a^5 b^3 x^7 - 1045}{589824 \left(bx^2 + a\right)^9 a^4 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b^5) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^4*b^5)
```

3.217 $\int \frac{x^8}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=202

$$\frac{35x}{65536a^5b^4(a+bx^2)}+\frac{35x}{98304a^4b^4(a+bx^2)^2}+\frac{7x}{24576a^3b^4(a+bx^2)^3}+\frac{x}{4096a^2b^4(a+bx^2)^4}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}-\frac{\quad}{288b^2}$$

```
[Out] -x^7/(18*b*(a + b*x^2)^9) - (7*x^5)/(288*b^2*(a + b*x^2)^8) - (5*x^3)/(576*
b^3*(a + b*x^2)^7) - (5*x)/(2304*b^4*(a + b*x^2)^6) + x/(4608*a*b^4*(a + b*
x^2)^5) + x/(4096*a^2*b^4*(a + b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a + b*x^2)
^3) + (35*x)/(98304*a^4*b^4*(a + b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a + b*x
^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))
```

Rubi [A] time = 0.115632, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{35x}{65536a^5b^4(a+bx^2)}+\frac{35x}{98304a^4b^4(a+bx^2)^2}+\frac{7x}{24576a^3b^4(a+bx^2)^3}+\frac{x}{4096a^2b^4(a+bx^2)^4}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}-\frac{\quad}{288b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^10,x]
```

```
[Out] -x^7/(18*b*(a + b*x^2)^9) - (7*x^5)/(288*b^2*(a + b*x^2)^8) - (5*x^3)/(576*
b^3*(a + b*x^2)^7) - (5*x)/(2304*b^4*(a + b*x^2)^6) + x/(4608*a*b^4*(a + b*
x^2)^5) + x/(4096*a^2*b^4*(a + b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a + b*x^2)
^3) + (35*x)/(98304*a^4*b^4*(a + b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a + b*x
^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^7}{18b\left(a+bx^2\right)^9} + \frac{7\int \frac{x^6}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} + \frac{35\int \frac{x^4}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} + \frac{5\int \frac{x^2}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{5\int \frac{1}{\left(a+bx^2\right)^6}\,dx}{2304b^4}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a+bx^2\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a+bx^2\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a+bx^2\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a+bx^2\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a+bx^2\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0562671, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(10458a^2b^6x^{12}+23202a^3b^5x^{10}+32768a^4b^4x^8-23202a^5b^3x^6-10458a^6b^2x^4-2730a^7bx^2-315a^8+2730ab^7x^{14}+315b^8x^{16}\right)}{\left(a+bx^2\right)^9}+315\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{589824a^{11/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 + 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(11/2)*b^(9/2))
```

Maple [A] time = 0.011, size = 122, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{35\,a^3x}{65536\,b^4}-\frac{455\,a^2x^3}{98304\,b^3}-\frac{581\,ax^5}{32768\,b^2}-\frac{1289\,x^7}{32768\,b}+\frac{x^9}{18\,a}+\frac{1289\,bx^{11}}{32768\,a^2}+\frac{581\,b^2x^{13}}{32768\,a^3}+\frac{455\,b^3x^{15}}{98304\,a^4}+\frac{35\,b^4x^{17}}{65536\,a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^10,x)
```

```
[Out] (-35/65536*a^3*x/b^4-455/98304*a^2*x^3/b^3-581/32768*a*x^5/b^2-1289/32768*x
^7/b+1/18/a*x^9+1289/32768*b/a^2*x^11+581/32768*b^2/a^3*x^13+455/98304*b^3/
a^4*x^15+35/65536*b^4/a^5*x^17)/(b*x^2+a)^9+35/65536/a^5/b^4/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25955, size = 1523, normalized size = 7.54

$$\left[\frac{630\,ab^{9}x^{17} + 5460\,a^{2}b^{8}x^{15} + 20916\,a^{3}b^{7}x^{13} + 46404\,a^{4}b^{6}x^{11} + 65536\,a^{5}b^{5}x^{9} - 46404\,a^{6}b^{4}x^{7} - 20916\,a^{7}b^{3}x^{5} - 5460\,a^{8}b^{2}}{1179648\left(a^{6}b^{14}x^{18} + 9\,a^{7}b^{13}x^{16} + 36\,a^{8}b^{12}x^{14} + 8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 + 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^6*b^14*x^18 + 9*a^7*b^13*x^16 + 36*a^8*b^12*x^14
 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11*b^9*x^8 + 84*a^12*b^8*x
^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 + 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^6*b^14*x^18 + 9*a^7*b^13
*x^16 + 36*a^8*b^12*x^14 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11
*b^9*x^8 + 84*a^12*b^8*x^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5)]
```

Sympy [A] time = 7.20997, size = 291, normalized size = 1.44

$$-\frac{35\sqrt{-\frac{1}{a^{11}b^{9}}}\log\left(-a^{6}b^{4}\sqrt{-\frac{1}{a^{11}b^{9}}} + x\right)}{131072} + \frac{35\sqrt{-\frac{1}{a^{11}b^{9}}}\log\left(a^{6}b^{4}\sqrt{-\frac{1}{a^{11}b^{9}}} + x\right)}{131072} + \frac{-315a^{8}x -}{589824a^{14}b^{4} + 5308416a^{13}b^{5}x^{2} + 2123366}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**11*b**9))*log(-a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ 35*sqrt(-1/(a**11*b**9))*log(a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 + 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**14*b**4 + 5308416*a**13*b
**5*x**2 + 21233664*a**12*b**6*x**4 + 49545216*a**11*b**7*x**6 + 74317824*a
**10*b**8*x**8 + 74317824*a**9*b**9*x**10 + 49545216*a**8*b**10*x**12 + 212
33664*a**7*b**11*x**14 + 5308416*a**6*b**12*x**16 + 589824*a**5*b**13*x**18
)
```

Giac [A] time = 1.457, size = 173, normalized size = 0.86

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}a^5b^4} + \frac{315 b^8x^{17} + 2730 ab^7x^{15} + 10458 a^2b^6x^{13} + 23202 a^3b^5x^{11} + 32768 a^4b^4x^9 - 23202 a^5b^3x^7 - 1045}{589824 \left(bx^2 + a\right)^9 a^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*b^4) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 + 32768*a^4*b^
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^5*b^4)
```

3.218 $\int \frac{x^6}{(a+bx^2)^{10}} dx$

Optimal. Leaf size=203

$$\frac{55x}{65536a^6b^3(a+bx^2)}+\frac{55x}{98304a^5b^3(a+bx^2)^2}+\frac{11x}{24576a^4b^3(a+bx^2)^3}+\frac{11x}{28672a^3b^3(a+bx^2)^4}+\frac{11x}{32256a^2b^3(a+bx^2)^5}$$

```
[Out] -x^5/(18*b*(a + b*x^2)^9) - (5*x^3)/(288*b^2*(a + b*x^2)^8) - (5*x)/(1344*b
^3*(a + b*x^2)^7) + (5*x)/(16128*a*b^3*(a + b*x^2)^6) + (11*x)/(32256*a^2*b
^3*(a + b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a + b*x^2)^4) + (11*x)/(24576*a^
4*b^3*(a + b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a + b*x^2)^2) + (55*x)/(65536
*a^6*b^3*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*b^
(7/2))
```

Rubi [A] time = 0.111235, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{55x}{65536a^6b^3(a+bx^2)}+\frac{55x}{98304a^5b^3(a+bx^2)^2}+\frac{11x}{24576a^4b^3(a+bx^2)^3}+\frac{11x}{28672a^3b^3(a+bx^2)^4}+\frac{11x}{32256a^2b^3(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^10,x]
```

```
[Out] -x^5/(18*b*(a + b*x^2)^9) - (5*x^3)/(288*b^2*(a + b*x^2)^8) - (5*x)/(1344*b
^3*(a + b*x^2)^7) + (5*x)/(16128*a*b^3*(a + b*x^2)^6) + (11*x)/(32256*a^2*b
^3*(a + b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a + b*x^2)^4) + (11*x)/(24576*a^
4*b^3*(a + b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a + b*x^2)^2) + (55*x)/(65536
*a^6*b^3*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*b^
(7/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^5}{18b\left(a+bx^2\right)^9}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}+\frac{5\int \frac{x^2}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{55\int \frac{1}{\left(a+bx^2\right)^6}}{16128ab^3}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{11x}{32256a^2b^3\left(a\right.}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{11x}{32256a^2b^3\left(a\right.}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{11x}{32256a^2b^3\left(a\right.}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{11x}{32256a^2b^3\left(a\right.}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{11x}{32256a^2b^3\left(a\right.}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}+\frac{11x}{32256a^2b^3\left(a\right.}$$

Mathematica [A] time = 0.0614697, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(115038a^2b^6x^{12}+255222a^3b^5x^{10}+360448a^4b^4x^8+334602a^5b^3x^6-115038a^6b^2x^4-30030a^7bx^2-3465a^8+30030ab^7x^{14}+3465b^8x^{16}\right)}{\left(a+bx^2\right)^9}+3465\tan^{-1}\left(\right.}{4128768a^{13/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 + 334
602*a^5*b^3*x^6 + 360448*a^4*b^4*x^8 + 255222*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(13/2)*b^(7/2))
```

Maple [A] time = 0.012, size = 122, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{55\,a^2x}{65536\,b^3}-\frac{715\,ax^3}{98304\,b^2}-\frac{913\,x^5}{32768\,b}+\frac{18589\,x^7}{229376\,a}+\frac{11\,bx^9}{126\,a^2}+\frac{14179\,b^2x^{11}}{229376\,a^3}+\frac{913\,b^3x^{13}}{32768\,a^4}+\frac{715\,b^4x^{15}}{98304\,a^5}+\frac{55\,b^5x^{17}}{65536\,a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^10,x)
```

```
[Out] (-55/65536*a^2*x/b^3-715/98304*a*x^3/b^2-913/32768*x^5/b+18589/229376/a*x^7
+11/126*b/a^2*x^9+14179/229376*b^2/a^3*x^11+913/32768*b^3/a^4*x^13+715/9830
4*b^4/a^5*x^15+55/65536/a^6*b^5*x^17)/(b*x^2+a)^9+55/65536/a^6/b^3/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34863, size = 1551, normalized size = 7.64

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}+510444\,a^4b^6x^{11}+720896\,a^5b^5x^9+669204\,a^6b^4x^7-230076\,a^7b^3x^5-}{8257536\left(a^7b^{13}x^{18}+9\,a^8b^{12}x^{16}+36\,a^9b^{11}x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 + 51
0444*a^4*b^6*x^11 + 720896*a^5*b^5*x^9 + 669204*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^7*b^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9
*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^11*b^9*x^10 + 126*a^12*b^8*x^8 + 84*
a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*b^5*x^2 + a^16*b^4), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 + 255222*a^4*b^6*x^
11 + 360448*a^5*b^5*x^9 + 334602*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^7*b
^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^1
1*b^9*x^10 + 126*a^12*b^8*x^8 + 84*a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*
b^5*x^2 + a^16*b^4)]
```

Sympy [A] time = 6.999, size = 291, normalized size = 1.43

$$-\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\log\left(-a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\log\left(a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}+\frac{-3465a}{4128768a^{15}b^3+37158912a^{14}b^4x^2+148}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**13*b**7))*log(-a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ 55*sqrt(-1/(a**13*b**7))*log(a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 + 334602*a**5*b
**3*x**7 + 360448*a**4*b**4*x**9 + 255222*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**15*b**3 + 37158
912*a**14*b**4*x**2 + 148635648*a**13*b**5*x**4 + 346816512*a**12*b**6*x**6
 + 520224768*a**11*b**7*x**8 + 520224768*a**10*b**8*x**10 + 346816512*a**9*
b**9*x**12 + 148635648*a**8*b**10*x**14 + 37158912*a**7*b**11*x**16 + 41287
68*a**6*b**12*x**18)
```

Giac [A] time = 2.23001, size = 173, normalized size = 0.85

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^6b^3}+\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}+255222\,a^3b^5x^{11}+360448\,a^4b^4x^9+334602\,a^5b^3x^7}{4128768\left(bx^2+a\right)^9a^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*a^3*b^5*x^11 + 360448*
a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^6*b^3)
```

3.219 $\int \frac{x^4}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=204

$$\frac{143x}{65536a^7b^2\left(a+bx^2\right)}+\frac{143x}{98304a^6b^2\left(a+bx^2\right)^2}+\frac{143x}{122880a^5b^2\left(a+bx^2\right)^3}+\frac{143x}{143360a^4b^2\left(a+bx^2\right)^4}+\frac{143x}{161280a^3b^2\left(a+bx^2\right)}$$

```
[Out] -x^3/(18*b*(a + b*x^2)^9) - x/(96*b^2*(a + b*x^2)^8) + x/(1344*a*b^2*(a + b
*x^2)^7) + (13*x)/(16128*a^2*b^2*(a + b*x^2)^6) + (143*x)/(161280*a^3*b^2*(
a + b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a + b*x^2)^4) + (143*x)/(122880*a^
5*b^2*(a + b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a + b*x^2)^2) + (143*x)/(655
36*a^7*b^2*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/2)
*b^(5/2))
```

Rubi [A] time = 0.108083, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{143x}{65536a^7b^2\left(a+bx^2\right)}+\frac{143x}{98304a^6b^2\left(a+bx^2\right)^2}+\frac{143x}{122880a^5b^2\left(a+bx^2\right)^3}+\frac{143x}{143360a^4b^2\left(a+bx^2\right)^4}+\frac{143x}{161280a^3b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^10,x]
```



```
[Out] -x^3/(18*b*(a + b*x^2)^9) - x/(96*b^2*(a + b*x^2)^8) + x/(1344*a*b^2*(a + b
*x^2)^7) + (13*x)/(16128*a^2*b^2*(a + b*x^2)^6) + (143*x)/(161280*a^3*b^2*(
a + b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a + b*x^2)^4) + (143*x)/(122880*a^
5*b^2*(a + b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a + b*x^2)^2) + (143*x)/(655
36*a^7*b^2*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/2)
*b^(5/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^3}{18b\left(a+bx^2\right)^9}+\frac{\int \frac{x^2}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344ab^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143\int \frac{1}{\left(a+bx^2\right)}}{16128a^2b}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143}{161280a^3b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143}{161280a^3b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143}{161280a^3b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143}{161280a^3b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143}{161280a^3b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143}{161280a^3b^2}$$

Mathematica [A] time = 0.0616886, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(1495494a^2b^6x^{12}+3317886a^3b^5x^{10}+4685824a^4b^4x^8+4349826a^5b^3x^6+2633274a^6b^2x^4-390390a^7bx^2-45045a^8+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+450}{20643840a^{15/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(15/2)*b^(5/2))
```

Maple [A] time = 0.013, size = 122, normalized size = 0.6

$$\frac{1}{\left(bx^{2}+a\right)^{9}}\left(-\frac{143\,ax}{65536\,b^{2}}-\frac{1859\,x^{3}}{98304\,b}+\frac{20899\,x^{5}}{163840\,a}+\frac{241657\,bx^{7}}{1146880\,a^{2}}+\frac{143\,b^{2}x^{9}}{630\,a^{3}}+\frac{184327\,b^{3}x^{11}}{1146880\,a^{4}}+\frac{11869\,b^{4}x^{13}}{163840\,a^{5}}+\frac{1859\,b^{5}x^{15}}{98304\,a^{6}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^10,x)
```

```
[Out] (-143/65536*a*x/b^2-1859/98304*x^3/b+20899/163840/a*x^5+241657/1146880*b/a^
2*x^7+143/630*b^2/a^3*x^9+184327/1146880*b^3/a^4*x^11+11869/163840*b^4/a^5*
x^13+1859/98304/a^6*b^5*x^15+143/65536/a^7*b^6*x^17)/(b*x^2+a)^9+143/65536/
a^7/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36829, size = 1581, normalized size = 7.75

$$\left[\frac{90090\,ab^{9}x^{17}+780780\,a^{2}b^{8}x^{15}+2990988\,a^{3}b^{7}x^{13}+6635772\,a^{4}b^{6}x^{11}+9371648\,a^{5}b^{5}x^{9}+8699652\,a^{6}b^{4}x^{7}+5266548\,a}{41287680\left(a^{8}b^{12}x^{18}+9\,a^{9}b^{11}x^{16}+36\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13
+ 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 526654
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^8*b^12*x^18 + 9*a^9*b^11*x^
16 + 36*a^10*b^10*x^14 + 84*a^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^
7*x^8 + 84*a^14*b^6*x^6 + 36*a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 33
17886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^8*b^12*x^18 + 9*a^9*b^11*x^16 + 36*a^10*b^10*x^14 + 84*a
^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^7*x^8 + 84*a^14*b^6*x^6 + 36*
a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3)]
```

Sympy [A] time = 7.15065, size = 291, normalized size = 1.43

$$-\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(-a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{-450\ldots}{20643840a^{16}b^2+185794560a^{15}b^3x^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**15*b**5))*log(-a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/131072
 + 143*sqrt(-1/(a**15*b**5))*log(a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*
a**5*b**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**16*
b**2 + 185794560*a**15*b**3*x**2 + 743178240*a**14*b**4*x**4 + 1734082560*a
**13*b**5*x**6 + 2601123840*a**12*b**6*x**8 + 2601123840*a**11*b**7*x**10 +
 1734082560*a**10*b**8*x**12 + 743178240*a**9*b**9*x**14 + 185794560*a**8*b
**10*x**16 + 20643840*a**7*b**11*x**18)
```

Giac [A] time = 3.37528, size = 173, normalized size = 0.85

$$\frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^7b^2}+\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}+1495494\,a^2b^6x^{13}+3317886\,a^3b^5x^{11}+4685824\,a^4b^4x^9+434982\ldots}{20643840\left(bx^2+a\right)^9a^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4
685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^7*b^2)
```

3.220 $\int \frac{x^2}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=205

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\left(a+bx^2\right)}+\frac{715x}{98304a^7b\left(a+bx^2\right)^2}+\frac{143x}{24576a^6b\left(a+bx^2\right)^3}+\frac{143x}{28672a^5b\left(a+bx^2\right)^4}+\frac{}{32256a}$$

```
[Out] -x/(18*b*(a + b*x^2)^9) + x/(288*a*b*(a + b*x^2)^8) + (5*x)/(1344*a^2*b*(a
+ b*x^2)^7) + (65*x)/(16128*a^3*b*(a + b*x^2)^6) + (143*x)/(32256*a^4*b*(a
+ b*x^2)^5) + (143*x)/(28672*a^5*b*(a + b*x^2)^4) + (143*x)/(24576*a^6*b*(a
 + b*x^2)^3) + (715*x)/(98304*a^7*b*(a + b*x^2)^2) + (715*x)/(65536*a^8*b*(
a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(17/2)*b^(3/2))
```

Rubi [A] time = 0.105971, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\left(a+bx^2\right)}+\frac{715x}{98304a^7b\left(a+bx^2\right)^2}+\frac{143x}{24576a^6b\left(a+bx^2\right)^3}+\frac{143x}{28672a^5b\left(a+bx^2\right)^4}+\frac{}{32256a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^10,x]
```


```
[Out] -x/(18*b*(a + b*x^2)^9) + x/(288*a*b*(a + b*x^2)^8) + (5*x)/(1344*a^2*b*(a
+ b*x^2)^7) + (65*x)/(16128*a^3*b*(a + b*x^2)^6) + (143*x)/(32256*a^4*b*(a
+ b*x^2)^5) + (143*x)/(28672*a^5*b*(a + b*x^2)^4) + (143*x)/(24576*a^6*b*(a
 + b*x^2)^3) + (715*x)/(98304*a^7*b*(a + b*x^2)^2) + (715*x)/(65536*a^8*b*(
a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(17/2)*b^(3/2))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{1}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96ab}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344a^2b}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{715\int \frac{1}{\left(a+bx^2\right.}}{16128a^3}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{143}{32256a^4b\,(a}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{143}{32256a^4b\,(a}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{143}{32256a^4b\,(a}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{143}{32256a^4b\,(a}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{143}{32256a^4b\,(a}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{143}{32256a^4b\,(a}$$

Mathematica [A] time = 0.0547511, size = 138, normalized size = 0.67

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(1495494a^2b^6x^{12}+3317886a^3b^5x^{10}+4685824a^4b^4x^8+4349826a^5b^3x^6+2633274a^6b^2x^4+985866a^7bx^2-45045a^8+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+450}{4128768a^{17/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 + 985866*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(17/2)*b^(3/2))
```

Maple [A] time = 0.011, size = 124, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^9}\left(-\frac{715\,x}{65536\,b} + \frac{23473\,x^3}{98304\,a} + \frac{20899\,bx^5}{32768\,a^2} + \frac{241657\,b^2x^7}{229376\,a^3} + \frac{143\,b^3x^9}{126\,a^4} + \frac{184327\,b^4x^{11}}{229376\,a^5} + \frac{11869\,b^5x^{13}}{32768\,a^6} + \frac{9295\,b^6}{98304\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^10,x)
```

```
[Out] (-715/65536*x/b+23473/98304/a*x^3+20899/32768*b/a^2*x^5+241657/229376*b^2/a
^3*x^7+143/126*b^3/a^4*x^9+184327/229376*b^4/a^5*x^11+11869/32768/a^6*b^5*x
^13+9295/98304/a^7*b^6*x^15+715/65536/a^8*b^7*x^17)/(b*x^2+a)^9+715/65536/a
^8/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29457, size = 1580, normalized size = 7.71

$$\left[\frac{90090\, ab^9x^{17} + 780780\, a^2b^8x^{15} + 2990988\, a^3b^7x^{13} + 6635772\, a^4b^6x^{11} + 9371648\, a^5b^5x^9 + 8699652\, a^6b^4x^7 + 5266548\, a}{8257536\left(a^9b^{11}x^{18} + 9\, a^{10}b^{10}x^{16} + 3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13 +
 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 5266548
*a^7*b^3*x^5 + 1971732*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^9*b^11*x^18 + 9*a^10*b^10*x
^16 + 36*a^11*b^9*x^14 + 84*a^12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^
6*x^8 + 84*a^15*b^5*x^6 + 36*a^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2), 1/4
128768*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 331
7886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^7
*b^3*x^5 + 985866*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a))/(a^9*b^11*x^18 + 9*a^10*b^10*x^16 + 36*a^11*b^9*x^14 + 84*a^
12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^6*x^8 + 84*a^15*b^5*x^6 + 36*a
^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2)]
```

Sympy [A] time = 7.63239, size = 286, normalized size = 1.4

$$-\frac{715\sqrt{-\frac{1}{a^{17}b^3}}\log\left(-a^9b\sqrt{-\frac{1}{a^{17}b^3}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^{17}b^3}}\log\left(a^9b\sqrt{-\frac{1}{a^{17}b^3}}+x\right)}{131072}+\frac{-45045a^8x+98}{4128768a^{17}b+37158912a^{16}b^2x^2+148635}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**17*b**3))*log(-a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 +
715*sqrt(-1/(a**17*b**3))*log(a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 + (-
45045*a**8*x + 985866*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*a**5*b
**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494*a**2*
b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**17*b + 371
58912*a**16*b**2*x**2 + 148635648*a**15*b**3*x**4 + 346816512*a**14*b**4*x*
*6 + 520224768*a**13*b**5*x**8 + 520224768*a**12*b**6*x**10 + 346816512*a**
11*b**7*x**12 + 148635648*a**10*b**8*x**14 + 37158912*a**9*b**9*x**16 + 412
8768*a**8*b**10*x**18)
```

<u>Giac</u> [A] time = 2.21387, size = 173, normalized size = 0.84

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}a^8 b} + \frac{45045 b^8 x^{17} + 390390 ab^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + 434982}{4128768 \left(bx^2 + a\right)^9 a^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b) + 1/4128768*(45045*b^8*x^
17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4685
824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 + 985866*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^8*b)
```

3.221 $\int \frac{1}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=181

$$\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\frac{2431x}{32256a^5\left(a+bx^2\right)^5}+\frac{11}{16128a^4}$$

```
[Out] x/(18*a*(a + b*x^2)^9) + (17*x)/(288*a^2*(a + b*x^2)^8) + (85*x)/(1344*a^3*
(a + b*x^2)^7) + (1105*x)/(16128*a^4*(a + b*x^2)^6) + (2431*x)/(32256*a^5*(
a + b*x^2)^5) + (2431*x)/(28672*a^6*(a + b*x^2)^4) + (2431*x)/(24576*a^7*(a
 + b*x^2)^3) + (12155*x)/(98304*a^8*(a + b*x^2)^2) + (12155*x)/(65536*a^9*(
a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(19/2)*Sqrt[b])
```

Rubi [A] time = 0.0996333, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\frac{2431x}{32256a^5\left(a+bx^2\right)^5}+\frac{11}{16128a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-10),x]
```

```
[Out] x/(18*a*(a + b*x^2)^9) + (17*x)/(288*a^2*(a + b*x^2)^8) + (85*x)/(1344*a^3*
(a + b*x^2)^7) + (1105*x)/(16128*a^4*(a + b*x^2)^6) + (2431*x)/(32256*a^5*(
a + b*x^2)^5) + (2431*x)/(28672*a^6*(a + b*x^2)^4) + (2431*x)/(24576*a^7*(a
 + b*x^2)^3) + (12155*x)/(98304*a^8*(a + b*x^2)^2) + (12155*x)/(65536*a^9*(
a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(19/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{10}}\,dx = \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17\int \frac{1}{\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344a^3}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{12155\int \frac{1}{\left(a+bx^2\right)^6}}{16128a^4}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431x}{32256a^5(a+b\ldots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431x}{32256a^5(a+b\ldots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431x}{32256a^5(a+b\ldots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431x}{32256a^5(a+b\ldots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431x}{32256a^5(a+b\ldots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431x}{32256a^5(a+b\ldots}$$

Mathematica [A] time = 0.0974405, size = 131, normalized size = 0.72

$$\frac{\frac{25423398a^2b^6x^{13}+56404062a^3b^5x^{11}+79659008a^4b^4x^9+73947042a^5b^3x^7+44765658a^6b^2x^5+16759722a^7bx^3+3363003a^8x+6636630ab^7x^{15}+765765b^8x^{17}}{a^9\left(a+bx^2\right)^9}+\ldots}{4128768}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-10),x]
```

```
[Out] ((3363003*a^8*x + 16759722*a^7*b*x^3 + 44765658*a^6*b^2*x^5 + 73947042*a^5*
b^3*x^7 + 79659008*a^4*b^4*x^9 + 56404062*a^3*b^5*x^11 + 25423398*a^2*b^6*x
^13 + 6636630*a*b^7*x^15 + 765765*b^8*x^17)/(a^9*(a + b*x^2)^9) + (765765*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(19/2)*Sqrt[b]))/4128768
```

Maple [A] time = 0.005, size = 156, normalized size = 0.9

$$\frac{x}{18\,a\left(bx^2+a\right)^9} + \frac{17\,x}{288\,a^2\left(bx^2+a\right)^8} + \frac{85\,x}{1344\,a^3\left(bx^2+a\right)^7} + \frac{1105\,x}{16128\,a^4\left(bx^2+a\right)^6} + \frac{2431\,x}{32256\,a^5\left(bx^2+a\right)^5} + \frac{2431}{28672\,a^6\,(b\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^10,x)
```

```
[Out] 1/18*x/a/(b*x^2+a)^9+17/288*x/a^2/(b*x^2+a)^8+85/1344*x/a^3/(b*x^2+a)^7+110
5/16128*x/a^4/(b*x^2+a)^6+2431/32256*x/a^5/(b*x^2+a)^5+2431/28672*x/a^6/(b*
x^2+a)^4+2431/24576*x/a^7/(b*x^2+a)^3+12155/98304*x/a^8/(b*x^2+a)^2+12155/6
5536*x/a^9/(b*x^2+a)+12155/65536/a^9/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.33232, size = 1612, normalized size = 8.91

$$\left[\frac{1531530\,ab^9x^{17} + 13273260\,a^2b^8x^{15} + 50846796\,a^3b^7x^{13} + 112808124\,a^4b^6x^{11} + 159318016\,a^5b^5x^9 + 147894084\,a^6b^4x^7}{8257536\left(a^{10}b^{10}x^{18} + 9\,a^{11}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(1531530*a*b^9*x^17 + 13273260*a^2*b^8*x^15 + 50846796*a^3*b^7*x
^13 + 112808124*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 147894084*a^6*b^4*x^
7 + 89531316*a^7*b^3*x^5 + 33519444*a^8*b^2*x^3 + 6726006*a^9*b*x - 765765*
(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*
x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^
9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^10*b^10*x^1
8 + 9*a^11*b^9*x^16 + 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^
10 + 126*a^15*b^5*x^8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2
+ a^19*b), 1/4128768*(765765*a*b^9*x^17 + 6636630*a^2*b^8*x^15 + 25423398*a
^3*b^7*x^13 + 56404062*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 73947042*a^6*b
^4*x^7 + 44765658*a^7*b^3*x^5 + 16759722*a^8*b^2*x^3 + 3363003*a^9*b*x + 76
5765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4
*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2
 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^10*b^10*x^18 + 9*a^11*b^9*x^16
+ 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^10 + 126*a^15*b^5*x^
8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2 + a^19*b)]
```

Sympy [A] time = 7.302, size = 272, normalized size = 1.5

$$-\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(-a^{10}\sqrt{-\frac{1}{a^{19}b}} + x\right)}{131072} + \frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(a^{10}\sqrt{-\frac{1}{a^{19}b}} + x\right)}{131072} + \frac{3363003a^8x + 1675972}{4128768a^{18} + 37158912a^{17}bx^2 + 1486356}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a**19*b))*log(-a**10*sqrt(-1/(a**19*b)) + x)/131072 + 12155
*sqrt(-1/(a**19*b))*log(a**10*sqrt(-1/(a**19*b)) + x)/131072 + (3363003*a**
8*x + 16759722*a**7*b*x**3 + 44765658*a**6*b**2*x**5 + 73947042*a**5*b**3*x
**7 + 79659008*a**4*b**4*x**9 + 56404062*a**3*b**5*x**11 + 25423398*a**2*b*
*6*x**13 + 6636630*a*b**7*x**15 + 765765*b**8*x**17)/(4128768*a**18 + 37158
912*a**17*b*x**2 + 148635648*a**16*b**2*x**4 + 346816512*a**15*b**3*x**6 +
520224768*a**14*b**4*x**8 + 520224768*a**13*b**5*x**10 + 346816512*a**12*b*
*6*x**12 + 148635648*a**11*b**7*x**14 + 37158912*a**10*b**8*x**16 + 4128768
*a**9*b**9*x**18)
```

Giac [A] time = 2.79747, size = 165, normalized size = 0.91

$$\frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^9} + \frac{765765\,b^8x^{17} + 6636630\,ab^7x^{15} + 25423398\,a^2b^6x^{13} + 56404062\,a^3b^5x^{11} + 79659008\,a^4b^4x^9 \cdots}{4128768\left(bx^2+a\right)^9a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9) + 1/4128768*(765765*b^8*x
^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 56404062*a^3*b^5*x^11 +
79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 44765658*a^6*b^2*x^5 + 167597
22*a^7*b*x^3 + 3363003*a^8*x)/((b*x^2 + a)^9*a^9)
```

3.222 $\int \frac{1}{x^2(a+bx^2)^{10}} dx$

Optimal. Leaf size=209

$$\frac{230945}{196608a^9x(a+bx^2)}+\frac{46189}{98304a^8x(a+bx^2)^2}+\frac{46189}{172032a^7x(a+bx^2)^3}+\frac{46189}{258048a^6x(a+bx^2)^4}+\frac{4199}{32256a^5x(a+bx^2)^5}+$$

```
[Out] -230945/(65536*a^10*x) + 1/(18*a*x*(a + b*x^2)^9) + 19/(288*a^2*x*(a + b*x^
2)^8) + 323/(4032*a^3*x*(a + b*x^2)^7) + 1615/(16128*a^4*x*(a + b*x^2)^6) +
 4199/(32256*a^5*x*(a + b*x^2)^5) + 46189/(258048*a^6*x*(a + b*x^2)^4) + 46
189/(172032*a^7*x*(a + b*x^2)^3) + 46189/(98304*a^8*x*(a + b*x^2)^2) + 2309
45/(196608*a^9*x*(a + b*x^2)) - (230945*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(65536*a^(21/2))
```

Rubi [A] time = 0.130814, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{230945}{196608a^9x(a+bx^2)}+\frac{46189}{98304a^8x(a+bx^2)^2}+\frac{46189}{172032a^7x(a+bx^2)^3}+\frac{46189}{258048a^6x(a+bx^2)^4}+\frac{4199}{32256a^5x(a+bx^2)^5}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^10),x]
```



```
[Out] -230945/(65536*a^10*x) + 1/(18*a*x*(a + b*x^2)^9) + 19/(288*a^2*x*(a + b*x^
2)^8) + 323/(4032*a^3*x*(a + b*x^2)^7) + 1615/(16128*a^4*x*(a + b*x^2)^6) +
 4199/(32256*a^5*x*(a + b*x^2)^5) + 46189/(258048*a^6*x*(a + b*x^2)^4) + 46
189/(172032*a^7*x*(a + b*x^2)^3) + 46189/(98304*a^8*x*(a + b*x^2)^2) + 2309
45/(196608*a^9*x*(a + b*x^2)) - (230945*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(65536*a^(21/2))
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19\int \frac{1}{x^2\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323\int \frac{1}{x^2\left(a+bx^2\right)^8}\,dx}{288a^2}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615\int \frac{1}{x^2\left(a+bx^2\right)^7}\,dx}{1344a^3}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} + \frac{20995\int}{1}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} + \frac{}{32256a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} + \frac{}{32256a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} + \frac{}{32256a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} + \frac{}{32256a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} + \frac{}{32256a}$$

$$= -\frac{230945}{65536a^{10}x} + \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+b\right.}$$

$$= -\frac{230945}{65536a^{10}x} + \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+b\right.}$$

Mathematica [A] time = 0.0944645, size = 147, normalized size = 0.7

$$-\frac{\frac{\sqrt{a}\left(483044562a^2b^7x^{14}+1071677178a^3b^6x^{12}+1513521152a^4b^5x^{10}+1404993798a^5b^4x^8+850547502a^6b^3x^6+318434718a^7b^2x^4+63897057a^8bx^2+4128768a^9\right.}{x\left(a+bx^2\right)^9}}{4128768a^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(4128768*a^9 + 63897057*a^8*b*x^2 + 318434718*a^7*b^2*x^4 + 850
547502*a^6*b^3*x^6 + 1404993798*a^5*b^4*x^8 + 1513521152*a^4*b^5*x^10 + 107
1677178*a^3*b^6*x^12 + 483044562*a^2*b^7*x^14 + 126095970*a*b^8*x^16 + 1454
9535*b^9*x^18))/(x*(a + b*x^2)^9)) - 14549535*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
```

```
rt[a]])/(4128768*a^(21/2))
```

Maple [A] time = 0.019, size = 206, normalized size = 1.

$$-\frac{1}{a^{10}x}-\frac{424415\,bx}{65536\,a^{2}\left(bx^{2}+a\right)^{9}}-\frac{4042835\,b^{2}x^{3}}{98304\,a^{3}\left(bx^{2}+a\right)^{9}}-\frac{3997865\,b^{3}x^{5}}{32768\,a^{4}\left(bx^{2}+a\right)^{9}}-\frac{49153835\,b^{4}x^{7}}{229376\,a^{5}\left(bx^{2}+a\right)^{9}}-\frac{30313\,b^{5}x^{9}}{126\,a^{6}\left(bx^{2}+a\right)^{9}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^10,x)
```

```
[Out] -1/a^10/x-424415/65536/a^2*b/(b*x^2+a)^9*x-4042835/98304/a^3*b^2/(b*x^2+a)^
9*x^3-3997865/32768/a^4*b^3/(b*x^2+a)^9*x^5-49153835/229376/a^5*b^4/(b*x^2+
a)^9*x^7-30313/126/a^6*b^5/(b*x^2+a)^9*x^9-40270037/229376/a^7*b^6/(b*x^2+a
)^9*x^11-2654039/32768/a^8*b^7/(b*x^2+a)^9*x^13-2117549/98304/a^9*b^8/(b*x^
2+a)^9*x^15-165409/65536/a^10*b^9/(b*x^2+a)^9*x^17-230945/65536/a^10*b/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54778, size = 1686, normalized size = 8.07

$$\left[-\frac{29099070\,b^{9}x^{18}+252191940\,ab^{8}x^{16}+966089124\,a^{2}b^{7}x^{14}+2143354356\,a^{3}b^{6}x^{12}+3027042304\,a^{4}b^{5}x^{10}+280998759}{8257536}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(29099070*b^9*x^18 + 252191940*a*b^8*x^16 + 966089124*a^2*b^7*x
^14 + 2143354356*a^3*b^6*x^12 + 3027042304*a^4*b^5*x^10 + 2809987596*a^5*b^
4*x^8 + 1701095004*a^6*b^3*x^6 + 636869436*a^7*b^2*x^4 + 127794114*a^8*b*x^
2 + 8257536*a^9 - 14549535*(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*
a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7
*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a
^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*
a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x), -1/4128768*(14549535*b^9*x^18 + 1260
95970*a*b^8*x^16 + 483044562*a^2*b^7*x^14 + 1071677178*a^3*b^6*x^12 + 15135
21152*a^4*b^5*x^10 + 1404993798*a^5*b^4*x^8 + 850547502*a^6*b^3*x^6 + 31843
4718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 + 4128768*a^9 + 14549535*(b^9*x^19 +
```

```
9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a
^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15
 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^
7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x)]
```

Sympy [A] time = 118.924, size = 280, normalized size = 1.34

$$\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(-\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072}-\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072}-\frac{4128768a^9+63897057a^8bx^2+318434\cdots}{4128768a^{19}x+37158912a^{18}bx^3+1486356\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-b/a**21)*log(-a**11*sqrt(-b/a**21)/b + x)/131072 - 230945*sqrt
(-b/a**21)*log(a**11*sqrt(-b/a**21)/b + x)/131072 - (4128768*a**9 + 6389705
7*a**8*b*x**2 + 318434718*a**7*b**2*x**4 + 850547502*a**6*b**3*x**6 + 14049
93798*a**5*b**4*x**8 + 1513521152*a**4*b**5*x**10 + 1071677178*a**3*b**6*x*
*12 + 483044562*a**2*b**7*x**14 + 126095970*a*b**8*x**16 + 14549535*b**9*x*
*18)/(4128768*a**19*x + 37158912*a**18*b*x**3 + 148635648*a**17*b**2*x**5 +
 346816512*a**16*b**3*x**7 + 520224768*a**15*b**4*x**9 + 520224768*a**14*b*
*5*x**11 + 346816512*a**13*b**6*x**13 + 148635648*a**12*b**7*x**15 + 371589
12*a**11*b**8*x**17 + 4128768*a**10*b**9*x**19)
```

Giac [A] time = 2.36857, size = 181, normalized size = 0.87

$$-\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{10}}-\frac{1}{a^{10}x}-\frac{10420767\,b^9x^{17}+88937058\,ab^8x^{15}+334408914\,a^2b^7x^{13}+724860666\,a^3b^6x^{11}+\cdots}{412\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^10) - 1/(a^10*x) - 1/412
8768*(10420767*b^9*x^17 + 88937058*a*b^8*x^15 + 334408914*a^2*b^7*x^13 + 72
4860666*a^3*b^6*x^11 + 993296384*a^4*b^5*x^9 + 884769030*a^5*b^4*x^7 + 5037
30990*a^6*b^3*x^5 + 169799070*a^7*b^2*x^3 + 26738145*a^8*b*x)/((b*x^2 + a)^
9*a^10)
```

3.223 $\int \frac{1}{x^4(a+bx^2)^{10}} dx$

Optimal. Leaf size=220

$$\frac{1616615b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{46189}{73728a^7x^3\left(a+bx^2\right)^3}+\frac{4199}{12288a^6x^3\left(a+bx^2\right.}$$

```
[Out] -1616615/(196608*a^10*x^3) + (1616615*b)/(65536*a^11*x) + 1/(18*a*x^3*(a +
b*x^2)^9) + 7/(96*a^2*x^3*(a + b*x^2)^8) + 19/(192*a^3*x^3*(a + b*x^2)^7) +
 323/(2304*a^4*x^3*(a + b*x^2)^6) + 323/(1536*a^5*x^3*(a + b*x^2)^5) + 4199
/(12288*a^6*x^3*(a + b*x^2)^4) + 46189/(73728*a^7*x^3*(a + b*x^2)^3) + 4618
9/(32768*a^8*x^3*(a + b*x^2)^2) + 323323/(65536*a^9*x^3*(a + b*x^2)) + (161
6615*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(23/2))
```

Rubi [A] time = 0.140752, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{1616615b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{46189}{73728a^7x^3\left(a+bx^2\right)^3}+\frac{4199}{12288a^6x^3\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] -1616615/(196608*a^10*x^3) + (1616615*b)/(65536*a^11*x) + 1/(18*a*x^3*(a +
b*x^2)^9) + 7/(96*a^2*x^3*(a + b*x^2)^8) + 19/(192*a^3*x^3*(a + b*x^2)^7) +
 323/(2304*a^4*x^3*(a + b*x^2)^6) + 323/(1536*a^5*x^3*(a + b*x^2)^5) + 4199
/(12288*a^6*x^3*(a + b*x^2)^4) + 46189/(73728*a^7*x^3*(a + b*x^2)^3) + 4618
9/(32768*a^8*x^3*(a + b*x^2)^2) + 323323/(65536*a^9*x^3*(a + b*x^2)) + (161
6615*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(23/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7\int \frac{1}{x^4\left(a+bx^2\right)^9}\,dx}{6a}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{133\int \frac{1}{x^4\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323\int \frac{1}{x^4\left(a+bx^2\right)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{1615\int}{7}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{}{1536a^5}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{}{1536a^5}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{}{1536a^5}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{}{1536a^5}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{}{1536a^5}$$

$$= -\frac{1616615}{196608a^{10}x^3}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a-\right.}$$

$$= -\frac{1616615}{196608a^{10}x^3}+\frac{1616615b}{65536a^{11}x}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+$$

$$= -\frac{1616615}{196608a^{10}x^3}+\frac{1616615b}{65536a^{11}x}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+$$

Mathematica [A] time = 0.0834459, size = 157, normalized size = 0.71

$$\frac{\frac{\sqrt{a}\left(483044562a^2b^8x^{16}+1071677178a^3b^7x^{14}+1513521152a^4b^6x^{12}+1404993798a^5b^5x^{10}+850547502a^6b^4x^8+318434718a^7b^3x^6+63897057a^8b^2x^4+4128768a^9\right.}{x^3\left(a+bx^2\right)^9}}{589824a^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] ((Sqrt[a]*(-196608*a^10 + 4128768*a^9*b*x^2 + 63897057*a^8*b^2*x^4 + 318434
718*a^7*b^3*x^6 + 850547502*a^6*b^4*x^8 + 1404993798*a^5*b^5*x^10 + 1513521
```

```
152*a^4*b^6*x^12 + 1071677178*a^3*b^7*x^14 + 483044562*a^2*b^8*x^16 + 12609
5970*a*b^9*x^18 + 14549535*b^10*x^20))/(x^3*(a + b*x^2)^9) + 14549535*b^(3/
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*a^(23/2))
```

Maple [A] time = 0.02, size = 219, normalized size = 1.

$$-\frac{1}{3\,a^{10}x^{3}}+10\,\frac{b}{a^{11}x}+\frac{1987865\,b^{2}x}{65536\,a^{3}\left(bx^{2}+a\right)^{9}}+\frac{20435525\,b^{3}x^{3}}{98304\,a^{4}\left(bx^{2}+a\right)^{9}}+\frac{21103775\,b^{4}x^{5}}{32768\,a^{5}\left(bx^{2}+a\right)^{9}}+\frac{38143787\,b^{5}x^{7}}{32768\,a^{6}\left(bx^{2}+a\right)^{9}}+\frac{2401}{18\,a^{7}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^10,x)
```

```
[Out] -1/3/a^10/x^3+10*b/a^11/x+1987865/65536/a^3*b^2/(b*x^2+a)^9*x+20435525/9830
4/a^4*b^3/(b*x^2+a)^9*x^3+21103775/32768/a^5*b^4/(b*x^2+a)^9*x^5+38143787/3
2768/a^6*b^5/(b*x^2+a)^9*x^7+24013/18/a^7*b^6/(b*x^2+a)^9*x^9+32405717/3276
8/a^8*b^7/(b*x^2+a)^9*x^11+15137633/32768/a^9*b^8/(b*x^2+a)^9*x^13+12201403
/98304/a^10*b^9/(b*x^2+a)^9*x^15+961255/65536/a^11*b^10/(b*x^2+a)^9*x^17+16
16615/65536/a^11*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32256, size = 1777, normalized size = 8.08

$$\left[\frac{29099070\,b^{10}x^{20}+252191940\,ab^{9}x^{18}+966089124\,a^{2}b^{8}x^{16}+2143354356\,a^{3}b^{7}x^{14}+3027042304\,a^{4}b^{6}x^{12}+28099875960}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(29099070*b^10*x^20 + 252191940*a*b^9*x^18 + 966089124*a^2*b^8*x
^16 + 2143354356*a^3*b^7*x^14 + 3027042304*a^4*b^6*x^12 + 2809987596*a^5*b^
5*x^10 + 1701095004*a^6*b^4*x^8 + 636869436*a^7*b^3*x^6 + 127794114*a^8*b^2
*x^4 + 8257536*a^9*b*x^2 - 393216*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19
 + 36*a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11
+ 84*a^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^11*b^9*x^21 + 9*a^12*b^8
*x^19 + 36*a^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*
b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3), 1/
589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^16
```

```
+ 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*x^
10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4 +
 4128768*a^9*b*x^2 - 196608*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19 + 36*
a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11 + 84*a
^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(b/a)*arctan(x
*sqrt(b/a)))/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a^13*b^7*x^17 + 84*a^14*
b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^1
8*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 3.04979, size = 200, normalized size = 0.91

$$\frac{1616615\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\, \sqrt{ab}a^{11}} + \frac{30\, bx^2 - a}{3\, a^{11}x^3} + \frac{8651295\, b^{10}x^{17} + 73208418\, ab^9x^{15} + 272477394\, a^2b^8x^{13} + 583302906\, a^3b^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11) + 1/3*(30*b*x^2 -
a)/(a^11*x^3) + 1/589824*(8651295*b^10*x^17 + 73208418*a*b^9*x^15 + 2724773
94*a^2*b^8*x^13 + 583302906*a^3*b^7*x^11 + 786857984*a^4*b^6*x^9 + 68658816
6*a^5*b^5*x^7 + 379867950*a^6*b^4*x^5 + 122613150*a^7*b^3*x^3 + 17890785*a^
8*b^2*x)/((b*x^2 + a)^9*a^11)
```

3.224 $\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=233

$$-\frac{7436429b^2}{65536a^{12}x}-\frac{7436429b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{25/2}}+\frac{7436429b}{196608a^{11}x^3}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+bx^2\right)^2}+\frac{965}{73728a^7x^5}$$

```
[Out] -7436429/(327680*a^10*x^5) + (7436429*b)/(196608*a^11*x^3) - (7436429*b^2)/
(65536*a^12*x) + 1/(18*a*x^5*(a + b*x^2)^9) + 23/(288*a^2*x^5*(a + b*x^2)^8
) + 23/(192*a^3*x^5*(a + b*x^2)^7) + 437/(2304*a^4*x^5*(a + b*x^2)^6) + 742
9/(23040*a^5*x^5*(a + b*x^2)^5) + 7429/(12288*a^6*x^5*(a + b*x^2)^4) + 9657
7/(73728*a^7*x^5*(a + b*x^2)^3) + 1062347/(294912*a^8*x^5*(a + b*x^2)^2) +
1062347/(65536*a^9*x^5*(a + b*x^2)) - (7436429*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(65536*a^(25/2))
```

Rubi [A] time = 0.157845, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{7436429b^2}{65536a^{12}x}-\frac{7436429b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{25/2}}+\frac{7436429b}{196608a^{11}x^3}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+bx^2\right)^2}+\frac{965}{73728a^7x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^10),x]
```


```
[Out] -7436429/(327680*a^10*x^5) + (7436429*b)/(196608*a^11*x^3) - (7436429*b^2)/
(65536*a^12*x) + 1/(18*a*x^5*(a + b*x^2)^9) + 23/(288*a^2*x^5*(a + b*x^2)^8
) + 23/(192*a^3*x^5*(a + b*x^2)^7) + 437/(2304*a^4*x^5*(a + b*x^2)^6) + 742
9/(23040*a^5*x^5*(a + b*x^2)^5) + 7429/(12288*a^6*x^5*(a + b*x^2)^4) + 9657
7/(73728*a^7*x^5*(a + b*x^2)^3) + 1062347/(294912*a^8*x^5*(a + b*x^2)^2) +
1062347/(65536*a^9*x^5*(a + b*x^2)) - (7436429*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(65536*a^(25/2))
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23\int \frac{1}{x^6\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{161\int \frac{1}{x^6\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437\int \frac{1}{x^6\left(a+bx^2\right)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} + \frac{7429}{}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} + \frac{}{23040}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} + \frac{}{23040}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} + \frac{}{23040}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} + \frac{}{23040}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} + \frac{}{23040}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a\right.}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{}{192a}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{}{192a}$$

Mathematica [A] time = 0.0875366, size = 169, normalized size = 0.73

$$-\frac{\frac{\sqrt{a}\left(11110024926a^2b^9x^{18}+24648575094a^3b^8x^{16}+34810986496a^4b^7x^{14}+32314857354a^5b^6x^{12}+19562592546a^6b^5x^{10}+7323998514a^7b^4x^8+1469632311a^8b\right.}{x^5\left(a+bx^2\right)^9}}{2949120a^{25/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(589824*a^11 - 4521984*a^10*b*x^2 + 94961664*a^9*b^2*x^4 + 1469
632311*a^8*b^3*x^6 + 7323998514*a^7*b^4*x^8 + 19562592546*a^6*b^5*x^10 + 32
314857354*a^5*b^6*x^12 + 34810986496*a^4*b^7*x^14 + 24648575094*a^3*b^8*x^1
6 + 11110024926*a^2*b^9*x^18 + 2900207310*a*b^10*x^20 + 334639305*b^11*x^22
))/(x^5*(a + b*x^2)^9)) - 334639305*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
949120*a^(25/2))
```

Maple [A] time = 0.022, size = 230, normalized size = 1.

$$-\frac{1}{5\,a^{10}x^{5}}-55\,\frac{b^{2}}{a^{12}x}+\frac{10\,b}{3\,a^{11}x^{3}}-\frac{6981491\,b^{3}x}{65536\,a^{4}\left(bx^{2}+a\right)^{9}}-\frac{74539223\,b^{4}x^{3}}{98304\,a^{5}\left(bx^{2}+a\right)^{9}}-\frac{394553929\,b^{5}x^{5}}{163840\,a^{6}\left(bx^{2}+a\right)^{9}}-\frac{725918941\,b^{6}x^{7}}{163840\,a^{7}\left(bx^{2}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^10,x)
```

```
[Out] -1/5/a^10/x^5-55*b^2/a^12/x+10/3*b/a^11/x^3-6981491/65536/a^4*b^3/(b*x^2+a)
^9*x-74539223/98304/a^5*b^4/(b*x^2+a)^9*x^3-394553929/163840/a^6*b^5/(b*x^2
+a)^9*x^5-725918941/163840/a^7*b^6/(b*x^2+a)^9*x^7-463199/90/a^8*b^7/(b*x^2
+a)^9*x^9-631790371/163840/a^9*b^8/(b*x^2+a)^9*x^11-297702839/163840/a^10*b
^9/(b*x^2+a)^9*x^13-48340777/98304/a^11*b^10/(b*x^2+a)^9*x^15-3831949/65536
/a^12*b^11/(b*x^2+a)^9*x^17-7436429/65536/a^12*b^3/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43982, size = 1901, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/5898240*(669278610*b^11*x^22 + 5800414620*a*b^10*x^20 + 22220049852*a^2
*b^9*x^18 + 49297150188*a^3*b^8*x^16 + 69621972992*a^4*b^7*x^14 + 646297147
08*a^5*b^6*x^12 + 39125185092*a^6*b^5*x^10 + 14647997028*a^7*b^4*x^8 + 2939
264622*a^8*b^3*x^6 + 189923328*a^9*b^2*x^4 - 9043968*a^10*b*x^2 + 1179648*a
^11 - 334639305*(b^11*x^23 + 9*a*b^10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x
```

```
^17 + 126*a^4*b^7*x^15 + 126*a^5*b^6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^
9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*
a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 +
36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5), -1/2949120*(334639305*b^11*x^22
 + 2900207310*a*b^10*x^20 + 11110024926*a^2*b^9*x^18 + 24648575094*a^3*b^8*
x^16 + 34810986496*a^4*b^7*x^14 + 32314857354*a^5*b^6*x^12 + 19562592546*a^
6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 1469632311*a^8*b^3*x^6 + 94961664*a^9
*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11 + 334639305*(b^11*x^23 + 9*a*b^
10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x^17 + 126*a^4*b^7*x^15 + 126*a^5*b^
6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sq
rt(b/a)*arctan(x*sqrt(b/a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7
*x^19 + 84*a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*
b^3*x^11 + 36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.09083, size = 215, normalized size = 0.92

$$-\frac{7436429\, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{12}} - \frac{825\, b^2x^4 - 50\,abx^2 + 3\,a^2}{15\,a^{12}x^5} - \frac{172437705\, b^{11}x^{17} + 1450223310\, ab^{10}x^{15} + 5358651102\, a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^12) - 1/15*(825*b^2*x
^4 - 50*a*b*x^2 + 3*a^2)/(a^12*x^5) - 1/2949120*(172437705*b^11*x^17 + 1450
223310*a*b^10*x^15 + 5358651102*a^2*b^9*x^13 + 11372226678*a^3*b^8*x^11 + 1
5178104832*a^4*b^7*x^9 + 13066540938*a^5*b^6*x^7 + 7101970722*a^6*b^5*x^5 +
 2236176690*a^7*b^4*x^3 + 314167095*a^8*b^3*x)/((b*x^2 + a)^9*a^12)
```

3.225 $\int \frac{x^3}{a-bx^2}\,dx$

Optimal. Leaf size=28

$$-\frac{a\log\left(a-bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

```
[Out] -x^2/(2*b) - (a*Log[a - b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0220877, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$-\frac{a\log\left(a-bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2),x]
```

```
[Out] -x^2/(2*b) - (a*Log[a - b*x^2])/(2*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a-bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a-bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{b}-\frac{a}{b(-a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{x^2}{2b}-\frac{a\log\left(a-bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0058522, size = 28, normalized size = 1.

$$-\frac{a\log\left(a-bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2),x]
```

```
[Out] -x^2/(2*b) - (a*Log[a - b*x^2])/(2*b^2)
```

Maple [A] time = 0.002, size = 26, normalized size = 0.9

$$-\frac{x^2}{2\,b}-\frac{a\ln\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a),x)
```

```
[Out] -1/2*x^2/b-1/2/b^2*a*ln(b*x^2-a)
```

Maxima [A] time = 1.52083, size = 34, normalized size = 1.21

$$-\frac{x^2}{2\,b}-\frac{a\log\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/b - 1/2*a*log(b*x^2 - a)/b^2
```

Fricas [A] time = 1.21094, size = 50, normalized size = 1.79

$$-\frac{bx^2+a\log\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2 + a*log(b*x^2 - a))/b^2
```

Sympy [A] time = 0.296177, size = 22, normalized size = 0.79

$$-\frac{a\log\left(-a+bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a),x)
```

```
[Out] -a*log(-a + b*x**2)/(2*b**2) - x**2/(2*b)
```

Giac [A] time = 2.58651, size = 35, normalized size = 1.25

$$-\frac{x^2}{2\,b}-\frac{a\log\left(\left|bx^2-a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*x^2/b - 1/2*a*log(abs(b*x^2 - a))/b^2
```

3.226 $\int \frac{x^2}{a-bx^2} dx$

Optimal. Leaf size=31

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

```
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.012385, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {321, 208}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2),x]
```

```
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a-bx^2}\,dx &= -\frac{x}{b}+\frac{a\int \frac{1}{a-bx^2}\,dx}{b}\\ &= -\frac{x}{b}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0089459, size = 31, normalized size = 1.

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2),x]
```

```
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$-\frac{x}{b}+\frac{a}{b}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a),x)
```

```
[Out] -x/b+1/b*a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33562, size = 165, normalized size = 5.32

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{\frac{a}{b}}+a}{bx^2-a}\right)-2\,x}{2\,b},\,-\frac{\sqrt{-\frac{a}{b}}\arctan\left(\frac{bx\sqrt{-\frac{a}{b}}}{a}\right)+x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/b)*log((b*x^2 + 2*b*x*sqrt(a/b) + a)/(b*x^2 - a)) - 2*x)/b, -(
sqrt(-a/b)*arctan(b*x*sqrt(-a/b)/a) + x)/b]
```

Sympy [A] time = 0.294116, size = 49, normalized size = 1.58

$$-\frac{\sqrt{\frac{a}{b^3}}\log\left(-b\sqrt{\frac{a}{b^3}}+x\right)}{2}+\frac{\sqrt{\frac{a}{b^3}}\log\left(b\sqrt{\frac{a}{b^3}}+x\right)}{2}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(a/b**3)*log(-b*sqrt(a/b**3) + x)/2 + sqrt(a/b**3)*log(b*sqrt(a/b**3)
+ x)/2 - x/b
```

Giac [A] time = 1.7587, size = 39, normalized size = 1.26

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - x/b
```

3.227 $\int \frac{x}{a-bx^2} dx$

Optimal. Leaf size=16

$$-\frac{\log \left(a-bx^2\right)}{2b}$$

[Out] -Log[a - b*x^2]/(2*b)

Rubi [A] time = 0.0028838, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {260}

$$-\frac{\log \left(a-bx^2\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2),x]

[Out] -Log[a - b*x^2]/(2*b)

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a-bx^2} dx = -\frac{\log \left(a-bx^2\right)}{2b}$$

Mathematica [A] time = 0.0022229, size = 16, normalized size = 1.

$$-\frac{\log \left(a-bx^2\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b*x^2),x]

[Out] -Log[a - b*x^2]/(2*b)

Maple [A] time = 0., size = 16, normalized size = 1.

$$-\frac{\ln \left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a),x)
```

```
[Out] -1/2/b*ln(b*x^2-a)
```

Maxima [A] time = 1.55631, size = 20, normalized size = 1.25

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 - a)/b
```

Fricas [A] time = 1.2447, size = 31, normalized size = 1.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 - a)/b
```

Sympy [A] time = 0.108907, size = 12, normalized size = 0.75

$$-\frac{\log\left(-a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a),x)
```

```
[Out] -log(-a + b*x**2)/(2*b)
```

Giac [A] time = 1.694, size = 22, normalized size = 1.38

$$-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b*x^2 - a))/b
```

3.228 $\int \frac{1}{a-bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.006682, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-bx^2} dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.0037806, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$\text{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31384, size = 151, normalized size = 6.29

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx^2+2\sqrt{ab}x+a}{bx^2-a}\right)}{2\,ab},\,-\frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a))/(a*b), -sqrt(-a
*b)*arctan(sqrt(-a*b)*x/a)/(a*b)]
```

Sympy [B] time = 0.130379, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{ab}}\log\left(-a\sqrt{\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{\frac{1}{ab}}\log\left(a\sqrt{\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a),x)
```

```
[Out] -sqrt(1/(a*b))*log(-a*sqrt(1/(a*b)) + x)/2 + sqrt(1/(a*b))*log(a*sqrt(1/(a*
b)) + x)/2
```

Giac [A] time = 1.55397, size = 24, normalized size = 1.

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -arctan(b*x/sqrt(-a*b))/sqrt(-a*b)
```

3.229 $\int \frac{1}{x(a-bx^2)} dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}$$

```
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Rubi [A] time = 0.0122526, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a - bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)} dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right)}{2a} + \frac{b\,\text{Subst}\left(\int \frac{1}{a-bx} dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.006051, size = 23, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Maple [A] time = 0.005, size = 23, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln\left(bx^2 - a\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a),x)
```

```
[Out] ln(x)/a-1/2/a*ln(b*x^2-a)
```

Maxima [A] time = 2.50863, size = 34, normalized size = 1.48

$$-\frac{\log\left(bx^2 - a\right)}{2\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 - a)/a + 1/2*log(x^2)/a
```

Fricas [A] time = 1.26249, size = 49, normalized size = 2.13

$$-\frac{\log\left(bx^2 - a\right) - 2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 - a) - 2*log(x))/a
```

Sympy [A] time = 0.192758, size = 15, normalized size = 0.65

$$\frac{\log(x)}{a} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a),x)
```

```
[Out] log(x)/a - log(-a/b + x**2)/(2*a)
```

Giac [A] time = 1.8525, size = 35, normalized size = 1.52

$$\frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(\left|bx^2 - a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 - a))/a
```

3.230 $\int \frac{1}{x^2(a-bx^2)}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0121497, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {325, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-bx^2\right)}\,dx &= -\frac{1}{ax}+\frac{b\int \frac{1}{a-bx^2}\,dx}{a}\\ &= -\frac{1}{ax}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0109404, size = 33, normalized size = 1.

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Maple [A] time = 0.004, size = 29, normalized size = 0.9

$$\frac{b}{a}\operatorname{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a),x)
```

```
[Out] b/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))-1/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2156, size = 173, normalized size = 5.24

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)-2}{2\,ax}, -\frac{x\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) - 2)/(a*x)
, -(x*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 1)/(a*x)]
```

Sympy [B] time = 0.329602, size = 58, normalized size = 1.76

$$-\frac{\sqrt{\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}+\frac{\sqrt{\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(b/a**3)*log(-a**2*sqrt(b/a**3)/b + x)/2 + sqrt(b/a**3)*log(a**2*sqrt(
b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A] time = 1.93039, size = 42, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/(a*x)
```

3.231 $\int \frac{1}{x^3(a-bx^2)}\,dx$

Optimal. Leaf size=35

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Rubi [A] time = 0.0235559, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a-bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{b}{a^2x}+\frac{b^2}{a^2(a-bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0082722, size = 35, normalized size = 1.

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$-\frac{1}{2\,a{x}^{2}}+\frac{b\ln \left(x\right)}{{a}^{2}}-\frac{b\ln \left(b{x}^{2}-a\right)}{2\,{a}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a),x)
```

```
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/2*b/a^2*ln(b*x^2-a)
```

Maxima [A] time = 1.58463, size = 47, normalized size = 1.34

$$-\frac{b\log \left(bx^{2}-a\right)}{2\,a^{2}}+\frac{b\log \left(x^{2}\right)}{2\,a^{2}}-\frac{1}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(b*x^2 - a)/a^2 + 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Fricas [A] time = 1.26507, size = 81, normalized size = 2.31

$$-\frac{bx^{2}\log \left(bx^{2}-a\right)-2\,bx^{2}\log \left(x\right)+a}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*log(b*x^2 - a) - 2*b*x^2*log(x) + a)/(a^2*x^2)
```

Sympy [A] time = 0.417309, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^{2}}+\frac{b\log \left(x\right)}{a^{2}}-\frac{b\log \left(-\frac{a}{b}+x^{2}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) + b*log(x)/a**2 - b*log(-a/b + x**2)/(2*a**2)
```

Giac [A] time = 2.96638, size = 58, normalized size = 1.66

$$\frac{b\log\left(x^2\right)}{2\,a^2} - \frac{b\log\left(\left|bx^2 - a\right|\right)}{2\,a^2} - \frac{bx^2 + a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2)/a^2 - 1/2*b*log(abs(b*x^2 - a))/a^2 - 1/2*(b*x^2 + a)/(a^2*x
^2)
```

3.232 $\int \frac{x^3}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=35

$$\frac{a}{2b^2\left(a-bx^2\right)}+\frac{\log\left(a-bx^2\right)}{2b^2}$$

```
[Out] a/(2*b^2*(a - b*x^2)) + Log[a - b*x^2]/(2*b^2)
```

Rubi [A] time = 0.0269475, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{a}{2b^2\left(a-bx^2\right)}+\frac{\log\left(a-bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a - b*x^2)) + Log[a - b*x^2]/(2*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a-bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a-bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a}{b(-a+bx)^2}+\frac{1}{b(-a+bx)}\right)dx,x,x^2\right)\\
&= \frac{a}{2b^2\left(a-bx^2\right)}+\frac{\log\left(a-bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.012539, size = 29, normalized size = 0.83

$$\frac{\frac{a}{a-bx^2}+\log\left(a-bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^2,x]
```

```
[Out] (a/(a - b*x^2) + Log[a - b*x^2])/(2*b^2)
```

Maple [A] time = 0.007, size = 34, normalized size = 1.

$$-\frac{a}{2\,b^{2}\left(b x^{2}-a\right)}+\frac{\ln\left(b x^{2}-a\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b^2*a/(b*x^2-a)+1/2/b^2*ln(b*x^2-a)
```

Maxima [A] time = 1.09328, size = 47, normalized size = 1.34

$$-\frac{a}{2\left(b^{3} x^{2}-a b^{2}\right)}+\frac{\log\left(b x^{2}-a\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a/(b^3*x^2 - a*b^2) + 1/2*log(b*x^2 - a)/b^2
```

Fricas [A] time = 1.24491, size = 76, normalized size = 2.17

$$\frac{\left(b x^{2}-a\right)\log\left(b x^{2}-a\right)-a}{2\left(b^{3} x^{2}-a b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 - a)*log(b*x^2 - a) - a)/(b^3*x^2 - a*b^2)
```

Sympy [A] time = 0.352681, size = 29, normalized size = 0.83

$$-\frac{a}{-2ab^{2}+2b^{3}x^{2}}+\frac{\log\left(-a+bx^{2}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**2,x)
```

```
[Out] -a/(-2*a*b**2 + 2*b**3*x**2) + log(-a + b*x**2)/(2*b**2)
```

Giac [A] time = 2.0632, size = 72, normalized size = 2.06

$$-\frac{\frac{\log\left(\frac{|bx^2-a|}{(bx^2-a)^2|b|}\right)}{b}+\frac{a}{(bx^2-a)b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(b*x^2 - a)/((b*x^2 - a)^2*abs(b)))/b + a/((b*x^2 - a)*b))/b
```

3.233 $\int \frac{x^2}{\left(a-bx^2\right)^2} \, dx$

Optimal. Leaf size=46

$$\frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

```
[Out] x/(2*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.0135534, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {288, 208}

$$\frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^2,x]
```

```
[Out] x/(2*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a-bx^2\right)^2} \, dx &= \frac{x}{2b\left(a-bx^2\right)}-\frac{\int \frac{1}{a-bx^2} \, dx}{2b}\\ &= \frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.029005, size = 47, normalized size = 1.02

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(bx^2-a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^2,x]
```

```
[Out] -x/(2*b*(-a + b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.006, size = 38, normalized size = 0.8

$$-\frac{x}{2\,b\left(bx^2-a\right)}-\frac{1}{2\,b}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2-a)-1/2/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30328, size = 263, normalized size = 5.72

$$\left[-\frac{2\,abx-\left(bx^2-a\right)\sqrt{ab}\log\left(\frac{bx^2-2\,\sqrt{ab}x+a}{bx^2-a}\right)}{4\left(ab^3x^2-a^2b^2\right)},-\frac{abx-\left(bx^2-a\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{2\left(ab^3x^2-a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 - 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a*b^3*x^2 - a^2*b^2), -1/2*(a*b*x - (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a*b^3*x^2 - a^2*b^2)]
```

Sympy [A] time = 0.352352, size = 71, normalized size = 1.54

$$-\frac{x}{-2ab+2b^2x^2}+\frac{\sqrt{\frac{1}{ab^3}}\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}-\frac{\sqrt{\frac{1}{ab^3}}\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a*b + 2*b**2*x**2) + sqrt(1/(a*b**3))*log(-a*b*sqrt(1/(a*b**3)) + x)
/4 - sqrt(1/(a*b**3))*log(a*b*sqrt(1/(a*b**3)) + x)/4
```

Giac [A] time = 2.56562, size = 53, normalized size = 1.15

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}b} - \frac{x}{2\left(bx^2 - a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - 1/2*x/((b*x^2 - a)*b)
```

3.234 $\int \frac{x}{(a-bx^2)^2} \, dx$

Optimal. Leaf size=17

$$\frac{1}{2b(a-bx^2)}$$

[Out] 1/(2*b*(a - b*x^2))

Rubi [A] time = 0.0028674, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2)^2,x]

[Out] 1/(2*b*(a - b*x^2))

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^2} \, dx = \frac{1}{2b(a-bx^2)}$$

Mathematica [A] time = 0.002445, size = 17, normalized size = 1.

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b*x^2)^2,x]

[Out] 1/(2*b*(a - b*x^2))

Maple [A] time = 0., size = 17, normalized size = 1.

$$-\frac{1}{2\,b\,(bx^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2-a)
```

Maxima [A] time = 2.08676, size = 22, normalized size = 1.29

$$-\frac{1}{2\left(bx^2-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

Fricas [A] time = 1.21364, size = 30, normalized size = 1.76

$$-\frac{1}{2\left(b^2x^2-ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 - a*b)
```

Sympy [A] time = 0.30581, size = 15, normalized size = 0.88

$$-\frac{1}{-2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a*b + 2*b**2*x**2)
```

Giac [A] time = 1.58455, size = 22, normalized size = 1.29

$$-\frac{1}{2\left(bx^2-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

3.235 $\int \frac{1}{\left(a-bx^2\right)^2} dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}$$

```
[Out] x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0122472, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-bx^2\right)^2} dx &= \frac{x}{2a\left(a-bx^2\right)}+\frac{\int \frac{1}{a-bx^2} dx}{2a}\\ &= \frac{x}{2a\left(a-bx^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0163193, size = 47, normalized size = 1.02

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{x}{2a\left(bx^2-a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-2),x]
```

```
[Out] -x/(2*a*(-a + b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$-\frac{x}{2\,a\left(bx^{2}-a\right)}+\frac{1}{2\,a}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^2,x)
```

```
[Out] -1/2*x/a/(b*x^2-a)+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25269, size = 263, normalized size = 5.72

$$\left[-\frac{2\,abx-\left(bx^{2}-a\right)\sqrt{ab}\log\left(\frac{bx^{2}+2\,\sqrt{ab}x+a}{bx^{2}-a}\right)}{4\left(a^{2}b^{2}x^{2}-a^{3}b\right)},-\frac{abx+\left(bx^{2}-a\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{2\left(a^{2}b^{2}x^{2}-a^{3}b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a^2*b^2*x^2 - a^3*b), -1/2*(a*b*x + (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a^2*b^2*x^2 - a^3*b)]
```

Sympy [A] time = 0.375821, size = 71, normalized size = 1.54

$$-\frac{x}{-2a^{2}+2abx^{2}}-\frac{\sqrt{\frac{1}{a^{3}b}}\log\left(-a^{2}\sqrt{\frac{1}{a^{3}b}}+x\right)}{4}+\frac{\sqrt{\frac{1}{a^{3}b}}\log\left(a^{2}\sqrt{\frac{1}{a^{3}b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a**2 + 2*a*b*x**2) - sqrt(1/(a**3*b))*log(-a**2*sqrt(1/(a**3*b)) + x
)/4 + sqrt(1/(a**3*b))*log(a**2*sqrt(1/(a**3*b)) + x)/4
```

Giac [A] time = 2.05377, size = 53, normalized size = 1.15

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}a} - \frac{x}{2\left(bx^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

3.236 $\int \frac{1}{x\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=40

$$-\frac{\log\left(a-bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a-bx^2\right)}$$

```
[Out] 1/(2*a*(a - b*x^2)) + Log[x]/a^2 - Log[a - b*x^2]/(2*a^2)
```

Rubi [A] time = 0.0279616, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$-\frac{\log\left(a-bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a - b*x^2)) + Log[x]/a^2 - Log[a - b*x^2]/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a-bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x}+\frac{b}{a(a-bx)^2}+\frac{b}{a^2(a-bx)}\right)dx, x, x^2\right)\\
&= \frac{1}{2a\left(a-bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a-bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0160346, size = 35, normalized size = 0.88

$$\frac{\frac{a}{a-bx^2}-\log\left(a-bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^2),x]
```

```
[Out] (a/(a - b*x^2) + 2*Log[x] - Log[a - b*x^2])/(2*a^2)
```

Maple [A] time = 0.008, size = 39, normalized size = 1.

$$\frac{\ln(x)}{a^2} - \frac{1}{2a(bx^2 - a)} - \frac{\ln(bx^2 - a)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^2,x)
```

```
[Out] ln(x)/a^2-1/2/a/(b*x^2-a)-1/2/a^2*ln(b*x^2-a)
```

Maxima [A] time = 1.20726, size = 55, normalized size = 1.38

$$-\frac{1}{2(abx^2 - a^2)} - \frac{\log(bx^2 - a)}{2a^2} + \frac{\log(x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b*x^2 - a^2) - 1/2*log(b*x^2 - a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A] time = 1.24117, size = 108, normalized size = 2.7

$$-\frac{(bx^2 - a)\log(bx^2 - a) - 2(bx^2 - a)\log(x) + a}{2(a^2bx^2 - a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 - a)*log(b*x^2 - a) - 2*(b*x^2 - a)*log(x) + a)/(a^2*b*x^2 - a
^3)
```

Sympy [A] time = 0.447608, size = 34, normalized size = 0.85

$$-\frac{1}{-2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(-a/b + x**2)/(2*a**2)
```

Giac [A] time = 2.76258, size = 69, normalized size = 1.72

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2 - a\right|\right)}{2\,a^2} + \frac{bx^2 - 2\,a}{2\left(bx^2 - a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 - a))/a^2 + 1/2*(b*x^2 - 2*a)/((b*x^2
- a)*a^2)
```

3.237 $\int \frac{1}{x^2 \left(a-bx^2\right)^2} \, dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(5/2))
```

Rubi [A] time = 0.0184034, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(5/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(a-bx^2)^2}\,dx = \frac{1}{2ax(a-bx^2)} + \frac{3\int \frac{1}{x^2(a-bx^2)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)} + \frac{(3b)\int \frac{1}{a-bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.0355046, size = 56, normalized size = 0.97

$$-\frac{bx}{2a^2(bx^2-a)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(-a + b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

Maple [A] time = 0.01, size = 47, normalized size = 0.8

$$-\frac{1}{a^2x} - \frac{b}{a^2}\left(\frac{x}{2\,bx^2-2\,a} - \frac{3}{2}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-b/a^2*(1/2*x/(b*x^2-a)-3/2/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32387, size = 288, normalized size = 4.97

$$\left[-\frac{6\,bx^2 - 3\left(bx^3 - ax\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right) - 4\,a}{4\left(a^2bx^3 - a^3x\right)}, -\frac{3\,bx^2 + 3\left(bx^3 - ax\right)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right) - 2\,a}{2\left(a^2bx^3 - a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 - a*x)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a
)/(b*x^2 - a)) - 4*a)/(a^2*b*x^3 - a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 - a*x)*
sqrt(-b/a)*arctan(x*sqrt(-b/a)) - 2*a)/(a^2*b*x^3 - a^3*x)]
```

Sympy [A] time = 0.483957, size = 83, normalized size = 1.43

$$-\frac{3\sqrt{\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{3\sqrt{\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{-2a+3bx^2}{-2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**2,x)
```

```
[Out] -3*sqrt(b/a**5)*log(-a**3*sqrt(b/a**5)/b + x)/4 + 3*sqrt(b/a**5)*log(a**3*s
qrt(b/a**5)/b + x)/4 - (-2*a + 3*b*x**2)/(-2*a**3*x + 2*a**2*b*x**3)
```

Giac [A] time = 1.97136, size = 68, normalized size = 1.17

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}a^2}-\frac{3\,bx^2-2\,a}{2\left(bx^3-ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/2*(3*b*x^2 - 2*a)/((b*x^
3 - a*x)*a^2)
```

3.238 $\int \frac{1}{x^3\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=52

$$\frac{b}{2a^2\left(a-bx^2\right)}-\frac{b\log\left(a-bx^2\right)}{a^3}+\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*x^
2])/a^3
```

Rubi [A] time = 0.0382507, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{b}{2a^2\left(a-bx^2\right)}-\frac{b\log\left(a-bx^2\right)}{a^3}+\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^2),x]
```

```
[Out] -1/(2*a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*x^
2])/a^3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a-bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^2(a-bx)^2}+\frac{2b^2}{a^3(a-bx)}\right)\,dx,x,x^2\right)\\ &= -\frac{1}{2a^2x^2}+\frac{b}{2a^2\left(a-bx^2\right)}+\frac{2b\log(x)}{a^3}-\frac{b\log\left(a-bx^2\right)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.030803, size = 44, normalized size = 0.85

$$\frac{\frac{ab}{a-bx^2}-2b\log\left(a-bx^2\right)-\frac{a}{x^2}+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^2),x]
```

```
[Out] (-(a/x^2) + (a*b)/(a - b*x^2) + 4*b*Log[x] - 2*b*Log[a - b*x^2])/(2*a^3)
```

Maple [A] time = 0.011, size = 51, normalized size = 1.

$$-\frac{1}{2\,a^{2}x^{2}}+2\,\frac{b\ln\left(x\right)}{a^{3}}-\frac{b}{2\,a^{2}\left(bx^{2}-a\right)}-\frac{b\ln\left(bx^{2}-a\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+2*b*ln(x)/a^3-1/2*b/a^2/(b*x^2-a)-b/a^3*ln(b*x^2-a)
```

Maxima [A] time = 1.9092, size = 77, normalized size = 1.48

$$-\frac{2\,bx^{2}-a}{2\left(a^{2}bx^{4}-a^{3}x^{2}\right)}-\frac{b\log\left(bx^{2}-a\right)}{a^{3}}+\frac{b\log\left(x^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 - a)/(a^2*b*x^4 - a^3*x^2) - b*log(b*x^2 - a)/a^3 + b*log(x^2
)/a^3
```

Fricas [A] time = 1.23887, size = 157, normalized size = 3.02

$$-\frac{2\,abx^{2}-a^{2}+2\left(b^{2}x^{4}-abx^{2}\right)\log\left(bx^{2}-a\right)-4\left(b^{2}x^{4}-abx^{2}\right)\log\left(x\right)}{2\left(a^{3}bx^{4}-a^{4}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 - a^2 + 2*(b^2*x^4 - a*b*x^2)*log(b*x^2 - a) - 4*(b^2*x^4 -
 a*b*x^2)*log(x))/(a^3*b*x^4 - a^4*x^2)
```

Sympy [A] time = 0.592225, size = 49, normalized size = 0.94

$$-\frac{-a+2bx^{2}}{-2a^{3}x^{2}+2a^{2}bx^{4}}+\frac{2b\log\left(x\right)}{a^{3}}-\frac{b\log\left(-\frac{a}{b}+x^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**2,x)
```

```
[Out] -(-a + 2*b*x**2)/(-2*a**3*x**2 + 2*a**2*b*x**4) + 2*b*log(x)/a**3 - b*log(-
a/b + x**2)/a**3
```

Giac [A] time = 1.86492, size = 76, normalized size = 1.46

$$\frac{b \log\left(x^2\right)}{a^3} - \frac{b \log\left(\left|bx^2 - a\right|\right)}{a^3} - \frac{2\,bx^2 - a}{2\,\left(bx^4 - ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*log(x^2)/a^3 - b*log(abs(b*x^2 - a))/a^3 - 1/2*(2*b*x^2 - a)/((b*x^4 - a*
x^2)*a^2)
```

3.239 $\int \frac{x^3}{\left(a-bx^2\right)^3}\,dx$

Optimal. Leaf size=20

$$\frac{x^4}{4a\left(a-bx^2\right)^2}$$

```
[Out] x^4/(4*a*(a - b*x^2)^2)
```

Rubi [A] time = 0.003635, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {264}

$$\frac{x^4}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^3,x]

[Out] x^4/(4*a*(a - b*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a-bx^2\right)^3}\,dx=\frac{x^4}{4a\left(a-bx^2\right)^2}$$

Mathematica [A] time = 0.0085554, size = 25, normalized size = 1.25

$$-\frac{a-2bx^2}{4b^2\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^3,x]

[Out] -(a - 2*b*x^2)/(4*b^2*(a - b*x^2)^2)
```

Maple [A] time = 0.006, size = 35, normalized size = 1.8

$$\frac{1}{2\,b^2\left(bx^2-a\right)}+\frac{a}{4\,b^2\left(bx^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^3,x)
```

```
[Out] 1/2/b^2/(b*x^2-a)+1/4/b^2*a/(b*x^2-a)^2
```

Maxima [A] time = 2.67729, size = 51, normalized size = 2.55

$$\frac{2\,bx^2 - a}{4\,\left(b^4x^4 - 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A] time = 1.2255, size = 72, normalized size = 3.6

$$\frac{2\,bx^2 - a}{4\,\left(b^4x^4 - 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] time = 0.632648, size = 34, normalized size = 1.7

$$\frac{-a + 2bx^2}{4a^2b^2 - 8ab^3x^2 + 4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**3,x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*b**2 - 8*a*b**3*x**2 + 4*b**4*x**4)
```

Giac [A] time = 2.219, size = 35, normalized size = 1.75

$$\frac{2\,bx^2 - a}{4\,\left(bx^2 - a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x^2 - a)/((b*x^2 - a)^2*b^2)
```

3.240 $\int \frac{x^2}{\left(a-bx^2\right)^3} dx$

Optimal. Leaf size=67

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab\left(a-bx^2\right)}+\frac{x}{4b\left(a-bx^2\right)^2}$$

```
[Out] x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0197181, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {288, 199, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab\left(a-bx^2\right)}+\frac{x}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^3,x]
```

```
[Out] x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^3}\,dx = \frac{x}{4b\left(a-bx^2\right)^2} - \frac{\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{4b}$$
$$= \frac{x}{4b\left(a-bx^2\right)^2} - \frac{x}{8ab\left(a-bx^2\right)} - \frac{\int \frac{1}{a-bx^2}\,dx}{8ab}$$
$$= \frac{x}{4b\left(a-bx^2\right)^2} - \frac{x}{8ab\left(a-bx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.0293294, size = 56, normalized size = 0.84

$$\frac{x\left(a+bx^2\right)}{8ab\left(a-bx^2\right)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2))/(8*a*b*(a - b*x^2)^2) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(8*a^(
3/2)*b^(3/2))
```

Maple [A] time = 0.006, size = 52, normalized size = 0.8

$$-\frac{1}{\left(bx^2-a\right)^2}\left(-\frac{x^3}{8\,a}-\frac{x}{8\,b}\right)-\frac{1}{8\,ab}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^3,x)
```

```
[Out] -(-1/8/a*x^3-1/8*x/b)/(b*x^2-a)^2-1/8/b/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29872, size = 394, normalized size = 5.88

$$\left[\frac{2\,ab^2x^3 + 2\,a^2bx + \left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{ab}\log\left(\frac{bx^2 - 2\,\sqrt{ab}x + a}{bx^2 - a}\right)}{16\,\left(a^2b^4x^4 - 2\,a^3b^3x^2 + a^4b^2\right)}, \frac{ab^2x^3 + a^2bx + \left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right.}{8\,\left(a^2b^4x^4 - 2\,a^3b^3x^2 + a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 + 2*a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*log(
(b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^
4*b^2), 1/8*(a*b^2*x^3 + a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-a*b)*a
rctan(sqrt(-a*b)*x/a))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [B] time = 0.47619, size = 104, normalized size = 1.55

$$\frac{\sqrt{\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{\frac{1}{a^3b^3}} + x\right)}{16} - \frac{\sqrt{\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{\frac{1}{a^3b^3}} + x\right)}{16} + \frac{ax + bx^3}{8a^3b - 16a^2b^2x^2 + 8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**3,x)
```

```
[Out] sqrt(1/(a**3*b**3))*log(-a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - sqrt(1/(a**3*
b**3))*log(a**2*b*sqrt(1/(a**3*b**3)) + x)/16 + (a*x + b*x**3)/(8*a**3*b -
16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

Giac [A] time = 2.14536, size = 72, normalized size = 1.07

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}ab} + \frac{bx^3 + ax}{8\,\left(bx^2 - a\right)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a*b) + 1/8*(b*x^3 + a*x)/((b*x^2 - a
)^2*a*b)
```

3.241 $\int \frac{x}{\left(a-bx^2\right)^3}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4b\left(a-bx^2\right)^2}$$

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

Rubi [A] time = 0.0030325, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^3,x]
```

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a-bx^2\right)^3}\,dx = \frac{1}{4b\left(a-bx^2\right)^2}$$

Mathematica [A] time = 0.0020358, size = 17, normalized size = 1.

$$\frac{1}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^3,x]
```

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

Maple [A] time = 0.001, size = 17, normalized size = 1.

$$\frac{1}{4\,b\left(bx^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^3,x)
```

```
[Out] 1/4/b/(b*x^2-a)^2
```

Maxima [A] time = 1.51478, size = 22, normalized size = 1.29

$$\frac{1}{4\left(bx^2-a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

Fricas [A] time = 1.1873, size = 50, normalized size = 2.94

$$\frac{1}{4\left(b^3x^4-2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4/(b^3*x^4 - 2*a*b^2*x^2 + a^2*b)
```

Sympy [B] time = 0.389559, size = 26, normalized size = 1.53

$$\frac{1}{4a^2b-8ab^2x^2+4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**3,x)
```

```
[Out] 1/(4*a**2*b - 8*a*b**2*x**2 + 4*b**3*x**4)
```

Giac [A] time = 2.89824, size = 22, normalized size = 1.29

$$\frac{1}{4\left(bx^2-a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

3.242 $\int \frac{1}{\left(a-bx^2\right)^3} \, dx$

Optimal. Leaf size=64

$$\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a-bx^2\right)^2}$$

```
[Out] x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0179858, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {199, 208}

$$\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-bx^2\right)^3} \, dx &= \frac{x}{4a\left(a-bx^2\right)^2}+\frac{3\int \frac{1}{\left(a-bx^2\right)^2} \, dx}{4a}\\
&= \frac{x}{4a\left(a-bx^2\right)^2}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\int \frac{1}{a-bx^2} \, dx}{8a^2}\\
&= \frac{x}{4a\left(a-bx^2\right)^2}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0383059, size = 56, normalized size = 0.88

$$\frac{5ax-3bx^3}{8a^2\left(a-bx^2\right)^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3),x]
```

```
[Out] (5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.004, size = 61, normalized size = 1.

$$\frac{x}{4\,a\left(bx^2-a\right)^2}+\frac{3}{4\,a}\left(-\frac{x}{2\,a\left(bx^2-a\right)}+\frac{1}{2\,a}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(b*x^2-a)^2+3/4/a*(-1/2*x/a/(b*x^2-a)+1/2/a/(a*b)^(1/2)*arctanh(b*x
/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28842, size = 404, normalized size = 6.31

$$\left[-\frac{6\,ab^2x^3-10\,a^2bx-3\left(b^2x^4-2\,abx^2+a^2\right)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{16\left(a^3b^3x^4-2\,a^4b^2x^2+a^5b\right)},-\frac{3\,ab^2x^3-5\,a^2bx+3\left(b^2x^4-2\,abx^2+a^2\right)\sqrt{-ab}\,\mathrm{arct}}{8\left(a^3b^3x^4-2\,a^4b^2x^2+a^5b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*a*b^2*x^3 - 10*a^2*b*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2
+ a^5*b), -1/8*(3*a*b^2*x^3 - 5*a^2*b*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqr
t(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] time = 0.489597, size = 99, normalized size = 1.55

$$-\frac{3\sqrt{\frac{1}{a^5b}}\log\left(-a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{\frac{1}{a^5b}}\log\left(a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}-\frac{-5ax+3bx^3}{8a^4-16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(1/(a**5*b))*log(-a**3*sqrt(1/(a**5*b)) + x)/16 + 3*sqrt(1/(a**5*b))
*log(a**3*sqrt(1/(a**5*b)) + x)/16 - (-5*a*x + 3*b*x**3)/(8*a**4 - 16*a**3*
b*x**2 + 8*a**2*b**2*x**4)
```

Giac [A] time = 1.61313, size = 66, normalized size = 1.03

$$-\frac{3\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}a^2}-\frac{3bx^3-5ax}{8\left(bx^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x^
2 - a)^2*a^2)
```

3.243 $\int \frac{1}{x(a-bx^2)^3} dx$

Optimal. Leaf size=57

$$\frac{1}{2a^2(a-bx^2)} - \frac{\log(a-bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{4a(a-bx^2)^2}$$

```
[Out] 1/(4*a*(a - b*x^2)^2) + 1/(2*a^2*(a - b*x^2)) + Log[x]/a^3 - Log[a - b*x^2]
/(2*a^3)
```

Rubi [A] time = 0.0376401, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{1}{2a^2(a-bx^2)} - \frac{\log(a-bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a - b*x^2)^2) + 1/(2*a^2*(a - b*x^2)) + Log[x]/a^3 - Log[a - b*x^2]
/(2*a^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a-bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x} + \frac{b}{a(a-bx)^3} + \frac{b}{a^2(a-bx)^2} + \frac{b}{a^3(a-bx)}\right) dx, x, x^2\right) \\
&= \frac{1}{4a(a-bx^2)^2} + \frac{1}{2a^2(a-bx^2)} + \frac{\log(x)}{a^3} - \frac{\log(a-bx^2)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.032024, size = 45, normalized size = 0.79

$$\frac{\frac{a(3a-2bx^2)}{(a-bx^2)^2} - 2\log(a-bx^2) + 4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^3),x]
```

```
[Out] ((a*(3*a - 2*b*x^2))/(a - b*x^2)^2 + 4*Log[x] - 2*Log[a - b*x^2])/(4*a^3)
```

Maple [A] time = 0.011, size = 55, normalized size = 1.

$$\frac{\ln(x)}{a^3} - \frac{1}{2a^2(bx^2 - a)} + \frac{1}{4a(bx^2 - a)^2} - \frac{\ln(bx^2 - a)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^3,x)
```

```
[Out] ln(x)/a^3-1/2/a^2/(b*x^2-a)+1/4/a/(b*x^2-a)^2-1/2/a^3*ln(b*x^2-a)
```

Maxima [A] time = 2.5395, size = 84, normalized size = 1.47

$$-\frac{2bx^2 - 3a}{4(a^2b^2x^4 - 2a^3bx^2 + a^4)} - \frac{\log(bx^2 - a)}{2a^3} + \frac{\log(x^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 - 3*a)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 - a)
/a^3 + 1/2*log(x^2)/a^3
```

Fricas [A] time = 1.21716, size = 197, normalized size = 3.46

$$-\frac{2abx^2 - 3a^2 + 2(b^2x^4 - 2abx^2 + a^2)\log(bx^2 - a) - 4(b^2x^4 - 2abx^2 + a^2)\log(x)}{4(a^3b^2x^4 - 2a^4bx^2 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x^2 - 3*a^2 + 2*(b^2*x^4 - 2*a*b*x^2 + a^2)*log(b*x^2 - a) - 4*
(b^2*x^4 - 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 - 2*a^4*b*x^2 + a^5)
```

Sympy [A] time = 0.589428, size = 56, normalized size = 0.98

$$-\frac{-3a + 2bx^2}{4a^4 - 8a^3bx^2 + 4a^2b^2x^4} + \frac{\log(x)}{a^3} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**3,x)
```

```
[Out] -(-3*a + 2*b*x**2)/(4*a**4 - 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**
3 - log(-a/b + x**2)/(2*a**3)
```

Giac [A] time = 2.57083, size = 85, normalized size = 1.49

$$\frac{\log\left(x^2\right)}{2\,a^3} - \frac{\log\left(\left|bx^2 - a\right|\right)}{2\,a^3} + \frac{3\,b^2x^4 - 8\,abx^2 + 6\,a^2}{4\left(bx^2 - a\right)^2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 - a))/a^3 + 1/4*(3*b^2*x^4 - 8*a*b*x^2
 + 6*a^2)/((b*x^2 - a)^2*a^3)
```

3.244 $\int \frac{1}{x^2(a-bx^2)^3}\,dx$

Optimal. Leaf size=78

$$\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] time = 0.0258514, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^3}\,dx = \frac{1}{4ax\left(a-bx^2\right)^2} + \frac{5\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx}{4a}$$
$$= \frac{1}{4ax\left(a-bx^2\right)^2} + \frac{5}{8a^2x\left(a-bx^2\right)} + \frac{15\int \frac{1}{x^2\left(a-bx^2\right)}\,dx}{8a^2}$$
$$= -\frac{15}{8a^3x} + \frac{1}{4ax\left(a-bx^2\right)^2} + \frac{5}{8a^2x\left(a-bx^2\right)} + \frac{(15b)\int \frac{1}{a-bx^2}\,dx}{8a^3}$$
$$= -\frac{15}{8a^3x} + \frac{1}{4ax\left(a-bx^2\right)^2} + \frac{5}{8a^2x\left(a-bx^2\right)} + \frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.0432956, size = 69, normalized size = 0.88

$$\frac{-8a^2+25abx^2-15b^2x^4}{8a^3x\left(a-bx^2\right)^2} + \frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] (-8*a^2 + 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a - b*x^2)^2) + (15*Sqrt[b]*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Maple [A] time = 0.01, size = 56, normalized size = 0.7

$$-\frac{1}{a^3x} - \frac{b}{a^3}\left(\frac{1}{\left(bx^2-a\right)^2}\left(\frac{7\,bx^3}{8} - \frac{9\,ax}{8}\right) - \frac{15}{8}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-1/a^3*b*((7/8*b*x^3-9/8*a*x)/(b*x^2-a)^2-15/8/(a*b)^(1/2)*arctanh(
b*x/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26044, size = 428, normalized size = 5.49

$$\left[-\frac{30\,b^2x^4 - 50\,abx^2 - 15\,\left(b^2x^5 - 2\,abx^3 + a^2x\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right) + 16\,a^2}{16\,\left(a^3b^2x^5 - 2\,a^4bx^3 + a^5x\right)}, -\frac{15\,b^2x^4 - 25\,abx^2 + 15\,\left(b^2x^5 - 2\,abx^3\right.}{8\,\left(a^3b^2x^5 - 2\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 - 50*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*sqrt(b/a
)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) + 16*a^2)/(a^3*b^2*x^5 - 2
*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 15*(b^2*x^5 - 2*a*b*x^
3 + a^2*x)*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*
x^3 + a^5*x)]
```

Sympy [A] time = 0.706262, size = 107, normalized size = 1.37

$$-\frac{15\sqrt{\frac{b}{a^7}}\log\left(-\frac{a^4\sqrt{\frac{b}{a^7}}}{b}+x\right)}{16}+\frac{15\sqrt{\frac{b}{a^7}}\log\left(\frac{a^4\sqrt{\frac{b}{a^7}}}{b}+x\right)}{16}-\frac{8a^2-25abx^2+15b^2x^4}{8a^5x-16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**3,x)
```

```
[Out] -15*sqrt(b/a**7)*log(-a**4*sqrt(b/a**7)/b + x)/16 + 15*sqrt(b/a**7)*log(a**
4*sqrt(b/a**7)/b + x)/16 - (8*a**2 - 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*x
- 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Giac [A] time = 1.93461, size = 82, normalized size = 1.05

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}a^3}-\frac{7\,b^2x^3-9\,abx}{8\,\left(bx^2-a\right)^2a^3}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3) - 1/8*(7*b^2*x^3 - 9*a*b*x)
/((b*x^2 - a)^2*a^3) - 1/(a^3*x)
```

3.245 $\int \frac{1}{x^3\left(a-bx^2\right)^3}\,dx$

Optimal. Leaf size=69

$$\frac{b}{a^3\left(a-bx^2\right)}+\frac{b}{4a^2\left(a-bx^2\right)^2}-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

```
[Out] -1/(2*a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log[x
])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)
```

Rubi [A] time = 0.0509335, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{b}{a^3\left(a-bx^2\right)}+\frac{b}{4a^2\left(a-bx^2\right)^2}-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] -1/(2*a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log[x
])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a-bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{a^2(a-bx)^3}+\frac{2b^2}{a^3(a-bx)^2}+\frac{3b^2}{a^4(a-bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}+\frac{b}{a^3\left(a-bx^2\right)}+\frac{3b\log(x)}{a^4}-\frac{3b\log\left(a-bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.0526544, size = 60, normalized size = 0.87

$$\frac{\frac{a\left(-2a^2+9abx^2-6b^2x^4\right)}{\left(ax-bx^3\right)^2}-6b\log\left(a-bx^2\right)+12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] ((a*(-2*a^2 + 9*a*b*x^2 - 6*b^2*x^4))/(a*x - b*x^3)^2 + 12*b*Log[x] - 6*b*L
og[a - b*x^2])/(4*a^4)
```

Maple [A] time = 0.012, size = 68, normalized size = 1.

$$-\frac{1}{2\,a^3x^2}+3\,\frac{b\ln(x)}{a^4}-\frac{b}{a^3\left(bx^2-a\right)}+\frac{b}{4\,a^2\left(bx^2-a\right)^2}-\frac{3\,b\ln\left(bx^2-a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+3*b*ln(x)/a^4-b/a^3/(b*x^2-a)+1/4*b/a^2/(b*x^2-a)^2-3/2*b/a^4*
ln(b*x^2-a)
```

Maxima [A] time = 2.05402, size = 107, normalized size = 1.55

$$-\frac{6\,b^2x^4-9\,abx^2+2\,a^2}{4\left(a^3b^2x^6-2\,a^4bx^4+a^5x^2\right)}-\frac{3\,b\log\left(bx^2-a\right)}{2\,a^4}+\frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 - 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 - 2*a^4*b*x^4 + a^5*x^2)
- 3/2*b*log(b*x^2 - a)/a^4 + 3/2*b*log(x^2)/a^4
```

Fricas [A] time = 1.26325, size = 247, normalized size = 3.58

$$-\frac{6\,ab^2x^4-9\,a^2bx^2+2\,a^3+6\left(b^3x^6-2\,ab^2x^4+a^2bx^2\right)\log\left(bx^2-a\right)-12\left(b^3x^6-2\,ab^2x^4+a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6-2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3 + 6*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 - a) - 12*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 - 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A] time = 0.831102, size = 78, normalized size = 1.13

$$-\frac{2a^2-9abx^2+6b^2x^4}{4a^5x^2-8a^4bx^4+4a^3b^2x^6}+\frac{3b\log(x)}{a^4}-\frac{3b\log\left(-\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(2*a**2 - 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 - 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) + 3*b*log(x)/a**4 - 3*b*log(-a/b + x**2)/(2*a**4)
```

Giac [A] time = 1.7146, size = 113, normalized size = 1.64

$$\frac{3\,b\log\left(x^2\right)}{2\,a^4}-\frac{3\,b\log\left(\left|bx^2-a\right|\right)}{2\,a^4}+\frac{9\,b^3x^4-22\,ab^2x^2+14\,a^2b}{4\left(bx^2-a\right)^2a^4}-\frac{3\,bx^2+a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*b*log(x^2)/a^4 - 3/2*b*log(abs(b*x^2 - a))/a^4 + 1/4*(9*b^3*x^4 - 22*a*
b^2*x^2 + 14*a^2*b)/((b*x^2 - a)^2*a^4) - 1/2*(3*b*x^2 + a)/(a^4*x^2)
```

3.246 $\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx$

Optimal. Leaf size=36

$$\frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}$$

```
[Out] a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)
```

Rubi [A] time = 0.0295651, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^5,x]
```

```
[Out] a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a-bx)^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a}{b(a-bx)^5}-\frac{1}{b(a-bx)^4}\right)dx,x,x^2\right)\\ &= \frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}\end{aligned}$$

Mathematica [A] time = 0.0096997, size = 25, normalized size = 0.69

$$-\frac{a-4bx^2}{24b^2\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^5,x]
```

```
[Out] -(a - 4*b*x^2)/(24*b^2*(a - b*x^2)^4)
```

Maple [A] time = 0.007, size = 35, normalized size = 1.

$$\frac{a}{8\,b^2\left(bx^2-a\right)^4}+\frac{1}{6\,b^2\left(bx^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^5,x)
```

```
[Out] 1/8/b^2*a/(b*x^2-a)^4+1/6/b^2/(b*x^2-a)^3
```

Maxima [A] time = 1.9082, size = 81, normalized size = 2.25

$$\frac{4\,bx^2-a}{24\,\left(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

Fricas [A] time = 1.21492, size = 116, normalized size = 3.22

$$\frac{4\,bx^2-a}{24\,\left(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

Sympy [A] time = 0.732412, size = 58, normalized size = 1.61

$$\frac{-a+4bx^2}{24a^4b^2-96a^3b^3x^2+144a^2b^4x^4-96ab^5x^6+24b^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**5,x)
```

```
[Out] (-a + 4*b*x**2)/(24*a**4*b**2 - 96*a**3*b**3*x**2 + 144*a**2*b**4*x**4 - 96
*a*b**5*x**6 + 24*b**6*x**8)
```

Giac [A] time = 2.33869, size = 53, normalized size = 1.47

$$\frac{\frac{4}{(bx^2-a)^3 b} + \frac{3a}{(bx^2-a)^4 b}}{24 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*(4/((b*x^2 - a)^3*b) + 3*a/((b*x^2 - a)^4*b))/b
```

3.247 $\int \frac{x^2}{(a-bx^2)^5} dx$

Optimal. Leaf size=109

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{x}{48ab\left(a-bx^2\right)^3}+\frac{x}{8b\left(a-bx^2\right)^4}$$

```
[Out] x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*
x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.0368752, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {288, 199, 208}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{x}{48ab\left(a-bx^2\right)^3}+\frac{x}{8b\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^5,x]
```

```
[Out] x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*
x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(7/2)*b^(3/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^5}\,dx = \frac{x}{8b\left(a-bx^2\right)^4} - \frac{\int \frac{1}{\left(a-bx^2\right)^4}\,dx}{8b}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5\int \frac{1}{\left(a-bx^2\right)^3}\,dx}{48ab}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5x}{192a^2b\left(a-bx^2\right)^2} - \frac{5\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{64a^2b}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5x}{192a^2b\left(a-bx^2\right)^2} - \frac{5x}{128a^3b\left(a-bx^2\right)} - \frac{5\int \frac{1}{a-bx^2}\,dx}{128a^3b}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5x}{192a^2b\left(a-bx^2\right)^2} - \frac{5x}{128a^3b\left(a-bx^2\right)} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.0477683, size = 81, normalized size = 0.74

$$\frac{73a^2bx^3+15a^3x-55ab^2x^5+15b^3x^7}{384a^3b\left(a-bx^2\right)^4} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^5,x]
```

```
[Out] (15*a^3*x + 73*a^2*b*x^3 - 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a^3*b*(a - b*x^2
)^4) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))
```

Maple [A] time = 0.009, size = 72, normalized size = 0.7

$$-\frac{1}{\left(bx^2-a\right)^4}\left(-\frac{5\,b^2x^7}{128\,a^3}+\frac{55\,bx^5}{384\,a^2}-\frac{73\,x^3}{384\,a}-\frac{5\,x}{128\,b}\right)-\frac{5}{128\,a^3b}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^5,x)
```

```
[Out] -(-5/128*b^2/a^3*x^7+55/384*b/a^2*x^5-73/384/a*x^3-5/128*x/b)/(b*x^2-a)^4-5
/128/a^3/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28953, size = 684, normalized size = 6.28

$$\left[\frac{30\,ab^4x^7 - 110\,a^2b^3x^5 + 146\,a^3b^2x^3 + 30\,a^4bx + 15\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\sqrt{ab}\log\left(\frac{bx^2 - 2\,\sqrt{ab}x + a}{bx^2 - a}\right)}{768\left(a^4b^6x^8 - 4\,a^5b^5x^6 + 6\,a^6b^4x^4 - 4\,a^7b^3x^2 + a^8b^2\right)}, 15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 + 30*a^4*b*x + 15*
(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*log((
b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^4*b^6*x^8 - 4*a^5*b^5*x^6 + 6*a
^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 - 55*a^2*b^3*x^5
 + 73*a^3*b^2*x^3 + 15*a^4*b*x + 15*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
- 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^4*b^6*x^8 - 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2)]
```

Sympy [A] time = 0.851353, size = 160, normalized size = 1.47

$$\frac{5\sqrt{\frac{1}{a^7b^3}}\log\left(-a^4b\sqrt{\frac{1}{a^7b^3}} + x\right)}{256} - \frac{5\sqrt{\frac{1}{a^7b^3}}\log\left(a^4b\sqrt{\frac{1}{a^7b^3}} + x\right)}{256} + \frac{15a^3x + 73a^2bx^3 - 55ab^2x^5 + 15b^3x^7}{384a^7b - 1536a^6b^2x^2 + 2304a^5b^3x^4 - 1536a^4b^4x^6 + 384a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**5,x)
```

```
[Out] 5*sqrt(1/(a**7*b**3))*log(-a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - 5*sqrt(1/(
a**7*b**3))*log(a**4*b*sqrt(1/(a**7*b**3)) + x)/256 + (15*a**3*x + 73*a**2*
b*x**3 - 55*a*b**2*x**5 + 15*b**3*x**7)/(384*a**7*b - 1536*a**6*b**2*x**2 +
 2304*a**5*b**3*x**4 - 1536*a**4*b**4*x**6 + 384*a**3*b**5*x**8)
```

Giac [A] time = 2.57477, size = 104, normalized size = 0.95

$$\frac{5\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}a^3b} + \frac{15\,b^3x^7 - 55\,ab^2x^5 + 73\,a^2bx^3 + 15\,a^3x}{384\left(bx^2 - a\right)^4a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3*b) + 1/384*(15*b^3*x^7 - 55*a*
b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/((b*x^2 - a)^4*a^3*b)
```

3.248 $\int \frac{x}{(a-bx^2)^5} dx$

Optimal. Leaf size=17

$$\frac{1}{8b(a-bx^2)^4}$$

```
[Out] 1/(8*b*(a - b*x^2)^4)
```

Rubi [A] time = 0.0027673, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^5,x]

[Out] 1/(8*b*(a - b*x^2)^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^5} dx = \frac{1}{8b(a-bx^2)^4}$$

Mathematica [A] time = 0.0030155, size = 17, normalized size = 1.

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^5,x]

[Out] 1/(8*b*(a - b*x^2)^4)
```

Maple [A] time = 0., size = 17, normalized size = 1.

$$\frac{1}{8b(bx^2-a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^5,x)
```

```
[Out] 1/8/b/(b*x^2-a)^4
```

Maxima [A] time = 2.10508, size = 22, normalized size = 1.29

$$\frac{1}{8\left(bx^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

Fricas [B] time = 1.16939, size = 93, normalized size = 5.47

$$\frac{1}{8\left(b^5x^8-4\,ab^4x^6+6\,a^2b^3x^4-4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8/(b^5*x^8 - 4*a*b^4*x^6 + 6*a^2*b^3*x^4 - 4*a^3*b^2*x^2 + a^4*b)
```

Sympy [B] time = 0.730105, size = 49, normalized size = 2.88

$$\frac{1}{8a^4b-32a^3b^2x^2+48a^2b^3x^4-32ab^4x^6+8b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**5,x)
```

```
[Out] 1/(8*a**4*b - 32*a**3*b**2*x**2 + 48*a**2*b**3*x**4 - 32*a*b**4*x**6 + 8*b*
*5*x**8)
```

Giac [A] time = 2.82081, size = 22, normalized size = 1.29

$$\frac{1}{8\left(bx^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

3.249 $\int \frac{1}{(a-bx^2)^5} \, dx$

Optimal. Leaf size=100

$$\frac{35x}{128a^4(a-bx^2)}+\frac{35x}{192a^3(a-bx^2)^2}+\frac{7x}{48a^2(a-bx^2)^3}+\frac{35\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{x}{8a(a-bx^2)^4}$$

```
[Out] x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a -
 b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
])/(128*a^(9/2)*Sqrt[b])
```

Rubi [A] time = 0.0336952, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {199, 208}

$$\frac{35x}{128a^4(a-bx^2)}+\frac{35x}{192a^3(a-bx^2)^2}+\frac{7x}{48a^2(a-bx^2)^3}+\frac{35\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{x}{8a(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5),x]
```

```
[Out] x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a -
 b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
])/(128*a^(9/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^5}\,dx = \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7\int \frac{1}{\left(a-bx^2\right)^4}\,dx}{8a}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35\int \frac{1}{\left(a-bx^2\right)^3}\,dx}{48a^2}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{64a^3}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35x}{128a^4\left(a-bx^2\right)} + \frac{35\int \frac{1}{a-bx^2}\,dx}{128a^4}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35x}{128a^4\left(a-bx^2\right)} + \frac{35\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}$$

Mathematica [A] time = 0.0438093, size = 79, normalized size = 0.79

$$\frac{\frac{\sqrt{a}x\left(-511a^2bx^2+279a^3+385ab^2x^4-105b^3x^6\right)}{\left(a-bx^2\right)^4}+\frac{105\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5),x]
```

```
[Out] ((Sqrt[a]*x*(279*a^3 - 511*a^2*b*x^2 + 385*a*b^2*x^4 - 105*b^3*x^6))/(a - b
*x^2)^4 + (105*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(384*a^(9/2))
```

Maple [A] time = 0.004, size = 107, normalized size = 1.1

$$\frac{x}{8\,a\left(bx^2-a\right)^4}+\frac{7}{8\,a}\left(-\frac{x}{6\,a\left(bx^2-a\right)^3}-\frac{5}{6\,a}\left(-\frac{x}{4\,a\left(bx^2-a\right)^2}-\frac{3}{4\,a}\left(-\frac{x}{2\,a\left(bx^2-a\right)}+\frac{1}{2\,a}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^5,x)
```

```
[Out] 1/8*x/a/(b*x^2-a)^4+7/8/a*(-1/6*x/a/(b*x^2-a)^3-5/6/a*(-1/4*x/a/(b*x^2-a)^2
-3/4/a*(-1/2*x/a/(b*x^2-a)+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26721, size = 694, normalized size = 6.94

$$\left[-\frac{210\,ab^4x^7 - 770\,a^2b^3x^5 + 1022\,a^3b^2x^3 - 558\,a^4bx - 105\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\sqrt{ab}\log\left(\frac{bx^2+2\sqrt{a}}{bx^2-}\right.}{768\left(a^5b^5x^8 - 4\,a^6b^4x^6 + 6\,a^7b^3x^4 - 4\,a^8b^2x^2 + a^9b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(210*a*b^4*x^7 - 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 - 558*a^4*b*x -
 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^5*b^5*x^8 - 4*a^6*b^4*x^6
+ 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b), -1/384*(105*a*b^4*x^7 - 385*a^2*b
^3*x^5 + 511*a^3*b^2*x^3 - 279*a^4*b*x + 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^5*b^5*x
^8 - 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b)]
```

Sympy [A] time = 0.981893, size = 146, normalized size = 1.46

$$-\frac{35\sqrt{\frac{1}{a^9b}}\log\left(-a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}+\frac{35\sqrt{\frac{1}{a^9b}}\log\left(a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}-\frac{-279a^3x+511a^2bx^3-385ab^2x^5+105b^3x^7}{384a^8-1536a^7bx^2+2304a^6b^2x^4-1536a^5b^3x^6+384a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**5,x)
```

```
[Out] -35*sqrt(1/(a**9*b))*log(-a**5*sqrt(1/(a**9*b)) + x)/256 + 35*sqrt(1/(a**9*
b))*log(a**5*sqrt(1/(a**9*b)) + x)/256 - (-279*a**3*x + 511*a**2*b*x**3 - 3
85*a*b**2*x**5 + 105*b**3*x**7)/(384*a**8 - 1536*a**7*b*x**2 + 2304*a**6*b*
*2*x**4 - 1536*a**5*b**3*x**6 + 384*a**4*b**4*x**8)
```

Giac [A] time = 2.52578, size = 96, normalized size = 0.96

$$-\frac{35\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\sqrt{-ab}a^4}-\frac{105\,b^3x^7-385\,ab^2x^5+511\,a^2bx^3-279\,a^3x}{384\left(bx^2-a\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] -35/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^4) - 1/384*(105*b^3*x^7 - 385*
a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/((b*x^2 - a)^4*a^4)
```

3.250 $\int \frac{1}{x\left(a-bx^2\right)^5} \, dx$

Optimal. Leaf size=91

$$\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{8a\left(a-bx^2\right)^4}$$

```
[Out] 1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) +
 1/(2*a^4*(a - b*x^2)) + Log[x]/a^5 - Log[a - b*x^2]/(2*a^5)
```

Rubi [A] time = 0.063872, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{8a\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^5),x]
```

```
[Out] 1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) +
 1/(2*a^4*(a - b*x^2)) + Log[x]/a^5 - Log[a - b*x^2]/(2*a^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a-bx^2\right)^5} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^5} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^5x}+\frac{b}{a(a-bx)^5}+\frac{b}{a^2(a-bx)^4}+\frac{b}{a^3(a-bx)^3}+\frac{b}{a^4(a-bx)^2}+\frac{b}{a^5(a-bx)}\right) \, dx, x, x^2\right)\\ &= \frac{1}{8a\left(a-bx^2\right)^4}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{\log(x)}{a^5}-\frac{\log\left(a-bx^2\right)}{2a^5}\end{aligned}$$

Mathematica [A] time = 0.0322645, size = 67, normalized size = 0.74

$$\frac{\frac{a\left(-52a^2bx^2+25a^3+42ab^2x^4-12b^3x^6\right)}{\left(a-bx^2\right)^4}-12\log\left(a-bx^2\right)+24\log(x)}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^5),x]
```

```
[Out] ((a*(25*a^3 - 52*a^2*b*x^2 + 42*a*b^2*x^4 - 12*b^3*x^6))/(a - b*x^2)^4 + 24
*Log[x] - 12*Log[a - b*x^2])/(24*a^5)
```

Maple [A] time = 0.013, size = 87, normalized size = 1.

$$\frac{\ln(x)}{a^5} - \frac{1}{2\,a^4\left(bx^2-a\right)} + \frac{1}{8\,a\left(bx^2-a\right)^4} + \frac{1}{4\,a^3\left(bx^2-a\right)^2} - \frac{1}{6\,a^2\left(bx^2-a\right)^3} - \frac{\ln\left(bx^2-a\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^5,x)
```

```
[Out] ln(x)/a^5-1/2/a^4/(b*x^2-a)+1/8/a/(b*x^2-a)^4+1/4/a^3/(b*x^2-a)^2-1/6/a^2/(
b*x^2-a)^3-1/2/a^5*ln(b*x^2-a)
```

Maxima [A] time = 1.36549, size = 143, normalized size = 1.57

$$-\frac{12\,b^3x^6 - 42\,ab^2x^4 + 52\,a^2bx^2 - 25\,a^3}{24\left(a^4b^4x^8 - 4\,a^5b^3x^6 + 6\,a^6b^2x^4 - 4\,a^7bx^2 + a^8\right)} - \frac{\log\left(bx^2-a\right)}{2\,a^5} + \frac{\log\left(x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*b^3*x^6 - 42*a*b^2*x^4 + 52*a^2*b*x^2 - 25*a^3)/(a^4*b^4*x^8 - 4*
a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 - a)/a^5 +
 1/2*log(x^2)/a^5
```

Fricas [B] time = 1.25652, size = 379, normalized size = 4.16

$$-\frac{12\,ab^3x^6 - 42\,a^2b^2x^4 + 52\,a^3bx^2 - 25\,a^4 + 12\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\log\left(bx^2-a\right) - 24\left(b^4x^8 - 4\right.}{24\left(a^5b^4x^8 - 4\,a^6b^3x^6 + 6\,a^7b^2x^4 - 4\,a^8bx^2 + a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a*b^3*x^6 - 42*a^2*b^2*x^4 + 52*a^3*b*x^2 - 25*a^4 + 12*(b^4*x^8
- 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(b*x^2 - a) - 24*(b^4
*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^
8 - 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 - 4*a^8*b*x^2 + a^9)
```

Sympy [A] time = 1.40765, size = 104, normalized size = 1.14

$$-\frac{-25a^3+52a^2bx^2-42ab^2x^4+12b^3x^6}{24a^8-96a^7bx^2+144a^6b^2x^4-96a^5b^3x^6+24a^4b^4x^8}+\frac{\log(x)}{a^5}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**5,x)
```

```
[Out] -(-25*a**3 + 52*a**2*b*x**2 - 42*a*b**2*x**4 + 12*b**3*x**6)/(24*a**8 - 96*
a**7*b*x**2 + 144*a**6*b**2*x**4 - 96*a**5*b**3*x**6 + 24*a**4*b**4*x**8) +
 log(x)/a**5 - log(-a/b + x**2)/(2*a**5)
```

Giac [A] time = 2.94426, size = 115, normalized size = 1.26

$$\frac{\log\left(x^2\right)}{2\,a^5}-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^5}+\frac{25\,b^4x^8-112\,ab^3x^6+192\,a^2b^2x^4-152\,a^3bx^2+50\,a^4}{24\left(bx^2-a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^5 - 1/2*log(abs(b*x^2 - a))/a^5 + 1/24*(25*b^4*x^8 - 112*a*b
^3*x^6 + 192*a^2*b^2*x^4 - 152*a^3*b*x^2 + 50*a^4)/((b*x^2 - a)^4*a^5)
```

3.251 $\int \frac{1}{x^2(a-bx^2)^5} dx$

Optimal. Leaf size=118

$$\frac{105}{128a^4x(a-bx^2)}+\frac{21}{64a^3x(a-bx^2)^2}+\frac{3}{16a^2x(a-bx^2)^3}+\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{1}{8ax(a-bx^2)^4}$$

```
[Out] -315/(128*a^5*x) + 1/(8*a*x*(a - b*x^2)^4) + 3/(16*a^2*x*(a - b*x^2)^3) + 2
1/(64*a^3*x*(a - b*x^2)^2) + 105/(128*a^4*x*(a - b*x^2)) + (315*Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2))
```

Rubi [A] time = 0.0487778, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{105}{128a^4x(a-bx^2)}+\frac{21}{64a^3x(a-bx^2)^2}+\frac{3}{16a^2x(a-bx^2)^3}+\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{1}{8ax(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] -315/(128*a^5*x) + 1/(8*a*x*(a - b*x^2)^4) + 3/(16*a^2*x*(a - b*x^2)^3) + 2
1/(64*a^3*x*(a - b*x^2)^2) + 105/(128*a^4*x*(a - b*x^2)) + (315*Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^5}\,dx = \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{9\int \frac{1}{x^2\left(a-bx^2\right)^4}\,dx}{8a}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21\int \frac{1}{x^2\left(a-bx^2\right)^3}\,dx}{16a^2}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx}{64a^3}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + \frac{315\int \frac{1}{x^2\left(a-bx^2\right)}\,dx}{128a^4}$$

$$= -\frac{315}{128a^5x} + \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + \frac{(315b)\int}{128}$$

$$= -\frac{315}{128a^5x} + \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + \frac{315\sqrt{b}\,\mathrm{ta}}{128}$$

Mathematica [A] time = 0.0530645, size = 92, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(-1533a^2b^2x^4+837a^3bx^2-128a^4+1155ab^3x^6-315b^4x^8\right)}{x\left(a-bx^2\right)^4}+315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] ((Sqrt[a]*(-128*a^4 + 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 - 3
15*b^4*x^8))/(x*(a - b*x^2)^4) + 315*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(11/2))
```

Maple [A] time = 0.013, size = 78, normalized size = 0.7

$$-\frac{1}{a^5x}-\frac{b}{a^5}\left(\frac{1}{\left(bx^2-a\right)^4}\left(\frac{187\,b^3x^7}{128}-\frac{643\,ab^2x^5}{128}+\frac{765\,a^2bx^3}{128}-\frac{325\,a^3x}{128}\right)-\frac{315}{128}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^5,x)
```

```
[Out] -1/a^5/x-1/a^5*b*((187/128*b^3*x^7-643/128*a*b^2*x^5+765/128*a^2*b*x^3-325/
128*a^3*x)/(b*x^2-a)^4-315/128/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28183, size = 721, normalized size = 6.11

$$\left[-\frac{630\,b^4x^8 - 2310\,ab^3x^6 + 3066\,a^2b^2x^4 - 1674\,a^3bx^2 + 256\,a^4 - 315\left(b^4x^9 - 4\,ab^3x^7 + 6\,a^2b^2x^5 - 4\,a^3bx^3 + a^4x\right)\sqrt{\frac{b}{a}}\,\mathrm{lo}}{256\left(a^5b^4x^9 - 4\,a^6b^3x^7 + 6\,a^7b^2x^5 - 4\,a^8bx^3 + a^9x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 - 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 - 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x
)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)))/(a^5*b^4*x^9 -
4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8 -
 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x^9
 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x)*sqrt(-b/a)*arctan(x*s
qrt(-b/a)))/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^
9*x)]
```

Sympy [A] time = 1.84662, size = 155, normalized size = 1.31

$$-\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(-\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}+\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}-\frac{128a^4 - 837a^3bx^2 + 1533a^2b^2x^4 - 1155ab^3x^6 + 315b^4x^{8}}{128a^9x - 512a^8bx^3 + 768a^7b^2x^5 - 512a^6b^3x^7 + 128a^5b^4x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**5,x)
```

```
[Out] -315*sqrt(b/a**11)*log(-a**6*sqrt(b/a**11)/b + x)/256 + 315*sqrt(b/a**11)*l
og(a**6*sqrt(b/a**11)/b + x)/256 - (128*a**4 - 837*a**3*b*x**2 + 1533*a**2*
b**2*x**4 - 1155*a*b**3*x**6 + 315*b**4*x**8)/(128*a**9*x - 512*a**8*b*x**3
 + 768*a**7*b**2*x**5 - 512*a**6*b**3*x**7 + 128*a**5*b**4*x**9)
```

Giac [A] time = 2.37003, size = 112, normalized size = 0.95

$$-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}a^5} - \frac{1}{a^5x} - \frac{187\,b^4x^7 - 643\,ab^3x^5 + 765\,a^2b^2x^3 - 325\,a^3bx}{128\left(bx^2 - a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] -315/128*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^5) - 1/(a^5*x) - 1/128*(187
*b^4*x^7 - 643*a*b^3*x^5 + 765*a^2*b^2*x^3 - 325*a^3*b*x)/((b*x^2 - a)^4*a^
5)
```

3.252 $\int \frac{1}{x^3(a-bx^2)^5}\,dx$

Optimal. Leaf size=106

$$\frac{2b}{a^5(a-bx^2)}+\frac{3b}{4a^4(a-bx^2)^2}+\frac{b}{3a^3(a-bx^2)^3}+\frac{b}{8a^2(a-bx^2)^4}-\frac{5b\log(a-bx^2)}{2a^6}+\frac{5b\log(x)}{a^6}-\frac{1}{2a^5x^2}$$

```
[Out] -1/(2*a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b)/
(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b*L
og[a - b*x^2])/(2*a^6)
```

Rubi [A] time = 0.0870354, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{2b}{a^5(a-bx^2)}+\frac{3b}{4a^4(a-bx^2)^2}+\frac{b}{3a^3(a-bx^2)^3}+\frac{b}{8a^2(a-bx^2)^4}-\frac{5b\log(a-bx^2)}{2a^6}+\frac{5b\log(x)}{a^6}-\frac{1}{2a^5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] -1/(2*a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b)/
(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b*L
og[a - b*x^2])/(2*a^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a-bx^2)^5}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^5}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{a^5x^2}+\frac{5b}{a^6x}+\frac{b^2}{a^2(a-bx)^5}+\frac{2b^2}{a^3(a-bx)^4}+\frac{3b^2}{a^4(a-bx)^3}+\frac{4b^2}{a^5(a-bx)^2}+\frac{5b^2}{a^6(a-bx}\right.\right.$$

$$=-\frac{1}{2a^5x^2}+\frac{b}{8a^2(a-bx^2)^4}+\frac{b}{3a^3(a-bx^2)^3}+\frac{3b}{4a^4(a-bx^2)^2}+\frac{2b}{a^5(a-bx^2)}+\frac{5b\log(x)}{a^6}-\frac{5b\log}{}$$

Mathematica [A] time = 0.0631878, size = 83, normalized size = 0.78

$$\frac{\frac{a(-260a^2b^2x^4+125a^3bx^2-12a^4+210ab^3x^6-60b^4x^8)}{x^2(a-bx^2)^4}-60b\log(a-bx^2)+120b\log(x)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] ((a*(-12*a^4 + 125*a^3*b*x^2 - 260*a^2*b^2*x^4 + 210*a*b^3*x^6 - 60*b^4*x^8
))/(x^2*(a - b*x^2)^4) + 120*b*Log[x] - 60*b*Log[a - b*x^2])/(24*a^6)
```

Maple [A] time = 0.015, size = 102, normalized size = 1.

$$-\frac{1}{2a^5x^2}+5\frac{b\ln(x)}{a^6}-2\frac{b}{a^5(bx^2-a)}-\frac{b}{3a^3(bx^2-a)^3}+\frac{b}{8a^2(bx^2-a)^4}+\frac{3b}{4a^4(bx^2-a)^2}-\frac{5b\ln(bx^2-a)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^5,x)
```

```
[Out] -1/2/a^5/x^2+5*b*ln(x)/a^6-2*b/a^5/(b*x^2-a)-1/3*b/a^3/(b*x^2-a)^3+1/8*b/a^
2/(b*x^2-a)^4+3/4*b/a^4/(b*x^2-a)^2-5/2*b/a^6*ln(b*x^2-a)
```

Maxima [A] time = 2.12544, size = 166, normalized size = 1.57

$$-\frac{60b^4x^8-210ab^3x^6+260a^2b^2x^4-125a^3bx^2+12a^4}{24(a^5b^4x^{10}-4a^6b^3x^8+6a^7b^2x^6-4a^8bx^4+a^9x^2)}-\frac{5b\log(bx^2-a)}{2a^6}+\frac{5b\log(x^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(60*b^4*x^8 - 210*a*b^3*x^6 + 260*a^2*b^2*x^4 - 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 - 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 - 4*a^8*b*x^4 + a^9*x^2) -
 5/2*b*log(b*x^2 - a)/a^6 + 5/2*b*log(x^2)/a^6
```

Fricas [B] time = 1.27491, size = 440, normalized size = 4.15

$$-\frac{60ab^4x^8-210a^2b^3x^6+260a^3b^2x^4-125a^4bx^2+12a^5+60(b^5x^{10}-4ab^4x^8+6a^2b^3x^6-4a^3b^2x^4+a^4bx^2)\log(bx^2-}{24(a^6b^4x^{10}-4a^7b^3x^8+6a^8b^2x^6-4a^9bx^4+a^{10}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 - 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 - 125*a^4*b*x^2 + 1
2*a^5 + 60*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 - a) - 120*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b
```

```
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 - 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 -
 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [A] time = 2.81119, size = 126, normalized size = 1.19

$$-\frac{12a^4 - 125a^3bx^2 + 260a^2b^2x^4 - 210ab^3x^6 + 60b^4x^8}{24a^9x^2 - 96a^8bx^4 + 144a^7b^2x^6 - 96a^6b^3x^8 + 24a^5b^4x^{10}} + \frac{5b\log(x)}{a^6} - \frac{5b\log\left(-\frac{a}{b} + x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(12*a**4 - 125*a**3*b*x**2 + 260*a**2*b**2*x**4 - 210*a*b**3*x**6 + 60*b**
4*x**8)/(24*a**9*x**2 - 96*a**8*b*x**4 + 144*a**7*b**2*x**6 - 96*a**6*b**3*
x**8 + 24*a**5*b**4*x**10) + 5*b*log(x)/a**6 - 5*b*log(-a/b + x**2)/(2*a**6
)
```

Giac [A] time = 2.55494, size = 143, normalized size = 1.35

$$\frac{5\,b\log\left(x^2\right)}{2\,a^6} - \frac{5\,b\log\left(\left|bx^2 - a\right|\right)}{2\,a^6} - \frac{5\,bx^2 + a}{2\,a^6x^2} + \frac{125\,b^5x^8 - 548\,ab^4x^6 + 912\,a^2b^3x^4 - 688\,a^3b^2x^2 + 202\,a^4b}{24\left(bx^2 - a\right)^4 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/2*b*log(x^2)/a^6 - 5/2*b*log(abs(b*x^2 - a))/a^6 - 1/2*(5*b*x^2 + a)/(a^6
*x^2) + 1/24*(125*b^5*x^8 - 548*a*b^4*x^6 + 912*a^2*b^3*x^4 - 688*a^3*b^2*x
^2 + 202*a^4*b)/((b*x^2 - a)^4*a^6)
```

3.253 $\int \frac{1}{x(1+bx^2)}\,dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rubi [A] time = 0.0090262, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)),x]

[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1+bx}\,dx, x, x^2\right)\\
&= \log(x) - \frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0041693, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{2}\log\left(bx^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(bx^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1),x)
```

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Maxima [A] time = 2.05713, size = 23, normalized size = 1.53

$$-\frac{1}{2}\log\left(bx^2 + 1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 1.24878, size = 41, normalized size = 2.73

$$-\frac{1}{2}\log\left(bx^2 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

Sympy [A] time = 0.120866, size = 12, normalized size = 0.8

$$\log(x) - \frac{\log\left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

Giac [A] time = 2.17215, size = 24, normalized size = 1.6

$$\frac{1}{2}\log\left(x^{2}\right)-\frac{1}{2}\log\left(\left|bx^{2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

3.254 $\int \frac{1}{x(-1+bx^2)} dx$

Optimal. Leaf size=18

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rubi [A] time = 0.0101157, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x^2)),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-1+bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)} dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right)\right)+\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{-1+bx} dx, x, x^2\right)\\
&= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0046615, size = 18, normalized size = 1.

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$-\ln(x)+\frac{\ln\left(bx^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1),x)
```

```
[Out] -ln(x)+1/2*ln(b*x^2-1)
```

Maxima [A] time = 2.79984, size = 23, normalized size = 1.28

$$\frac{1}{2}\log\left(bx^2-1\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - 1/2*log(x^2)
```

Fricas [A] time = 1.25919, size = 39, normalized size = 2.17

$$\frac{1}{2}\log\left(bx^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

Sympy [A] time = 0.124142, size = 12, normalized size = 0.67

$$-\log(x)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

Giac [A] time = 2.66321, size = 24, normalized size = 1.33

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

3.255 $\int \frac{1}{x^3(1+bx^2)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rubi [A] time = 0.0169842, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + b*x^2)),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0052488, size = 26, normalized size = 1.

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + b*x^2)),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Maple [A] time = 0.004, size = 23, normalized size = 0.9

$$-\frac{1}{2x^2} - b\ln(x) + \frac{b\ln\left(bx^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+1),x)
```

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Maxima [A] time = 2.81431, size = 32, normalized size = 1.23

$$\frac{1}{2}b\log\left(bx^2+1\right) - \frac{1}{2}b\log\left(x^2\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + 1) - 1/2*b*log(x^2) - 1/2/x^2
```

Fricas [A] time = 1.21282, size = 72, normalized size = 2.77

$$\frac{bx^2\log\left(bx^2+1\right) - 2bx^2\log(x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

Sympy [A] time = 0.343845, size = 22, normalized size = 0.85

$$-b\log(x) + \frac{b\log\left(x^2+\frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+1),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

Giac [A] time = 1.79871, size = 43, normalized size = 1.65

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

3.256 $\int \frac{1}{x^3(-1+bx^2)}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rubi [A] time = 0.017959, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-1 + b*x^2)),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(-1+bx^2\right)}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)dx,x,x^2\right)\\&=\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.004213, size = 27, normalized size = 1.

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-1 + b*x^2)),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Maple [A] time = 0.007, size = 23, normalized size = 0.9

$$\frac{1}{2\,x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(b x^{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2-1),x)
```

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

Maxima [A] time = 2.2964, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b\log\left(b x^{2}-1\right)-\frac{1}{2}\,b\log\left(x^{2}\right)+\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 - 1) - 1/2*b*log(x^2) + 1/2/x^2
```

Fricas [A] time = 1.24557, size = 72, normalized size = 2.67

$$\frac{b x^{2}\log\left(b x^{2}-1\right)-2\,b x^{2}\log\left(x\right)+1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

Sympy [A] time = 0.193962, size = 22, normalized size = 0.81

$$-b\log\left(x\right)+\frac{b\log\left(x^{2}-\frac{1}{b}\right)}{2}+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2-1),x)
```

```
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

<u>Giac</u> [A] time = 1.85536, size = 43, normalized size = 1.59

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

3.257 $\int \frac{1}{-1+a+ax^2} dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

```
[Out] -(ArcTanh[(Sqrt[a]*x)/Sqrt[1 - a]]/Sqrt[(1 - a)*a])
```

Rubi [A] time = 0.0267875, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*x)/Sqrt[1 - a]]/Sqrt[(1 - a)*a])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-1+a+ax^2} dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Mathematica [A] time = 0.0101971, size = 28, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-1}}\right)}{\sqrt{a-1}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[-1 + a]]/(Sqrt[-1 + a]*Sqrt[a])
```

Maple [A] time = 0.005, size = 20, normalized size = 0.7

$$\arctan\left(ax\frac{1}{\sqrt{(a-1)\,a}}\right)\frac{1}{\sqrt{(a-1)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a-1),x)
```

```
[Out] 1/((a-1)*a)^(1/2)*arctan(a*x/((a-1)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18032, size = 189, normalized size = 6.3

$$\left[-\frac{\sqrt{-a^2+a}\log\left(\frac{ax^2-2\sqrt{-a^2+a}x-a+1}{ax^2+a-1}\right)}{2\left(a^2-a\right)}, \frac{\arctan\left(\frac{\sqrt{a^2-a}x}{a-1}\right)}{\sqrt{a^2-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + a)*log((a*x^2 - 2*sqrt(-a^2 + a)*x - a + 1)/(a*x^2 + a -
1))/(a^2 - a), arctan(sqrt(a^2 - a)*x/(a - 1))/sqrt(a^2 - a)]
```

Sympy [B] time = 0.149289, size = 83, normalized size = 2.77

$$-\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(-a\sqrt{-\frac{1}{a(a-1)}}+x+\sqrt{-\frac{1}{a(a-1)}}\right)}{2}+\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(a\sqrt{-\frac{1}{a(a-1)}}+x-\sqrt{-\frac{1}{a(a-1)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a-1),x)
```

```
[Out] -sqrt(-1/(a*(a - 1)))*log(-a*sqrt(-1/(a*(a - 1))) + x + sqrt(-1/(a*(a - 1))
))/2 + sqrt(-1/(a*(a - 1)))*log(a*sqrt(-1/(a*(a - 1))) + x - sqrt(-1/(a*(a
- 1))))/2
```

Giac [A] time = 2.22977, size = 31, normalized size = 1.03

$$\frac{\arctan\left(\frac{ax}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="giac")
```

```
[Out] arctan(a*x/sqrt(a^2 - a))/sqrt(a^2 - a)
```

3.258 $\int \frac{1}{-c-d+(c-d)x^2} \, dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

```
[Out] -(ArcTanh[(Sqrt[c - d]*x)/Sqrt[c + d]]/(Sqrt[c - d]*Sqrt[c + d]))
```

Rubi [A] time = 0.0326792, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {208}

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[c - d]*x)/Sqrt[c + d]]/(Sqrt[c - d]*Sqrt[c + d]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-c-d+(c-d)x^2} \, dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{c-d}x}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Mathematica [A] time = 0.0157183, size = 44, normalized size = 1.19

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c - d]*x)/Sqrt[-c - d]]/(Sqrt[-c - d]*Sqrt[c - d])
```

Maple [A] time = 0.007, size = 33, normalized size = 0.9

$$-\text{Artanh}\left((c-d)\,x\frac{1}{\sqrt{(c+d)\,(c-d)}}\right)\frac{1}{\sqrt{(c+d)\,(c-d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c-d+(c-d)*x^2),x)
```

```
[Out] -1/((c+d)*(c-d))^(1/2)*arctanh((c-d)*x/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29292, size = 217, normalized size = 5.86

$$\left[\frac{\log\left(\frac{(c-d)x^2-2\sqrt{c^2-d^2}x+c+d}{(c-d)x^2-c-d}\right)}{2\sqrt{c^2-d^2}}, \frac{\sqrt{-c^2+d^2}\arctan\left(\frac{\sqrt{-c^2+d^2}x}{c+d}\right)}{c^2-d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((c - d)*x^2 - 2*sqrt(c^2 - d^2)*x + c + d)/((c - d)*x^2 - c - d))
/sqrt(c^2 - d^2), sqrt(-c^2 + d^2)*arctan(sqrt(-c^2 + d^2)*x/(c + d))/(c^2
- d^2)]
```

Sympy [B] time = 0.212743, size = 87, normalized size = 2.35

$$\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(-c\sqrt{\frac{1}{(c-d)(c+d)}}-d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}-\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(c\sqrt{\frac{1}{(c-d)(c+d)}}+d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x**2),x)
```

```
[Out] sqrt(1/((c - d)*(c + d)))*log(-c*sqrt(1/((c - d)*(c + d))) - d*sqrt(1/((c -
 d)*(c + d))) + x)/2 - sqrt(1/((c - d)*(c + d)))*log(c*sqrt(1/((c - d)*(c +
 d))) + d*sqrt(1/((c - d)*(c + d))) + x)/2
```

Giac [A] time = 2.75656, size = 45, normalized size = 1.22

$$\frac{\arctan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="giac")
```

```
[Out] arctan((c*x - d*x)/sqrt(-c^2 + d^2))/sqrt(-c^2 + d^2)
```

3.259 $\int \frac{1}{x(1+bx^2)^2}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2(bx^2+1)} - \frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

```
[Out] 1/(2*(1 + b*x^2)) + Log[x] - Log[1 + b*x^2]/2
```

Rubi [A] time = 0.0199275, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2(bx^2+1)} - \frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 + b*x^2)) + Log[x] - Log[1 + b*x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+bx^2)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x} - \frac{b}{(1+bx)^2} - \frac{b}{1+bx}\right)dx, x, x^2\right)\\
&= \frac{1}{2(1+bx^2)} + \log(x) - \frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0125481, size = 25, normalized size = 0.89

$$\frac{1}{2bx^2+2} - \frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)^2),x]
```

```
[Out] (2 + 2*b*x^2)^(-1) + Log[x] - Log[1 + b*x^2]/2
```

Maple [A] time = 0.009, size = 25, normalized size = 0.9

$$\frac{1}{2\,b x^2+2}+\ln(x)-\frac{\ln\left(b x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1)^2,x)
```

```
[Out] 1/2/(b*x^2+1)+ln(x)-1/2*ln(b*x^2+1)
```

Maxima [A] time = 2.5868, size = 38, normalized size = 1.36

$$\frac{1}{2\left(b x^2+1\right)}-\frac{1}{2}\,\log\left(b x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(b*x^2 + 1) - 1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 1.24831, size = 100, normalized size = 3.57

$$-\frac{\left(b x^2+1\right)\log\left(b x^2+1\right)-2\left(b x^2+1\right)\log(x)-1}{2\left(b x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + 1)*log(b*x^2 + 1) - 2*(b*x^2 + 1)*log(x) - 1)/(b*x^2 + 1)
```

Sympy [A] time = 0.344172, size = 22, normalized size = 0.79

$$\log(x)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}+\frac{1}{2bx^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2 + 1/(2*b*x**2 + 2)
```

Giac [A] time = 2.26561, size = 49, normalized size = 1.75

$$\frac{bx^2+2}{2\left(bx^2+1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2)/(b*x^2 + 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

3.260 $\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{2\left(1-bx^2\right)}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

```
[Out] 1/(2*(1 - b*x^2)) + Log[x] - Log[1 - b*x^2]/2
```

Rubi [A] time = 0.0194258, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2\left(1-bx^2\right)}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 - b*x^2)) + Log[x] - Log[1 - b*x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{b}{(-1+bx)^2}-\frac{b}{-1+bx}\right)dx,x,x^2\right)\\
&= \frac{1}{2\left(1-bx^2\right)}+\log(x)-\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0121764, size = 26, normalized size = 0.87

$$\frac{1}{2-2bx^2}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)^2),x]
```

```
[Out] (2 - 2*b*x^2)^(-1) + Log[x] - Log[1 - b*x^2]/2
```

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$\ln(x) - \frac{\ln\left(bx^2-1\right)}{2} - \frac{1}{2\,bx^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1)^2,x)
```

```
[Out] ln(x)-1/2*ln(b*x^2-1)-1/2/(b*x^2-1)
```

Maxima [A] time = 2.48238, size = 38, normalized size = 1.27

$$-\frac{1}{2\left(bx^2-1\right)} - \frac{1}{2}\log\left(bx^2-1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(b*x^2 - 1) - 1/2*log(b*x^2 - 1) + 1/2*log(x^2)
```

Fricas [A] time = 1.40382, size = 100, normalized size = 3.33

$$-\frac{\left(bx^2-1\right)\log\left(bx^2-1\right) - 2\left(bx^2-1\right)\log(x) + 1}{2\left(bx^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 - 1)*log(b*x^2 - 1) - 2*(b*x^2 - 1)*log(x) + 1)/(b*x^2 - 1)
```

Sympy [A] time = 0.354297, size = 22, normalized size = 0.73

$$\log(x) - \frac{\log\left(x^2 - \frac{1}{b}\right)}{2} - \frac{1}{2bx^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1)**2,x)
```

```
[Out] log(x) - log(x**2 - 1/b)/2 - 1/(2*b*x**2 - 2)
```

Giac [A] time = 2.77783, size = 49, normalized size = 1.63

$$\frac{bx^2-2}{2\left(bx^2-1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 - 2)/(b*x^2 - 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 - 1))
```

3.261 $\int \frac{1}{a+(b-ac)x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

```
[Out] ArcTan[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rubi [A] time = 0.0253734, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b-ac}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Mathematica [A] time = 0.0150337, size = 36, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

Maple [A] time = 0.005, size = 34, normalized size = 1.

$$\mathrm{Artanh}\left((ac-b)\,x\frac{1}{\sqrt{a\,(ac-b)}}\right)\frac{1}{\sqrt{a\,(ac-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(-a*c+b)*x^2),x)
```

```
[Out] 1/(a*(a*c-b))^(1/2)*arctanh((a*c-b)*x/(a*(a*c-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52992, size = 219, normalized size = 6.44

$$\left[\frac{\log\left(\frac{(ac-b)x^2+2\sqrt{a^2c-ab}x+a}{(ac-b)x^2-a}\right)}{2\sqrt{a^2c-ab}}, -\frac{\sqrt{-a^2c+ab}\arctan\left(\frac{\sqrt{-a^2c+abx}}{a}\right)}{a^2c-ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((a*c - b)*x^2 + 2*sqrt(a^2*c - a*b)*x + a)/((a*c - b)*x^2 - a))/s
qrt(a^2*c - a*b), -sqrt(-a^2*c + a*b)*arctan(sqrt(-a^2*c + a*b)*x/a)/(a^2*c
 - a*b)]
```

Sympy [B] time = 0.23586, size = 60, normalized size = 1.76

$$-\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(-a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(1/(a*(a*c - b)))*log(-a*sqrt(1/(a*(a*c - b))) + x)/2 + sqrt(1/(a*(a*c
 - b)))*log(a*sqrt(1/(a*(a*c - b))) + x)/2
```

Giac [A] time = 2.77855, size = 50, normalized size = 1.47

$$-\frac{\arctan\left(\frac{acx-bx}{\sqrt{-a^2c+ab}}\right)}{\sqrt{-a^2c+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((a*c*x - b*x)/sqrt(-a^2*c + a*b))/sqrt(-a^2*c + a*b)
```

3.262 $\int \frac{1}{a-(b-ac)x^2} dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

```
[Out] ArcTanh[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rubi [A] time = 0.0111079, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-(b-ac)x^2} dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-ac}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Mathematica [A] time = 0.0097602, size = 36, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

Maple [A] time = 0.003, size = 34, normalized size = 1.

$$\arctan\left((ac-b)x\frac{1}{\sqrt{a(ac-b)}}\right)\frac{1}{\sqrt{a(ac-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(-a*c+b)*x^2),x)
```

```
[Out] 1/(a*(a*c-b))^(1/2)*arctan((a*c-b)*x/(a*(a*c-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46587, size = 219, normalized size = 6.44

$$\left[-\frac{\sqrt{-a^2c+ab}\log\left(\frac{(ac-b)x^2-2\sqrt{-a^2c+ab}x-a}{(ac-b)x^2+a}\right)}{2\left(a^2c-ab\right)}, \frac{\arctan\left(\frac{\sqrt{a^2c-ab}x}{a}\right)}{\sqrt{a^2c-ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2*c + a*b)*log(((a*c - b)*x^2 - 2*sqrt(-a^2*c + a*b)*x - a)/(
(a*c - b)*x^2 + a))/(a^2*c - a*b), arctan(sqrt(a^2*c - a*b)*x/a)/sqrt(a^2*c
 - a*b)]
```

Sympy [B] time = 0.256464, size = 66, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(-a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(-1/(a*(a*c - b)))*log(-a*sqrt(-1/(a*(a*c - b))) + x)/2 + sqrt(-1/(a*(
a*c - b)))*log(a*sqrt(-1/(a*(a*c - b))) + x)/2
```

Giac [A] time = 2.48133, size = 49, normalized size = 1.44

$$\frac{\arctan\left(\frac{acx-bx}{\sqrt{a^2c-ab}}\right)}{\sqrt{a^2c-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] arctan((a*c*x - b*x)/sqrt(a^2*c - a*b))/sqrt(a^2*c - a*b)
```

3.263 $\int \frac{1}{c(a-d)-(b-c)x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

```
[Out] ArcTanh[(Sqrt[b - c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[b - c]*Sqrt[c]*Sqrt[a
- d])
```

Rubi [A] time = 0.0533083, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[b - c]*Sqrt[c]*Sqrt[a
- d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-c}x}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{b-c}\sqrt{c}\sqrt{a-d}}$$

Mathematica [A] time = 0.0202594, size = 50, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-b}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{c-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[-b + c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[c]*Sqrt[-b + c]*Sqrt[a
 - d])
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$\operatorname{Artanh}\left((b-c)\,x\frac{1}{\sqrt{c\,(a-d)\,(b-c)}}\right)\frac{1}{\sqrt{c\,(a-d)\,(b-c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a-d)-(b-c)*x^2),x)
```

```
[Out] 1/(c*(a-d)*(b-c))^(1/2)*arctanh((b-c)*x/(c*(a-d)*(b-c))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.52494, size = 370, normalized size = 7.4

$$\left[\frac{\log\left(\frac{(b-c)x^2+ac-cd+2\sqrt{abc-ac^2-(bc-c^2)d}x}{(b-c)x^2-ac+cd}\right)}{2\sqrt{abc-ac^2-(bc-c^2)d}},\ \frac{\sqrt{-abc+ac^2+(bc-c^2)d}\arctan\left(-\frac{\sqrt{-abc+ac^2+(bc-c^2)d}x}{ac-cd}\right)}{abc-ac^2-(bc-c^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((b - c)*x^2 + a*c - c*d + 2*sqrt(a*b*c - a*c^2 - (b*c - c^2)*d)*x
)/((b - c)*x^2 - a*c + c*d))/sqrt(a*b*c - a*c^2 - (b*c - c^2)*d), sqrt(-a*b
*c + a*c^2 + (b*c - c^2)*d)*arctan(-sqrt(-a*b*c + a*c^2 + (b*c - c^2)*d)*x/
(a*c - c*d))/(a*b*c - a*c^2 - (b*c - c^2)*d)]
```

Sympy [B] time = 0.352276, size = 104, normalized size = 2.08

$$-\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(-ac\sqrt{\frac{1}{c(a-d)(b-c)}}+cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(ac\sqrt{\frac{1}{c(a-d)(b-c)}}-cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x**2),x)
```

```
[Out] -sqrt(1/(c*(a - d)*(b - c)))*log(-a*c*sqrt(1/(c*(a - d)*(b - c))) + c*d*sqr
t(1/(c*(a - d)*(b - c))) + x)/2 + sqrt(1/(c*(a - d)*(b - c)))*log(a*c*sqrt(
1/(c*(a - d)*(b - c))) - c*d*sqrt(1/(c*(a - d)*(b - c))) + x)/2
```

Giac [A] time = 2.47143, size = 78, normalized size = 1.56

$$-\frac{\arctan\left(\frac{bx-cx}{\sqrt{-abc+ac^2+bcd-c^2d}}\right)}{\sqrt{-abc+ac^2+bcd-c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((b*x - c*x)/sqrt(-a*b*c + a*c^2 + b*c*d - c^2*d))/sqrt(-a*b*c + a*c
^2 + b*c*d - c^2*d)
```

3.264 $\int x^{7/2} \left(a + bx^2\right) dx$

Optimal. Leaf size=21

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Rubi [A] time = 0.0043362, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(a + bx^2\right) dx &= \int \left(ax^{7/2} + bx^{11/2}\right) dx\\ &= \frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0049709, size = 21, normalized size = 1.

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{18\,bx^2 + 26\,a}{117}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a),x)
```

```
[Out] 2/117*x^(9/2)*(9*b*x^2+13*a)
```

Maxima [A] time = 2.0432, size = 18, normalized size = 0.86

$$\frac{2}{13}\,bx^{\frac{13}{2}} + \frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

Fricas [A] time = 1.46267, size = 49, normalized size = 2.33

$$\frac{2}{117}\left(9\,bx^{6} + 13\,ax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/117*(9*b*x^6 + 13*a*x^4)*sqrt(x)
```

Sympy [A] time = 5.79703, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{9}{2}}}{9} + \frac{2bx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(9/2)/9 + 2*b*x**(13/2)/13
```

Giac [A] time = 2.72801, size = 18, normalized size = 0.86

$$\frac{2}{13}\,bx^{\frac{13}{2}} + \frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

3.265 $\int x^{5/2}(a+bx^2)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Rubi [A] time = 0.0043558, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx^2)\,dx &= \int \left(ax^{5/2}+bx^{9/2}\right)dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0045701, size = 21, normalized size = 1.

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{14\,bx^2+22\,a}{77}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a),x)
```

```
[Out] 2/77*x^(7/2)*(7*b*x^2+11*a)
```

Maxima [A] time = 1.25353, size = 18, normalized size = 0.86

$$\frac{2}{11}bx^{\frac{11}{2}} + \frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Fricas [A] time = 1.47502, size = 47, normalized size = 2.24

$$\frac{2}{77}\left(7\,bx^5 + 11\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*x^5 + 11*a*x^3)*sqrt(x)
```

Sympy [A] time = 2.72535, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11
```

Giac [A] time = 2.77619, size = 18, normalized size = 0.86

$$\frac{2}{11}bx^{\frac{11}{2}} + \frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

3.266 $\int x^{3/2}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Rubi [A] time = 0.0044014, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)\,dx &= \int \left(ax^{3/2}+bx^{7/2}\right)\,dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0044832, size = 21, normalized size = 1.

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{10\,bx^2+18\,a}{45}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a),x)
```

```
[Out] 2/45*x^(5/2)*(5*b*x^2+9*a)
```

Maxima [A] time = 1.89795, size = 18, normalized size = 0.86

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Fricas [A] time = 1.4849, size = 46, normalized size = 2.19

$$\frac{2}{45}\left(5\,bx^{4} + 9\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b*x^4 + 9*a*x^2)*sqrt(x)
```

Sympy [A] time = 0.939886, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9
```

Giac [A] time = 2.03944, size = 18, normalized size = 0.86

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

3.267 $\int \sqrt{x}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

Rubi [A] time = 0.0042427, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)\,dx &= \int \left(a\sqrt{x}+bx^{5/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0041893, size = 21, normalized size = 1.

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{6\,bx^2+14\,a}{21}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*x^(1/2),x)
```

```
[Out] 2/21*x^(3/2)*(3*b*x^2+7*a)
```

Maxima [A] time = 2.46075, size = 18, normalized size = 0.86

$$\frac{2}{7}bx^{\frac{7}{2}}+\frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Fricas [A] time = 1.43141, size = 43, normalized size = 2.05

$$\frac{2}{21}\left(3\,bx^{3}+7\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(x)
```

Sympy [A] time = 1.14283, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7
```

Giac [A] time = 1.40552, size = 18, normalized size = 0.86

$$\frac{2}{7}bx^{\frac{7}{2}}+\frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

3.268 $\int \frac{a+bx^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

Rubi [A] time = 0.0038726, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+bx^{3/2}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0042909, size = 19, normalized size = 1.

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{2\,bx^2+10\,a}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/5*x^(1/2)*(b*x^2+5*a)
```

Maxima [A] time = 1.89907, size = 18, normalized size = 0.95

$$\frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Fricas [A] time = 1.48494, size = 36, normalized size = 1.89

$$\frac{2}{5}\left(bx^2 + 5\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^2 + 5*a)*sqrt(x)
```

Sympy [A] time = 0.226014, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5
```

Giac [A] time = 3.11987, size = 18, normalized size = 0.95

$$\frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

3.269 $\int \frac{a+bx^2}{x^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Rubi [A] time = 0.0042045, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{3/2}}\,dx&=\int \left(\frac{a}{x^{3/2}}+b\sqrt{x}\right)dx\\&=-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0053236, size = 19, normalized size = 1.

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$-\frac{-2\,bx^2+6\,a}{3}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(3/2),x)
```

```
[Out] -2/3*(-b*x^2+3*a)/x^(1/2)
```

Maxima [A] time = 1.74622, size = 18, normalized size = 0.95

$$\frac{2}{3}bx^{\frac{3}{2}} - \frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Fricas [A] time = 1.56427, size = 36, normalized size = 1.89

$$\frac{2\left(bx^2 - 3a\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - 3*a)/sqrt(x)
```

Sympy [A] time = 0.507086, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3
```

Giac [A] time = 2.49301, size = 18, normalized size = 0.95

$$\frac{2}{3}bx^{\frac{3}{2}} - \frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

3.270 $\int \frac{a+bx^2}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Rubi [A] time = 0.0039198, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.0060218, size = 19, normalized size = 1.

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$-\frac{-6\,bx^2+2\,a}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(5/2),x)
```

```
[Out] -2/3*(-3*b*x^2+a)/x^(3/2)
```

Maxima [A] time = 2.0212, size = 18, normalized size = 0.95

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Fricas [A] time = 1.44335, size = 36, normalized size = 1.89

$$\frac{2\left(3\,bx^2-a\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b*x^2 - a)/x^(3/2)
```

Sympy [A] time = 0.639299, size = 17, normalized size = 0.89

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x)
```

Giac [A] time = 2.63459, size = 18, normalized size = 0.95

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

3.271 $\int \frac{a+bx^2}{x^{7/2}} \, dx$

Optimal. Leaf size=19

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Rubi [A] time = 0.0039203, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{7/2}} \, dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}\right) dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0048721, size = 19, normalized size = 1.

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$-\frac{10\,bx^2+2\,a}{5}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(7/2),x)
```

```
[Out] -2/5*(5*b*x^2+a)/x^(5/2)
```

Maxima [A] time = 2.62221, size = 18, normalized size = 0.95

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

Fricas [A] time = 1.48795, size = 38, normalized size = 2.

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

Sympy [A] time = 1.29671, size = 19, normalized size = 1.

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(7/2),x)
```

```
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x)
```

Giac [A] time = 2.76476, size = 18, normalized size = 0.95

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

3.272 $\int x^{7/2} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

```
[Out] (2*a^2*x^(9/2))/9 + (4*a*b*x^(13/2))/13 + (2*b^2*x^(17/2))/17
```

Rubi [A] time = 0.008615, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(9/2))/9 + (4*a*b*x^(13/2))/13 + (2*b^2*x^(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^{7/2}+2abx^{11/2}+b^2x^{15/2}\right) dx\\ &= \frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.008592, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221a^2+306abx^2+117b^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(9/2)*(221*a^2 + 306*a*b*x^2 + 117*b^2*x^4))/1989
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{234\,b^2x^4+612\,abx^2+442\,a^2}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2,x)
```

```
[Out] 2/1989*x^(9/2)*(117*b^2*x^4+306*a*b*x^2+221*a^2)
```

Maxima [A] time = 2.73022, size = 32, normalized size = 0.89

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

Fricas [A] time = 1.43009, size = 78, normalized size = 2.17

$$\frac{2}{1989}\left(117\,b^2x^8+306\,abx^6+221\,a^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^2*x^8 + 306*a*b*x^6 + 221*a^2*x^4)*sqrt(x)
```

Sympy [A] time = 11.4758, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{9}{2}}}{9}+\frac{4abx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17
```

Giac [A] time = 1.51576, size = 32, normalized size = 0.89

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

3.273 $\int x^{5/2} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*b^2*x^(15/2))/15
```

Rubi [A] time = 0.0084177, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*b^2*x^(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{9/2}+b^2x^{13/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.007443, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(7/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{154\,b^2x^4+420\,abx^2+330\,a^2}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2,x)
```

```
[Out] 2/1155*x^(7/2)*(77*b^2*x^4+210*a*b*x^2+165*a^2)
```

Maxima [A] time = 1.50366, size = 32, normalized size = 0.89

$$\frac{2}{15}\,b^{2}x^{\frac{15}{2}} + \frac{4}{11}\,abx^{\frac{11}{2}} + \frac{2}{7}\,a^{2}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Fricas [A] time = 1.21277, size = 77, normalized size = 2.14

$$\frac{2}{1155}\left(77\,b^{2}x^{7} + 210\,abx^{5} + 165\,a^{2}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*x^7 + 210*a*b*x^5 + 165*a^2*x^3)*sqrt(x)
```

Sympy [A] time = 5.69772, size = 34, normalized size = 0.94

$$\frac{2a^{2}x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{11}{2}}}{11} + \frac{2b^{2}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15
```

Giac [A] time = 2.75891, size = 32, normalized size = 0.89

$$\frac{2}{15}\,b^{2}x^{\frac{15}{2}} + \frac{4}{11}\,abx^{\frac{11}{2}} + \frac{2}{7}\,a^{2}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

3.274 $\int x^{3/2} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}$$

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(13/2))/13
```

Rubi [A] time = 0.0085225, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(13/2))/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^{3/2}+2abx^{7/2}+b^2x^{11/2}\right) dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0072432, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(5/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{90\,b^2x^4+260\,abx^2+234\,a^2}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2,x)
```

```
[Out] 2/585*x^(5/2)*(45*b^2*x^4+130*a*b*x^2+117*a^2)
```

Maxima [A] time = 1.98848, size = 32, normalized size = 0.89

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Fricas [A] time = 1.24236, size = 76, normalized size = 2.11

$$\frac{2}{585}\left(45\,b^2x^6+130\,abx^4+117\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*x^6 + 130*a*b*x^4 + 117*a^2*x^2)*sqrt(x)
```

Sympy [A] time = 2.56054, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
```

Giac [A] time = 1.79306, size = 32, normalized size = 0.89

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

3.275 $\int \sqrt{x} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(11/2))/11
```

Rubi [A] time = 0.0082896, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x}\left(a+bx^2\right)^2 \, dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+b^2x^{9/2}\right) dx \\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2} \end{aligned}$$

Mathematica [A] time = 0.007235, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{42\,b^2x^4+132\,abx^2+154\,a^2}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*b^2*x^4+66*a*b*x^2+77*a^2)
```

Maxima [A] time = 2.45789, size = 32, normalized size = 0.89

$$\frac{2}{11}\,b^2x^{\frac{11}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Fricas [A] time = 1.30838, size = 70, normalized size = 1.94

$$\frac{2}{231}\left(21\,b^2x^5 + 66\,abx^3 + 77\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(x)
```

Sympy [A] time = 1.62027, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{3}{2}}}{3} + \frac{4abx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2),x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(11/2)/11
```

Giac [A] time = 3.11783, size = 32, normalized size = 0.89

$$\frac{2}{11}\,b^2x^{\frac{11}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

3.276 $\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}$$

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*b^2*x^(9/2))/9
```

Rubi [A] time = 0.0087443, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*b^2*x^(9/2))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+b^2x^{7/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0075841, size = 30, normalized size = 0.88

$$\frac{2}{45}\sqrt{x}\left(45a^2+18abx^2+5b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/45
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{10\,b^2x^4+36\,abx^2+90\,a^2}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/45*x^(1/2)*(5*b^2*x^4+18*a*b*x^2+45*a^2)
```

Maxima [A] time = 2.2178, size = 32, normalized size = 0.94

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Fricas [A] time = 1.16672, size = 65, normalized size = 1.91

$$\frac{2}{45}\left(5\,b^2x^4 + 18\,abx^2 + 45\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(x)
```

Sympy [A] time = 0.730421, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
```

Giac [A] time = 2.61957, size = 32, normalized size = 0.94

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

3.277 $\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(7/2))/7
```

Rubi [A] time = 0.0084777, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(7/2))/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+b^2x^{5/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0091576, size = 30, normalized size = 0.88

$$\frac{2\left(-21a^2+14abx^2+3b^2x^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (2*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*Sqrt[x])
```

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$-\frac{-6\,b^2x^4-28\,abx^2+42\,a^2}{21}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2),x)
```

```
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)/x^(1/2)
```

Maxima [A] time = 2.22246, size = 32, normalized size = 0.94

$$\frac{2}{7}\,b^{2}x^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Fricas [A] time = 1.2249, size = 65, normalized size = 1.91

$$\frac{2\left(3\,b^{2}x^{4} + 14\,abx^{2} - 21\,a^{2}\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)/sqrt(x)
```

Sympy [A] time = 0.97361, size = 32, normalized size = 0.94

$$-\frac{2a^{2}}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^{2}x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2)/7
```

Giac [A] time = 3.03508, size = 32, normalized size = 0.94

$$\frac{2}{7}\,b^{2}x^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

3.278 $\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*b^2*x^(5/2))/5
```

Rubi [A] time = 0.0082254, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*b^2*x^(5/2))/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+b^2x^{3/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0091456, size = 30, normalized size = 0.88

$$\frac{2\left(-5a^2+30abx^2+3b^2x^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(5/2),x]
```

```
[Out] (2*(-5*a^2 + 30*a*b*x^2 + 3*b^2*x^4))/(15*x^(3/2))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$-\frac{-6\,b^2x^4-60\,abx^2+10\,a^2}{15}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2),x)
```

```
[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)/x^(3/2)
```

Maxima [A] time = 1.22346, size = 32, normalized size = 0.94

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Fricas [A] time = 1.1983, size = 63, normalized size = 1.85

$$\frac{2\left(3\,b^2x^4 + 30\,abx^2 - 5\,a^2\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)/x^(3/2)
```

Sympy [A] time = 1.21377, size = 32, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5
```

Giac [A] time = 2.34887, size = 32, normalized size = 0.94

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

3.279 $\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rubi [A] time = 0.008181, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+b^2\sqrt{x}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0087118, size = 30, normalized size = 0.88

$$\frac{2\left(-3a^2-30abx^2+5b^2x^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^(5/2))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$-\frac{-10\,b^2x^4+60\,abx^2+6\,a^2}{15}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2),x)
```

```
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)/x^(5/2)
```

Maxima [A] time = 1.93498, size = 34, normalized size = 1.

$$\frac{2}{3}\,b^2x^{\frac{3}{2}} - \frac{2\left(10\,abx^2 + a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Fricas [A] time = 1.30228, size = 63, normalized size = 1.85

$$\frac{2\left(5\,b^2x^4 - 30\,abx^2 - 3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)/x^(5/2)
```

Sympy [A] time = 1.82759, size = 32, normalized size = 0.94

$$-\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)/3
```

Giac [A] time = 2.65104, size = 34, normalized size = 1.

$$\frac{2}{3}\,b^2x^{\frac{3}{2}} - \frac{2\left(10\,abx^2 + a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

3.280 $\int x^{7/2} \left(a + bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{6}{13}a^2bx^{13/2} + \frac{2}{9}a^3x^{9/2} + \frac{6}{17}ab^2x^{17/2} + \frac{2}{21}b^3x^{21/2}$$

```
[Out] (2*a^3*x^(9/2))/9 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*x^(17/2))/17 + (2*b^3*
x^(21/2))/21
```

Rubi [A] time = 0.0125644, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{13}a^2bx^{13/2} + \frac{2}{9}a^3x^{9/2} + \frac{6}{17}ab^2x^{17/2} + \frac{2}{21}b^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(9/2))/9 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*x^(17/2))/17 + (2*b^3*
x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(a + bx^2\right)^3 \, dx &= \int \left(a^3x^{7/2} + 3a^2bx^{11/2} + 3ab^2x^{15/2} + b^3x^{19/2}\right) dx \\ &= \frac{2}{9}a^3x^{9/2} + \frac{6}{13}a^2bx^{13/2} + \frac{6}{17}ab^2x^{17/2} + \frac{2}{21}b^3x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0103996, size = 41, normalized size = 0.8

$$\frac{2x^{9/2}\left(3213a^2bx^2 + 1547a^3 + 2457ab^2x^4 + 663b^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(9/2)*(1547*a^3 + 3213*a^2*b*x^2 + 2457*a*b^2*x^4 + 663*b^3*x^6))/1392
3
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$\frac{1326\,b^3x^6 + 4914\,ab^2x^4 + 6426\,a^2bx^2 + 3094\,a^3}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/13923*x^(9/2)*(663*b^3*x^6+2457*a*b^2*x^4+3213*a^2*b*x^2+1547*a^3)
```

Maxima [A] time = 1.99198, size = 47, normalized size = 0.92

$$\frac{2}{21}\,b^3x^{\frac{21}{2}} + \frac{6}{17}\,ab^2x^{\frac{17}{2}} + \frac{6}{13}\,a^2bx^{\frac{13}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

Fricas [A] time = 1.19382, size = 109, normalized size = 2.14

$$\frac{2}{13923}\left(663\,b^3x^{10} + 2457\,ab^2x^8 + 3213\,a^2bx^6 + 1547\,a^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^3*x^10 + 2457*a*b^2*x^8 + 3213*a^2*b*x^6 + 1547*a^3*x^4)*sqr
t(x)
```

Sympy [A] time = 21.0104, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{9}{2}}}{9} + \frac{6a^2bx^{\frac{13}{2}}}{13} + \frac{6ab^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(9/2)/9 + 6*a**2*b*x**(13/2)/13 + 6*a*b**2*x**(17/2)/17 + 2*b**3*
x**(21/2)/21
```

Giac [A] time = 1.47747, size = 47, normalized size = 0.92

$$\frac{2}{21}\,b^3x^{\frac{21}{2}} + \frac{6}{17}\,ab^2x^{\frac{17}{2}} + \frac{6}{13}\,a^2bx^{\frac{13}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

3.281 $\int x^{5/2} \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*x^(15/2))/5 + (2*b^3*x
^(19/2))/19
```

Rubi [A] time = 0.012456, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*x^(15/2))/5 + (2*b^3*x
^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^{5/2}+3a^2bx^{9/2}+3ab^2x^{13/2}+b^3x^{17/2}\right)dx\\&=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0094307, size = 41, normalized size = 0.8

$$\frac{2x^{7/2}\left(1995a^2bx^2+1045a^3+1463ab^2x^4+385b^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(7/2)*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^2*x^4 + 385*b^3*x^6))/7315
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$\frac{770\,b^3x^6+2926\,ab^2x^4+3990\,a^2bx^2+2090\,a^3}{7315}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/7315*x^(7/2)*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)
```

Maxima [A] time = 2.38975, size = 47, normalized size = 0.92

$$\frac{2}{19}\,b^{3}x^{\frac{19}{2}}+\frac{2}{5}\,ab^{2}x^{\frac{15}{2}}+\frac{6}{11}\,a^{2}bx^{\frac{11}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

Fricas [A] time = 1.16818, size = 107, normalized size = 2.1

$$\frac{2}{7315}\left(385\,b^{3}x^{9}+1463\,ab^{2}x^{7}+1995\,a^{2}bx^{5}+1045\,a^{3}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*x^9 + 1463*a*b^2*x^7 + 1995*a^2*b*x^5 + 1045*a^3*x^3)*sqrt(
x)
```

Sympy [A] time = 11.2705, size = 49, normalized size = 0.96

$$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{6a^{2}bx^{\frac{11}{2}}}{11}+\frac{2ab^{2}x^{\frac{15}{2}}}{5}+\frac{2b^{3}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*b**2*x**(15/2)/5 + 2*b**3*x
**(19/2)/19
```

Giac [A] time = 2.97581, size = 47, normalized size = 0.92

$$\frac{2}{19}\,b^{3}x^{\frac{19}{2}}+\frac{2}{5}\,ab^{2}x^{\frac{15}{2}}+\frac{6}{11}\,a^{2}bx^{\frac{11}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

3.282 $\int x^{3/2} \left(a + bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^2bx^{9/2} + \frac{2}{5}a^3x^{5/2} + \frac{6}{13}ab^2x^{13/2} + \frac{2}{17}b^3x^{17/2}$$

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(13/2))/13 + (2*b^3*x^
(17/2))/17
```

Rubi [A] time = 0.0125412, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{3}a^2bx^{9/2} + \frac{2}{5}a^3x^{5/2} + \frac{6}{13}ab^2x^{13/2} + \frac{2}{17}b^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(13/2))/13 + (2*b^3*x^
(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(a + bx^2\right)^3 \, dx &= \int \left(a^3x^{3/2} + 3a^2bx^{7/2} + 3ab^2x^{11/2} + b^3x^{15/2}\right) dx \\ &= \frac{2}{5}a^3x^{5/2} + \frac{2}{3}a^2bx^{9/2} + \frac{6}{13}ab^2x^{13/2} + \frac{2}{17}b^3x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0096036, size = 41, normalized size = 0.8

$$\frac{2x^{5/2}\left(1105a^2bx^2 + 663a^3 + 765ab^2x^4 + 195b^3x^6\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(5/2)*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*x^4 + 195*b^3*x^6))/3315
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$\frac{390\,b^3x^6 + 1530\,ab^2x^4 + 2210\,a^2bx^2 + 1326\,a^3}{3315}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/3315*x^(5/2)*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)
```

Maxima [A] time = 2.17352, size = 47, normalized size = 0.92

$$\frac{2}{17}\,b^3x^{\frac{17}{2}} + \frac{6}{13}\,ab^2x^{\frac{13}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)
```

Fricas [A] time = 1.30844, size = 104, normalized size = 2.04

$$\frac{2}{3315}\left(195\,b^3x^8 + 765\,ab^2x^6 + 1105\,a^2bx^4 + 663\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^3*x^8 + 765*a*b^2*x^6 + 1105*a^2*b*x^4 + 663*a^3*x^2)*sqrt(x)
```

Sympy [A] time = 5.72263, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{13}{2}}}{13} + \frac{2b^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(13/2)/13 + 2*b**3*x*
*(17/2)/17
```

Giac [A] time = 2.25685, size = 47, normalized size = 0.92

$$\frac{2}{17}\,b^3x^{\frac{17}{2}} + \frac{6}{13}\,ab^2x^{\frac{13}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)
```

3.283 $\int \sqrt{x} \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(15/2))/15
```

Rubi [A] time = 0.0127798, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^3 \, dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3ab^2x^{9/2}+b^3x^{13/2}\right) dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0097537, size = 41, normalized size = 0.8

$$\frac{2x^{3/2}\left(495a^2bx^2+385a^3+315ab^2x^4+77b^3x^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4 + 77*b^3*x^6))/1155
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$\frac{154\,b^3x^6+630\,ab^2x^4+990\,a^2bx^2+770\,a^3}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)
```

Maxima [A] time = 2.30333, size = 47, normalized size = 0.92

$$\frac{2}{15}\,b^3x^{\frac{15}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

Fricas [A] time = 1.2759, size = 99, normalized size = 1.94

$$\frac{2}{1155}\left(77\,b^3x^7 + 315\,ab^2x^5 + 495\,a^2bx^3 + 385\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(x)
```

Sympy [A] time = 2.11113, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{3}{2}}}{3} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*x**(1/2),x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(15/2)/15
```

Giac [A] time = 2.9259, size = 47, normalized size = 0.92

$$\frac{2}{15}\,b^3x^{\frac{15}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

3.284 $\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(13/2)
)/13
```

Rubi [A] time = 0.0115788, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(13/2)
)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3ab^2x^{7/2}+b^3x^{11/2}\right)dx\\ &= 2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.009848, size = 41, normalized size = 0.84

$$\frac{2}{195}\sqrt{x}\left(117a^2bx^2+195a^3+65ab^2x^4+15b^3x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x^6))/195
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$\frac{30\,b^3x^6+130\,ab^2x^4+234\,a^2bx^2+390\,a^3}{195}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2/195*x^(1/2)*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)
```

Maxima [A] time = 2.33593, size = 47, normalized size = 0.96

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Fricas [A] time = 1.25466, size = 93, normalized size = 1.9

$$\frac{2}{195}\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(x)
```

Sympy [A] time = 2.2027, size = 48, normalized size = 0.98

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(13/
2)/13
```

Giac [A] time = 2.1557, size = 47, normalized size = 0.96

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

3.285 $\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(11/2))
/11
```

Rubi [A] time = 0.0116159, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(11/2))
/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3ab^2x^{5/2}+b^3x^{9/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0107062, size = 41, normalized size = 0.87

$$\frac{2\left(77a^2bx^2-77a^3+33ab^2x^4+7b^3x^6\right)}{77\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(3/2),x]
```

```
[Out] (2*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6))/(77*Sqrt[x])
```

Maple [A] time = 0.006, size = 38, normalized size = 0.8

$$-\frac{-14\,b^{3}x^{6}-66\,ab^{2}x^{4}-154\,a^{2}bx^{2}+154\,a^{3}}{77}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(3/2),x)
```

```
[Out] -2/77*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)/x^(1/2)
```

Maxima [A] time = 1.94537, size = 47, normalized size = 1.

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+2\,a^{2}bx^{\frac{3}{2}}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Fricas [A] time = 1.15043, size = 88, normalized size = 1.87

$$\frac{2\left(7\,b^{3}x^{6}+33\,ab^{2}x^{4}+77\,a^{2}bx^{2}-77\,a^{3}\right)}{77\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/sqrt(x)
```

Sympy [A] time = 2.40461, size = 46, normalized size = 0.98

$$-\frac{2a^{3}}{\sqrt{x}}+2a^{2}bx^{\frac{3}{2}}+\frac{6ab^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(7/2)/7 + 2*b**3*x**(11/2
)/11
```

Giac [A] time = 2.23314, size = 47, normalized size = 1.

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+2\,a^{2}bx^{\frac{3}{2}}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

3.286 $\int \frac{(a+bx^2)^3}{x^{5/2}} \, dx$

Optimal. Leaf size=49

$$6a^2b\sqrt{x} - \frac{2a^3}{3x^{3/2}} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{9}b^3x^{9/2}$$

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/
2))/9
```

Rubi [A] time = 0.011726, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$6a^2b\sqrt{x} - \frac{2a^3}{3x^{3/2}} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/
2))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{5/2}} \, dx &= \int \left(\frac{a^3}{x^{5/2}} + \frac{3a^2b}{\sqrt{x}} + 3ab^2x^{3/2} + b^3x^{7/2}\right) dx \\
&= -\frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{9}b^3x^{9/2}
\end{aligned}$$

Mathematica [A] time = 0.0112587, size = 41, normalized size = 0.84

$$\frac{2\left(135a^2bx^2 - 15a^3 + 27ab^2x^4 + 5b^3x^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(5/2),x]
```

```
[Out] (2*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^6))/(45*x^(3/2))
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$-\frac{-10 b^3x^6-54 ab^2x^4-270 a^2bx^2+30 a^3}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(5/2),x)
```

```
[Out] -2/45*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)/x^(3/2)
```

Maxima [A] time = 1.88921, size = 47, normalized size = 0.96

$$\frac{2}{9} b^3x^{\frac{9}{2}}+\frac{6}{5} ab^2x^{\frac{5}{2}}+6 a^2b\sqrt{x}-\frac{2 a^3}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Fricas [A] time = 1.21198, size = 89, normalized size = 1.82

$$\frac{2 \left(5 b^3x^6+27 ab^2x^4+135 a^2bx^2-15 a^3\right)}{45 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/x^(3/2)
```

Sympy [A] time = 3.03934, size = 48, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(
9/2)/9
```

Giac [A] time = 2.89049, size = 47, normalized size = 0.96

$$\frac{2}{9} b^3x^{\frac{9}{2}}+\frac{6}{5} ab^2x^{\frac{5}{2}}+6 a^2b\sqrt{x}-\frac{2 a^3}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

3.287 $\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2)
)/7
```

Rubi [A] time = 0.0121488, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2)
)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3ab^2\sqrt{x}+b^3x^{5/2}\right)dx\\ &= -\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0108319, size = 41, normalized size = 0.87

$$\frac{2\left(-105a^2bx^2-7a^3+35ab^2x^4+5b^3x^6\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (2*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6))/(35*x^(5/2))
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$-\frac{-10 b^{3} x^{6}-70 a b^{2} x^{4}+210 a^{2} b x^{2}+14 a^{3}}{35} x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(7/2),x)
```

```
[Out] -2/35*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)/x^(5/2)
```

Maxima [A] time = 2.45956, size = 49, normalized size = 1.04

$$\frac{2}{7} b^{3} x^{\frac{7}{2}} + 2 a b^{2} x^{\frac{3}{2}} - \frac{2 \left(15 a^{2} b x^{2} + a^{3}\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Fricas [A] time = 1.22153, size = 88, normalized size = 1.87

$$\frac{2 \left(5 b^{3} x^{6} + 35 a b^{2} x^{4} - 105 a^{2} b x^{2} - 7 a^{3}\right)}{35 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/x^(5/2)
```

Sympy [A] time = 3.84514, size = 46, normalized size = 0.98

$$-\frac{2a^{3}}{5x^{\frac{5}{2}}} - \frac{6a^{2}b}{\sqrt{x}} + 2ab^{2}x^{\frac{3}{2}} + \frac{2b^{3}x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7/
2)/7
```

Giac [A] time = 1.61519, size = 49, normalized size = 1.04

$$\frac{2}{7} b^{3} x^{\frac{7}{2}} + 2 a b^{2} x^{\frac{3}{2}} - \frac{2 \left(15 a^{2} b x^{2} + a^{3}\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

3.288 $\int \frac{x^{7/2}}{a+bx^2} dx$

Optimal. Leaf size=215

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}+\frac{a^{5/4}\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}b^{9/4}}$$

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(5/2))/(5*b) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4)) + (a^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.198196, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}+\frac{a^{5/4}\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(5/2))/(5*b) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4)) + (a^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx^2}\,dx}{b}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} - \cdots$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}} + \cdots$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\cdots\right.}{2\sqrt{2}b^{9/4}}$$

Mathematica [A] time = 0.0650102, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-10\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+1}{20b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(5/2) - 10*Sqrt[2]*a^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(20*b^(9/4))
```

Maple [A] time = 0.012, size = 152, normalized size = 0.7

$$\frac{2}{5b}x^{\frac{5}{2}}-2\frac{a\sqrt{x}}{b^2}+\frac{a\sqrt{2}}{4b^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{a\sqrt{2}}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a),x)
```

```
[Out] 2/5*x^(5/2)/b-2*a*x^(1/2)/b^2+1/4*a/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)
^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b
*a)^(1/2)))+1/2*a/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)+1)+1/2*a/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/
2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3777, size = 393, normalized size = 1.83

$$\frac{20\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab^7\sqrt{x}\left(-\frac{a^5}{b^9}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^5}{b^9}}+a^2xb^7\left(-\frac{a^5}{b^9}\right)^{\frac{3}{4}}}}{a^5}\right)+5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}+a\sqrt{x}\right)-5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\right.\right.}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*b^2*(-a^5/b^9)^(1/4)*arctan(-(a*b^7*sqrt(x)*(-a^5/b^9)^(3/4) - sqr
t(b^4*sqrt(-a^5/b^9) + a^2*x)*b^7*(-a^5/b^9)^(3/4))/a^5) + 5*b^2*(-a^5/b^9)
^(1/4)*log(b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) - 5*b^2*(-a^5/b^9)^(1/4)*log(-
b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) + 4*(b*x^2 - 5*a)*sqrt(x))/b^2
```

Sympy [A] time = 166.208, size = 192, normalized size = 0.89

$$\begin{cases} \tilde{\infty} x^{\frac{5}{2}} \\ \frac{2x^{\frac{9}{2}}}{9a} \\ \frac{2x^{\frac{5}{2}}}{5b} \\ -\frac{\sqrt[4]{-1}a^{\frac{5}{4}}b^{17}\left(\frac{1}{b}\right)^{\frac{77}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2} + \frac{\sqrt[4]{-1}a^{\frac{5}{4}}b^{17}\left(\frac{1}{b}\right)^{\frac{77}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2} - \sqrt[4]{-1}a^{\frac{5}{4}}b^{17}\left(\frac{1}{b}\right)^{\frac{77}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right) - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{\frac{5}{2}}}{5b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a), Eq(b, 0))
, (2*x**(5/2)/(5*b), Eq(a, 0)), (-(-1)**(1/4)*a**(5/4)*b**17*(1/b)**(77/4)*
log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 + (-1)**(1/4)*a**(5/4)*
b**17*(1/b)**(77/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 - (-
1)**(1/4)*a**(5/4)*b**17*(1/b)**(77/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(
1/b)**(1/4))) - 2*a*sqrt(x)/b**2 + 2*x**(5/2)/(5*b), True))
```

Giac [A] time = 1.73939, size = 265, normalized size = 1.23

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^3} + \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^3} + \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x +\right.}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*(a*b^3)^(1/4
)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*(a*b
^3)^(1/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 2/5*(b^
4*x^(5/2) - 5*a*b^3*sqrt(x))/b^5
```

3.289 $\int \frac{x^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=204

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}-\frac{a^{3/4}\tan^{-1}\left(\sqrt{\right.}{\sqrt{2}b}$$

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(7/4)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*b^(7/4)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4))
```

Rubi [A] time = 0.152454, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}-\frac{a^{3/4}\tan^{-1}\left(\sqrt{\right.}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(7/4)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*b^(7/4)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b}$$

$$= \frac{2x^{3/2}}{3b} - \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2x^{3/2}}{3b} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= \frac{2x^{3/2}}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a^{3/4}\,\text{Subst}\left(\int \cdots\right)}{2\cdots}$$

$$= \frac{2x^{3/2}}{3b} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}} - \frac{a^{3/4}\,\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{2x^{3/2}}{3b} + \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}} + \cdots$$

Mathematica [A] time = 0.0253083, size = 78, normalized size = 0.38

$$\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{7/4}}-\frac{(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{7/4}}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(7/
4) - ((-a)^(3/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(7/4)
```

Maple [A] time = 0.006, size = 143, normalized size = 0.7

$$\frac{2}{3b}x^{\frac{3}{2}}-\frac{a\sqrt{2}}{4b^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{a\sqrt{2}}{2b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{a\sqrt{2}}{2b^2}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a),x)
```

```
[Out] 2/3*x^(3/2)/b-1/4/b^2*a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2
^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/
b^2*a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b^2
*a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41165, size = 377, normalized size = 1.85

$$\frac{12\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^2\sqrt{x}\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}-\sqrt{-a^3b^3\sqrt{-\frac{a^3}{b^7}}+a^4xb^2}\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}}{a^3}\right)-3\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}+a^2\sqrt{x}\right)+3\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(-b^5\left(\right.\right.}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*b*(-a^3/b^7)^(1/4)*arctan(-(a^2*b^2*sqrt(x)*(-a^3/b^7)^(1/4) - sqrt
(-a^3*b^3*sqrt(-a^3/b^7) + a^4*x)*b^2*(-a^3/b^7)^(1/4))/a^3) - 3*b*(-a^3/b^
```

```
7)^(1/4)*log(b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 3*b*(-a^3/b^7)^(1/4)*log
(-b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 4*x^(3/2))/b
```

Sympy [A] time = 25.5627, size = 180, normalized size = 0.88

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ \frac{(-1)^{\frac{3}{4}} a^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^8\left(\frac{1}{b}\right)^{\frac{25}{4}}} - \frac{(-1)^{\frac{3}{4}} a^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^8\left(\frac{1}{b}\right)^{\frac{25}{4}}} - \frac{(-1)^{\frac{3}{4}} a^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{15}\left(\frac{1}{b}\right)^{\frac{53}{4}}} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0))
, (2*x**(3/2)/(3*b), Eq(a, 0)), ((-1)**(3/4)*a**(3/4)*log(-(-1)**(1/4)*a**(
1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**8*(1/b)**(25/4)) - (-1)**(3/4)*a**(3/4)*
atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**8*(1/b)**(25/4)) - (-
1)**(3/4)*a**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**1
5*(1/b)**(53/4)) + 2*x**(3/2)/(3*b), True))
```

Giac [A] time = 2.24192, size = 240, normalized size = 1.18

$$\frac{2\,x^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + \right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/b - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*
(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4
```

3.290 $\int \frac{x^{3/2}}{a+bx^2} dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}$$

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4))
```

Rubi [A] time = 0.162816, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{a+bx^2}\,dx &= \frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{3/2}}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\,\text{Subst}\left(\int\right.}{}\\
&= \frac{2\sqrt{x}}{b}+\frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\,\text{Subst}\left(\int \frac{1}{-1-}\right.}{\sqrt{}}\\
&= \frac{2\sqrt{x}}{b}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-
\end{aligned}$$

Mathematica [A] time = 0.0387865, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt{2}}{4b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x] - Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(4*b^(5/4))
```

Maple [A] time = 0.006, size = 140, normalized size = 0.7

$$2\,\frac{\sqrt{x}}{b}-\frac{\sqrt{2}}{4\,b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{\sqrt{2}}{2\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}}{2\,b}\sqrt[4]{\frac{a}{b}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a),x)
```

```
[Out] 2*x^(1/2)/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)
+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/b*(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b*(1/b*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35279, size = 312, normalized size = 1.54

$$-\frac{4\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2\sqrt{-\frac{a}{b^5}}+x}\,b^4\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}-b^4\sqrt{x}\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}}{a}\right)+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*(-a/b^5)^(1/4)*arctan((sqrt(b^2*sqrt(-a/b^5) + x)*b^4*(-a/b^5)^(3
/4) - b^4*sqrt(x)*(-a/b^5)^(3/4))/a) + b*(-a/b^5)^(1/4)*log(b*(-a/b^5)^(1/4
) + sqrt(x)) - b*(-a/b^5)^(1/4)*log(-b*(-a/b^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/b
```

Sympy [A] time = 9.4927, size = 177, normalized size = 0.88

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{\sqrt[4]{-1}\sqrt[4]{a}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^3\left(\frac{1}{b}\right)^{\frac{7}{4}}} + \frac{\sqrt[4]{-1}\sqrt[4]{a}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^3\left(\frac{1}{b}\right)^{\frac{7}{4}}} + \frac{\sqrt[4]{-1}\sqrt[4]{a}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{10}\left(\frac{1}{b}\right)^{\frac{35}{4}}} + \frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a), Eq(b, 0)),
 (2*sqrt(x)/b, Eq(a, 0)), (-(-1)**(1/4)*a**(1/4)*log((-1)**(1/4)*a**(1/4)*(
1/b)**(1/4) + sqrt(x))/(2*b**3*(1/b)**(7/4)) + (-1)**(1/4)*a**(1/4)*atan((-
1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**3*(1/b)**(7/4)) + (-1)**(1/4
)*a**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**10*(1/b)
**(35/4)) + 2*sqrt(x)/b, True))
```

Giac [A] time = 1.86621, size = 240, normalized size = 1.19

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{}\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2*sqrt(x)/b
```

3.291 $\int \frac{\sqrt{x}}{a+bx^2}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) +
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(1/4)*b^(3/4))
```

Rubi [A] time = 0.144383, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) +
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{a+bx^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x \ldots}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}\\
&= \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\ldots}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}b^{3/4}}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{\ldots}\right.}{2\sqrt{2}\sqrt[4]{a}b\ldots}
\end{aligned}$$

Mathematica [A] time = 0.0242538, size = 54, normalized size = 0.28

$$\frac{a\left(\tan^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)\right)}{(-a)^{5/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] (a*(ArcTan[(a*b^(1/4)*Sqrt[x])/(-a)^(5/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/(-a)
^(1/4)]))/((-a)^(5/4)*b^(3/4))
```

Maple [A] time = 0.006, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4b}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}}{2b}\arctan\left(\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/
2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/b/(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2/b/(1/b*a)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35074, size = 332, normalized size = 1.73

$$-2\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab\sqrt{-\frac{1}{ab^3}}+x}b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}-b\sqrt{x}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*(-1/(a*b^3))^(1/4)*arctan(sqrt(-a*b*sqrt(-1/(a*b^3)) + x)*b*(-1/(a*b^3))
^(1/4) - b*sqrt(x)*(-1/(a*b^3))^(1/4)) + 1/2*(-1/(a*b^3))^(1/4)*log(a*b^2*(
-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(-1/(a*b^3))^(1/4)*log(-a*b^2*(-1/(a*b^3
))^(3/4) + sqrt(x))
```

Sympy [A] time = 4.46716, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ -\frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}b^2\left(\frac{1}{b}\right)^{\frac{5}{4}}}+\frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}b^2\left(\frac{1}{b}\right)^{\frac{5}{4}}}+\frac{(-1)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{a}b^2\left(\frac{1}{b}\right)^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)),
 (-2/(b*sqrt(x)), Eq(a, 0)), (-(-1)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)*
*(1/4) + sqrt(x))/(2*a**(1/4)*b**2*(1/b)**(5/4)) + (-1)**(3/4)*log((-1)**(1
/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b**2*(1/b)**(5/4)) + (-1)*
*(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b**2*(1/
b)**(5/4)), True))
```

Giac [A] time = 3.01733, size = 246, normalized size = 1.28

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{3}}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{3}}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right.}{4\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3)
```

3.292 $\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) -
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rubi [A] time = 0.146102, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) -
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{a}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\right.}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.0340394, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
```

```
Sqrt[b]*x] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(3/4)*b^(1/4))
```

Maple [A] time = 0.004, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{\sqrt{2}}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 1/4*(1/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/
2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*(1/b*a)^(1/4)/a*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*(1/b*a)^(1/4)/a*2^(1/2)*a
rctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33363, size = 329, normalized size = 1.71

$$2\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2\sqrt{-\frac{1}{a^3b}}+x}a^2b\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b))^(1/4)*arctan(sqrt(a^2*sqrt(-1/(a^3*b)) + x)*a^2*b*(-1/(a^3*b
))^(3/4) - a^2*b*sqrt(x)*(-1/(a^3*b))^(3/4)) + 1/2*(-1/(a^3*b))^(1/4)*log(a
*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*(-1/(a^3*b))^(1/4)*log(-a*(-1/(a^3*b))
^(1/4) + sqrt(x))
```

Sympy [A] time = 8.71447, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ -\frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b^{11}\left(\frac{1}{b}\right)^{\frac{43}{4}}} + \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b^{11}\left(\frac{1}{b}\right)^{\frac{43}{4}}} - \frac{\sqrt[4]{-1}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{3}{4}}b^{11}\left(\frac{1}{b}\right)^{\frac{43}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a, Eq(b, 0)), (-2
/(3*b*x**(3/2)), Eq(a, 0)), (-(-1)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**
(1/4) + sqrt(x))/(2*a**(3/4)*b**11*(1/b)**(43/4)) + (-1)**(1/4)*log((-1)**(
1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)*b**11*(1/b)**(43/4)) - (-
1)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(3/4)*b**11
*(1/b)**(43/4)), True))
```

Giac [A] time = 2.60393, size = 246, normalized size = 1.28

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab} + \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab} + \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right.}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(a*b
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b)
```

3.293 $\int \frac{1}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{\sqrt{2}a^{5/4}}$$

```
[Out] -2/(a*Sqrt[x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4))
```

Rubi [A] time = 0.166747, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{\sqrt{2}a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] -2/(a*Sqrt[x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{b\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a} - \sqrt[4]{b}\,\text{Subst}\left(\int\right.$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}} - \sqrt[4]{b}\,\text{Subst}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{2\sqrt{2}a^{5/4}}$$

Mathematica [C] time = 0.0053555, size = 27, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a)])/(a*Sqrt[x])
```

Maple [A] time = 0.007, size = 140, normalized size = 0.7

$$-\frac{\sqrt{2}}{4a}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{2a}\arctan\left(\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/4/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/a/(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/a/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/a/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3265, size = 343, normalized size = 1.7

$$\frac{4\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}-\sqrt{-a^3b\sqrt{-\frac{b}{a^5}}+b^2x}\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}}{b}\right)-ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)+ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-a^4\left(-\frac{b}{a^5}\right)\right.}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-b/a^5)^(1/4)*arctan(-(a*b*sqrt(x)*(-b/a^5)^(1/4) - sqrt(-a^3*b
*sqrt(-b/a^5) + b^2*x)*a*(-b/a^5)^(1/4))/b) - a*x*(-b/a^5)^(1/4)*log(a^4*(-
b/a^5)^(3/4) + b*sqrt(x)) + a*x*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)^(3/4) + b*
```

```
sqrt(x)) - 4*sqrt(x))/(a*x)
```

Sympy [A] time = 17.5696, size = 180, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{a\sqrt{x}} + \frac{(-1)^{\frac{3}{4}}b^3\left(\frac{1}{b}\right)^{\frac{11}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}}b^3\left(\frac{1}{b}\right)^{\frac{11}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}}b^3\left(\frac{1}{b}\right)^{\frac{11}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*b**3*(1/b)**(1
1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)) - (-1)*
*(3/4)*b**3*(1/b)**(11/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/
(2*a**(5/4)) - (-1)**(3/4)*b**3*(1/b)**(11/4)*atan((-1)**(3/4)*sqrt(x)/(a**
(1/4)*(1/b)**(1/4)))/a**(5/4), True))
```

Giac [A] time = 3.06703, size = 257, normalized size = 1.27

$$-\frac{2}{a\sqrt{x}} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^2} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^2} + \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+ \dots\right)}{4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/(a*sqrt(x)) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) +
 1/4*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^2*b^2) - 1/4*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^2)
```

3.294 $\int \frac{1}{x^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=204

$$\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}a^{7/4}}$$

```
[Out] -2/(3*a*x^(3/2)) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(7/4)) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)) - (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4))
```

Rubi [A] time = 0.161096, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(7/4)) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)) - (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx = -\frac{2}{3ax^{3/2}} - \frac{b\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}} + \frac{b^{3/4}\,\text{S}\ldots}{\ldots}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}} - \frac{b^{3/4}\,\text{Su}\ldots}{\ldots}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\ldots\right.}{2\sqrt{2}a^{7/4}}$$

Mathematica [C] time = 0.0058437, size = 29, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^2)/a)])/(3*a*x^(3/2))
```

Maple [A] time = 0.007, size = 143, normalized size = 0.7

$$-\frac{b\sqrt{2}}{4\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{b\sqrt{2}}{2\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{b\sqrt{2}}{2\,a^2}\sqrt[4]{\frac{a}{b}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/4*b/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a
)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b/a^2*(1/b*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b/a^2*(1/b*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/3/a/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37002, size = 387, normalized size = 1.9

$$-\frac{12\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5b\sqrt{x}\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}-\sqrt{a^4\sqrt{-\frac{b^3}{a^7}}+b^2x}\,a^5\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}}{b^3}\right)+3\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-3\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(-\right.}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-b^3/a^7)^(1/4)*arctan(-(a^5*b*sqrt(x)*(-b^3/a^7)^(3/4) - s
qrt(a^4*sqrt(-b^3/a^7) + b^2*x)*a^5*(-b^3/a^7)^(3/4))/b^3) + 3*a*x^2*(-b^3/
a^7)^(1/4)*log(a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) - 3*a*x^2*(-b^3/a^7)^(1/4)
```

```
*log(-a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) + 4*sqrt(x))/(a*x^2)
```

Sympy [A] time = 76.1118, size = 184, normalized size = 0.9

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} + \frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}b^{12}\left(\frac{1}{b}\right)^{\frac{51}{4}}} - \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}b^{12}\left(\frac{1}{b}\right)^{\frac{51}{4}}} + \frac{\sqrt[4]{-1}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}b^{12}\left(\frac{1}{b}\right)^{\frac{51}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a*x**(3/2)), Eq(b, 0)
), (-2/(7*b*x**(7/2)), Eq(a, 0)), (-2/(3*a*x**(3/2)) + (-1)**(1/4)*log(-(-1
)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)*b**12*(1/b)**(51/4))
- (-1)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)*
b**12*(1/b)**(51/4)) + (-1)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)
**(1/4)))/(a**(7/4)*b**12*(1/b)**(51/4)), True))
```

Giac [A] time = 2.20772, size = 240, normalized size = 1.18

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 - 2/3/(a*x^(3/2)
)
```

3.295 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=215

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{\sqrt{2}a^{9/4}}$$

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rubi [A] time = 0.175071, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{\sqrt{2}a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx = -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{a}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^2\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{b^{5/}}{}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}} - \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}} + -\frac{b}{}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} + \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}a^{9/4}}$$

Mathematica [C] time = 0.0065314, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^2)/a)])/(5*a*x^(5/2))
```

Maple [A] time = 0.008, size = 152, normalized size = 0.7

$$\frac{b\sqrt{2}}{4\,a^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{b\sqrt{2}}{2\,a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{b\sqrt{2}}{2\,a^2}\arctan\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a),x)
```

```
[Out] 1/4*b/a^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*b/a^2/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*b/a^2/(1/b*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/5/a/x^(5/2)+2*b/a^2/x^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.434, size = 433, normalized size = 2.01

$$-\frac{20\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^4\sqrt{x}\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}-\sqrt{-a^5b^5\sqrt{-\frac{b^5}{a^9}}+b^8x}\,a^2\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}}{b^5}\right)-5\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}}+b^4\sqrt{x}\right)+5\,a^2x^3\left(-\right.}{10\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-b^5/a^9)^(1/4)*arctan(-(a^2*b^4*sqrt(x)*(-b^5/a^9)^(1/4
) - sqrt(-a^5*b^5*sqrt(-b^5/a^9) + b^8*x)*a^2*(-b^5/a^9)^(1/4))/b^5) - 5*a^
2*x^3*(-b^5/a^9)^(1/4)*log(a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) + 5*a^2*x^3*
(-b^5/a^9)^(1/4)*log(-a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) - 4*(5*b*x^2 - a)
*sqrt(x))/(a^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.07966, size = 270, normalized size = 1.26

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right.}{4\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^3*b) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
```

```
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) +
 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

3.296 $\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\tan^{-1}}{4}$$

```
[Out] (5*Sqrt[x])/(2*b^2) - x^(5/2)/(2*b*(a + b*x^2)) + (5*a^(1/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(9/4))
 - (5*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.166363, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\tan^{-1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^2,x]
```


```
[Out] (5*Sqrt[x])/(2*b^2) - x^(5/2)/(2*b*(a + b*x^2)) + (5*a^(1/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(9/4))
 - (5*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*b^(9/4))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\int \frac{x^{3/2}}{a+bx^2}\,dx}{4b}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{(5a)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{5/2}} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx\right.}{8b^{5/2}}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}} + \frac{5\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\right.}{8\sqrt{2}}$$

Mathematica [A] time = 0.10183, size = 221, normalized size = 0.96

$$\frac{\frac{32b^{5/4}x^{5/2}}{a+bx^2} + \frac{40a\sqrt[4]{b}\sqrt{x}}{a+bx^2} + 5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + 10\sqrt{2}\sqrt[4]{a}\,\text{ta}}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] ((40*a*b^(1/4)*Sqrt[x])/(a + b*x^2) + (32*b^(5/4)*x^(5/2))/(a + b*x^2) + 10
*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 10*Sqrt[2]
*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 5*Sqrt[2]*a^(1/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 5*Sqrt[2]*a^(1
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*b^(9/4)
)
```

Maple [A] time = 0.011, size = 158, normalized size = 0.7

$$2\,\frac{\sqrt{x}}{b^2} + \frac{a}{2\,b^2\left(bx^2+a\right)}\sqrt{x} - \frac{5\,\sqrt{2}}{16\,b^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) - \frac{5\,\sqrt{2}}{8\,b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/2/b^2*a*x^(1/2)/(b*x^2+a)-5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln
((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2
^(1/2)+(1/b*a)^(1/2)))-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x^(1/2)+1)-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4
)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34265, size = 443, normalized size = 1.93

$$-\frac{20\left(b^{3}x^{2}+ab^{2}\right)\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{4}\sqrt{-\frac{a}{b^{9}}}+x}b^{7}\left(-\frac{a}{b^{9}}\right)^{\frac{3}{4}}-b^{7}\sqrt{x}\left(-\frac{a}{b^{9}}\right)^{\frac{3}{4}}}{a}\right)+5\left(b^{3}x^{2}+ab^{2}\right)\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}\log\left(5\,b^{2}\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)-5\,(b}{8\left(b^{3}x^{2}+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(b^3*x^2 + a*b^2)*(-a/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-a/b^9) + x
)*b^7*(-a/b^9)^(3/4) - b^7*sqrt(x)*(-a/b^9)^(3/4))/a) + 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(-5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 4*(4*b*x^2 + 5*a)*
sqrt(x))/(b^3*x^2 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.55286, size = 265, normalized size = 1.15

$$-\frac{5\,\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}}-\frac{5\,\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}}-\frac{5\,\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\ldots\right.}{16\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^3 - 5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 - 5/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 5/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 1/2*a*sqrt(x
)/((b*x^2 + a)*b^2) + 2*sqrt(x)/b^2
```

3.297 $\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

```
[Out] -x^(3/2)/(2*b*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rubi [A] time = 0.148484, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^2,x]
```


```
[Out] -x^(3/2)/(2*b*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2b}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}-\frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^2}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^2}+\frac{3\,\mathrm{Sul}}{}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\,\mathrm{Subst}}{}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0124453, size = 43, normalized size = 0.2

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{a}-\frac{1}{a+bx^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(-(a + b*x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)
]/a))/b
```

Maple [A] time = 0.009, size = 149, normalized size = 0.7

$$-\frac{1}{2b\left(bx^2+a\right)}x^{\frac{3}{2}}+\frac{3\sqrt{2}}{16b^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\sqrt{2}}{8b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2)))+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37307, size = 447, normalized size = 2.05

$$-\frac{12\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab^3\sqrt{-\frac{1}{ab^7}}+x}b^2\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}-b^2\sqrt{x}\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*arctan(sqrt(-a*b^3*sqrt(-1/(a*b
^7)) + x)*b^2*(-1/(a*b^7))^(1/4) - b^2*sqrt(x)*(-1/(a*b^7))^(1/4)) - 3*(b^2
```

```
*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x)) + 3*
(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(-a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(b^2*x^2 + a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.75973, size = 269, normalized size = 1.23

$$-\frac{x^{\frac{3}{2}}}{2\left(bx^2+a\right)b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((b*x^2 + a)*b) + 3/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 3/8*sqrt(2)*(a*b^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(a*b^4) - 3/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a*b^4) + 3/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a*b^4)
```

3.298 $\int \frac{x^{3/2}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{}{2b}$$

```
[Out] -Sqrt[x]/(2*b*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rubi [A] time = 0.148017, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[x]/(2*b*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2b}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{ab}}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{ab}}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{a}b^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{a}b^{3/2}}-\frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+$$

Mathematica [A] time = 0.0951198, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8\sqrt[4]{b}\sqrt{x}}{a+bx^2}}{16b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/a^(3/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/a^(3/4))/(16*b^(5/4))
```

Maple [A] time = 0.008, size = 158, normalized size = 0.7

$$-\frac{1}{2b\left(bx^2+a\right)}\sqrt{x}+\frac{\sqrt{2}}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{\sqrt{2}}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x^(1/2)/b/(b*x^2+a)+1/16/b*(1/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2)))+1/8/b*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
+1/8/b*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45489, size = 467, normalized size = 2.14

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^2\sqrt{-\frac{1}{a^3b^5}}+xa^2b^4\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{4}}}-a^2b^4\sqrt{x}\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{4}}\right)+\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{1}{a^3\cdots}\right.\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*arctan(sqrt(a^2*b^2*sqrt(-1/(a^
3*b^5)) + x)*a^2*b^4*(-1/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(x)*(-1/(a^3*b^5))^
(3/4)) + (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(a*b*(-1/(a^3*b^5))^(1/4)
+ sqrt(x)) - (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(-a*b*(-1/(a^3*b^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(b^2*x^2 + a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.98469, size = 269, normalized size = 1.23

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right.}{16\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) - 1/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
 - 1/2*sqrt(x)/((b*x^2 + a)*b)
```

3.299 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{}{2a\left(\right.}$$

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
4*Sqrt[2]*a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rubi [A] time = 0.147091, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{}{2a\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^2,x]
```



```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
4*Sqrt[2]*a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4))
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx = \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)}-\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a\sqrt{b}}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a\sqrt{b}}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8ab}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8ab}+\frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}+\frac{\text{Subst}\left(\int \frac{1}{-1-}\right.}{4}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\text{lo}}{}$$

Mathematica [C] time = 0.0048495, size = 29, normalized size = 0.13

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2)
```

Maple [A] time = 0.007, size = 158, normalized size = 0.7

$$\frac{1}{2\,a\left(bx^2+a\right)}x^{\frac{3}{2}}+\frac{\sqrt{2}}{16\,ab}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)+1/16/a/b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*
x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/
2)))+1/8/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+
1/8/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39898, size = 467, normalized size = 2.14

$$-\frac{4\left(abx^2+a^2\right)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b\sqrt{-\frac{1}{a^5b^3}}+x}\,ab\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}-ab\sqrt{x}\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\right)-\left(abx^2+a^2\right)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{1}{a^5\cdots}\right.\right.}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*arctan(sqrt(-a^3*b*sqrt(-1/(a^
5*b^3)) + x)*a*b*(-1/(a^5*b^3))^(1/4) - a*b*sqrt(x)*(-1/(a^5*b^3))^(1/4)) -
 (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4) + sq
rt(x)) + (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-1/(a^5*b^3))^(
```

```
3/4) + sqrt(x)) - 4*x^(3/2))/(a*b*x^2 + a^2)
```

Sympy [A] time = 125.801, size = 619, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b,
0)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-4*(-1)**(1/4)*a**(85/4)*b**22*x**(
3/2)*(1/b)**(85/4)/(-8*(-1)**(1/4)*a**(93/4)*b**22*(1/b)**(85/4) - 8*(-1)**
(1/4)*a**(89/4)*b**23*x**2*(1/b)**(85/4)) - a**22*log(-(-1)**(1/4)*a**(1/4)
*(1/b)**(1/4) + sqrt(x))/(-8*(-1)**(1/4)*a**(93/4)*b**22*(1/b)**(85/4) - 8*
(-1)**(1/4)*a**(89/4)*b**23*x**2*(1/b)**(85/4)) + a**22*log((-1)**(1/4)*a**
(1/4)*(1/b)**(1/4) + sqrt(x))/(-8*(-1)**(1/4)*a**(93/4)*b**22*(1/b)**(85/4)
 - 8*(-1)**(1/4)*a**(89/4)*b**23*x**2*(1/b)**(85/4)) + 2*a**22*atan((-1)**(
3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(-8*(-1)**(1/4)*a**(93/4)*b**22*(1/b)
**(85/4) - 8*(-1)**(1/4)*a**(89/4)*b**23*x**2*(1/b)**(85/4)) - a**21*b*x**2
*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(-8*(-1)**(1/4)*a**(93/4
)*b**22*(1/b)**(85/4) - 8*(-1)**(1/4)*a**(89/4)*b**23*x**2*(1/b)**(85/4)) +
 a**21*b*x**2*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(-8*(-1)**(1
/4)*a**(93/4)*b**22*(1/b)**(85/4) - 8*(-1)**(1/4)*a**(89/4)*b**23*x**2*(1/b
)**(85/4)) + 2*a**21*b*x**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)
))/(-8*(-1)**(1/4)*a**(93/4)*b**22*(1/b)**(85/4) - 8*(-1)**(1/4)*a**(89/4)*
b**23*x**2*(1/b)**(85/4)), True))
```

Giac [A] time = 2.16401, size = 269, normalized size = 1.23

$$\frac{x^{\frac{3}{2}}}{2\left(bx^2+a\right)a}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\right.}{16\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((b*x^2 + a)*a) + 1/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/8*sqrt(2)*(a*b
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^2*b^3) - 1/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

3.300 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rubi [A] time = 0.151183, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{3/2}}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^{3/2}\sqrt{b}}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^{3/2}\sqrt{b}}-\frac{3\,\text{S}}{}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\,\text{Subs}}{}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{}\right.}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Mathematica [A] time = 0.0981756, size = 199, normalized size = 0.91

$$\frac{\frac{8a^{3/4}\sqrt{x}}{a+bx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}}{16a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(16*a^(7/4))
```

Maple [A] time = 0.008, size = 149, normalized size = 0.7

$$\frac{1}{2\,a\left(bx^2+a\right)}\sqrt{x}+\frac{3\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{3\sqrt{2}}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/a/(b*x^2+a)+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*
x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/
2)))+3/8/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+
3/8/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43559, size = 441, normalized size = 2.02

$$\frac{12\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4\sqrt{-\frac{1}{a^7b}}+xa^5b\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}}-a^5b\sqrt{x}\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}\right)+3\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\right.}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*arctan(sqrt(a^4*sqrt(-1/(a^7*b))
 + x)*a^5*b*(-1/(a^7*b))^(3/4) - a^5*b*sqrt(x)*(-1/(a^7*b))^(3/4)) + 3*(a*b
*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) - 3*(a
*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(a*b*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.67319, size = 269, normalized size = 1.23

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\right.}{16a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^2*b) + 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 3/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b)
 + 1/2*sqrt(x)/((b*x^2 + a)*a)
```

3.301 $\int \frac{1}{x^{3/2}(a+bx^2)^2} dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}}{4\sqrt{}}$$

```
[Out] -5/(2*a^2*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*b^(1/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)
) + (5*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4))
```

Rubi [A] time = 0.187815, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -5/(2*a^2*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*b^(1/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)
) + (5*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a}$$
$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{(5b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}$$
$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$
$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^2} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,\right.}{4a^2}$$
$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x\right.}{8a^2}$$
$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{5\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}} + \frac{5\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{8\sqrt{2}a^{9/4}}$$
$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}} - \frac{5\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}} - \frac{5\sqrt[4]{b}\log\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.0054701, size = 27, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -((b*x^2)/a)])/(a^2*Sqrt[x])
```

Maple [A] time = 0.013, size = 158, normalized size = 0.7

$$-\frac{b}{2\,a^2\left(bx^2+a\right)}x^{\frac{3}{2}} - \frac{5\sqrt{2}}{16\,a^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{5\sqrt{2}}{8\,a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*b/a^2*x^(3/2)/(b*x^2+a)-5/16/a^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(
1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a
)^(1/2)))-5/8/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2
)+1)-5/8/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-
2/a^2/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38925, size = 512, normalized size = 2.23

$$\frac{20\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,a^2b\sqrt{x}\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5b\sqrt{-\frac{b}{a^9}}+15625\,b^2x}a^2\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}}{125\,b}\right)-5\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log\left(125\right.}{8\left(a^2bx^3+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*arctan(-1/125*(125*a^2*b*sqrt(x)
*(-b/a^9)^(1/4) - sqrt(-15625*a^5*b*sqrt(-b/a^9) + 15625*b^2*x)*a^2*(-b/a^9
)^(1/4))/b) - 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(125*a^7*(-b/a^9)^(3/
4) + 125*b*sqrt(x)) + 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(-125*a^7*(-b
/a^9)^(3/4) + 125*b*sqrt(x)) - 4*(5*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^3 + a^3*
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.82679, size = 284, normalized size = 1.23

$$-\frac{5\,bx^2+4\,a}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/((b*x^(5/2) + a*sqrt(x))*a^2) - 5/8*sqrt(2)*(a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*
b^2) - 5/8*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 5/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 5/16*sqrt(2)*(a*b^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2)
```

3.302 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\tan}{}$$

```
[Out] -7/(6*a^2*x^(3/2)) + 1/(2*a*x^(3/2)*(a + b*x^2)) + (7*b^(3/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) - (7*b^(3/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) + (7*b^(3/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11
/4)) - (7*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4))
```

Rubi [A] time = 0.176691, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -7/(6*a^2*x^(3/2)) + 1/(2*a*x^(3/2)*(a + b*x^2)) + (7*b^(3/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) - (7*b^(3/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) + (7*b^(3/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11
/4)) - (7*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}dx = \frac{1}{2ax^{3/2}\left(a+bx^2\right)} + \frac{7\int \frac{1}{x^{5/2}\left(a+bx^2\right)}dx}{4a}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}\left(a+bx^2\right)} - \frac{(7b)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}dx}{4a^2}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}\left(a+bx^2\right)} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}\left(a+bx^2\right)} - \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{4a^{5/2}} - \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}dx,\right.}{4a^{5/2}}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}\left(a+bx^2\right)} - \frac{\left(7\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx, x, \sqrt{x}\right)}{8a^{5/2}} - \frac{\left(7\sqrt{b}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{8a}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}} - \frac{7b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{8\sqrt{2}a^{11/4}}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}} - \frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}} + \frac{7b^{3/4}}{}$$

Mathematica [C] time = 0.0060163, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -((b*x^2)/a)])/(3*a^2*x^(3/2))
```

Maple [A] time = 0.011, size = 161, normalized size = 0.7

$$-\frac{b}{2a^2\left(bx^2+a\right)}\sqrt{x} - \frac{7b\sqrt{2}}{16a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) - \frac{7b\sqrt{2}}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*b/a^2*x^(1/2)/(b*x^2+a)-7/16*b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)
^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b
*a)^(1/2)))-7/8*b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)+1)-7/8*b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/
2)-1)-2/3/a^2/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45044, size = 518, normalized size = 2.25

$$-\frac{84\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^8b\sqrt{x}\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}-\sqrt{a^6\sqrt{-\frac{b^3}{a^{11}}}+b^2x}\,a^8\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}}{b^3}\right)+21\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\cdots\right.}{24\left(a^2bx^4+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*arctan(-(a^8*b*sqrt(x)*(-
b^3/a^11)^(3/4) - sqrt(a^6*sqrt(-b^3/a^11) + b^2*x)*a^8*(-b^3/a^11)^(3/4))/
b^3) + 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(7*a^3*(-b^3/a^11)^(1/
4) + 7*b*sqrt(x)) - 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(-7*a^3*(
-b^3/a^11)^(1/4) + 7*b*sqrt(x)) + 4*(7*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^4 + a
^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.93922, size = 265, normalized size = 1.15

$$-\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3}-\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3}-\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\cdots\right.}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^3 - 7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^3 - 7/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 + 7/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 - 1/2*b*sqrt(x
)/((b*x^2 + a)*a^2) - 2/3/(a^2*x^(3/2))
```

3.303 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=243

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{9b^{5/4}\tan^{-1}}{4\sqrt{}}$$

```
[Out] -9/(10*a^2*x^(5/2)) + (9*b)/(2*a^3*Sqrt[x]) + 1/(2*a*x^(5/2)*(a + b*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]
*a^(13/4)) + (9*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)) - (9*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rubi [A] time = 0.190601, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{9b^{5/4}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2),x]
```


```
[Out] -9/(10*a^2*x^(5/2)) + (9*b)/(2*a^3*Sqrt[x]) + 1/(2*a*x^(5/2)*(a + b*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]
*a^(13/4)) + (9*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)) - (9*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{9\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} - \frac{(9b)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^3}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^3} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{(9b)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^3} + \frac{(9b)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{9b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}} - \frac{9b^{5/4}\log\left(\sqrt{a}+\right.}{8}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}} + \frac{9b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}$$

Mathematica [C] time = 0.0066367, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -((b*x^2)/a)])/(5*a^2*x^(5/2))
```

Maple [A] time = 0.015, size = 172, normalized size = 0.7

$$\frac{b^2}{2\,a^3\left(bx^2+a\right)}x^{\frac{3}{2}} + \frac{9\,b\sqrt{2}}{16\,a^3}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{9\,b\sqrt{2}}{8\,a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*b^2/a^3*x^(3/2)/(b*x^2+a)+9/16*b/a^3/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/
b*a)^(1/2)))+9/8*b/a^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
```

```
^(1/2)+1)+9/8*b/a^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
/2)-1)-2/5/a^2/x^(5/2)+4*b/a^3/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43287, size = 605, normalized size = 2.49

$$-\frac{180\left(a^{3}bx^{5}+a^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\,a^{3}b^{4}\sqrt{x}\left(-\frac{b^{5}}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^{7}b^{5}\sqrt{-\frac{b^{5}}{a^{13}}}+531441\,b^{8}x}\,a^{3}\left(-\frac{b^{5}}{a^{13}}\right)^{\frac{1}{4}}}{729\,b^{5}}\right)-45\left(a^{3}bx^{5}+a^{4}x^{3}\right)\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)^(1/4)*arctan(-1/729*(729*a^3*b
^4*sqrt(x)*(-b^5/a^13)^(1/4) - sqrt(-531441*a^7*b^5*sqrt(-b^5/a^13) + 53144
1*b^8*x)*a^3*(-b^5/a^13)^(1/4))/b^5) - 45*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)
^(1/4)*log(729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x)) + 45*(a^3*b*x^5 +
a^4*x^3)*(-b^5/a^13)^(1/4)*log(-729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x
)) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.32377, size = 297, normalized size = 1.22

$$\frac{b^{2}x^{\frac{3}{2}}}{2\left(bx^{2}+a\right)a^{3}}+\frac{9\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}b}+\frac{9\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}b}-\frac{9\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^(3/2)/((b*x^2 + a)*a^3) + 9/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 9/8*sqrt(2)*
(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^4*b) - 9/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(a^4*b) + 9/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 2/5*(10*b*x^2 - a)/(a^3*x^(5/2))
```

3.304 $\int \frac{x^{7/2}}{(a+bx^2)^3} dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{3/4}b^{9/4}}$$

```
[Out] -x^(5/2)/(4*b*(a + b*x^2)^2) - (5*Sqrt[x])/(16*b^2*(a + b*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4))
 - (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(3/4)*b^(9/4)) + (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rubi [A] time = 0.1774, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] -x^(5/2)/(4*b*(a + b*x^2)^2) - (5*Sqrt[x])/(16*b^2*(a + b*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4))
 - (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(3/4)*b^(9/4)) + (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} + \frac{5\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{a}b^2} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{32\sqrt{a}b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{a}b^{5/2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\right.}{64\sqrt{a}b^{5/2}}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} - \frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}} + \frac{5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}a^{3/4}b^{9/4}}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}} - \frac{5\log\left(\sqrt{a}-\right.}{64}$$

Mathematica [A] time = 0.106279, size = 242, normalized size = 1.01

$$\frac{-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}} + \frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}} - \frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}} - \frac{256b^{5/4}x^{5/2}}{\left(a+bx^2\right)^2} + }{384b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-160*a*b^(1/4)*Sqrt[x])/(a + b*x^2)^2 - (256*b^(5/4)*x^(5/2))/(a + b*x^2)
^2 + (40*b^(1/4)*Sqrt[x])/(a + b*x^2) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/a^(3/4) - (15*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (15*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(384*b^(9/4))
```

Maple [A] time = 0.012, size = 170, normalized size = 0.7

$$2\,\frac{1}{\left(bx^2+a\right)^2}\left(-\frac{9\,x^{5/2}}{32\,b} - \frac{5\,a\sqrt{x}}{32\,b^2}\right) + \frac{5\,\sqrt{2}}{128\,ab^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{5\,\sqrt{2}}{64\,ab^2}\sqrt[4]{\frac{a}{b}}\,\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(-9/32*x^(5/2)/b-5/32*a*x^(1/2)/b^2)/(b*x^2+a)^2+5/128/b^2*(1/b*a)^(1/4)/
a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+5/64/b^2*(1/b*a)^(1/4)/a*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+5/64/b^2*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47267, size = 599, normalized size = 2.51

$$\frac{20\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^4\sqrt{-\frac{1}{a^3b^9}}+xa^2b^7\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}}-a^2b^7\sqrt{x}\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}\right)+5\left(b^4x^4+2\,ab^3x^2+\right.}{64\left(b^4x^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*arctan(sqrt
(a^2*b^4*sqrt(-1/(a^3*b^9)) + x)*a^2*b^7*(-1/(a^3*b^9))^(3/4) - a^2*b^7*sqr
t(x)*(-1/(a^3*b^9))^(3/4)) + 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b
^9))^(1/4)*log(a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(b^4*x^4 + 2*a*b^3
*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*log(-a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt
(x)) - 4*(9*b*x^2 + 5*a)*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.435, size = 282, normalized size = 1.18

$$\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\right.}}{128\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b^3) + 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) + 5/128*sqrt(2)*(a*b
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) - 5/128*
sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*
b^3) - 1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/((b*x^2 + a)^2*b^2)
```

3.305 $\int \frac{x^{5/2}}{(a+bx^2)^3} dx$

Optimal. Leaf size=242

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{5/4}b^{7/4}}$$

```
[Out] -x^(3/2)/(4*b*(a + b*x^2)^2) + (3*x^(3/2))/(16*a*b*(a + b*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4))
 + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(5/4)*b^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rubi [A] time = 0.16977, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{5/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] -x^(3/2)/(4*b*(a + b*x^2)^2) + (3*x^(3/2))/(16*a*b*(a + b*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4))
 + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(5/4)*b^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}-\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32ab^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32ab^{3/2}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64ab^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx\right.}{64ab^2}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\right.}}{64\sqrt{2}a^{5/4}b^{7/4}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\right.}{64\sqrt{2}}$$

Mathematica [C] time = 0.0153598, size = 45, normalized size = 0.19

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^2}-\frac{1}{\left(a+bx^2\right)^2}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(-(a + b*x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)
]/a^2))/(5*b)
```

Maple [A] time = 0.013, size = 169, normalized size = 0.7

$$2\frac{1}{\left(bx^2+a\right)^2}\left(\frac{3\,x^{7/2}}{32\,a}-1/32\,\frac{x^{3/2}}{b}\right)+\frac{3\,\sqrt{2}}{128\,ab^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\,\sqrt{2}}{64\,ab^2}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(3/32/a*x^(7/2)-1/32*x^(3/2)/b)/(b*x^2+a)^2+3/128/b^2/a/(1/b*a)^(1/4)*2^(
1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^
```

```
(1/2)*2^(1/2)+(1/b*a)^(1/2)))+3/64/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/64/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35648, size = 613, normalized size = 2.53

$$-\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x}ab^2\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}-ab^2\sqrt{x}\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\right)-3\left(ab^3x^4+2\,a^2\cdots\right.}{64\left(a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*arctan(s
qrt(-a^3*b^3*sqrt(-1/(a^5*b^7)) + x)*a*b^2*(-1/(a^5*b^7))^(1/4) - a*b^2*sqr
t(x)*(-1/(a^5*b^7))^(1/4)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5
*b^7))^(1/4)*log(a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) + 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-1/(a^5*b^7))^(3/4
) + sqrt(x)) - 4*(3*b*x^3 - a*x)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*
b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.28955, size = 286, normalized size = 1.18

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\left(bx^2+a\right)^2ab}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/((b*x^2 + a)^2*a*b) + 3/64*sqrt(2)*(a*b^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2
*b^4) + 3/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 3/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 3/128*sqrt(2)*(a*b^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4)
```

3.306 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{7/4}b^{5/4}}$$

```
[Out] -Sqrt[x]/(4*b*(a + b*x^2)^2) + Sqrt[x]/(16*a*b*(a + b*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) - (
3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a
^(7/4)*b^(5/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.163816, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] -Sqrt[x]/(4*b*(a + b*x^2)^2) + Sqrt[x]/(16*a*b*(a + b*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) - (
3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a
^(7/4)*b^(5/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\int \frac{1}{\sqrt{x}(a+bx^2)^2}\,dx}{8b}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{32ab}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{3/2}b} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{\right.}}{32a^{3/2}b}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{3/2}b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\right.}{64a^{3/2}b^{3/2}}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{64\sqrt{2}a^{7/4}b^{5/4}}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} - \frac{3\log\left(\sqrt{a}-\right.}{64}$$

Mathematica [A] time = 0.10785, size = 223, normalized size = 0.92

$$\frac{\frac{8\sqrt[4]{b}\sqrt{x}}{a^2+abx^2} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}} - \frac{32\sqrt[4]{b}\sqrt{x}}{(a+bx^2)^2}}{128b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-32*b^(1/4)*Sqrt[x])/(a + b*x^2)^2 + (8*b^(1/4)*Sqrt[x])/(a^2 + a*b*x^2)
- (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sq
rt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4))/(
128*b^(5/4))
```

Maple [A] time = 0.011, size = 169, normalized size = 0.7

$$2\frac{1}{(bx^2+a)^2}\left(1/32\frac{x^{5/2}}{a} - \frac{3\sqrt{x}}{32b}\right) + \frac{3\sqrt{2}}{128a^2b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3\sqrt{2}}{64a^2b}\sqrt[4]{\frac{a}{b}}\,\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32/a*x^(5/2)-3/32*x^(1/2)/b)/(b*x^2+a)^2+3/128/b/a^2*(1/b*a)^(1/4)*2^(
1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^
(1/2)*2^(1/2)+(1/b*a)^(1/2)))+3/64/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/64/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44062, size = 608, normalized size = 2.51

$$\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x}a^5b^4\left(-\frac{1}{a^7b^5}\right)^{\frac{3}{4}}-a^5b^4\sqrt{x}\left(-\frac{1}{a^7b^5}\right)^{\frac{3}{4}}\right)+3\left(ab^3x^4+2\,a^2b^2x^2\right.}{64\left(ab^3x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*arctan(sq
rt(a^4*b^2*sqrt(-1/(a^7*b^5)) + x)*a^5*b^4*(-1/(a^7*b^5))^(3/4) - a^5*b^4*s
qrt(x)*(-1/(a^7*b^5))^(3/4)) + 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a
^7*b^5))^(1/4)*log(a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*log(-a^2*b*(-1/(a^7*b^5))^(1/4)
+ sqrt(x)) + 4*(b*x^2 - 3*a)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.3881, size = 285, normalized size = 1.18

$$\frac{3\,\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2}+\frac{3\,\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2}+\frac{3\,\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\right.}{128\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^2*b^2) + 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/128*sqrt(2)*
(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) -
3/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^2) + 1/16*(b*x^(5/2) - 3*a*sqrt(x))/((b*x^2 + a)^2*a*b)
```

3.307 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^3}\,dx$

<u>**Optimal**</u>. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{9/4}b^{3/4}}$$

```
[Out] x^(3/2)/(4*a*(a + b*x^2)^2) + (5*x^(3/2))/(16*a^2*(a + b*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
+ (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))
```

<u>**Rubi**</u> **[A]** time = 0.164189, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{9/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] x^(3/2)/(4*a*(a + b*x^2)^2) + (5*x^(3/2))/(16*a^2*(a + b*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
+ (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))
```

<u>Rule 290</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

<u>Rule 329</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 297</u>

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^3}\,dx = \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{5\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{5\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}-\frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^2\sqrt{b}}+\frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^2\sqrt{b}}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64a^2b}+\frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x\right.}{64a^2b}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{64\sqrt{2}a^{9/4}b^{3/4}}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\right.}}{64\sqrt{2}a^{9}}$$

Mathematica [C] time = 0.0048228, size = 29, normalized size = 0.12

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^3)
```

Maple [A] time = 0.007, size = 175, normalized size = 0.7

$$\frac{1}{4\,a\left(bx^2+a\right)^2}x^{\frac{3}{2}}+\frac{5}{16\,a^2\left(bx^2+a\right)}x^{\frac{3}{2}}+\frac{5\sqrt{2}}{128\,a^2b}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\sqrt{2}}{64\,a^2b}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/16*x^(3/2)/a^2/(b*x^2+a)+5/128/a^2/b/(1/b*a)^(1
/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(
1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+5/64/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+5/64/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(
```

```
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45506, size = 602, normalized size = 2.52

$$-\frac{20\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^5b\sqrt{-\frac{1}{a^9b^3}}+x}a^2b\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\right)-5\left(a^2b^2x^4+2\,a^3b\right.}{64\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*arctan(sqr
t(-a^5*b*sqrt(-1/(a^9*b^3)) + x)*a^2*b*(-1/(a^9*b^3))^(1/4) - a^2*b*sqrt(x)
*(-1/(a^9*b^3))^(1/4)) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))
^(1/4)*log(a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) + 5*(a^2*b^2*x^4 + 2*a^3
*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt
(x)) - 4*(5*b*x^3 + 9*a*x)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.49284, size = 282, normalized size = 1.18

$$\frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^2}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/((b*x^2 + a)^2*a^2) + 5/64*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
^3*b^3) + 5/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 5/128*sqrt(2)*(a*b^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 5/128*sqrt(2)*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

3.308 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=239

$$\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

```
[Out] Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*Sqrt[x])/(16*a^2*(a + b*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/
4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*S
qrt[2]*a^(11/4)*b^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi [A] time = 0.172223, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*Sqrt[x])/(16*a^2*(a + b*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/
4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*S
qrt[2]*a^(11/4)*b^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3}\,dx = \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{5/2}} + \frac{21\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x\right.}{32a^{5/2}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{5/2}\sqrt{b}} + \frac{21\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}\right.}{64a^{5/2}\sqrt{}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{64\sqrt{2}a^{11/4}\sqrt[4]{}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21\log\left(\right.}{}$$

Mathematica [A] time = 0.0835067, size = 220, normalized size = 0.92

$$\frac{\frac{32a^{7/4}\sqrt{x}}{\left(a+bx^2\right)^2} + \frac{56a^{3/4}\sqrt{x}}{a+bx^2} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.}{\sqrt[4]{b}}}{128a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((32*a^(7/4)*Sqrt[x])/(a + b*x^2)^2 + (56*a^(3/4)*Sqrt[x])/(a + b*x^2) - (4
2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (42*Sqrt
[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (21*Sqrt[2]*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(
128*a^(11/4))
```

Maple [A] time = 0.007, size = 166, normalized size = 0.7

$$\frac{1}{4a\left(bx^2+a\right)^2}\sqrt{x} + \frac{7}{16a^2\left(bx^2+a\right)}\sqrt{x} + \frac{21\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21\sqrt{2}}{64a^3}\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 1/4*x^(1/2)/a/(b*x^2+a)^2+7/16*x^(1/2)/a^2/(b*x^2+a)+21/128/a^3*(1/b*a)^(1/
4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1
/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+21/64/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+21/64/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4554, size = 576, normalized size = 2.41

$$\frac{84\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6\sqrt{-\frac{1}{a^{11}b}}+x}a^8b\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}-a^8b\sqrt{x}\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}\right)+21\left(a^2b^2x^4+2\,a^3bx^2+a\right.}{64\left(a^2b^2x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(84*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*arctan(sqrt(
a^6*sqrt(-1/(a^11*b)) + x)*a^8*b*(-1/(a^11*b))^(3/4) - a^8*b*sqrt(x)*(-1/(a
^11*b))^(3/4)) + 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*l
og(a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4
)*(-1/(a^11*b))^(1/4)*log(-a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) + 4*(7*b*x^2
+ 11*a)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.45155, size = 282, normalized size = 1.18

$$\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b}+\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b}+\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x\right.}{128\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b) + 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 21/128*sqrt(2)*(
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) - 21/
128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^3*b) + 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/((b*x^2 + a)^2*a^2)
```

3.309 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=251

$$\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{13/4}}$$

```
[Out] -45/(16*a^3*Sqrt[x]) + 1/(4*a*Sqrt[x]*(a + b*x^2)^2) + 9/(16*a^2*Sqrt[x]*(a
 + b*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32
*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(32*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)) + (45*b^(1/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4))
```

Rubi [A] time = 0.190952, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] -45/(16*a^3*Sqrt[x]) + 1/(4*a*Sqrt[x]*(a + b*x^2)^2) + 9/(16*a^2*Sqrt[x]*(a
 + b*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32
*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(32*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)) + (45*b^(1/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx = \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx}{8a}$$

$$= \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} + \frac{45\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{32a^2}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} - \frac{(45b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} - \frac{(45b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} + \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^3} - \frac{(45\cdots}{\cdots}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} - \frac{45\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^3} - \frac{45\,\text{Su}\cdots}{\cdots}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} - \frac{45\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}} + \frac{45\cdots}{\cdots}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} + \frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}} - \frac{45\sqrt[4]{b}\tan^{-1}\left(1\cdots\right.}{32\sqrt{2}a\cdots}$$

Mathematica [C] time = 0.0059544, size = 27, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -((b*x^2)/a)])/(a^3*Sqrt[x])
```

Maple [A] time = 0.017, size = 178, normalized size = 0.7

$$-\frac{13\,b^2}{16\,a^3\left(bx^2+a\right)^2}x^{\frac{7}{2}} - \frac{17\,b}{16\,a^2\left(bx^2+a\right)^2}x^{\frac{3}{2}} - \frac{45\sqrt{2}}{128\,a^3}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{45\sqrt{2}}{64\,a^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] -13/16*b^2/a^3/(b*x^2+a)^2*x^(7/2)-17/16*b/a^2/(b*x^2+a)^2*x^(3/2)-45/128/a
^3/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))
```

```
/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-45/64/a^3/(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-45/64/a^3/(1/b*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/a^3/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41627, size = 672, normalized size = 2.68

$$\frac{180\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,a^3b\sqrt{x}\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b\sqrt{-\frac{b}{a^{13}}}+8303765625\,b^2xa^3}\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}}{91125\,b}\right)-45\left(a^{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*arctan(-1/911
25*(91125*a^3*b*sqrt(x)*(-b/a^13)^(1/4) - sqrt(-8303765625*a^7*b*sqrt(-b/a^
13) + 8303765625*b^2*x)*a^3*(-b/a^13)^(1/4))/b) - 45*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(91125*a^10*(-b/a^13)^(3/4) + 91125*b*sqrt
(x)) + 45*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(-91125*a^
10*(-b/a^13)^(3/4) + 91125*b*sqrt(x)) - 4*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2
)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.14213, size = 297, normalized size = 1.18

$$-\frac{2}{a^3\sqrt{x}}-\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2}-\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2}+\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\cdots\right.}{128\,a^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2/(a^3*sqrt(x)) - 45/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 45/64*sqrt(2)*(a*b^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*
b^2) + 45/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(a^4*b^2) - 45/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*(13*b^2*x^(7/2) + 17*a*b*x^(3/2)
)/((b*x^2 + a)^2*a^3)
```

3.310 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}}-\frac{77b^3}{}$$

```
[Out] -77/(48*a^3*x^(3/2)) + 1/(4*a*x^(3/2)*(a + b*x^2)^2) + 11/(16*a^2*x^(3/2)*(
a + b*x^2)) + (77*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(3
2*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(32*Sqrt[2]*a^(15/4)) + (77*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4))
```

Rubi [A] time = 0.184596, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}}-\frac{77b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^3),x]
```



```
[Out] -77/(48*a^3*x^(3/2)) + 1/(4*a*x^(3/2)*(a + b*x^2)^2) + 11/(16*a^2*x^(3/2)*(
a + b*x^2)) + (77*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(3
2*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(32*Sqrt[2]*a^(15/4)) + (77*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4))
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx}{8a}$$
$$= \frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{32a^2}$$
$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^3}$$
$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^3}$$
$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{7/2}}-\ (7$$
$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{\left(77\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{\right.}{64a^{7/2}}$$
$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}$$
$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\tan}{3}$$

Mathematica [C] time = 0.0065073, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -((b*x^2)/a)])/(3*a^3*x^(3/2))
```

Maple [A] time = 0.016, size = 181, normalized size = 0.7

$$-\frac{15\,b^2}{16\,a^3\left(bx^2+a\right)^2}x^{\frac{5}{2}}-\frac{19\,b}{16\,a^2\left(bx^2+a\right)^2}\sqrt{x}-\frac{77\,b\sqrt{2}}{128\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{77}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] -15/16/a^3*b^2/(b*x^2+a)^2*x^(5/2)-19/16/a^2*b/(b*x^2+a)^2*x^(1/2)-77/128/a
^4*b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2
```

```
))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-77/64/a^4*b*(1/b*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-77/64/a^4*b*(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/3/a^3/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48447, size = 645, normalized size = 2.57

$$-\frac{924\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{11}b\sqrt{x}\left(-\frac{b^3}{a^{15}}\right)^{\frac{3}{4}}-\sqrt{a^8\sqrt{-\frac{b^3}{a^{15}}}+b^2x}\,a^{11}\left(-\frac{b^3}{a^{15}}\right)^{\frac{3}{4}}}{b^3}\right)+231\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*arctan(
-(a^11*b*sqrt(x)*(-b^3/a^15)^(3/4) - sqrt(a^8*sqrt(-b^3/a^15) + b^2*x)*a^11
*(-b^3/a^15)^(3/4))/b^3) + 231*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/
a^15)^(1/4)*log(77*a^4*(-b^3/a^15)^(1/4) + 77*b*sqrt(x)) - 231*(a^3*b^2*x^6
 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*log(-77*a^4*(-b^3/a^15)^(1/4) +
 77*b*sqrt(x)) + 4*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)*sqrt(x))/(a^3*b^2*x^
6 + 2*a^4*b*x^4 + a^5*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.11764, size = 281, normalized size = 1.12

$$-\frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4}-\frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4}-\frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x\right.}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/a^4 - 77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^4 - 77/128*sqrt(2)*(a*b^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 + 77/128*sqrt(2)
*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 - 1/16
*(15*b^2*x^(5/2) + 19*a*b*sqrt(x))/((b*x^2 + a)^2*a^3) - 2/3/(a^3*x^(3/2))
```

3.311 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{117b}{}$$

```
[Out] -117/(80*a^3*x^(5/2)) + (117*b)/(16*a^4*Sqrt[x]) + 1/(4*a*x^(5/2)*(a + b*x^
2)^2) + 13/(16*a^2*x^(5/2)*(a + b*x^2)) - (117*b^(5/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17
/4)) - (117*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(64*Sqrt[2]*a^(17/4))
```

Rubi [A] time = 0.211701, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{117b}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] -117/(80*a^3*x^(5/2)) + (117*b)/(16*a^4*Sqrt[x]) + 1/(4*a*x^(5/2)*(a + b*x^
2)^2) + 13/(16*a^2*x^(5/2)*(a + b*x^2)) - (117*b^(5/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17
/4)) - (117*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(64*Sqrt[2]*a^(17/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{117\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{(117b)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{\left(117b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{\left(117b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{\ldots}\right.}{16a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{\left(117b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,\ldots\right.}{32a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{(117b)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,d\ldots\right.}{64a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{117b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\ldots\right.}{64\sqrt{2}a^{17/4}}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}} + \ldots$$

Mathematica [C] time = 0.006324, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -((b*x^2)/a)])/(5*a^3*x^(5/2))
```

Maple [A] time = 0.017, size = 192, normalized size = 0.7

$$\frac{21\,b^3}{16\,a^4\left(bx^2+a\right)^2}x^{\frac{7}{2}} + \frac{25\,b^2}{16\,a^3\left(bx^2+a\right)^2}x^{\frac{3}{2}} + \frac{117\,b\sqrt{2}}{128\,a^4}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{117\,b\sqrt{\ldots}}{64\,a^4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] 21/16*b^3/a^4/(b*x^2+a)^2*x^(7/2)+25/16*b^2/a^3/(b*x^2+a)^2*x^(3/2)+117/128
*b/a^4/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+117/64*b/a^4/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+117/64*b/a^4/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/5/a^3/x^(5/2)+6*b/a
^4/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47201, size = 778, normalized size = 2.95

$$-\frac{2340\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613a^4b^4\sqrt{x}\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769a^9b^5\sqrt{-\frac{b^5}{a^{17}}}+2565164201769b^8xa^4\left(-\frac{b}{a}\right.}}{1601613b^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*arctan
(-1/1601613*(1601613*a^4*b^4*sqrt(x)*(-b^5/a^17)^(1/4) - sqrt(-256516420176
9*a^9*b^5*sqrt(-b^5/a^17) + 2565164201769*b^8*x)*a^4*(-b^5/a^17)^(1/4))/b^5
) - 585*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(1601613
*a^13*(-b^5/a^17)^(3/4) + 1601613*b^4*sqrt(x)) + 585*(a^4*b^2*x^7 + 2*a^5*b
*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(-1601613*a^13*(-b^5/a^17)^(3/4) + 160
1613*b^4*sqrt(x)) - 4*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^
3)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.29089, size = 313, normalized size = 1.19

$$\frac{117\,\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b}+\frac{117\,\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b}-\frac{117\,\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+\right.}{128\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(a^5*b) + 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b) - 117/128*sqrt(2
)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b) +
117/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(a^5*b) + 1/16*(21*b^3*x^(7/2) + 25*a*b^2*x^(3/2))/((b*x^2 + a)^2*a^4)
 + 2/5*(15*b*x^2 - a)/(a^4*x^(5/2))
```

3.312 $\int \frac{\sqrt{x}}{a-bx^2} dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}$$

```
[Out] -(ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))) + ArcTanh[(b^(1/4)*S
qrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))
```

Rubi [A] time = 0.0340864, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {329, 298, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] -(ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))) + ArcTanh[(b^(1/4)*S
qrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^2}{a-bx^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}$$

Mathematica [A] time = 0.0158488, size = 48, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) - \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] (-ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(1/4)*b^(3/4))
```

Maple [A] time = 0.007, size = 66, normalized size = 1.1

$$-\frac{1}{b}\arctan\left(\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{1}{2b}\ln\left(\left(\sqrt{x}+\sqrt[4]{\frac{a}{b}}\right)\left(\sqrt{x}-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x^2+a),x)
```

```
[Out] -1/b/(1/b*a)^(1/4)*arctan(x^(1/2)/(1/b*a)^(1/4))+1/2/b/(1/b*a)^(1/4)*ln((x^
(1/2)+(1/b*a)^(1/4))/(x^(1/2)-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.38314, size = 319, normalized size = 5.5

$$2\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{ab\sqrt{\frac{1}{ab^3}}+xb}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}} - b\sqrt{x}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\right) + \frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) - \frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(\frac{1}{a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2*(1/(a*b^3))^(1/4)*arctan(sqrt(a*b*sqrt(1/(a*b^3)) + x)*b*(1/(a*b^3))^(1/4
) - b*sqrt(x)*(1/(a*b^3))^(1/4)) + 1/2*(1/(a*b^3))^(1/4)*log(a*b^2*(1/(a*b^
3))^(3/4) + sqrt(x)) - 1/2*(1/(a*b^3))^(1/4)*log(-a*b^2*(1/(a*b^3))^(3/4) +
 sqrt(x))
```

Sympy [A] time = 4.39652, size = 128, normalized size = 2.21

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ \frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ -\frac{\log\left(-\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}b^2\left(\frac{1}{b}\right)^{\frac{5}{4}}} + \frac{\log\left(\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}b^2\left(\frac{1}{b}\right)^{\frac{5}{4}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{a}b^2\left(\frac{1}{b}\right)^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)),
 (2/(b*sqrt(x)), Eq(a, 0)), (-log(-a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(
1/4)*b**2*(1/b)**(5/4)) + log(a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*
b**2*(1/b)**(5/4)) - atan(sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b**2*(
1/b)**(5/4)), True))
```

Giac [B] time = 1.88666, size = 262, normalized size = 4.52

$$\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} + \frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} - \frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}} +\right.}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sqr
t(x))/(-a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a/b)^(1/4) - 2*sqrt(x))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(-
a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b))/(a*b^3) + 1
/4*sqrt(2)*(-a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b
))/(a*b^3)
```

3.313 $\int \frac{x^{7/2}}{1+x^2} dx$

Optimal. Leaf size=108

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rubi [A] time = 0.0610106, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{1+x^2}\,dx &= \frac{2x^{5/2}}{5}-\int \frac{x^{3/2}}{1+x^2}\,dx\\
&= -2\sqrt{x}+\frac{2x^{5/2}}{5}+\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -2\sqrt{x}+\frac{2x^{5/2}}{5}+2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}+\frac{2x^{5/2}}{5}+\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}+\frac{2x^{5/2}}{5}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{\text{Subs}}{}\\
&= -2\sqrt{x}+\frac{2x^{5/2}}{5}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{Su}}{}\\
&= -2\sqrt{x}+\frac{2x^{5/2}}{5}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0229026, size = 108, normalized size = 1.

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Maple [A] time = 0.007, size = 72, normalized size = 0.7

$$\frac{2}{5}x^{\frac{5}{2}} - 2\sqrt{x} + \frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right) + \frac{\sqrt{2}}{4}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1),x)
```

```
[Out] 2/5*x^(5/2)-2*x^(1/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/4*2^(1/2)*ln
((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+1/2*arctan(1+2^(1/2)*x^(1/2))
*2^(1/2)
```

Maxima [A] time = 3.18871, size = 113, normalized size = 1.05

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

Fricas [A] time = 1.33545, size = 382, normalized size = 3.54

$$\frac{2}{5}\left(x^2-5\right)\sqrt{x} - \sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right) - \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right) + \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 - 5)*sqrt(x) - sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1
) - sqrt(2)*sqrt(x) - 1) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(
x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4) - 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A] time = 5.34837, size = 105, normalized size = 0.97

$$\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1),x)
```

```
[Out] 2*x**(5/2)/5 - 2*sqrt(x) - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sq
rt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1
)/2 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2
```

<u>Giac</u> [A] time = 2.95182, size = 113, normalized size = 1.05

$$\frac{2}{5}\,x^{\frac{5}{2}} + \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} - 2\,\sqrt{x}\right)\right) + \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{4}\,\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

3.314 $\int \frac{x^{5/2}}{1+x^2} dx$

Optimal. Leaf size=101

$$\frac{2x^{3/2}}{3} - \frac{\log\left(x - \sqrt{2}\sqrt{x} + 1\right)}{2\sqrt{2}} + \frac{\log\left(x + \sqrt{2}\sqrt{x} + 1\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{x} + 1\right)}{\sqrt{2}}$$

```
[Out] (2*x^(3/2))/3 + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]
*Sqrt[x] + x]/(2*Sqrt[2])
```

Rubi [A] time = 0.0552592, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2x^{3/2}}{3} - \frac{\log\left(x - \sqrt{2}\sqrt{x} + 1\right)}{2\sqrt{2}} + \frac{\log\left(x + \sqrt{2}\sqrt{x} + 1\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{x} + 1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2),x]
```

```
[Out] (2*x^(3/2))/3 + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]
*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{1+x^2}\,dx &= \frac{2x^{3/2}}{3} - \int \frac{\sqrt{x}}{1+x^2}\,dx \\
&= \frac{2x^{3/2}}{3} - 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \cdots \\
&= \frac{2x^{3/2}}{3} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \cdots \\
&= \frac{2x^{3/2}}{3} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0059658, size = 24, normalized size = 0.24

$$
-\frac{2}{3}x^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^2\right)-1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2),x]
```

```
[Out] (-2*x^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^2]))/3
```

Maple [A] time = 0.006, size = 67, normalized size = 0.7

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)}{2}-\frac{\sqrt{2}\,\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)}{2}-\frac{\sqrt{2}\,\ln\left(\left(1+x-\sqrt{2}\,\sqrt{x}\right)\left(1+x+\sqrt{2}\,\sqrt{x}\right)^{-1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1),x)
```

```
[Out] 2/3*x^(3/2)-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(-1+2^(1/2)*x^(
1/2))*2^(1/2)-1/4*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 4.83252, size = 107, normalized size = 1.06

$$\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Fricas [A] time = 1.41728, size = 369, normalized size = 3.65

$$\frac{2}{3}x^{\frac{3}{2}}+\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) + sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) -
 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A] time = 1.97161, size = 99, normalized size = 0.98

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*
sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2
```

```
)*atan(sqrt(2)*sqrt(x) + 1)/2
```

Giac [A] time = 1.5132, size = 107, normalized size = 1.06

$$\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

3.315 $\int \frac{x^{3/2}}{1+x^2} dx$

Optimal. Leaf size=99

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

Rubi [A] time = 0.0550577, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{1+x^2}\,dx &= 2\sqrt{x}-\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= 2\sqrt{x}-2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{\text{Subst}\left(\int \frac{\sqrt{2}\cdot}{-1-\sqrt{}}\right.}{2\cdot}\\
&= 2\sqrt{x}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{}{-1\cdot}\right.}{}\\
&= 2\sqrt{x}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0139088, size = 99, normalized size = 1.

$$
2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
```

```
t[x] + x]/(2*Sqrt[2])
```

Maple [A] time = 0.004, size = 67, normalized size = 0.7

$$2\sqrt{x}-\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)-\frac{\sqrt{2}}{4}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)-\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1),x)
```

```
[Out] 2*x^(1/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/4*2^(1/2)*ln((1+x+2^(1/2
)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)
```

Maxima [A] time = 3.23855, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

Fricas [A] time = 1.37285, size = 366, normalized size = 3.7

$$\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1)
 + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*
sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*l
og(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 2*sqrt(x)
```

Sympy [A] time = 0.922942, size = 97, normalized size = 0.98

$$2\sqrt{x}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x) + sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqr
t(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*a
tan(sqrt(2)*sqrt(x) + 1)/2
```

Giac [A] time = 2.4465, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

3.316 $\int \frac{\sqrt{x}}{1+x^2} \, dx$

Optimal. Leaf size=92

$$\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rubi [A] time = 0.0548459, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{1+x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= -\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,d\ldots\right.}{2\sqrt{2}}\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, \ldots\right.}{\sqrt{2}}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0035997, size = 22, normalized size = 0.24

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2),x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -x^2])/3
```

Maple [A] time = 0.004, size = 62, normalized size = 0.7

$$\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{4}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2
)+1/4*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 2.17482, size = 100, normalized size = 1.09

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Fricas [A] time = 1.34809, size = 351, normalized size = 3.82

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A] time = 0.570055, size = 90, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2
```

Giac [A] time = 2.61486, size = 100, normalized size = 1.09

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

3.317 $\int \frac{1}{\sqrt{x}(1+x^2)} dx$

Optimal. Leaf size=92

$$-\frac{\log \left(x-\sqrt{2} \sqrt{x}+1\right)}{2 \sqrt{2}}+\frac{\log \left(x+\sqrt{2} \sqrt{x}+1\right)}{2 \sqrt{2}}-\frac{\tan ^{-1}\left(1-\sqrt{2} \sqrt{x}\right)}{\sqrt{2}}+\frac{\tan ^{-1}\left(\sqrt{2} \sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rubi [A] time = 0.0532149, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log \left(x-\sqrt{2} \sqrt{x}+1\right)}{2 \sqrt{2}}+\frac{\log \left(x+\sqrt{2} \sqrt{x}+1\right)}{2 \sqrt{2}}-\frac{\tan ^{-1}\left(1-\sqrt{2} \sqrt{x}\right)}{\sqrt{2}}+\frac{\tan ^{-1}\left(\sqrt{2} \sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right)-\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2\ldots}{-1-\sqrt{2}x\ldots}\ldots\right.}{2\sqrt{\ldots}}\\
&= -\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{Subst}\left(\int \ldots\right.}{\ldots}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0122664, size = 76, normalized size = 0.83

$$
\frac{-\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[x]] - Log[1 - S
qrt[2]*Sqrt[x] + x] + Log[1 + Sqrt[2]*Sqrt[x] + x])/(2*Sqrt[2])
```

Maple [A] time = 0.003, size = 62, normalized size = 0.7

$$
\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{4}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/x^(1/2),x)
```

```
[Out] 1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2
)+1/4*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 1.9687, size = 100, normalized size = 1.09

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Fricas [A] time = 1.36039, size = 351, normalized size = 3.82

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A] time = 0.786437, size = 90, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/x**(1/2),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2
```

Giac [A] time = 2.08909, size = 100, normalized size = 1.09

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

3.318 $\int \frac{1}{x^{3/2}(1+x^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2])
```

Rubi [A] time = 0.0563801, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx &= -\frac{2}{\sqrt{x}}-\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{2}{\sqrt{x}}-2\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{\sqrt{x}}+\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{\sqrt{x}}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{\mathrm{Subst}\left(\ldots\right.}{\ldots}\\
&= -\frac{2}{\sqrt{x}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\mathrm{Subs}\ldots}{\ldots}\\
&= -\frac{2}{\sqrt{x}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0048697, size = 20, normalized size = 0.2

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -x^2])/Sqrt[x]
```

Maple [A] time = 0.005, size = 67, normalized size = 0.7

$$-\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)-\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)-\frac{\sqrt{2}}{4}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right)-2\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1),x)
```

```
[Out] -1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/
2)-1/4*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))-2/x^(1/2)
```

Maxima [A] time = 2.2267, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

Fricas [A] time = 1.41135, size = 382, normalized size = 3.86

$$\frac{4\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}x\log\left(4\right.}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqr
t(x) - 1) + 4*sqrt(2)*x*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*x*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - sq
rt(2)*x*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x
```

Sympy [A] time = 1.69108, size = 97, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2 - 2/sqrt(x)
```

Giac [A] time = 2.67519, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

3.319 $\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -2/(3*x^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*S
qrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2
]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rubi [A] time = 0.054339, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] -2/(3*x^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*S
qrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2
]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx &= -\frac{2}{3x^{3/2}}-\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{2}{3x^{3/2}}-2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{3x^{3/2}}-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{3x^{3/2}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{\text{Subst}}{}\\
&= -\frac{2}{3x^{3/2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{Sub}}{}\\
&= -\frac{2}{3x^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0049411, size = 22, normalized size = 0.22

$$
-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-x^2\right)}{3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -x^2])/(3*x^(3/2))
```

Maple [A] time = 0.006, size = 67, normalized size = 0.7

$$-\frac{2}{3}x^{-\frac{3}{2}}-\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)-\frac{\sqrt{2}}{4}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)-\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1),x)
```

```
[Out] -2/3/x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/4*2^(1/2)*ln((1+x+2^(
1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)
```

Maxima [A] time = 3.33666, size = 107, normalized size = 1.06

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

Fricas [A] time = 1.36245, size = 405, normalized size = 4.01

$$\frac{12\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}x^2}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)
*sqrt(x) - 1) + 12*sqrt(2)*x^2*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) +
 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*x^2*log(4*sqrt(2)*sqrt(x) + 4*
x + 4) + 3*sqrt(2)*x^2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x^2
```

Sympy [A] time = 2.98806, size = 99, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 - 2/(3*x**(3/2))
```

Giac [A] time = 2.3185, size = 107, normalized size = 1.06

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

3.320 $\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=108

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -2/(5*x^(5/2)) + 2/Sqrt[x] - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rubi [A] time = 0.0573483, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)),x]
```


```
[Out] -2/(5*x^(5/2)) + 2/Sqrt[x] - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx &= -\frac{2}{5x^{5/2}}-\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{}\right.}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0049442, size = 22, normalized size = 0.2

$$
-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-x^2\right)}{5x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -x^2])/(5*x^(5/2))
```

Maple [A] time = 0.007, size = 72, normalized size = 0.7

$$-\frac{2}{5}x^{-\frac{5}{2}}+2\frac{1}{\sqrt{x}}+\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{4}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1),x)
```

```
[Out] -2/5/x^(5/2)+2/x^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(-1+
2^(1/2)*x^(1/2))*2^(1/2)+1/4*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*
x^(1/2)))
```

Maxima [A] time = 4.31251, size = 116, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

Fricas [A] time = 1.31501, size = 423, normalized size = 3.92

$$-\frac{20\sqrt{2}x^3\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}x^3\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+5\sqrt{2}\cdots}{20x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + 20*sqrt(2)*x^3*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x)
+ 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*x^3*log(4*sqrt(2)*sqrt(x) + 4
*x + 4) - 5*sqrt(2)*x^3*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^2 - 1)*s
qrt(x))/x^3
```

Sympy [A] time = 7.1108, size = 105, normalized size = 0.97

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}+\frac{2}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 + 2/sqrt(x) - 2/(5*x**(5/2))
```

Giac [A] time = 1.69134, size = 116, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

3.321 $\int \frac{x^{7/2}}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{x^{5/2}}{2\left(x^2+1\right)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] (5*Sqrt[x])/2 - x^(5/2)/(2*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*
Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (5*Log[1 - Sqrt[2]
*Sqrt[x] + x])/(8*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rubi [A] time = 0.0614054, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}}{2\left(x^2+1\right)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] (5*Sqrt[x])/2 - x^(5/2)/(2*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*
Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (5*Log[1 - Sqrt[2]
*Sqrt[x] + x])/(8*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5}{4}\int \frac{x^{3/2}}{1+x^2}\,dx\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{5}{8}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\cdots\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{\cdots}\right)}{4\sqrt{2}}\cdots\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\log\left(1\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.0462298, size = 121, normalized size = 0.99

$$\frac{1}{16}\left(\frac{32x^{5/2}}{x^2+1}+\frac{40\sqrt{x}}{x^2+1}+5\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)-5\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] ((40*Sqrt[x])/(1 + x^2) + (32*x^(5/2))/(1 + x^2) + 10*Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[x]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] + 5*Sqrt[2]*Log[1 -
 Sqrt[2]*Sqrt[x] + x] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

Maple [A] time = 0.01, size = 79, normalized size = 0.7

$$2\sqrt{x}+\frac{1}{2x^2+2}\sqrt{x}-\frac{5\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)-\frac{5\sqrt{2}}{16}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)-\frac{5\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^2,x)
```

```
[Out] 2*x^(1/2)+1/2*x^(1/2)/(x^2+1)-5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/16*2
^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))-5/8*arctan(1+2^(1/2)
*x^(1/2))*2^(1/2)
```

Maxima [A] time = 2.35135, size = 123, normalized size = 1.01

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

Fricas [A] time = 1.3944, size = 462, normalized size = 3.79

$$\frac{20\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+\right.}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*(x^2 + 1)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) + 8*(4*x^2 + 5)*sqrt(x))/(x^2 + 1)
```

Sympy [B] time = 13.795, size = 277, normalized size = 2.27

$$\frac{32x^{\frac{5}{2}}}{16x^2+16}+\frac{40\sqrt{x}}{16x^2+16}+\frac{5\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{5\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{10\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right.}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 32*x**(5/2)/(16*x**2 + 16) + 40*sqrt(x)/(16*x**2 + 16) + 5*sqrt(2)*x**2*log
(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2)*x**2*log(4*sqrt(2
)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x)
- 1)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 +
16) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)
*sqrt(x) - 1)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**
2 + 16)
```

Giac [A] time = 2.35365, size = 123, normalized size = 1.01

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

3.322 $\int \frac{x^{5/2}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=113

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] -x^(3/2)/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/
(8*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rubi [A] time = 0.0587743, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^2,x]
```


```
[Out] -x^(3/2)/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/
(8*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3\,\mathrm{Su}}{}\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0105864, size = 30, normalized size = 0.27

$$2x^{3/2}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-x^2\right)-\frac{1}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] 2*x^(3/2)*(-(1 + x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -x^2])
```

Maple [A] time = 0.008, size = 74, normalized size = 0.7

$$-\frac{1}{2x^2+2}x^{\frac{3}{2}}+\frac{3\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{3\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{3\sqrt{2}}{16}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^2,x)
```

```
[Out] -1/2*x^(3/2)/(x^2+1)+3/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(-1+2^
(1/2)*x^(1/2))*2^(1/2)+3/16*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x
^(1/2)))
```

Maxima [A] time = 4.39211, size = 116, normalized size = 1.03

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

Fricas [A] time = 1.37106, size = 447, normalized size = 3.96

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+\right.}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) + 8*x^(3/2))/(x^2 + 1)
```

Sympy [B] time = 7.90666, size = 264, normalized size = 2.34

$$-\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{ata}}{16x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**2,x)
```

```
[Out] -8*x**(3/2)/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**
2 + 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(
2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + 3*sqrt(2)*log(-4*sqrt(2)
*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x
+ 4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) +
6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A] time = 2.44266, size = 116, normalized size = 1.03

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

3.323 $\int \frac{x^{3/2}}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{\log \left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log \left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] -Sqrt[x]/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
+ Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rubi [A] time = 0.0585175, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{\log \left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log \left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] -Sqrt[x]/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
+ Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{\mathrm{Sub}}{}\ldots \\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{\mathrm{Su}}{}\ldots \\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+\ldots\right.}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.043658, size = 106, normalized size = 0.94

$$
\frac{1}{16}\left(-\frac{8\sqrt{x}}{x^2+1}-\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] ((-8*Sqrt[x])/(1 + x^2) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[x]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

Maple [A] time = 0.009, size = 74, normalized size = 0.7

$$-\frac{1}{2\,x^2+2}\sqrt{x}+\frac{\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{16}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)+\frac{\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^2,x)
```

```
[Out] -1/2*x^(1/2)/(x^2+1)+1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/16*2^(1/2)*ln
((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+1/8*arctan(1+2^(1/2)*x^(1/2))
*2^(1/2)
```

Maxima [A] time = 4.54629, size = 116, normalized size = 1.03

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

Fricas [A] time = 1.36982, size = 439, normalized size = 3.88

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right.}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) + sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 + 8*sqrt(x))/(x^2 + 1)
```

Sympy [B] time = 4.66203, size = 257, normalized size = 2.27

$$-\frac{8\sqrt{x}}{16x^2+16}-\frac{\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\operatorname{atan}\left(\right.}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -8*sqrt(x)/(16*x**2 + 16) - sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) + sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A] time = 2.33158, size = 116, normalized size = 1.03

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

3.324 $\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] x^(3/2)/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2]) -
 Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rubi [A] time = 0.0600187, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] x^(3/2)/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2]) -
 Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx &= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{1}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{\text{Subs}}{}\\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{\text{Sub}}{}\\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0036141, size = 22, normalized size = 0.19

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -x^2])/3
```

Maple [A] time = 0.005, size = 74, normalized size = 0.7

$$\frac{1}{2x^2+2}x^{\frac{3}{2}}+\frac{\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{16}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2*x^(3/2)/(x^2+1)+1/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(-1+2^(
1/2)*x^(1/2))*2^(1/2)+1/16*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^
(1/2)))
```

Maxima [A] time = 3.21816, size = 116, normalized size = 1.03

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

Fricas [A] time = 1.46522, size = 439, normalized size = 3.88

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right.}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 - 8*x^(3/2))/(x^2 + 1)
```

Sympy [B] time = 2.85631, size = 257, normalized size = 2.27

$$\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\operatorname{atan}\left(\sqrt{}\right.}{16x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**2,x)
```

```
[Out] 8*x**(3/2)/(16*x**2 + 16) + sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) - sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) - sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A] time = 1.88885, size = 116, normalized size = 1.03

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

3.325 $\int \frac{1}{\sqrt{x}(1+x^2)^2} dx$

Optimal. Leaf size=113

$$\frac{\sqrt{x}}{2(x^2+1)} - \frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}} + \frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] Sqrt[x]/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Ar
cTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(
8*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rubi [A] time = 0.0591118, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{2(x^2+1)} - \frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}} + \frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^2),x]
```


```
[Out] Sqrt[x]/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Ar
cTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(
8*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx &= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{3\,\text{S}}{}\\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}\\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\log\left(1+\right.}{8}
\end{aligned}$$

Mathematica [A] time = 0.0407927, size = 107, normalized size = 0.95

$$\frac{1}{16}\left(\frac{8\sqrt{x}}{x^2+1}-3\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] ((8*Sqrt[x])/(1 + x^2) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 6*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[x]] - 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + 3*Sq
rt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

Maple [A] time = 0.006, size = 74, normalized size = 0.7

$$\frac{1}{2x^2+2}\sqrt{x}+\frac{3\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{3\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{3\sqrt{2}}{16}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/(x^2+1)+3/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(-1+2^(
1/2)*x^(1/2))*2^(1/2)+3/16*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^
(1/2)))
```

Maxima [A] time = 3.68477, size = 116, normalized size = 1.03

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

Fricas [A] time = 1.41224, size = 447, normalized size = 3.96

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+\right.}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) - 8*sqrt(x))/(x^2 + 1)
```

Sympy [B] time = 3.78855, size = 264, normalized size = 2.34

$$\frac{8\sqrt{x}}{16x^2+16}-\frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/x**(1/2),x)
```

```
[Out] 8*sqrt(x)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2
+ 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(2)
*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - 3*sqrt(2)*log(-4*sqrt(2)*s
qrt(x) + 4*x + 4)/(16*x**2 + 16) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*
sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A] time = 2.10189, size = 116, normalized size = 1.03

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

3.326 $\int \frac{1}{x^{3/2}\left(1+x^2\right)^2} dx$

Optimal. Leaf size=122

$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] -5/(2*Sqrt[x]) + 1/(2*Sqrt[x]*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (5*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rubi [A] time = 0.0648586, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] -5/(2*Sqrt[x]) + 1/(2*Sqrt[x]*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (5*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}dx = \frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5}{4}\int \frac{1}{x^{3/2}\left(1+x^2\right)}dx$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{4}\int \frac{\sqrt{x}}{1+x^2}dx$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4}dx,x,\sqrt{x}\right)$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx,x,\sqrt{x}\right)-\frac{5}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx,x,\sqrt{x}\right)$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx,x,\sqrt{x}\right)-\frac{5}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}dx,x,\right.$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\,\text{Subst}\left(\int \frac{1}{-1-x^2}dx,x,\right.}{4\sqrt{2}}$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{5}{}$$

Mathematica [C] time = 0.004854, size = 20, normalized size = 0.16

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -x^2])/Sqrt[x]
```

Maple [A] time = 0.011, size = 79, normalized size = 0.7

$$-2\frac{1}{\sqrt{x}}-\frac{1}{2x^2+2}x^{\frac{3}{2}}-\frac{5\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)-\frac{5\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)-\frac{5\sqrt{2}}{16}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^2,x)
```

```
[Out] -2/x^(1/2)-1/2*x^(3/2)/(x^2+1)-5/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*ar
ctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/16*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x
+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 3.72872, size = 124, normalized size = 1.02

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

Fricas [A] time = 1.34484, size = 462, normalized size = 3.79

$$\frac{20\sqrt{2}\left(x^3+x\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^3+x\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{\cdots}\right.}{16\left(x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^3 + x)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*(x^3 + x)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) - 8*(5*x^2 + 4)*sqrt(x))/(x^3 + x)
```

Sympy [B] time = 6.46214, size = 366, normalized size = 3.

$$-\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}+\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\cdots\right.}{16x^{\frac{5}{2}}+16\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -5*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) + 16*sqr
t(x)) + 5*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**(5/2) +
16*sqrt(x)) - 5*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5
/2) + 16*sqrt(x)) + 5*sqrt(2)*sqrt(x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(16*x
**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(16*x*
*(5/2) + 16*sqrt(x)) - 40*x**2/(16*x**(5/2) + 16*sqrt(x)) - 32/(16*x**(5/2)
 + 16*sqrt(x))
```

Giac [A] time = 1.98657, size = 124, normalized size = 1.02

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

3.327 $\int \frac{1}{x^{5/2}\left(1+x^2\right)^2} dx$

Optimal. Leaf size=122

$$\frac{1}{2x^{3/2}\left(x^2+1\right)}-\frac{7}{6x^{3/2}}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] -7/(6*x^(3/2)) + 1/(2*x^(3/2)*(1 + x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rubi [A] time = 0.0608723, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2x^{3/2}\left(x^2+1\right)}-\frac{7}{6x^{3/2}}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] -7/(6*x^(3/2)) + 1/(2*x^(3/2)*(1 + x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx = \frac{1}{2x^{3/2}\left(1+x^2\right)} + \frac{7}{4}\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}\left(1+x^2\right)} - \frac{7}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}\left(1+x^2\right)} - \frac{7}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}\left(1+x^2\right)} - \frac{7}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \frac{7}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}\left(1+x^2\right)} - \frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,d\right.$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}\left(1+x^2\right)} + \frac{7\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{7\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{7\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,d\right.}{4\sqrt{2}}$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}\left(1+x^2\right)} + \frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{7\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{7\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}$$

Mathematica [C] time = 0.0051349, size = 22, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -x^2])/(3*x^(3/2))
```

Maple [A] time = 0.009, size = 79, normalized size = 0.7

$$-\frac{2}{3}x^{-\frac{3}{2}} - \frac{1}{2x^2+2}\sqrt{x} - \frac{7\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right) - \frac{7\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right) - \frac{7\sqrt{2}}{16}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^2,x)
```

```
[Out] -2/3/x^(3/2)-1/2*x^(1/2)/(x^2+1)-7/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-7/8*
arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-7/16*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1
+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 3.28638, size = 124, normalized size = 1.02

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{7}{16}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/6*(7*x^2 + 4)/(x^(7/
2) + x^(3/2))
```

Fricas [A] time = 1.34961, size = 478, normalized size = 3.92

$$\frac{84\sqrt{2}\left(x^4+x^2\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}\left(x^4+x^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}\right.}{48\left(x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(84*sqrt(2)*(x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
 sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + x^2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)*(x^4 + x^2)*l
og(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + x^2)*log(-4*sqrt(2)*sqr
t(x) + 4*x + 4) - 8*(7*x^2 + 4)*sqrt(x))/(x^4 + x^2)
```

Sympy [B] time = 13.6437, size = 366, normalized size = 3.

$$\frac{21\sqrt{2}x^{\frac{7}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{21\sqrt{2}x^{\frac{7}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\right.}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**2,x)
```

```
[Out] 21*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2) + 48*x**
(3/2)) - 21*sqrt(2)*x**(7/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2)
+ 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(48*x**(7/2)
 + 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(48*x**(7/2
) + 48*x**(3/2)) + 21*sqrt(2)*x**(3/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(4
8*x**(7/2) + 48*x**(3/2)) - 21*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x
) - 1)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(
x) + 1)/(48*x**(7/2) + 48*x**(3/2)) - 56*x**2/(48*x**(7/2) + 48*x**(3/2)) -
 32/(48*x**(7/2) + 48*x**(3/2))
```

Giac [A] time = 2.65837, size = 123, normalized size = 1.01

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{7}{16}\sqrt{2}\log\left(-\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
- 2/3/x^(3/2)
```

3.328 $\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx$

Optimal. Leaf size=131

$$\frac{1}{2x^{5/2}\left(x^2+1\right)}-\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

```
[Out] -9/(10*x^(5/2)) + 9/(2*Sqrt[x]) + 1/(2*x^(5/2)*(1 + x^2)) - (9*ArcTan[1 - S
qrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2])
+ (9*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (9*Log[1 + Sqrt[2]*Sqrt[x]
 + x])/(8*Sqrt[2])
```

Rubi [A] time = 0.065451, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2x^{5/2}\left(x^2+1\right)}-\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] -9/(10*x^(5/2)) + 9/(2*Sqrt[x]) + 1/(2*x^(5/2)*(1 + x^2)) - (9*ArcTan[1 - S
qrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2])
+ (9*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (9*Log[1 + Sqrt[2]*Sqrt[x]
 + x])/(8*Sqrt[2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{1}{2x^{5/2}\left(1+x^2\right)} + \frac{9}{4}\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{9}{10x^{5/2}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} - \frac{9}{4}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} + \frac{9}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} + \frac{9}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} - \frac{9}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{9}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} + \frac{9}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) + \frac{9}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x\cdots}\right.$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} + \frac{9\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{9\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}\left(1+x^2\right)} - \frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{9\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{9\log\left(1-\sqrt{2}\sqrt{x}\cdots\right.}{8\sqrt{2}}$$

Mathematica [C] time = 0.0055808, size = 22, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -x^2])/(5*x^(5/2))
```

Maple [A] time = 0.011, size = 84, normalized size = 0.6

$$-\frac{2}{5}x^{-\frac{5}{2}} + 4\frac{1}{\sqrt{x}} + \frac{1}{2x^2+2}x^{\frac{3}{2}} + \frac{9\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right) + \frac{9\sqrt{2}}{8}\arctan\left(-1+\sqrt{2}\sqrt{x}\right) + \frac{9\sqrt{2}}{16}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^2,x)
```

```
[Out] -2/5/x^(5/2)+4/x^(1/2)+1/2*x^(3/2)/(x^2+1)+9/8*arctan(1+2^(1/2)*x^(1/2))*2^
(1/2)+9/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+9/16*2^(1/2)*ln((1+x-2^(1/2)*x
^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 3.84609, size = 131, normalized size = 1.

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/10*(45*x^4 + 36*x^2 -
 4)/(x^(9/2) + x^(5/2))
```

Fricas [A] time = 1.35359, size = 495, normalized size = 3.78

$$-\frac{180\sqrt{2}\left(x^5+x^3\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\left(x^5+x^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\right.}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/80*(180*sqrt(2)*(x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1)
 - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + x^3)*arctan(1/2*sqrt(2)*sqrt(-
4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2)*(x^5 + x^3
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + x^3)*log(-4*sqrt(2)*
sqrt(x) + 4*x + 4) - 8*(45*x^4 + 36*x^2 - 4)*sqrt(x))/(x^5 + x^3)
```

Sympy [B] time = 32.2977, size = 384, normalized size = 2.93

$$\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{\right.}}{80x^{\frac{9}{2}}+80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2) + 80*x**
(5/2)) - 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2)
+ 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(80*x**(9/2)
 + 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(80*x**(9/2
) + 80*x**(5/2)) + 45*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(8
0*x**(9/2) + 80*x**(5/2)) - 45*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x
) - 1)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(
x) + 1)/(80*x**(9/2) + 80*x**(5/2)) + 360*x**4/(80*x**(9/2) + 80*x**(5/2))
+ 288*x**2/(80*x**(9/2) + 80*x**(5/2)) - 32/(80*x**(9/2) + 80*x**(5/2))
```

Giac [A] time = 2.02969, size = 132, normalized size = 1.01

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1) +
 2/5*(10*x^2 - 1)/x^(5/2)
```

3.329 $\int \frac{x^{7/2}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{x^{5/2}}{4\left(x^2+1\right)^2}-\frac{5\sqrt{x}}{16\left(x^2+1\right)}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

```
[Out] -x^(5/2)/(4*(1 + x^2)^2) - (5*Sqrt[x])/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
(5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rubi [A] time = 0.068182, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}}{4\left(x^2+1\right)^2}-\frac{5\sqrt{x}}{16\left(x^2+1\right)}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] -x^(5/2)/(4*(1 + x^2)^2) - (5*Sqrt[x])/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
(5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\left(1+x^2\right)^3}\,dx &= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}+\frac{5}{8}\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{16}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{5}{32}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{5}{64}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,\right.\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\right.}{32\sqrt{2}}\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5}{}
\end{aligned}$$

Mathematica [A] time = 0.0444045, size = 135, normalized size = 1.05

$$\frac{1}{384}\left(-\frac{256x^{5/2}}{\left(x^2+1\right)^2}+\frac{40\sqrt{x}}{x^2+1}-\frac{160\sqrt{x}}{\left(x^2+1\right)^2}-15\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+15\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-30\sqrt{2}\tan^{-1}\left(1-\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] ((-160*Sqrt[x])/(1 + x^2)^2 - (256*x^(5/2))/(1 + x^2)^2 + (40*Sqrt[x])/(1 +
 x^2) - 30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 30*Sqrt[2]*ArcTan[1 + Sqrt
[2]*Sqrt[x]] - 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + 15*Sqrt[2]*Log[1 +
 Sqrt[2]*Sqrt[x] + x])/384
```

Maple [A] time = 0.009, size = 82, normalized size = 0.6

$$2\,\frac{1}{\left(x^2+1\right)^2}\left(-\frac{9\,x^{5/2}}{32}-\frac{5\,\sqrt{x}}{32}\right)+\frac{5\,\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{5\,\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{5\,\sqrt{2}}{128}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^3,x)
```

```
[Out] 2*(-9/32*x^(5/2)-5/32*x^(1/2))/(x^2+1)^2+5/64*arctan(1+2^(1/2)*x^(1/2))*2^(
1/2)+5/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+5/128*2^(1/2)*ln((1+x+2^(1/2)*
x^(1/2))/(1+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 2.9388, size = 134, normalized size = 1.04

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{128}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.37088, size = 518, normalized size = 4.02

$$-\frac{20\,\sqrt{2}\left(x^4+2\,x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\,\sqrt{2}\left(x^4+2\,x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\,\sqrt{2}\sqrt{x}+\right.\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 8*(9*x^2 + 5)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

Sympy [B] time = 44.6923, size = 481, normalized size = 3.73

$$-\frac{72x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{40\sqrt{x}}{128x^4+256x^2+128}-\frac{5\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**3,x)
```

```
[Out] -72*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 40*sqrt(x)/(128*x**4 + 256*x**2
+ 128) - 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) - 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) + 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A] time = 2.0662, size = 127, normalized size = 0.98

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{128}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^2 + 1)^2
```

3.330 $\int \frac{x^{5/2}}{\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{3x^{3/2}}{16\left(x^2+1\right)}-\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

```
[Out] -x^(3/2)/(4*(1 + x^2)^2) + (3*x^(3/2))/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) +
(3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rubi [A] time = 0.0698324, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3x^{3/2}}{16\left(x^2+1\right)}-\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^3,x]
```


```
[Out] -x^(3/2)/(4*(1 + x^2)^2) + (3*x^(3/2))/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) +
(3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3}{8}\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} + \frac{3}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} + \frac{3}{16}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} - \frac{3}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{3}{32}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} + \frac{3}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) + \frac{3}{64}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} + \frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{32\sqrt{2}}$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}$$

Mathematica [C] time = 0.0122691, size = 32, normalized size = 0.25

$$\frac{2}{5}x^{3/2}\left({}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-x^2\right) - \frac{1}{(x^2+1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(-(1 + x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -x^2]))/5
```

Maple [A] time = 0.009, size = 82, normalized size = 0.6

$$2\frac{1}{(x^2+1)^2}\left(\frac{3x^{7/2}}{32} - 1/32\,x^{3/2}\right) + \frac{3\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right) + \frac{3\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right) + \frac{3\sqrt{2}}{128}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^3,x)
```

```
[Out] 2*(3/32*x^(7/2)-1/32*x^(3/2))/(x^2+1)^2+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1
/2)+3/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/128*2^(1/2)*ln((1+x-2^(1/2)*x
^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 2.54206, size = 134, normalized size = 1.04

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{3}{128}\sqrt{2}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.6743, size = 518, normalized size = 4.02

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+\cdots}\right.}{128\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(3*x^3 - x)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

Sympy [B] time = 29.0798, size = 481, normalized size = 3.73

$$\frac{24x^{\frac{7}{2}}}{128x^4+256x^2+128}-\frac{8x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\cdots}{128\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 24*x**(7/2)/(128*x**4 + 256*x**2 + 128) - 8*x**(3/2)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) + 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) - 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 12
8) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sq
rt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A] time = 1.57622, size = 127, normalized size = 0.98

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{128}\sqrt{2}\log\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^2 + 1)^2
```

3.331 $\int \frac{x^{3/2}}{\left(1+x^2\right)^3} dx$

Optimal. Leaf size=129

$$\frac{\sqrt{x}}{16\left(x^2+1\right)}-\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

```
[Out] -Sqrt[x]/(4*(1 + x^2)^2) + Sqrt[x]/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*S
qrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (3*L
og[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x]
)/(64*Sqrt[2])
```

Rubi [A] time = 0.0679411, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{16\left(x^2+1\right)}-\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] -Sqrt[x]/(4*(1 + x^2)^2) + Sqrt[x]/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*S
qrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (3*L
og[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x]
)/(64*Sqrt[2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(1+x^2\right)^3}\,dx = -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{1}{8}\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{16}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{32}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{64}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx\right.$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\right.}{32\sqrt{2}}$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3}{}$$

Mathematica [A] time = 0.0301475, size = 121, normalized size = 0.94

$$\frac{1}{128}\left(\frac{8\sqrt{x}}{x^2+1}-\frac{32\sqrt{x}}{\left(x^2+1\right)^2}-3\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+6\sqrt{2}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] ((-32*Sqrt[x])/(1 + x^2)^2 + (8*Sqrt[x])/(1 + x^2) - 6*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[x]] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] - 3*Sqrt[2]*Log[1 -
 Sqrt[2]*Sqrt[x] + x] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/128
```

Maple [A] time = 0.007, size = 82, normalized size = 0.6

$$2\,\frac{1}{\left(x^2+1\right)^2}\left(1/32\,x^{5/2}-\frac{3\,\sqrt{x}}{32}\right)+\frac{3\,\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{3\,\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{3\,\sqrt{2}}{128}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^3,x)
```

```
[Out] 2*(1/32*x^(5/2)-3/32*x^(1/2))/(x^2+1)^2+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1
/2)+3/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/128*2^(1/2)*ln((1+x+2^(1/2)*x
^(1/2))/(1+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 2.93074, size = 131, normalized size = 1.02

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{128}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.50976, size = 516, normalized size = 4.

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4\cdots}\right.}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(x^2 - 3)*sqrt(x))/(x^4 + 2*x^2
 + 1)
```

Sympy [B] time = 17.4848, size = 481, normalized size = 3.73

$$\frac{8x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{24\sqrt{x}}{128x^4+256x^2+128}-\frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**3,x)
```

```
[Out] 8*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 24*sqrt(x)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) + 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 1
28) - 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 1
28) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 128
) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sqr
t(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A] time = 1.77861, size = 124, normalized size = 0.96

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{128}\sqrt{2}\,\mathrm{lo}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^2 + 1)^2
```

3.332 $\int \frac{\sqrt{x}}{(1+x^2)^3} dx$

Optimal. Leaf size=129

$$\frac{5x^{3/2}}{16(x^2+1)}+\frac{x^{3/2}}{4(x^2+1)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

```
[Out] x^(3/2)/(4*(1 + x^2)^2) + (5*x^(3/2))/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[2
]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (
5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] +
 x])/(64*Sqrt[2])
```

Rubi [A] time = 0.0690822, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5x^{3/2}}{16(x^2+1)}+\frac{x^{3/2}}{4(x^2+1)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] x^(3/2)/(4*(1 + x^2)^2) + (5*x^(3/2))/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[2
]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (
5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] +
 x])/(64*Sqrt[2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(1+x^2\right)^3}\,dx = \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5}{8}\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5}{16}\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}-\frac{5}{32}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{5}{32}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5}{64}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{5}{64}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x\right.$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1\right.}{32\sqrt{2}}$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{5\,\mathrm{l}}{}$$

Mathematica [C] time = 0.004153, size = 22, normalized size = 0.17

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -x^2])/3
```

Maple [A] time = 0.006, size = 86, normalized size = 0.7

$$\frac{1}{4\left(x^2+1\right)^2}x^{\frac{3}{2}}+\frac{5}{16x^2+16}x^{\frac{3}{2}}+\frac{5\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{5\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{5\sqrt{2}}{128}\ln\left(\left(1+x-\sqrt{2}\sqrt{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^3,x)
```

```
[Out] 1/4*x^(3/2)/(x^2+1)^2+5/16*x^(3/2)/(x^2+1)+5/64*arctan(1+2^(1/2)*x^(1/2))*2
^(1/2)+5/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+5/128*2^(1/2)*ln((1+x-2^(1/2
)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 3.79966, size = 134, normalized size = 1.04

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{128}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.38735, size = 521, normalized size = 4.04

$$-\frac{20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+\cdots}\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
```

```
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^3 + 9*x)*sqrt(x))/(x^4 + 2
*x^2 + 1)
```

Sympy [B] time = 10.7292, size = 481, normalized size = 3.73

$$\frac{40x^{\frac{7}{2}}}{128x^4+256x^2+128}+\frac{72x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{5\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**3,x)
```

```
[Out] 40*x**(7/2)/(128*x**4 + 256*x**2 + 128) + 72*x**(3/2)/(128*x**4 + 256*x**2
+ 128) + 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) - 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A] time = 1.72214, size = 127, normalized size = 0.98

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{128}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^2 + 1)^2
```

3.333 $\int \frac{1}{\sqrt{x}(1+x^2)^3} \, dx$

Optimal. Leaf size=129

$$\frac{7\sqrt{x}}{16(x^2+1)}+\frac{\sqrt{x}}{4(x^2+1)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}}$$

```
[Out] Sqrt[x]/(4*(1 + x^2)^2) + (7*Sqrt[x])/(16*(1 + x^2)) - (21*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (21*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
 (21*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (21*Log[1 + Sqrt[2]*Sqrt[
x] + x])/(64*Sqrt[2])
```

Rubi [A] time = 0.0682552, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16(x^2+1)}+\frac{\sqrt{x}}{4(x^2+1)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^3),x]
```


```
[Out] Sqrt[x]/(4*(1 + x^2)^2) + (7*Sqrt[x])/(16*(1 + x^2)) - (21*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (21*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
 (21*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (21*Log[1 + Sqrt[2]*Sqrt[
x] + x])/(64*Sqrt[2])
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)^3}\,dx &= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7}{8}\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx \\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{16}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{32}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{21}{32}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{64}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{21}{64}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,d\right. \\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}-\frac{21\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{21\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{21\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\right.}{32\sqrt{}} \\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{21\log\left(1-\sqrt{2}\sqrt{x}+x\right.}{64\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0313471, size = 121, normalized size = 0.94

$$
\frac{1}{128}\left(\frac{56\sqrt{x}}{x^2+1}+\frac{32\sqrt{x}}{\left(x^2+1\right)^2}-21\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+21\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+42\sqrt{2}\tan\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] ((32*Sqrt[x])/(1 + x^2)^2 + (56*Sqrt[x])/(1 + x^2) - 42*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[x]] + 42*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] - 21*Sqrt[2]*Log[
1 - Sqrt[2]*Sqrt[x] + x] + 21*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/128
```

Maple [A] time = 0.004, size = 86, normalized size = 0.7

$$\frac{1}{4\left(x^{2}+1\right)^{2}}\sqrt{x}+\frac{7}{16x^{2}+16}\sqrt{x}+\frac{21\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{21\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{21\sqrt{2}}{128}\ln\left(\left(1+x+\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/x^(1/2),x)
```

```
[Out] 1/4*x^(1/2)/(x^2+1)^2+7/16*x^(1/2)/(x^2+1)+21/64*arctan(1+2^(1/2)*x^(1/2))*
2^(1/2)+21/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+21/128*2^(1/2)*ln((1+x+2^(
1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 2.96133, size = 134, normalized size = 1.04

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{21}{128}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.47195, size = 522, normalized size = 4.05

$$-\frac{84\sqrt{2}\left(x^{4}+2x^{2}+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}\left(x^{4}+2x^{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)
*(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + 2*x
^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(7*x^2 + 11)*sqrt(x))/(x^4 +
```

```
2*x^2 + 1)
```

Sympy [B] time = 14.4115, size = 481, normalized size = 3.73

$$\frac{56x^{\frac{5}{2}}}{128x^4+256x^2+128}+\frac{88\sqrt{x}}{128x^4+256x^2+128}-\frac{21\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{21\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/x**(1/2),x)
```

```
[Out] 56*x**(5/2)/(128*x**4 + 256*x**2 + 128) + 88*sqrt(x)/(128*x**4 + 256*x**2 +
 128) - 21*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 21*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**
2 + 128) - 42*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 42*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
+ 256*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
56*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*
x**2 + 128) - 21*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 21*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 12
8) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A] time = 3.14375, size = 127, normalized size = 0.98

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{21}{128}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^2 + 1)^2
```

3.334 $\int \frac{1}{x^{3/2}\left(1+x^2\right)^3} \, dx$

Optimal. Leaf size=138

$$\frac{9}{16\sqrt{x}\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}$$

```
[Out] -45/(16*Sqrt[x]) + 1/(4*Sqrt[x]*(1 + x^2)^2) + 9/(16*Sqrt[x]*(1 + x^2)) + (
45*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (45*ArcTan[1 + Sqrt[2]*Sqrt[
x]])/(32*Sqrt[2]) - (45*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (45*Lo
g[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rubi [A] time = 0.0703358, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16\sqrt{x}\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^3),x]
```

```
[Out] -45/(16*Sqrt[x]) + 1/(4*Sqrt[x]*(1 + x^2)^2) + 9/(16*Sqrt[x]*(1 + x^2)) + (
45*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (45*ArcTan[1 + Sqrt[2]*Sqrt[
x]])/(32*Sqrt[2]) - (45*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (45*Lo
g[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}dx = \frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{8}\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}dx$$
$$= \frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}dx$$
$$= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45}{32}\int \frac{\sqrt{x}}{1+x^2}dx$$
$$= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45}{16}\text{Subst}\left(\int \frac{x^2}{1+x^4}dx,x,\sqrt{x}\right)$$
$$= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx,x,\sqrt{x}\right)-\frac{45}{32}\text{Subst}\left(\int \frac{1}{1}\right.$$
$$= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx,x,\sqrt{x}\right)-\frac{45}{64}\text{Subs}$$
$$= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{45\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}$$
$$= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-$$

Mathematica [C] time = 0.0046598, size = 20, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -x^2])/Sqrt[x]
```

Maple [A] time = 0.01, size = 87, normalized size = 0.6

$$-2\frac{1}{\sqrt{x}}-2\frac{1}{\left(x^2+1\right)^2}\left(\frac{13\,x^{7/2}}{32}+\frac{17\,x^{3/2}}{32}\right)-\frac{45\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)-\frac{45\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)-\frac{45\sqrt{2}}{128}\ln\left(\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^3,x)
```

```
[Out] -2/x^(1/2)-2*(13/32*x^(7/2)+17/32*x^(3/2))/(x^2+1)^2-45/64*arctan(1+2^(1/2)
*x^(1/2))*2^(1/2)-45/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-45/128*2^(1/2)*l
n((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 4.07082, size = 138, normalized size = 1.

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(45*x^4 +
 81*x^2 + 32)/(x^(9/2) + 2*x^(5/2) + sqrt(x))
```

Fricas [A] time = 1.33975, size = 537, normalized size = 3.89

$$180\sqrt{2}\left(x^5+2x^3+x\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\left(x^5+2x^3+x\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(1/2*sq
rt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2
)*(x^5 + 2*x^3 + x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + 2*
x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(45*x^4 + 81*x^2 + 32)*sqrt(
x))/(x^5 + 2*x^3 + x)
```

Sympy [B] time = 22.5495, size = 653, normalized size = 4.73

$$-\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\right.}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**3,x)
```

```
[Out] -45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*
x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x +
 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2)*x**(9/2)*atan(
sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(
2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sq
rt(x)) - 90*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2
) + 256*x**(5/2) + 128*sqrt(x)) + 90*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 180*sqrt(2)*x**(5
/2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) -
 180*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2
) + 128*sqrt(x)) - 45*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(12
8*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*sqrt(x)*log(4*sqrt(2)
```

```
*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2
)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt
(x)) - 90*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**
(5/2) + 128*sqrt(x)) - 360*x**4/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x))
 - 648*x**2/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 256/(128*x**(9/2)
 + 256*x**(5/2) + 128*sqrt(x))
```

Giac [A] time = 2.474, size = 134, normalized size = 0.97

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x) - 1/
16*(13*x^(7/2) + 17*x^(3/2))/(x^2 + 1)^2
```

3.335 $\int \frac{1}{x^{5/2}\left(1+x^2\right)^3} \, dx$

Optimal. Leaf size=138

$$\frac{11}{16x^{3/2}\left(x^2+1\right)}-\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-$$

```
[Out] -77/(48*x^(3/2)) + 1/(4*x^(3/2)*(1 + x^2)^2) + 11/(16*x^(3/2)*(1 + x^2)) +
(77*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (77*ArcTan[1 + Sqrt[2]*Sqrt
[x]])/(32*Sqrt[2]) + (77*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (77*L
og[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rubi [A] time = 0.070822, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11}{16x^{3/2}\left(x^2+1\right)}-\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] -77/(48*x^(3/2)) + 1/(4*x^(3/2)*(1 + x^2)^2) + 11/(16*x^(3/2)*(1 + x^2)) +
(77*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (77*ArcTan[1 + Sqrt[2]*Sqrt
[x]])/(32*Sqrt[2]) + (77*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (77*L
og[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{8}\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx$$
$$= \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} + \frac{77}{32}\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} - \frac{77}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$
$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} - \frac{77}{16}\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} - \frac{77}{32}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \frac{77}{32}\,\mathrm{Subst}\left(\int \frac{1+}{1+}\right.$$
$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} - \frac{77}{64}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \frac{77}{64}\,\mathrm{Subst}\left(\right.$$
$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} + \frac{77\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} - \frac{77\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} -$$
$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}\left(1+x^2\right)^2} + \frac{11}{16x^{3/2}\left(1+x^2\right)} + \frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{77\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{77}{}$$

Mathematica [C] time = 0.0047892, size = 22, normalized size = 0.16

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -x^2])/(3*x^(3/2))
```

Maple [A] time = 0.01, size = 87, normalized size = 0.6

$$-\frac{2}{3}x^{-\frac{3}{2}} - 2\frac{1}{\left(x^2+1\right)^2}\left(\frac{15\,x^{5/2}}{32} + \frac{19\,\sqrt{x}}{32}\right) - \frac{77\,\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right) - \frac{77\,\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right) - \frac{77\,\sqrt{2}}{128}\ln\left(\left(1+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^3,x)
```

```
[Out] -2/3/x^(3/2)-2*(15/32*x^(5/2)+19/32*x^(1/2))/(x^2+1)^2-77/64*arctan(1+2^(1/
2)*x^(1/2))*2^(1/2)-77/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-77/128*2^(1/2)
*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 3.77732, size = 138, normalized size = 1.

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{77}{128}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/48*(77*x^4 +
 121*x^2 + 32)/(x^(11/2) + 2*x^(7/2) + x^(3/2))
```

Fricas [B] time = 1.41728, size = 555, normalized size = 4.02

$$\frac{924\sqrt{2}\left(x^6+2x^4+x^2\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+924\sqrt{2}\left(x^6+2x^4+x^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x)
+ x + 1) - sqrt(2)*sqrt(x) - 1) + 924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(1/
2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 231*s
qrt(2)*(x^6 + 2*x^4 + x^2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 231*sqrt(2)*(
x^6 + 2*x^4 + x^2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(77*x^4 + 121*x^2
+ 32)*sqrt(x))/(x^6 + 2*x^4 + x^2)
```

Sympy [B] time = 43.6697, size = 653, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 231*sqrt(2)*x**(11/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 76
8*x**(7/2) + 384*x**(3/2)) - 231*sqrt(2)*x**(11/2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(11
/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2))
 - 462*sqrt(2)*x**(11/2)*atan(sqrt(2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) + 462*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(7/2)*lo
g(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)
) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) + 231*sqrt(2)*x**(3/2)*log(-4*sqr
t(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 231
*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x) - 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(
2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 616*x**4/(3
84*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 968*x**2/(384*x**(11/2) + 768
*x**(7/2) + 384*x**(3/2)) - 256/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2
))
```

Giac [A] time = 2.67007, size = 134, normalized size = 0.97

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{77}{128}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(15*x^(5/
2) + 19*sqrt(x))/(x^2 + 1)^2 - 2/3/x^(3/2)
```

3.336 $\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=147

$$\frac{13}{16x^{5/2}\left(x^2+1\right)}+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}-\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\tan^{-1}}{3}$$

```
[Out] -117/(80*x^(5/2)) + 117/(16*Sqrt[x]) + 1/(4*x^(5/2)*(1 + x^2)^2) + 13/(16*x
^(5/2)*(1 + x^2)) - (117*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*Log[1 - Sqrt[2]*Sqrt[x] + x
])/(64*Sqrt[2]) - (117*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rubi [A] time = 0.0745039, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13}{16x^{5/2}\left(x^2+1\right)}+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}-\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\tan^{-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] -117/(80*x^(5/2)) + 117/(16*Sqrt[x]) + 1/(4*x^(5/2)*(1 + x^2)^2) + 13/(16*x
^(5/2)*(1 + x^2)) - (117*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*Log[1 - Sqrt[2]*Sqrt[x] + x
])/(64*Sqrt[2]) - (117*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = \frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{8}\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx$$
$$= \frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{32}\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{117}{80x^{5/2}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{16}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{117}{32}$$
$$= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{117\log(1+}{64}$$
$$= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\tan^{-1}(1+}{32\sqrt{2}}$$

Mathematica [C] time = 0.0051212, size = 22, normalized size = 0.15

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -x^2])/(5*x^(5/2))
```

Maple [A] time = 0.011, size = 92, normalized size = 0.6

$$-\frac{2}{5}x^{-\frac{5}{2}}+6\frac{1}{\sqrt{x}}+2\frac{1}{\left(x^2+1\right)^2}\left(\frac{21\,x^{7/2}}{32}+\frac{25\,x^{3/2}}{32}\right)+\frac{117\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{117\sqrt{2}}{64}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{117}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^3,x)
```

```
[Out] -2/5/x^(5/2)+6/x^(1/2)+2*(21/32*x^(7/2)+25/32*x^(3/2))/(x^2+1)^2+117/64*arc
tan(1+2^(1/2)*x^(1/2))*2^(1/2)+117/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+11
7/128*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))
```

Maxima [A] time = 1.93831, size = 144, normalized size = 0.98

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{117}{128}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/80*(585*x
^6 + 1053*x^4 + 416*x^2 - 32)/(x^(13/2) + 2*x^(9/2) + x^(5/2))
```

Fricas [A] time = 1.66257, size = 575, normalized size = 3.91

$$-\frac{2340\sqrt{2}\left(x^{7}+2x^{5}+x^{3}\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+2340\sqrt{2}\left(x^{7}+2x^{5}+x^{3}\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/640*(2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x
) + x + 1) - sqrt(2)*sqrt(x) - 1) + 2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan
(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 58
5*sqrt(2)*(x^7 + 2*x^5 + x^3)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 585*sqrt(2
)*(x^7 + 2*x^5 + x^3)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(585*x^6 + 1053
*x^4 + 416*x^2 - 32)*sqrt(x))/(x^7 + 2*x^5 + x^3)
```

Sympy [B] time = 97.5842, size = 678, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**3,x)
```

```
[Out] 585*sqrt(2)*x**(13/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) - 585*sqrt(2)*x**(13/2)*log(4*sqrt(2)*sqrt(x) +
 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**
(13/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5
/2)) + 1170*sqrt(2)*x**(13/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x)
+ 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) - 1170*sqrt(2)*x*
*(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt
(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 585*sqrt(2)*x**(5
/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*
x**(5/2)) - 585*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(
13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(5/2)*atan(sqrt(2)*
```

```
sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*
x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**
(5/2)) + 4680*x**6/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 8424*x*
*4/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 3328*x**2/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) - 256/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2))
```

Giac [A] time = 2.15828, size = 143, normalized size = 0.97

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{117}{128}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(21*x^
(7/2) + 25*x^(3/2))/(x^2 + 1)^2 + 2/5*(15*x^2 - 1)/x^(5/2)
```

3.337 $\int \frac{\sqrt{x}}{1-x^2} dx$

Optimal. Leaf size=15

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0074245, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {329, 298, 203, 206}

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 - x^2),x]

[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1-x^2} dx &= 2 \operatorname{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt{x}\right)\\
&= \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right)-\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right)\\
&= -\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0036275, size = 15, normalized size = 1.

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 - x^2),x]

[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [B] time = 0.007, size = 24, normalized size = 1.6

$$-\frac{1}{2}\ln\left(-1+\sqrt{x}\right)+\frac{1}{2}\ln\left(\sqrt{x}+1\right)-\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^2+1),x)

[Out] -1/2*ln(-1+x^(1/2))+1/2*ln(x^(1/2)+1)-arctan(x^(1/2))
```

Maxima [B] time = 2.09145, size = 31, normalized size = 2.07

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="maxima")

[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] time = 1.66467, size = 86, normalized size = 5.73

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="fricas")

[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] time = 0.299491, size = 26, normalized size = 1.73

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**2+1),x)

[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 - atan(sqrt(x))
```

Giac [B] time = 2.82384, size = 32, normalized size = 2.13

$$-\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="giac")

[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.338 $\int \frac{x^{2/3}}{1+x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{2}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x}}{x^{2/3}+1}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x^(1/3)]/2 + ArcTan[Sqrt[3] + 2*x^(1/3)]/2 + ArcTan[x^(
1/3)] - (Sqrt[3]*ArcTanh[(Sqrt[3]*x^(1/3))/(1 + x^(2/3))])/2
```

Rubi [A] time = 0.257018, antiderivative size = 100, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 295, 634, 618, 204, 628, 203}

$$\frac{1}{4}\sqrt{3}\log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{4}\sqrt{3}\log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(1 + x^2),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^(1/3)]/2 + ArcTan[Sqrt[3] + 2*x^(1/3)]/2 + ArcTan[x^(
1/3)] + (Sqrt[3]*Log[1 - Sqrt[3]*x^(1/3) + x^(2/3)])/4 - (Sqrt[3]*Log[1 + S
qrt[3]*x^(1/3) + x^(2/3)])/4
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{2/3}}{1+x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \sqrt[3]{x}\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{x}\right) + \text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right)\\
&= \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\,\text{Subs}\\
&= \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\log\left(1-\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{4}\sqrt{3}\log\left(1+\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{}\right.\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{x}\right) + \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\log\left(1-\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{4}\sqrt{3}\log\left(1\right.
\end{aligned}$$

Mathematica [C] time = 0.0047313, size = 22, normalized size = 0.3

$$\frac{3}{5}x^{5/3}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(1 + x^2),x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/6, 1, 11/6, -x^2])/5
```

Maple [A] time = 0.024, size = 69, normalized size = 1.

$$\arctan\left(\sqrt[3]{x}\right)+\frac{1}{2}\arctan\left(2\sqrt[3]{x}-\sqrt{3}\right)+\frac{1}{2}\arctan\left(2\sqrt[3]{x}+\sqrt{3}\right)+\frac{\sqrt{3}}{4}\ln\left(1+x^{\frac{2}{3}}-\sqrt[3]{x}\sqrt{3}\right)-\frac{\sqrt{3}}{4}\ln\left(1+x^{\frac{2}{3}}+\sqrt[3]{x}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2+1),x)
```

```
[Out] arctan(x^(1/3))+1/2*arctan(2*x^(1/3)-3^(1/2))+1/2*arctan(2*x^(1/3)+3^(1/2))
+1/4*ln(1+x^(2/3)-x^(1/3)*3^(1/2))*3^(1/2)-1/4*ln(1+x^(2/3)+x^(1/3)*3^(1/2)
```

```
)*3^(1/2)
```

Maxima [A] time = 2.03088, size = 92, normalized size = 1.26

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)+\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

Fricas [B] time = 1.744, size = 381, normalized size = 5.22

$$-\frac{1}{4}\sqrt{3}\log\left(16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)+\frac{1}{4}\sqrt{3}\log\left(-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)-\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*log(16*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) + 1/4*sqrt(3)*log(-1
6*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) - arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3
)*x^(1/3) + 16*x^(2/3) + 16) - 2*x^(1/3)) - arctan(-sqrt(3) + 2*sqrt(sqrt(3
)*x^(1/3) + x^(2/3) + 1) - 2*x^(1/3)) + arctan(x^(1/3))
```

Sympy [A] time = 2.34158, size = 94, normalized size = 1.29

$$\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}-4\sqrt{3}\sqrt[3]{x}+4\right)}{4}-\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}+4\sqrt{3}\sqrt[3]{x}+4\right)}{4}+\operatorname{atan}\left(\sqrt[3]{x}\right)+\frac{\operatorname{atan}\left(2\sqrt[3]{x}-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2\sqrt[3]{x}+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(x**2+1),x)
```

```
[Out] sqrt(3)*log(4*x**(2/3) - 4*sqrt(3)*x**(1/3) + 4)/4 - sqrt(3)*log(4*x**(2/3)
 + 4*sqrt(3)*x**(1/3) + 4)/4 + atan(x**(1/3)) + atan(2*x**(1/3) - sqrt(3))/
2 + atan(2*x**(1/3) + sqrt(3))/2
```

Giac [A] time = 1.50239, size = 92, normalized size = 1.26

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)+\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

3.339 $\int x^m \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=97

$$\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

```
[Out] (a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(3 + m))/(3 + m) + (10*a^3*b^2*x^(5 +
m))/(5 + m) + (10*a^2*b^3*x^(7 + m))/(7 + m) + (5*a*b^4*x^(9 + m))/(9 + m)
+ (b^5*x^(11 + m))/(11 + m)
```

Rubi [A] time = 0.0430813, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(3 + m))/(3 + m) + (10*a^3*b^2*x^(5 +
m))/(5 + m) + (10*a^2*b^3*x^(7 + m))/(7 + m) + (5*a*b^4*x^(9 + m))/(9 + m)
+ (b^5*x^(11 + m))/(11 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^m+5a^4bx^{2+m}+10a^3b^2x^{4+m}+10a^2b^3x^{6+m}+5ab^4x^{8+m}+b^5x^{10+m}\right) dx\\ &= \frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{3+m}}{3+m}+\frac{10a^3b^2x^{5+m}}{5+m}+\frac{10a^2b^3x^{7+m}}{7+m}+\frac{5ab^4x^{9+m}}{9+m}+\frac{b^5x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A] time = 0.0504206, size = 88, normalized size = 0.91

$$x^{m+1}\left(\frac{10a^2b^3x^6}{m+7}+\frac{10a^3b^2x^4}{m+5}+\frac{5a^4bx^2}{m+3}+\frac{a^5}{m+1}+\frac{5ab^4x^8}{m+9}+\frac{b^5x^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^5,x]
```

```
[Out] x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10*a^3*b^2*x^4)/(5 + m) +
 (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) + (b^5*x^10)/(11 + m))
```

Maple [B] time = 0.004, size = 432, normalized size = 4.5

$$\frac{x^{1+m}\left(b^5m^5x^{10}+25\,b^5m^4x^{10}+5\,ab^4m^5x^8+230\,b^5m^3x^{10}+135\,ab^4m^4x^8+950\,b^5m^2x^{10}+10\,a^2b^3m^5x^6+1310\,ab^4m^3x^8+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^5,x)
```

```
[Out] x^(1+m)*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*
a*b^4*m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b
^5*m*x^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^
5*x^4+3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*
b^3*m^2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b
^3*m*x^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+
2030*a^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790
*a^3*b^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+912
9*a^5*m+10395*a^5)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59317, size = 868, normalized size = 8.95

$$\frac{\left(\left(b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5\right)x^{11}+5\left(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+1122\,ab^4m^2+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*x^m/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [A] time = 4.06092, size = 1999, normalized size = 20.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**5,x)
```

```
[Out] Piecewise((-a**5/(10*x**10) - 5*a**4*b/(8*x**8) - 5*a**3*b**2/(3*x**6) - 5*
a**2*b**3/(2*x**4) - 5*a*b**4/(2*x**2) + b**5*log(x), Eq(m, -11)), (-a**5/(
8*x**8) - 5*a**4*b/(6*x**6) - 5*a**3*b**2/(2*x**4) - 5*a**2*b**3/x**2 + 5*a
*b**4*log(x) + b**5*x**2/2, Eq(m, -9)), (-a**5/(6*x**6) - 5*a**4*b/(4*x**4)
 - 5*a**3*b**2/x**2 + 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4,
Eq(m, -7)), (-a**5/(4*x**4) - 5*a**4*b/(2*x**2) + 10*a**3*b**2*log(x) + 5*a
**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6, Eq(m, -5)), (-a**5/(2*x**2)
+ 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**6/6
 + b**5*x**8/8, Eq(m, -3)), (a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x*
*4/2 + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**8/8 + b**5*x**10/10, Eq(m, -1)), (a
**5*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 35*a**5*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 470*a**5*m**3*x*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**5*m**2*x*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129
*a**5*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 10395*a**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 5*a**4*b*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 165*a**4*b*m**4*x**3*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
2030*a**4*b*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 11310*a**4*b*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 26765*a**4*b*m*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1
7325*a**4*b*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 10*a**3*b**2*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 310*a**3*b**2*m**4*x**5*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35
00*a**3*b**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 17300*a**3*b**2*m**2*x**5*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 34890*a**3*b**2*m*x*
*5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 20790*a**3*b**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 10*a**2*b**3*m**5*x**7*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 290*a**2*b**3*m**4*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3020*a**2*b**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 13660*a**2*b**3*m**2*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25770*a**2
*b**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 14850*a**2*b**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a*b**4*m**5*x**9*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 135*a*b**4*m*
*4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 1310*a*b**4*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 5610*a*b**4*m**2*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10205*a*b**4*m
*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 5775*a*b**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + b**5*m**5*x**11*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25*b**5*m**4*x**11*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 230*b
**5*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 950*b**5*m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b**5*m*x**11*x**m/(m**6 + 36*
```

```
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 945*b**5*x**1
1*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5), True))
```

Giac [B] time = 2.89287, size = 729, normalized size = 7.52

$$\frac{b^5m^5x^{11}x^m + 25\,b^5m^4x^{11}x^m + 5\,ab^4m^5x^9x^m + 230\,b^5m^3x^{11}x^m + 135\,ab^4m^4x^9x^m + 950\,b^5m^2x^{11}x^m + 10\,a^2b^3m^5x^7x^m + 13}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^11*x^m + 25*b^5*m^4*x^11*x^m + 5*a*b^4*m^5*x^9*x^m + 230*b^5*m^3
*x^11*x^m + 135*a*b^4*m^4*x^9*x^m + 950*b^5*m^2*x^11*x^m + 10*a^2*b^3*m^5*x
^7*x^m + 1310*a*b^4*m^3*x^9*x^m + 1689*b^5*m*x^11*x^m + 290*a^2*b^3*m^4*x^7
*x^m + 5610*a*b^4*m^2*x^9*x^m + 945*b^5*x^11*x^m + 10*a^3*b^2*m^5*x^5*x^m +
 3020*a^2*b^3*m^3*x^7*x^m + 10205*a*b^4*m*x^9*x^m + 310*a^3*b^2*m^4*x^5*x^m
 + 13660*a^2*b^3*m^2*x^7*x^m + 5775*a*b^4*x^9*x^m + 5*a^4*b*m^5*x^3*x^m + 3
500*a^3*b^2*m^3*x^5*x^m + 25770*a^2*b^3*m*x^7*x^m + 165*a^4*b*m^4*x^3*x^m +
 17300*a^3*b^2*m^2*x^5*x^m + 14850*a^2*b^3*x^7*x^m + a^5*m^5*x*x^m + 2030*a
^4*b*m^3*x^3*x^m + 34890*a^3*b^2*m*x^5*x^m + 35*a^5*m^4*x*x^m + 11310*a^4*b
*m^2*x^3*x^m + 20790*a^3*b^2*x^5*x^m + 470*a^5*m^3*x*x^m + 26765*a^4*b*m*x^
3*x^m + 3010*a^5*m^2*x*x^m + 17325*a^4*b*x^3*x^m + 9129*a^5*m*x*x^m + 10395
*a^5*x*x^m)/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 1039
5)
```

3.340 $\int x^m \left(a+bx^2\right)^4 \, dx$

Optimal. Leaf size=79

$$\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4a^3bx^{m+3}}{m+3}+\frac{a^4x^{m+1}}{m+1}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

```
[Out] (a^4*x^(1 + m))/(1 + m) + (4*a^3*b*x^(3 + m))/(3 + m) + (6*a^2*b^2*x^(5 + m
))/(5 + m) + (4*a*b^3*x^(7 + m))/(7 + m) + (b^4*x^(9 + m))/(9 + m)
```

Rubi [A] time = 0.0315087, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4a^3bx^{m+3}}{m+3}+\frac{a^4x^{m+1}}{m+1}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^4,x]
```

```
[Out] (a^4*x^(1 + m))/(1 + m) + (4*a^3*b*x^(3 + m))/(3 + m) + (6*a^2*b^2*x^(5 + m
))/(5 + m) + (4*a*b^3*x^(7 + m))/(7 + m) + (b^4*x^(9 + m))/(9 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^4 dx &= \int \left(a^4x^m+4a^3bx^{2+m}+6a^2b^2x^{4+m}+4ab^3x^{6+m}+b^4x^{8+m}\right) dx\\ &= \frac{a^4x^{1+m}}{1+m}+\frac{4a^3bx^{3+m}}{3+m}+\frac{6a^2b^2x^{5+m}}{5+m}+\frac{4ab^3x^{7+m}}{7+m}+\frac{b^4x^{9+m}}{9+m}\end{aligned}$$

Mathematica [A] time = 0.0323216, size = 72, normalized size = 0.91

$$x^{m+1}\left(\frac{6a^2b^2x^4}{m+5}+\frac{4a^3bx^2}{m+3}+\frac{a^4}{m+1}+\frac{4ab^3x^6}{m+7}+\frac{b^4x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^4,x]
```

```
[Out] x^(1 + m)*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

Maple [B] time = 0.005, size = 291, normalized size = 3.7

$$x^{1+m}\left(b^4m^4x^8+16\,b^4m^3x^8+4\,ab^3m^4x^6+86\,b^4m^2x^8+72\,ab^3m^3x^6+176\,b^4mx^8+6\,a^2b^2m^4x^4+416\,ab^3m^2x^6+105\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^4,x)
```

```
[Out] x^(1+m)*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62544, size = 571, normalized size = 7.23

$$\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+135\,ab^3\right)x^7+6\left(a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*x^m/(m^
5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 2.5335, size = 1221, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**4,x)
```

```
[Out] Piecewise((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a*
b**3/x**2 + b**4*log(x), Eq(m, -9)), (-a**4/(6*x**6) - a**3*b/x**4 - 3*a**2
*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2, Eq(m, -7)), (-a**4/(4*x**4) - 2
*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4, Eq(m, -5))
, (-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4
*x**6/6, Eq(m, -3)), (a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*
```

```
a*b**3*x**6/3 + b**4*x**8/8, Eq(m, -1)), (a**4*m**4*x*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 24*a**4*m**3*x*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 206*a**4*m**2*x*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**4*m*x*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*a**4*x*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a**3*b*m**4*x**3*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2*x**3*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b*x
**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a**2*b**
2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 12
0*a**2*b**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 780*a**2*b**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 1800*a**2*b**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 1134*a**2*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 72*a*b**3*m**3*x**7*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 416*a*b**3*m**2*x**7*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 888*a*b**3*m*x**7*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*x**7*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**4*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*m**3*x**9*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*m**2*x
**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*m
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b**4
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

Giac [B] time = 2.94744, size = 493, normalized size = 6.24

$$\frac{b^4m^4x^9x^m + 16\,b^4m^3x^9x^m + 4\,ab^3m^4x^7x^m + 86\,b^4m^2x^9x^m + 72\,ab^3m^3x^7x^m + 176\,b^4mx^9x^m + 6\,a^2b^2m^4x^5x^m + 416\,ab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] (b^4*m^4*x^9*x^m + 16*b^4*m^3*x^9*x^m + 4*a*b^3*m^4*x^7*x^m + 86*b^4*m^2*x^
9*x^m + 72*a*b^3*m^3*x^7*x^m + 176*b^4*m*x^9*x^m + 6*a^2*b^2*m^4*x^5*x^m +
416*a*b^3*m^2*x^7*x^m + 105*b^4*x^9*x^m + 120*a^2*b^2*m^3*x^5*x^m + 888*a*b
^3*m*x^7*x^m + 4*a^3*b*m^4*x^3*x^m + 780*a^2*b^2*m^2*x^5*x^m + 540*a*b^3*x^
7*x^m + 88*a^3*b*m^3*x^3*x^m + 1800*a^2*b^2*m*x^5*x^m + a^4*m^4*x*x^m + 656
*a^3*b*m^2*x^3*x^m + 1134*a^2*b^2*x^5*x^m + 24*a^4*m^3*x*x^m + 1832*a^3*b*m
*x^3*x^m + 206*a^4*m^2*x*x^m + 1260*a^3*b*x^3*x^m + 744*a^4*m*x*x^m + 945*a
^4*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.341 $\int x^m \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=61

$$\frac{3a^2bx^{m+3}}{m+3}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3 + m) + (3*a*b^2*x^(5 + m))
/(5 + m) + (b^3*x^(7 + m))/(7 + m)
```

Rubi [A] time = 0.022071, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3a^2bx^{m+3}}{m+3}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3 + m) + (3*a*b^2*x^(5 + m))
/(5 + m) + (b^3*x^(7 + m))/(7 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^m+3a^2bx^{2+m}+3ab^2x^{4+m}+b^3x^{6+m}\right) dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{3+m}}{3+m}+\frac{3ab^2x^{5+m}}{5+m}+\frac{b^3x^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.0296843, size = 56, normalized size = 0.92

$$x^{m+1}\left(\frac{3a^2bx^2}{m+3}+\frac{a^3}{m+1}+\frac{3ab^2x^4}{m+5}+\frac{b^3x^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*b^2*x^4)/(5 + m) + (b
^3*x^6)/(7 + m))
```

Maple [B] time = 0.004, size = 178, normalized size = 2.9

$$\frac{x^{1+m}\left(b^3m^3x^6+9\,b^3m^2x^6+3\,ab^2m^3x^4+23\,b^3mx^6+33\,ab^2m^2x^4+15\,b^3x^6+3\,a^2bm^3x^2+93\,ab^2mx^4+39\,a^2bm^2x^2+63\right.}{(7+m)\,(5+m)\,(3+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3,x)
```

```
[Out] x^(1+m)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)/(7+m)
/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66446, size = 347, normalized size = 5.69

$$\frac{\left(\left(b^3m^3 + 9\,b^3m^2 + 23\,b^3m + 15\,b^3\right)x^7 + 3\left(ab^2m^3 + 11\,ab^2m^2 + 31\,ab^2m + 21\,ab^2\right)x^5 + 3\left(a^2bm^3 + 13\,a^2bm^2 + 47\,a^2bm\right.\right.}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*x^m/(m^4 +
16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [A] time = 1.53952, size = 683, normalized size = 11.2

$$\begin{cases} -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3\log(x) \\ -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2\log(x) + \frac{b^3x^2}{2} \\ -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} \\ a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6} \\ \frac{a^3m^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15a^3m^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71a^3mxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{3a^2bm^3x^3x}{m^4+16m^3+86m^2+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) + b**3*lo
g(x), Eq(m, -7)), (-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a*b**2*log(x) + b
**3*x**2/2, Eq(m, -5)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2
 + b**3*x**4/4, Eq(m, -3)), (a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/
```

```
4 + b**3*x**6/6, Eq(m, -1)), (a**3*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 +
176*m + 105) + 15*a**3*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 71*a**3*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**3*x*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 3*a**2*b*m**3*x**3*x**m/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105) + 39*a**2*b*m**2*x**3*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 141*a**2*b*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 105*a**2*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 3*a*b**2*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 33*a*b**2*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 93*
a*b**2*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 63*a*b**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**3*m**3*x**7*x**m/(m**4
 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*b**3*m**2*x**7*x**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 23*b**3*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + 15*b**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105),
 True))
```

Giac [B] time = 2.1142, size = 302, normalized size = 4.95

$$\frac{b^3m^3x^7x^m + 9\,b^3m^2x^7x^m + 3\,ab^2m^3x^5x^m + 23\,b^3mx^7x^m + 33\,ab^2m^2x^5x^m + 15\,b^3x^7x^m + 3\,a^2bm^3x^3x^m + 93\,ab^2mx^5x^m + 3}{m^4 + 16\,m^3 + 86\,m^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m + 9*b^3*m^2*x^7*x^m + 3*a*b^2*m^3*x^5*x^m + 23*b^3*m*x^7*x
^m + 33*a*b^2*m^2*x^5*x^m + 15*b^3*x^7*x^m + 3*a^2*b*m^3*x^3*x^m + 93*a*b^2
*m*x^5*x^m + 39*a^2*b*m^2*x^3*x^m + 63*a*b^2*x^5*x^m + a^3*m^3*x*x^m + 141*
a^2*b*m*x^3*x^m + 15*a^3*m^2*x*x^m + 105*a^2*b*x^3*x^m + 71*a^3*m*x*x^m + 1
05*a^3*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.342 $\int x^m (a + bx^2)^2 \, dx$

Optimal. Leaf size=43

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+3}}{m+3} + \frac{b^2 x^{m+5}}{m+5}$$

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3 + m))/(3 + m) + (b^2*x^(5 + m))/(5 +
m)
```

Rubi [A] time = 0.0151615, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+3}}{m+3} + \frac{b^2 x^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3 + m))/(3 + m) + (b^2*x^(5 + m))/(5 +
m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^2\right)^2 dx &= \int \left(a^2 x^m + 2abx^{2+m} + b^2 x^{4+m}\right) dx \\ &= \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{3+m}}{3+m} + \frac{b^2 x^{5+m}}{5+m}\end{aligned}$$

Mathematica [A] time = 0.0194217, size = 40, normalized size = 0.93

$$x^{m+1} \left(\frac{a^2}{m+1} + \frac{2abx^2}{m+3} + \frac{b^2 x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Maple [B] time = 0.003, size = 93, normalized size = 2.2

$$\frac{x^{1+m} \left(b^2 m^2 x^4 + 4 b^2 m x^4 + 2 abm^2 x^2 + 3 b^2 x^4 + 12 abmx^2 + a^2 m^2 + 10 abx^2 + 8 a^2 m + 15 a^2\right)}{(5+m)(3+m)(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2,x)
```

```
[Out] x^(1+m)*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m
^2+10*a*b*x^2+8*a^2*m+15*a^2)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57655, size = 181, normalized size = 4.21

$$\frac{\left(\left(b^2m^2+4\,b^2m+3\,b^2\right)x^5+2\left(abm^2+6\,abm+5\,ab\right)x^3+\left(a^2m^2+8\,a^2m+15\,a^2\right)x\right)x^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [A] time = 0.841628, size = 306, normalized size = 7.12

$$\begin{cases} -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log\left(x\right) \\ -\frac{a^2}{2x^2}+2ab\log\left(x\right)+\frac{b^2x^2}{2} \\ a^2\log\left(x\right)+abx^2+\frac{b^2x^4}{4} \\ \frac{a^2m^2xx^m}{m^3+9m^2+23m+15}+\frac{8a^2mxx^m}{m^3+9m^2+23m+15}+\frac{15a^2xx^m}{m^3+9m^2+23m+15}+\frac{2abm^2x^3x^m}{m^3+9m^2+23m+15}+\frac{12abmx^3x^m}{m^3+9m^2+23m+15}+\frac{10abx^3x^m}{m^3+9m^2+23m+15}+\frac{b^2m^2x^5x^m}{m^3+9m^2+23m+15} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x), Eq(m, -5)), (-a**2/(2*x
**2) + 2*a*b*log(x) + b**2*x**2/2, Eq(m, -3)), (a**2*log(x) + a*b*x**2 + b*
*2*x**4/4, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
*2*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*x*x**m/(m**3 + 9*m**2 + 2
3*m + 15) + 2*a*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**
3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 10*a*b*x**3*x**m/(m**3 + 9*m**2 + 23*m
 + 15) + b**2*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*x*
*m/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 1
5), True))
```

Giac [B] time = 2.97042, size = 158, normalized size = 3.67

$$\frac{b^2m^2x^5x^m + 4\,b^2mx^5x^m + 2\,abm^2x^3x^m + 3\,b^2x^5x^m + 12\,abmx^3x^m + a^2m^2xx^m + 10\,abx^3x^m + 8\,a^2mxx^m + 15\,a^2xx^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^5*x^m + 4*b^2*m*x^5*x^m + 2*a*b*m^2*x^3*x^m + 3*b^2*x^5*x^m + 12
*a*b*m*x^3*x^m + a^2*m^2*x*x^m + 10*a*b*x^3*x^m + 8*a^2*m*x*x^m + 15*a^2*x*
x^m)/(m^3 + 9*m^2 + 23*m + 15)
```

3.343 $\int x^m \left(a + bx^2\right) dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+3}}{m+3}$$

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Rubi [A] time = 0.0073573, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^2\right) dx &= \int \left(ax^m + bx^{2+m}\right) dx\\ &= \frac{ax^{1+m}}{1+m} + \frac{bx^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.0123004, size = 25, normalized size = 1.

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Maple [A] time = 0.002, size = 35, normalized size = 1.4

$$\frac{x^{1+m}\left(bmx^2 + bx^2 + am + 3\,a\right)}{(3+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a),x)
```

```
[Out] x^(1+m)*(b*m*x^2+b*x^2+a*m+3*a)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53331, size = 72, normalized size = 2.88

$$\frac{\left((bm+b)x^3+(am+3\,a)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*x^m/(m^2 + 4*m + 3)
```

Sympy [A] time = 0.375347, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{2x^2}+b\log(x) & \text{for } m=-3 \\ a\log(x)+\frac{bx^2}{2} & \text{for } m=-1 \\ \frac{amxx^m}{m^2+4m+3}+\frac{3axx^m}{m^2+4m+3}+\frac{bmx^3x^m}{m^2+4m+3}+\frac{bx^3x^m}{m^2+4m+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a),x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*log(x), Eq(m, -3)), (a*log(x) + b*x**2/2, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 4*m + 3) + 3*a*x*x**m/(m**2 + 4*m + 3) + b*m*x**3
*x**m/(m**2 + 4*m + 3) + b*x**3*x**m/(m**2 + 4*m + 3), True))
```

Giac [A] time = 2.58023, size = 58, normalized size = 2.32

$$\frac{bmx^3x^m+bx^3x^m+amxx^m+3\,axx^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*m*x^3*x^m + b*x^3*x^m + a*m*x*x^m + 3*a*x*x^m)/(m^2 + 4*m + 3)
```

3.344 $\int \frac{x^m}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
+ m))
```

Rubi [A] time = 0.0082732, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
+ m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a+bx^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.0069063, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
*(1 + m))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{b x^{2}+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a),x)
```

```
[Out] int(x^m/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{b x^{2}+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}}{b x^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x^2 + a), x)
```

Sympy [C] time = 1.28015, size = 88, normalized size = 2.26

$$\frac{m x x^{m} \Phi\left(\frac{b x^{2} e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4 a \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{x x^{m} \Phi\left(\frac{b x^{2} e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4 a \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(4*a*gamma(m/2 + 3/2)) + x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{b x^{2}+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a), x)
```

3.345 $\int \frac{x^m}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(
1 + m))
```

Rubi [A] time = 0.0079825, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2}\,dx=\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

Mathematica [A] time = 0.0069396, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2,x)
```

```
[Out] int(x^m/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 7.12927, size = 374, normalized size = 9.59

$$-\frac{am^2xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2amxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{axx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2,x)
```

```
[Out] -a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m*x*
x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*x*
x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
```

```
amma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))
+ b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

3.346 $\int \frac{x^m}{(a+bx^2)^3} dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^3*(
1 + m))
```

Rubi [A] time = 0.0083129, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {364}

$$\frac{x^{m+1}{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^3*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^3} dx = \frac{x^{1+m}{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3(1+m)}$$

Mathematica [A] time = 0.0071185, size = 41, normalized size = 1.05

$$\frac{x^{m+1}{}_2F_1\left(3,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^3*(1 + m))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^3,x)
```

```
[Out] int(x^m/(b*x^2+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] time = 25.3879, size = 1556, normalized size = 39.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**3,x)
```

```
[Out] a**2*m**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a*
*3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x*x**m*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a
**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a**2*
m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gam
ma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - a**2*m*x*x**m*lerchph
i(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m
/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 +
```

```
 3/2)) + 8*a**2*m*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a*
*4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**2*x
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32
*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**
4*gamma(m/2 + 3/2)) + 10*a**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 +
3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)
) + 2*a*b*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2)
 + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*m**2*x**3*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3
/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2))
 - 2*a*b*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**
4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*b*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + 4*a*b*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(
m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2
+ 3/2)) + 6*a*b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2
) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 6*a*b*x**3*x**m*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + b**2*m**3*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x
**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*b**2*m**2*x*
*5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x
**4*gamma(m/2 + 3/2)) - b**2*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a,
 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*b**2*x**5*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*g
amma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma
(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^3, x)
```

3.347 $\int \frac{(cx)^{1+m}}{a+bx^2}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac(m+2)}$$

```
[Out] ((c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a
*c*(2 + m))
```

Rubi [A] time = 0.0118202, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$\frac{(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1 + m)/(a + b*x^2),x]
```

```
[Out] ((c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a
*c*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx=\frac{(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac(2+m)}$$

Mathematica [A] time = 0.0144331, size = 45, normalized size = 1.02

$$\frac{cx^2(cx)^m\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+2}{2}+1;-\frac{bx^2}{a}\right)}{a(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1 + m)/(a + b*x^2),x]
```

```
[Out] (c*x^2*(c*x)^m*Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, -((b*x^2)/a)]
)/(a*(2 + m))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(cx)^{1+m}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1+m)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(1+m)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m+1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m+1}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m + 1)/(b*x^2 + a), x)
```

Sympy [C] time = 5.3791, size = 92, normalized size = 2.09

$$\frac{cc^m m x^2 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{cc^m x^2 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1+m)/(b*x**2+a),x)
```

```
[Out] c*c**m*m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a*gamma(m/2 + 2)) + c*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m+1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

3.348 $\int \frac{(cx)^m}{a+bx^2}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*c*(1 + m))
```

Rubi [A] time = 0.0096857, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {364}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a + b*x^2),x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*c*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \frac{(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

Mathematica [A] time = 0.0054144, size = 42, normalized size = 0.95

$$\frac{x(cx)^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
*(1 + m))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2+a),x)
```

```
[Out] int((c*x)^m/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(b*x^2 + a), x)
```

Sympy [C] time = 1.28519, size = 95, normalized size = 2.16

$$\frac{c^m m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{c^m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**2+a),x)
```

```
[Out] c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(b*x^2 + a), x)
```

3.349 $\int \frac{(cx)^{-1+m}}{a+bx^2} dx$

Optimal. Leaf size=38

$$\frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{acm}$$

```
[Out] ((c*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -((b*x^2)/a)])/(a*c*m)
```

Rubi [A] time = 0.0085645, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$\frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{acm}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + m)/(a + b*x^2),x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -((b*x^2)/a)])/(a*c*m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-1+m}}{a+bx^2} dx = \frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{acm}$$

Mathematica [A] time = 0.0073654, size = 38, normalized size = 1.

$$\frac{x(cx)^{m-1} {}_2F_1\left(1,\frac{m}{2};\frac{m}{2}+1;-\frac{bx^2}{a}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-1 + m)*Hypergeometric2F1[1, m/2, 1 + m/2, -((b*x^2)/a)])/(a*m)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{(cx)^{-1+m}}{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+m)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(-1+m)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m-1}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 1)/(b*x^2 + a), x)
```

Sympy [C] time = 18.7562, size = 39, normalized size = 1.03

$$\frac{c^m m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a},1,\frac{m}{2}\right)\Gamma\left(\frac{m}{2}\right)}{4ac\Gamma\left(\frac{m}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+m)/(b*x**2+a),x)
```

```
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2)*gamma(m/2)/(4*a*c*ga
mma(m/2 + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

3.350 $\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx$

Optimal. Leaf size=47

$$-\frac{(cx)^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

```
[Out] -((((c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b*x^2)/a)]
)/(a*c*(1 - m)))
```

Rubi [A] time = 0.0118837, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$-\frac{(cx)^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-2 + m)/(a + b*x^2),x]
```

```
[Out] -((((c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b*x^2)/a)]
)/(a*c*(1 - m)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = -\frac{(cx)^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

Mathematica [A] time = 0.0114152, size = 44, normalized size = 0.94

$$\frac{x(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m-1}{2}+1;-\frac{bx^2}{a}\right)}{a(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-2 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-2 + m)*Hypergeometric2F1[1, (-1 + m)/2, 1 + (-1 + m)/2, -((b*x^2
)/a)])/(a*(-1 + m))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{(cx)^{-2+m}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-2+m)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(-2+m)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-2}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m-2}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 2)/(b*x^2 + a), x)
```

Sympy [C] time = 51.9566, size = 102, normalized size = 2.17

$$\frac{c^m m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}-\frac{1}{2}\right)\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}{4ac^2x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)} - \frac{c^m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}-\frac{1}{2}\right)\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}{4ac^2x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-2+m)/(b*x**2+a),x)
```

```
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/
2)/(4*a*c**2*x*gamma(m/2 + 1/2)) - c**m*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/2)/(4*a*c**2*x*gamma(m/2 + 1/2))
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-2}}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

3.351 $\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

```
[Out] -(((c*x)^(-2 + m)*Hypergeometric2F1[1, (-2 + m)/2, m/2, -((b*x^2)/a)])/(a*c
*(2 - m)))
```

Rubi [A] time = 0.01166, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$-\frac{(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-3 + m)/(a + b*x^2),x]
```

```
[Out] -(((c*x)^(-2 + m)*Hypergeometric2F1[1, (-2 + m)/2, m/2, -((b*x^2)/a)])/(a*c
*(2 - m)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = -\frac{(cx)^{-2+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

Mathematica [A] time = 0.0108855, size = 44, normalized size = 0.98

$$\frac{x(cx)^{m-3}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m-2}{2}+1;-\frac{bx^2}{a}\right)}{a(m-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-3 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-3 + m)*Hypergeometric2F1[1, (-2 + m)/2, 1 + (-2 + m)/2, -((b*x^2
)/a)])/(a*(-2 + m))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(cx)^{-3+m}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-3+m)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(-3+m)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-3}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m-3}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 3)/(b*x^2 + a), x)
```

Sympy [C] time = 141.816, size = 92, normalized size = 2.04

$$\frac{c^m m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a},1,\frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{4ac^3x^2\Gamma\left(\frac{m}{2}\right)}-\frac{c^m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a},1,\frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{2ac^3x^2\Gamma\left(\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-3+m)/(b*x**2+a),x)
```

```
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1)*gamma(m/2 - 1)/(
4*a*c**3*x**2*gamma(m/2)) - c**m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 - 1)*gamma(m/2 - 1)/(2*a*c**3*x**2*gamma(m/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-3}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

3.352 $\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{ax^2}{b}\right)}{m+1}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a*x^2)/b)])/(1 + m
)
```

Rubi [A] time = 0.0078092, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + (a*x^2)/b)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a*x^2)/b)])/(1 + m
)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ax^2}{b}\right)}{1+m}$$

Mathematica [A] time = 0.0085049, size = 38, normalized size = 1.06

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 + (a*x^2)/b)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((a*x^2)/b)])/(1
 + m)
```

Maple [C] time = 0.05, size = 92, normalized size = 2.6

$$\frac{1}{2}\left(\frac{a}{b}\right)^{-\frac{1}{2}-\frac{m}{2}}\left(2x^{1+m}\left(\frac{a}{b}\right)^{1/2+m/2}\left(2+2\frac{ax^{2}}{b}\right)^{-1}+2\frac{x^{1+m}\left(-1/4\,m^{2}+1/4\right)}{1+m}\left(\frac{a}{b}\right)^{1/2+m/2}\operatorname{LerchPhi}\left(-\frac{ax^{2}}{b},1,1/2+m/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+1/b*a*x^2)^2,x)
```

```
[Out] 1/2*(1/b*a)^(-1/2-1/2*m)*(2*x^(1+m)*(1/b*a)^(1/2+1/2*m)/(2+2/b*a*x^2)+2/(1+
m)*x^(1+m)*(1/b*a)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(-1/b*a*x^2,1,1/2+1/2
*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\frac{ax^{2}}{b}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(a*x^2/b + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^{2}x^{m}}{a^{2}x^{4}+2\,abx^{2}+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^m/(a^2*x^4 + 2*a*b*x^2 + b^2), x)
```

Sympy [C] time = 7.30806, size = 343, normalized size = 9.53

$$-\frac{am^{2}x^{3}x^{m}\Phi\left(\frac{ax^{2}e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{ax^{3}x^{m}\Phi\left(\frac{ax^{2}e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{bm^{2}xx^{m}\Phi\left(\frac{ax^{2}e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+a*x**2/b)**2,x)
```

```
[Out] -a*m**2*x**3*x**m*lerchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + a*x**3*x**m*l
erchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*
```

```
gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) - b*m**2*x*x**m*lerchphi(a*x**2*ex
p_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2)
+ 8*b*gamma(m/2 + 3/2)) + 2*b*m*x*x**m*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2
 + 3/2) + 8*b*gamma(m/2 + 3/2)) + b*x*x**m*lerchphi(a*x**2*exp_polar(I*pi)/
b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/
2 + 3/2)) + 2*b*x*x**m*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*ga
mma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(a*x^2/b + 1)^2, x)
```

3.353 $\int x^7\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] -(a^3*(a + b*x^2)^(3/2))/(3*b^4) + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a
*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)
```

Rubi [A] time = 0.0452027, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] -(a^3*(a + b*x^2)^(3/2))/(3*b^4) + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a
*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\sqrt{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.0258305, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{3/2}\left(24a^2bx^2-16a^3-30ab^2x^4+35b^3x^6\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-16*a^3 + 24*a^2*b*x^2 - 30*a*b^2*x^4 + 35*b^3*x^6))/(3
15*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-35\,b^{3}x^{6}+30\,ab^{2}x^{4}-24\,a^{2}bx^{2}+16\,a^{3}}{315\,b^{4}}\left(bx^{2}+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-35*b^3*x^6+30*a*b^2*x^4-24*a^2*b*x^2+16*a^3)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61312, size = 126, normalized size = 1.58

$$\frac{\left(35\,b^{4}x^{8}+5\,ab^{3}x^{6}-6\,a^{2}b^{2}x^{4}+8\,a^{3}bx^{2}-16\,a^{4}\right)\sqrt{bx^{2}+a}}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 5*a*b^3*x^6 - 6*a^2*b^2*x^4 + 8*a^3*b*x^2 - 16*a^4)*sqr
t(b*x^2 + a)/b^4
```

Sympy [A] time = 1.29625, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{16a^{4}\sqrt{a+bx^{2}}}{315b^{4}}+\frac{8a^{3}x^{2}\sqrt{a+bx^{2}}}{315b^{3}}-\frac{2a^{2}x^{4}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{6}\sqrt{a+bx^{2}}}{63b}+\frac{x^{8}\sqrt{a+bx^{2}}}{9} & \text{for}\: b\neq 0 \\ \frac{\sqrt{a}x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x*
*2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a +
b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)
)
```

Giac [A] time = 1.86378, size = 77, normalized size = 0.96

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/
2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)/b^4
```

3.354 $\int x^5\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}$$

```
[Out] (a^2*(a + b*x^2)^(3/2))/(3*b^3) - (2*a*(a + b*x^2)^(5/2))/(5*b^3) + (a + b*
x^2)^(7/2)/(7*b^3)
```

Rubi [A] time = 0.0327254, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(a + b*x^2)^(3/2))/(3*b^3) - (2*a*(a + b*x^2)^(5/2))/(5*b^3) + (a + b*
x^2)^(7/2)/(7*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+bx^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx,x,x^2\right)\\&=\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.018481, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{3/2}\left(8a^2-12abx^2+15b^2x^4\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2],x]

[Out] ((a + b*x^2)^(3/2)*(8*a^2 - 12*a*b*x^2 + 15*b^2*x^4))/(105*b^3)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.6

$$\frac{15\,b^2x^4 - 12\,abx^2 + 8\,a^2}{105\,b^3}\left(bx^2 + a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2),x)

[Out] 1/105*(b*x^2+a)^(3/2)*(15*b^2*x^4-12*a*b*x^2+8*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5173, size = 103, normalized size = 1.75

$$\frac{\left(15\,b^3x^6 + 3\,ab^2x^4 - 4\,a^2bx^2 + 8\,a^3\right)\sqrt{bx^2 + a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] 1/105*(15*b^3*x^6 + 3*a*b^2*x^4 - 4*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 0.672038, size = 87, normalized size = 1.47

$$\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3} - \frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{ax^4\sqrt{a+bx^2}}{35b} + \frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b \neq 0 \\ \frac{\sqrt{a}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2),x)

[Out] Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2
)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne
```

```
(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [A] time = 2.28285, size = 58, normalized size = 0.98

$$\frac{15\left(bx^{2}+a\right)^{\frac{7}{2}}-42\left(bx^{2}+a\right)^{\frac{5}{2}}a+35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)
*a^2)/b^3
```

3.355 $\int x^3\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^2\right)^{3/2}}{3b^2}$$

```
[Out] -(a*(a + b*x^2)^(3/2))/(3*b^2) + (a + b*x^2)^(5/2)/(5*b^2)
```

Rubi [A] time = 0.0216558, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(a + b*x^2)^(3/2))/(3*b^2) + (a + b*x^2)^(5/2)/(5*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx^2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{a+bx}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,x^2\right)\\&=-\frac{a\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{\left(a+bx^2\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0137205, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(3bx^2-2a\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2],x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-2*a + 3*b*x^2))/(15*b^2)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$-\frac{-3\,bx^2+2\,a}{15\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-3*b*x^2+2*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58821, size = 76, normalized size = 2.

$$\frac{\left(3\,b^2x^4+abx^2-2\,a^2\right)\sqrt{bx^2+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^4 + a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 0.303815, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15
*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

<u>Giac</u> [A] time = 1.96966, size = 39, normalized size = 1.03

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)/b^2
```

3.356 $\int x\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{(a+bx^2)^{3/2}}{3b}$$

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Rubi [A] time = 0.0030336, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2],x]
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{a+bx^2}\,dx = \frac{(a+bx^2)^{3/2}}{3b}$$

Mathematica [A] time = 0.0028413, size = 18, normalized size = 1.

$$\frac{(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2],x]
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{1}{3\,b}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(3/2)/b
```

Maxima [A] time = 2.2608, size = 19, normalized size = 1.06

$$\frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

Fricas [A] time = 1.47137, size = 34, normalized size = 1.89

$$\frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

Sympy [A] time = 0.160883, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (
sqrt(a)*x**2/2, True))
```

Giac [A] time = 2.48041, size = 19, normalized size = 1.06

$$\frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

3.357 $\int \frac{\sqrt{a+bx^2}}{x}\, dx$

Optimal. Leaf size=37

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.024309, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x,x]
```

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\ &= \sqrt{a+bx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \sqrt{a+bx^2}+\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= \sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.0079023, size = 37, normalized size = 1.

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x,x]
```

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Maple [A] time = 0.004, size = 39, normalized size = 1.1

$$\sqrt{bx^2+a}-\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x,x)
```

```
[Out] (b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59571, size = 200, normalized size = 5.41

$$\left[\frac{1}{2}\,\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+\sqrt{bx^2+a},\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(b*x
^2 + a), sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)]
```

Sympy [A] time = 1.40853, size = 56, normalized size = 1.51

$$-\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{a}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A] time = 2.44334, size = 45, normalized size = 1.22

$$\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)
```

3.358 $\int \frac{\sqrt{a+bx^2}}{x^3} dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -Sqrt[a + b*x^2]/(2*x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rubi [A] time = 0.0249238, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2}}{2x^2} + \frac{1}{4}b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2}}{2x^2} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)$$
$$= -\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.030973, size = 59, normalized size = 1.26

$$-\frac{bx^2\sqrt{\frac{bx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+a+bx^2}{2x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -(a + b*x^2 + b*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(2*x^
2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 63, normalized size = 1.3

$$-\frac{1}{2\,a x^2}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}} + \frac{b}{2\,a}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(3/2)-1/2*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)+1/2*b/a*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59829, size = 257, normalized size = 5.47

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}a}{4ax^2}, \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}a}{2ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) - sqrt(b*x^2 + a)*a)/(a*x^2)]
```

Sympy [A] time = 1.99129, size = 42, normalized size = 0.89

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(
a))
```

Giac [A] time = 2.88758, size = 58, normalized size = 1.23

$$\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^2+a}}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)/(b*x^2))
```

3.359 $\int \frac{\sqrt{a+bx^2}}{x^5} \, dx$

Optimal. Leaf size=71

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

```
[Out] -Sqrt[a + b*x^2]/(4*x^4) - (b*Sqrt[a + b*x^2])/(8*a*x^2) + (b^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rubi [A] time = 0.0390252, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -Sqrt[a + b*x^2]/(4*x^4) - (b*Sqrt[a + b*x^2])/(8*a*x^2) + (b^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2}}{4x^4}+\frac{1}{8}b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2}}{4x^4}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{16a}\\ &= -\frac{\sqrt{a+bx^2}}{4x^4}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{b\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{8a}\\ &= -\frac{\sqrt{a+bx^2}}{4x^4}-\frac{b\sqrt{a+bx^2}}{8ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.008291, size = 39, normalized size = 0.55

$$-\frac{b^2\left(a+bx^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^2}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -(b^2*(a + b*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^2)/a])/(3*a
^3)
```

Maple [A] time = 0.005, size = 85, normalized size = 1.2

$$-\frac{1}{4\,a x^4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{b}{8\,a^2x^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{b^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{b^2}{8\,a^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(3/2)+1/8*b/a^2/x^2*(b*x^2+a)^(3/2)+1/8*b^2/a^(3/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/8*b^2/a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6081, size = 313, normalized size = 4.41

$$\left[\frac{\sqrt{a}b^2x^4\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(abx^2+2a^2\right)\sqrt{bx^2+a}}{16a^2x^4}, -\frac{\sqrt{-a}b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(abx^2+2a^2\right)\sqrt{bx^2+a}}{8a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(a)*b^2*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/8*(sqrt(-a)*b^2*x^4*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^
4)]
```

Sympy [A] time = 3.62681, size = 92, normalized size = 1.3

$$-\frac{a}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{b^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(8*x**3*sqrt(a/(b*x**2
) + 1)) - b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + b**2*asinh(sqrt(a)/(sqrt(
b)*x))/(8*a**(3/2))
```

Giac [A] time = 2.66669, size = 84, normalized size = 1.18

$$-\frac{1}{8}b^2\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}+\sqrt{bx^2+a}a}{ab^2x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*b^2*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^2 + a)^(3/2
) + sqrt(b*x^2 + a)*a)/(a*b^2*x^4))
```

3.360 $\int \frac{\sqrt{a+bx^2}}{x^7} dx$

Optimal. Leaf size=95

$$\frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}} - \frac{b\sqrt{a+bx^2}}{24ax^4} - \frac{\sqrt{a+bx^2}}{6x^6}$$

```
[Out] -Sqrt[a + b*x^2]/(6*x^6) - (b*Sqrt[a + b*x^2])/(24*a*x^4) + (b^2*Sqrt[a + b
*x^2])/(16*a^2*x^2) - (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rubi [A] time = 0.05173, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}} - \frac{b\sqrt{a+bx^2}}{24ax^4} - \frac{\sqrt{a+bx^2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] -Sqrt[a + b*x^2]/(6*x^6) - (b*Sqrt[a + b*x^2])/(24*a*x^4) + (b^2*Sqrt[a + b
*x^2])/(16*a^2*x^2) - (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} + \frac{1}{12}b\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{32a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0077977, size = 39, normalized size = 0.41

$$\frac{b^3\left(a+bx^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx^2}{a}+1\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x^2)/a])/(3*a^
4)
```

Maple [A] time = 0.007, size = 105, normalized size = 1.1

$$-\frac{1}{6\,a x^6}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{b}{8\,a^2x^4}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{b^2}{16\,a^3x^2}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{b^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{b^3}{16\,a^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(3/2)+1/8*b/a^2/x^4*(b*x^2+a)^(3/2)-1/16*b^2/a^3/x^2*(
b*x^2+a)^(3/2)-1/16*b^3/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+1/16*
b^3/a^3*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63721, size = 367, normalized size = 3.86

$$\left[\frac{3\sqrt{a}b^3x^6\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3ab^2x^4-2a^2bx^2-8a^3\right)\sqrt{bx^2+a}}{96a^3x^6}, \frac{3\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3ab^2x^4-2a^2\right.}{48a^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b^2*x^4 - 2*a^2*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6), 1/48*
(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 - 2*a^2
*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [A] time = 5.58049, size = 117, normalized size = 1.23

$$-\frac{a}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{5\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*sqrt(b)/(24*x**5*sqrt(a/(b*x**
2) + 1)) + b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(16*a**2*x*
sqrt(a/(b*x**2) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

Giac [A] time = 1.84982, size = 108, normalized size = 1.14

$$\frac{1}{48}b^3\left(\frac{3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}-8\left(bx^2+a\right)^{\frac{3}{2}}a-3\sqrt{bx^2+a}a^2}{a^2b^3x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*b^3*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a
)^(5/2) - 8*(b*x^2 + a)^(3/2)*a - 3*sqrt(b*x^2 + a)*a^2)/(a^2*b^3*x^6))
```

3.361 $\int x^4\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=94

$$-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

```
[Out] -(a^2*x*Sqrt[a + b*x^2])/(16*b^2) + (a*x^3*Sqrt[a + b*x^2])/(24*b) + (x^5*S
qrt[a + b*x^2])/6 + (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] time = 0.0327913, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] -(a^2*x*Sqrt[a + b*x^2])/(16*b^2) + (a*x^3*Sqrt[a + b*x^2])/(24*b) + (x^5*S
qrt[a + b*x^2])/6 + (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\sqrt{a+bx^2}\,dx = \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{1}{6}a\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} - \frac{a^2\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b}$$
$$= -\frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{a^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b^2}$$
$$= -\frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b^2}$$
$$= -\frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

Mathematica [A] time = 0.0323827, size = 77, normalized size = 0.82

$$\sqrt{a+bx^2}\left(-\frac{a^2x}{16b^2} + \frac{ax^3}{24b} + \frac{x^5}{6}\right) + \frac{a^3\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*(-(a^2*x)/(16*b^2) + (a*x^3)/(24*b) + x^5/6) + (a^3*Log[b*x
 + Sqrt[b]*Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Maple [A] time = 0.007, size = 77, normalized size = 0.8

$$\frac{x^3}{6b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{ax}{8b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{a^2x}{16b^2}\sqrt{bx^2+a} + \frac{a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^3*(b*x^2+a)^(3/2)/b-1/8/b^2*a*x*(b*x^2+a)^(3/2)+1/16*a^2*x*(b*x^2+a)^
(1/2)/b^2+1/16/b^(5/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65298, size = 344, normalized size = 3.66

$$\left[\frac{3\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,b^3x^5+2\,ab^2x^3-3\,a^2bx\right)\sqrt{bx^2+a}}{96\,b^3},-\frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8\,b\right.}{48}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*b*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*
b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 5.29654, size = 117, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{bx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - a**(3/2)*x**3/(48*b*sqrt(1 + b*x
**2/a)) + 5*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(5/2)) + b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.55328, size = 86, normalized size = 0.91

$$\frac{1}{48}\left(2\left(4\,x^2+\frac{a}{b}\right)x^2-\frac{3\,a^2}{b^2}\right)\sqrt{bx^2+a}x-\frac{a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 + a/b)*x^2 - 3*a^2/b^2)*sqrt(b*x^2 + a)*x - 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.362 $\int x^2\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=70

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{1}{4}x^3\sqrt{a+bx^2}+\frac{ax\sqrt{a+bx^2}}{8b}$$

```
[Out] (a*x*Sqrt[a + b*x^2])/(8*b) + (x^3*Sqrt[a + b*x^2])/4 - (a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] time = 0.0207127, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{1}{4}x^3\sqrt{a+bx^2}+\frac{ax\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2],x]
```


```
[Out] (a*x*Sqrt[a + b*x^2])/(8*b) + (x^3*Sqrt[a + b*x^2])/4 - (a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^2}\,dx &= \frac{1}{4}x^3\sqrt{a+bx^2}+\frac{1}{4}a\int\frac{x^2}{\sqrt{a+bx^2}}\,dx\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0226565, size = 64, normalized size = 0.91

$$\sqrt{a+bx^2}\left(\frac{ax}{8b}+\frac{x^3}{4}\right)-\frac{a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*((a*x)/(8*b) + x^3/4) - (a^2*Log[b*x + Sqrt[b]*Sqrt[a + b*x
^2]])/(8*b^(3/2))
```

Maple [A] time = 0.004, size = 57, normalized size = 0.8

$$\frac{x}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{ax}{8\,b}\sqrt{bx^2+a}-\frac{a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)/b-1/8*a*x*(b*x^2+a)^(1/2)/b-1/8/b^(3/2)*a^2*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62835, size = 288, normalized size = 4.11

$$\left[\frac{a^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2x^3+abx\right)\sqrt{bx^2+a}}{16\,b^2},\frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(2\,b^2x^3+abx\right)\sqrt{bx^2}}{8\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b
^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2, 1/8*(a^2*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (2*b^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 3.40731, size = 92, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*sqrt(a)*x**3/(8*sqrt(1 + b*x**2/a))
 - a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + b*x**5/(4*sqrt(a)*sqrt(1 +
b*x**2/a))
```

Giac [A] time = 2.51079, size = 68, normalized size = 0.97

$$\frac{1}{8}\sqrt{bx^2+a}\left(2x^2+\frac{a}{b}\right)x+\frac{a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*x^2 + a/b)*x + 1/8*a^2*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

3.363 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rubi [A] time = 0.0099707, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\,dx &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0130181, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$\frac{x}{2}\sqrt{bx^2+a}+\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56214, size = 232, normalized size = 5.04

$$\left[\frac{2\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4b},\ \frac{\sqrt{bx^2+a}bx-a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

Sympy [A] time = 1.81339, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A] time = 2.85498, size = 50, normalized size = 1.09

$$\frac{1}{2}\sqrt{bx^2+a}x-\frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

3.364 $\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=42

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

```
[Out] -(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0116222, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 206}

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt{a+bx^2}}{x}+b\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= -\frac{\sqrt{a+bx^2}}{x}+b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= -\frac{\sqrt{a+bx^2}}{x}+\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0682273, size = 63, normalized size = 1.5

$$-\frac{-\sqrt{a}\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+a+bx^2}{x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -((a + b*x^2 - Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sq
rt[a]])/(x*Sqrt[a + b*x^2]))
```

Maple [A] time = 0.005, size = 54, normalized size = 1.3

$$-\frac{1}{ax}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{bx}{a}\sqrt{bx^2+a}+\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(3/2)+b/a*x*(b*x^2+a)^(1/2)+b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56335, size = 217, normalized size = 5.17

$$\left[\frac{\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,\sqrt{bx^2+a}}{2\,x},-\frac{\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqrt(b*
x^2 + a))/x, -(sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(b*x^2 +
 a))/x]
```

Sympy [A] time = 1.37001, size = 56, normalized size = 1.33

$$-\frac{\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{bx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)/(x*sqrt(1 + b*x**2/a)) + sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - b*x/(s
qrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.30917, size = 77, normalized size = 1.83

$$-\frac{1}{2}\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2a\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a*sqrt(b)/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)
```

3.365 $\int \frac{\sqrt{a+bx^2}}{x^4} \, dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

Rubi [A] time = 0.0044524, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^4} \, dx = -\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Mathematica [A] time = 0.0049409, size = 21, normalized size = 1.

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$-\frac{1}{3\,ax^3}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(b*x^2+a)^(3/2)/a/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44487, size = 43, normalized size = 2.05

$$-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

Sympy [B] time = 0.627931, size = 42, normalized size = 2.

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a
)
```

Giac [B] time = 2.82172, size = 80, normalized size = 3.81

$$\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}+a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) + a^2*b^(3/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^3
```

3.366 $\int \frac{\sqrt{a+bx^2}}{x^6} \, dx$

Optimal. Leaf size=44

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

```
[Out] -(a + b*x^2)^(3/2)/(5*a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)
```

Rubi [A] time = 0.0106586, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(5*a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^6} \, dx &= -\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}-\frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^4} \, dx}{5a}\\ &= -\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.0087928, size = 31, normalized size = 0.7

$$\frac{\left(a+bx^2\right)^{3/2}\left(2bx^2-3a\right)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-3*a + 2*b*x^2))/(15*a^2*x^5)
```

Maple [A] time = 0.003, size = 28, normalized size = 0.6

$$-\frac{-2\,b x^2+3\,a}{15\,a^2 x^5}\left(b x^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-2*b*x^2+3*a)/a^2/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52771, size = 84, normalized size = 1.91

$$\frac{\left(2\,b^2 x^4 - a b x^2 - 3\,a^2\right)\sqrt{b x^2 + a}}{15\,a^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*b^2*x^4 - a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a)/(a^2*x^5)
```

Sympy [A] time = 0.842172, size = 68, normalized size = 1.55

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*
a*x**2) + 2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

Giac [B] time = 1.46924, size = 151, normalized size = 3.43

$$\frac{4\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}+5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 ab^{\frac{5}{2}}+5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^2 b^{\frac{5}{2}}-a^3 b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2) - a^3*b
^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.367 $\int \frac{\sqrt{a+bx^2}}{x^8} dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

```
[Out] -(a + b*x^2)^(3/2)/(7*a*x^7) + (4*b*(a + b*x^2)^(3/2))/(35*a^2*x^5) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rubi [A] time = 0.0179749, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(7*a*x^7) + (4*b*(a + b*x^2)^(3/2))/(35*a^2*x^5) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^8} dx &= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}-\frac{(4b)\int \frac{\sqrt{a+bx^2}}{x^6} dx}{7a}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}+\frac{\left(8b^2\right)\int \frac{\sqrt{a+bx^2}}{x^4} dx}{35a^2}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0101219, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{3/2}\left(15a^2-12abx^2+8b^2x^4\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] -((a + b*x^2)^(3/2)*(15*a^2 - 12*a*b*x^2 + 8*b^2*x^4))/(105*a^3*x^7)
```

Maple [A] time = 0.003, size = 39, normalized size = 0.6

$$-\frac{8\,b^2x^4-12\,abx^2+15\,a^2}{105\,a^3x^7}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^8,x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(8*b^2*x^4-12*a*b*x^2+15*a^2)/a^3/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58286, size = 112, normalized size = 1.65

$$-\frac{\left(8\,b^3x^6-4\,ab^2x^4+3\,a^2bx^2+15\,a^3\right)\sqrt{bx^2+a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(8*b^3*x^6 - 4*a*b^2*x^4 + 3*a^2*b*x^2 + 15*a^3)*sqrt(b*x^2 + a)/(a^
3*x^7)
```

Sympy [B] time = 1.31844, size = 359, normalized size = 5.28

$$-\frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33a^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17a^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*
x**8 + 105*a**3*b**6*x**10) - 33*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*a**3*b*
*(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 3*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*a*b**(17/2)*x
**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**
3*b**6*x**10) - 8*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
+ 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
```

Giac [B] time = 2.13103, size = 186, normalized size = 2.74

$$\frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 ab^{\frac{7}{2}}+21\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 a^2b^{\frac{7}{2}}-7\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^3b^{\frac{7}{2}}+a^4 \cdots}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(7/2) + 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(7/2) - 7
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(7/2) + a^4*b^(7/2))/((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^7
```

3.368 $\int \frac{\sqrt{a+bx^2}}{x^{10}} dx$

Optimal. Leaf size=92

$$\frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

```
[Out] -(a + b*x^2)^(3/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(3/2))/(21*a^2*x^7) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^(3/2))/(315*a^4*x^
3)
```

Rubi [A] time = 0.0291518, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(3/2))/(21*a^2*x^7) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^(3/2))/(315*a^4*x^
3)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^{10}} dx &= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}-\frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^8} dx}{3a}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}+\frac{\left(8b^2\right)\int \frac{\sqrt{a+bx^2}}{x^6} dx}{21a^2}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}-\frac{\left(16b^3\right)\int \frac{\sqrt{a+bx^2}}{x^4} dx}{105a^3}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}+\frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}
\end{aligned}$$

Mathematica [A] time = 0.0120811, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{3/2}\left(30a^2bx^2-35a^3-24ab^2x^4+16b^3x^6\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-35*a^3 + 30*a^2*b*x^2 - 24*a*b^2*x^4 + 16*b^3*x^6))/(3
15*a^4*x^9)
```

Maple [A] time = 0.003, size = 50, normalized size = 0.5

$$-\frac{-16\,b^3x^6+24\,ab^2x^4-30\,a^2bx^2+35\,a^3}{315\,x^9a^4}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-16*b^3*x^6+24*a*b^2*x^4-30*a^2*b*x^2+35*a^3)/x^9/a
^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59386, size = 134, normalized size = 1.46

$$\frac{\left(16\,b^4x^8-8\,ab^3x^6+6\,a^2b^2x^4-5\,a^3bx^2-35\,a^4\right)\sqrt{bx^2+a}}{315\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(16*b^4*x^8 - 8*a*b^3*x^6 + 6*a^2*b^2*x^4 - 5*a^3*b*x^2 - 35*a^4)*sqr
t(b*x^2 + a)/(a^4*x^9)
```

Sympy [B] time = 1.85392, size = 575, normalized size = 6.25

$$-\frac{35a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{110a^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**1
0*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*a**6*b**(21/2)
*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*a**5*b**(23/2)*x**4*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x
**12 + 315*a**4*b**12*x**14) - 40*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/
(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**
4*b**12*x**14) + 5*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*
x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14)
+ 30*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a*
*6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 40*a*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*b**(33/2)*x**14*sqrt(a/(
b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14)
```

Giac [B] time = 2.4691, size = 224, normalized size = 2.43

$$\frac{32\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{9}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{9}{2}}-36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3\right.}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(9/2) + 189*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a*b^(9/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2)
- 36*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9/2) + 9*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^4*b^(9/2) - a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^
9
```

3.369 $\int x^7 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=80

$$\frac{3a^2\left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a^3\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{a\left(a+bx^2\right)^{9/2}}{3b^4}$$

```
[Out] -(a^3*(a + b*x^2)^(5/2))/(5*b^4) + (3*a^2*(a + b*x^2)^(7/2))/(7*b^4) - (a*(
a + b*x^2)^(9/2))/(3*b^4) + (a + b*x^2)^(11/2)/(11*b^4)
```

Rubi [A] time = 0.0476189, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a^3\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{a\left(a+bx^2\right)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(5/2))/(5*b^4) + (3*a^2*(a + b*x^2)^(7/2))/(7*b^4) - (a*(
a + b*x^2)^(9/2))/(3*b^4) + (a + b*x^2)^(11/2)/(11*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{3/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{a^3\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{3a^2\left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a\left(a+bx^2\right)^{9/2}}{3b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.0253097, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{5/2}\left(40a^2bx^2-16a^3-70ab^2x^4+105b^3x^6\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-16*a^3 + 40*a^2*b*x^2 - 70*a*b^2*x^4 + 105*b^3*x^6))/(
1155*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-105\,b^{3}x^{6}+70\,ab^{2}x^{4}-40\,a^{2}bx^{2}+16\,a^{3}}{1155\,b^{4}}\left(bx^{2}+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(3/2),x)
```

```
[Out] -1/1155*(b*x^2+a)^(5/2)*(-105*b^3*x^6+70*a*b^2*x^4-40*a^2*b*x^2+16*a^3)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55586, size = 154, normalized size = 1.92

$$\frac{\left(105\,b^{5}x^{10}+140\,ab^{4}x^{8}+5\,a^{2}b^{3}x^{6}-6\,a^{3}b^{2}x^{4}+8\,a^{4}bx^{2}-16\,a^{5}\right)\sqrt{bx^{2}+a}}{1155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^5*x^10 + 140*a*b^4*x^8 + 5*a^2*b^3*x^6 - 6*a^3*b^2*x^4 + 8*a^
4*b*x^2 - 16*a^5)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 3.69204, size = 133, normalized size = 1.66

$$\begin{cases} -\frac{16a^{5}\sqrt{a+bx^{2}}}{1155b^{4}}+\frac{8a^{4}x^{2}\sqrt{a+bx^{2}}}{1155b^{3}}-\frac{2a^{3}x^{4}\sqrt{a+bx^{2}}}{385b^{2}}+\frac{a^{2}x^{6}\sqrt{a+bx^{2}}}{231b}+\frac{4ax^{8}\sqrt{a+bx^{2}}}{33}+\frac{bx^{10}\sqrt{a+bx^{2}}}{11} & \text{for}\ b\neq 0\\ \frac{a^{\frac{3}{2}}x^{8}}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-16*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*a**4*x**2*sqrt(a + b*x
**2)/(1155*b**3) - 2*a**3*x**4*sqrt(a + b*x**2)/(385*b**2) + a**2*x**6*sqrt
(a + b*x**2)/(231*b) + 4*a*x**8*sqrt(a + b*x**2)/33 + b*x**10*sqrt(a + b*x*
*2)/11, Ne(b, 0)), (a**(3/2)*x**8/8, True))
```

Giac [B] time = 2.03377, size = 181, normalized size = 2.26

$$\frac{\frac{11\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)a}{b^3}+\frac{315\left(bx^2+a\right)^{\frac{11}{2}}-1540\left(bx^2+a\right)^{\frac{9}{2}}a+2970\left(bx^2+a\right)^{\frac{7}{2}}a^2-2772\left(bx^2+a\right)^{\frac{5}{2}}a^3+1155\left(bx^2+a\right.}{b^3}}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*(11*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a
)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*a/b^3 + (315*(b*x^2 + a)^(11/2) -
1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5
/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)/b^3)/b
```

3.370 $\int x^5 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=59

$$\frac{a^2 \left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{2a \left(a+bx^2\right)^{7/2}}{7b^3}$$

```
[Out] (a^2*(a + b*x^2)^(5/2))/(5*b^3) - (2*a*(a + b*x^2)^(7/2))/(7*b^3) + (a + b*
x^2)^(9/2)/(9*b^3)
```

Rubi [A] time = 0.0351328, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{2a \left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(5/2))/(5*b^3) - (2*a*(a + b*x^2)^(7/2))/(7*b^3) + (a + b*
x^2)^(9/2)/(9*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+bx^2\right)^{3/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a^2 \left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a \left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.017978, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{5/2} \left(8a^2-20abx^2+35b^2x^4\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(8*a^2 - 20*a*b*x^2 + 35*b^2*x^4))/(315*b^3)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.6

$$\frac{35 b^2x^4 - 20 abx^2 + 8 a^2}{315 b^3} \left(bx^2 + a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(5/2)*(35*b^2*x^4-20*a*b*x^2+8*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57227, size = 126, normalized size = 2.14

$$\frac{\left(35 b^4x^8 + 50 ab^3x^6 + 3 a^2b^2x^4 - 4 a^3bx^2 + 8 a^4\right)\sqrt{bx^2 + a}}{315 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 50*a*b^3*x^6 + 3*a^2*b^2*x^4 - 4*a^3*b*x^2 + 8*a^4)*sqr
t(b*x^2 + a)/b^3
```

Sympy [A] time = 2.07804, size = 109, normalized size = 1.85

$$\begin{cases} \frac{8a^4\sqrt{a+bx^2}}{315b^3} - \frac{4a^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{a^2x^4\sqrt{a+bx^2}}{105b} + \frac{10ax^6\sqrt{a+bx^2}}{63} + \frac{bx^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((8*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*a**3*x**2*sqrt(a + b*x**2
)/(315*b**2) + a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*a*x**6*sqrt(a + b*x*
```

```
*2)/63 + b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

Giac [B] time = 2.72273, size = 143, normalized size = 2.42

$$\frac{\frac{3\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)a}{b^2}+\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3}{b^2}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3
/2)*a^2)*a/b^2 + (35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x
^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)/b^2)/b
```

3.371 $\int x^3 \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}$$

```
[Out] -(a*(a + b*x^2)^(5/2))/(5*b^2) + (a + b*x^2)^(7/2)/(7*b^2)
```

Rubi [A] time = 0.0243898, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)^(5/2))/(5*b^2) + (a + b*x^2)^(7/2)/(7*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^{3/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{3/2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{\left(a+bx^2\right)^{7/2}}{7b^2}
\end{aligned}$$

Mathematica [A] time = 0.0137865, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(5bx^2-2a\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-2*a + 5*b*x^2))/(35*b^2)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$-\frac{-5\,bx^2+2\,a}{35\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-5*b*x^2+2*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55671, size = 97, normalized size = 2.55

$$\frac{\left(5\,b^3x^6+8\,ab^2x^4+a^2bx^2-2\,a^3\right)\sqrt{bx^2+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*x^6 + 8*a*b^2*x^4 + a^2*b*x^2 - 2*a^3)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 1.16344, size = 85, normalized size = 2.24

$$\begin{cases} -\frac{2a^3\sqrt{a+bx^2}}{35b^2}+\frac{a^2x^2\sqrt{a+bx^2}}{35b}+\frac{8ax^4\sqrt{a+bx^2}}{35}+\frac{bx^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(a + b*x**2)/(35*b**2) + a**2*x**2*sqrt(a + b*x**2)/
(35*b) + 8*a*x**4*sqrt(a + b*x**2)/35 + b*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)
), (a**(3/2)*x**4/4, True))
```

Giac [B] time = 1.44151, size = 105, normalized size = 2.76

$$\frac{\frac{7\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)a}{b}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2}{b}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*a/b + (15*(b*x^2 + a
)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)/b)/b
```

3.372 $\int x\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

Rubi [A] time = 0.0037953, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(3/2),x]
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Mathematica [A] time = 0.0030838, size = 18, normalized size = 1.

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(3/2),x]
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{1}{5\,b}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)/b
```

Maxima [A] time = 2.1725, size = 19, normalized size = 1.06

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

Fricas [B] time = 1.51756, size = 69, normalized size = 3.83

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/b
```

Sympy [A] time = 0.574784, size = 61, normalized size = 3.39

$$\begin{cases} \frac{a^2\sqrt{a+bx^2}}{5b}+\frac{2ax^2\sqrt{a+bx^2}}{5}+\frac{bx^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((a**2*sqrt(a + b*x**2)/(5*b) + 2*a*x**2*sqrt(a + b*x**2)/5 + b*x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (a**(3/2)*x**2/2, True))
```

Giac [A] time = 3.16103, size = 19, normalized size = 1.06

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

3.373 $\int \frac{(a+bx^2)^{3/2}}{x} \, dx$

Optimal. Leaf size=54

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}$$

```
[Out] a*Sqrt[a + b*x^2] + (a + b*x^2)^(3/2)/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]
```

Rubi [A] time = 0.0356325, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] a*Sqrt[a + b*x^2] + (a + b*x^2)^(3/2)/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{3}\left(a+bx^2\right)^{3/2}+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\
&= a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}+\frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.020053, size = 50, normalized size = 0.93

$$
\frac{1}{3}\sqrt{a+bx^2}\left(4a+bx^2\right)-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Maple [A] time = 0.003, size = 52, normalized size = 1.

$$
\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}-a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+a\sqrt{bx^2+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/3*(b*x^2+a)^(3/2)-a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+a*(b*x^2+
a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58719, size = 251, normalized size = 4.65

$$\left[\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{3}\left(bx^2+4\,a\right)\sqrt{bx^2+a},\ \sqrt{-a}a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{3}\left(bx^2+4\,a\right)\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(3/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/3*(b*x
^2 + 4*a)*sqrt(b*x^2 + a), sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/
3*(b*x^2 + 4*a)*sqrt(b*x^2 + a)]
```

Sympy [A] time = 1.86413, size = 78, normalized size = 1.44

$$\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}{3}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{\sqrt{a}bx^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x,x)
```

```
[Out] 4*a**(3/2)*sqrt(1 + b*x**2/a)/3 + a**(3/2)*log(b*x**2/a)/2 - a**(3/2)*log(s
qrt(1 + b*x**2/a) + 1) + sqrt(a)*b*x**2*sqrt(1 + b*x**2/a)/3
```

Giac [A] time = 1.37338, size = 65, normalized size = 1.2

$$\frac{a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}+\sqrt{bx^2+a}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*(b*x^2 + a)^(3/2) + sqr
t(b*x^2 + a)*a
```

3.374 $\int \frac{(a+bx^2)^{3/2}}{x^3} dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{3}{2}b\sqrt{a+bx^2}-\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] (3*b*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.0367053, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{3}{2}b\sqrt{a+bx^2}-\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{4}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{4}(3ab)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{2x^2}-\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0089778, size = 37, normalized size = 0.59

$$\frac{b\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^2)/a])/(5*a^2)
```

Maple [A] time = 0.003, size = 75, normalized size = 1.2

$$-\frac{1}{2\,ax^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{b}{2\,a}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,b}{2}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+\frac{3\,b}{2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(5/2)+1/2*b/a*(b*x^2+a)^(3/2)-3/2*b*a^(1/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x)+3/2*b*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61507, size = 284, normalized size = 4.51

$$\left[\frac{3\,\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,bx^2-a\right)\sqrt{bx^2+a}}{4\,x^2}, \frac{3\,\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,bx^2-a\right)\sqrt{bx^2+a}}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2, 1/2*(3*sqrt(-a)*b*x^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + (2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] time = 2.34298, size = 88, normalized size = 1.4

$$-\frac{3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{a^2}{2\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{a\sqrt{b}}{2x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - a**2/(2*sqrt(b)*x**3*sqrt(a/(b*
x**2) + 1)) + a*sqrt(b)/(2*x*sqrt(a/(b*x**2) + 1)) + b**(3/2)*x/sqrt(a/(b*x
**2) + 1)
```

Giac [A] time = 1.59864, size = 77, normalized size = 1.22

$$\frac{1}{2}\left(\frac{3\,a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,\sqrt{bx^2+a}-\frac{\sqrt{bx^2+a}a}{bx^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^2 + a) - sq
rt(b*x^2 + a)*a/(b*x^2))*b
```

3.375 $\int \frac{(a+bx^2)^{3/2}}{x^5} dx$

Optimal. Leaf size=68

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{(a+bx^2)^{3/2}}{4x^4}$$

```
[Out] (-3*b*Sqrt[a + b*x^2])/(8*x^2) - (a + b*x^2)^(3/2)/(4*x^4) - (3*b^2*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] time = 0.0395221, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{(a+bx^2)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2])/(8*x^2) - (a + b*x^2)^(3/2)/(4*x^4) - (3*b^2*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{8}(3b)\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{16}\left(3b^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{8}(3b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0354422, size = 76, normalized size = 1.12

$$
-\frac{2a^2+3b^2x^4\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+7abx^2+5b^2x^4}{8x^4\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] -(2*a^2 + 7*a*b*x^2 + 5*b^2*x^4 + 3*b^2*x^4*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqr
t[1 + (b*x^2)/a]])/(8*x^4*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 102, normalized size = 1.5

$$
-\frac{1}{4\,ax^4}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{b}{8\,a^2x^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{b^2}{8\,a^2}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,b^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{3\,b^2}{8\,a}\sqrt{bx^2+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(5/2)-1/8*b/a^2/x^2*(b*x^2+a)^(5/2)+1/8*b^2/a^2*(b*x^2
+a)^(3/2)-3/8*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/8*b^2/a*(
b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54177, size = 317, normalized size = 4.66

$$\left[\frac{3\sqrt{a}b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(5abx^2+2a^2\right)\sqrt{bx^2+a}}{16ax^4}, \frac{3\sqrt{-a}b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(5abx^2+2a^2\right)\sqrt{bx^2+a}}{8ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a*x^4), 1/8*(3*sqrt(-a)*b^2*x^4
*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a
*x^4)]
```

Sympy [A] time = 3.00593, size = 71, normalized size = 1.04

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{4x^3}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{8x}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(4*x**3) - 5*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(8*x) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
```

Giac [A] time = 2.6188, size = 82, normalized size = 1.21

$$\frac{1}{8}b^2\left(\frac{3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}-3\sqrt{bx^2+a}a}{b^2x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^2*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^2 + a)^(3/2)
 - 3*sqrt(b*x^2 + a)*a)/(b^2*x^4))
```

3.376 $\int \frac{(a+bx^2)^{3/2}}{x^7} dx$

Optimal. Leaf size=92

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{b\sqrt{a+bx^2}}{8x^4} - \frac{\left(a+bx^2\right)^{3/2}}{6x^6}$$

```
[Out] -(b*Sqrt[a + b*x^2])/(8*x^4) - (b^2*Sqrt[a + b*x^2])/(16*a*x^2) - (a + b*x^
2)^(3/2)/(6*x^6) + (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rubi [A] time = 0.0541872, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{b\sqrt{a+bx^2}}{8x^4} - \frac{\left(a+bx^2\right)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a + b*x^2])/(8*x^4) - (b^2*Sqrt[a + b*x^2])/(16*a*x^2) - (a + b*x^
2)^(3/2)/(6*x^6) + (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{6x^6} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{\left(a+bx^2\right)^{3/2}}{6x^6} + \frac{1}{16}b^2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{\left(a+bx^2\right)^{3/2}}{6x^6} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{32a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{\left(a+bx^2\right)^{3/2}}{6x^6} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{16a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{\left(a+bx^2\right)^{3/2}}{6x^6} + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0089841, size = 39, normalized size = 0.42

$$
\frac{b^3\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^2)/a])/(5*a^
4)
```

Maple [A] time = 0.007, size = 122, normalized size = 1.3

$$
-\frac{1}{6\,ax^6}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{b}{24\,a^2x^4}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{b^2}{48\,a^3x^2}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{b^3}{48\,a^3}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{b^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(5/2)+1/24*b/a^2/x^4*(b*x^2+a)^(5/2)+1/48*b^2/a^3/x^2*
(b*x^2+a)^(5/2)-1/48*b^3/a^3*(b*x^2+a)^(3/2)+1/16*b^3/a^(3/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)-1/16*b^3/a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68335, size = 371, normalized size = 4.03

$$\left[\frac{3\sqrt{a}b^3x^6\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(3ab^2x^4+14a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{96a^2x^6}, -\frac{3\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3ab^2x^4+1\right.}{48a^2x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(3*a*b^2*x^4 + 14*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6), -1/4
8*(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 + 14*
a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

Sympy [A] time = 5.28767, size = 119, normalized size = 1.29

$$-\frac{a^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{11a\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{17b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{b^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] -a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*a*sqrt(b)/(24*x**5*sqrt(a/
(b*x**2) + 1)) - 17*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - b**(5/2)/(16*
a*x*sqrt(a/(b*x**2) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2))
```

Giac [A] time = 1.56627, size = 108, normalized size = 1.17

$$-\frac{1}{48}b^3\left(\frac{3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}+8\left(bx^2+a\right)^{\frac{3}{2}}a-3\sqrt{bx^2+a}a^2}{ab^3x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*b^3*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x^2 + a)
^(5/2) + 8*(b*x^2 + a)^(3/2)*a - 3*sqrt(b*x^2 + a)*a^2)/(a*b^3*x^6))
```

3.377 $\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=116

$$\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{(a+bx^2)^{3/2}}{8x^8}$$

```
[Out] -(b*Sqrt[a + b*x^2])/(16*x^6) - (b^2*Sqrt[a + b*x^2])/(64*a*x^4) + (3*b^3*S
qrt[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^(3/2)/(8*x^8) - (3*b^4*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rubi [A] time = 0.0693875, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{(a+bx^2)^{3/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -(b*Sqrt[a + b*x^2])/(16*x^6) - (b^2*Sqrt[a + b*x^2])/(64*a*x^4) + (3*b^3*S
qrt[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^(3/2)/(8*x^8) - (3*b^4*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{1}{16}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{1}{32}b^2\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} - \frac{\left(3b^3\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{128a}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{\left(3b^4\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{256a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{\left(3b^3\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{128a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} - \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0093422, size = 39, normalized size = 0.34

$$
-\frac{b^4\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -(b^4*(a + b*x^2)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x^2)/a])/(5*a
^5)
```

Maple [A] time = 0.013, size = 142, normalized size = 1.2

$$
-\frac{1}{8ax^8}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{b}{16a^2x^6}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{b^2}{64a^3x^4}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{b^3}{128a^4x^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{b^4}{128a^4}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{3b^4}{128}\ln\left(\frac{1}{x}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(5/2)+1/16*b/a^2/x^6*(b*x^2+a)^(5/2)-1/64*b^2/a^3/x^4*
(b*x^2+a)^(5/2)-1/128*b^3/a^4/x^2*(b*x^2+a)^(5/2)+1/128*b^4/a^4*(b*x^2+a)^(
3/2)-3/128*b^4/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/128*b^4/a^3*
(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65344, size = 419, normalized size = 3.61

$$\left[\frac{3\,\sqrt{a}b^4x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,ab^3x^6-2\,a^2b^2x^4-24\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{256\,a^3x^8},\ \frac{3\,\sqrt{-a}b^4x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a
))/(a^3*x^8), 1/128*(3*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

Sympy [A] time = 8.1865, size = 148, normalized size = 1.28

$$-\frac{a^2}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{5a\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{13b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] -a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*a*sqrt(b)/(16*x**7*sqrt(a/(
b*x**2) + 1)) - 13*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(128*
a*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(7/2)/(128*a**2*x*sqrt(a/(b*x**2) + 1))
 - 3*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2))
```

Giac [A] time = 2.94811, size = 127, normalized size = 1.09

$$\frac{1}{128}\,b^4\left(\frac{3\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\left(bx^2+a\right)^{\frac{7}{2}}-11\left(bx^2+a\right)^{\frac{5}{2}}a-11\left(bx^2+a\right)^{\frac{3}{2}}a^2+3\sqrt{bx^2+a}a^3}{a^2b^4x^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*b^4*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 +
a)^(7/2) - 11*(b*x^2 + a)^(5/2)*a - 11*(b*x^2 + a)^(3/2)*a^2 + 3*sqrt(b*x^2
 + a)*a^3)/(a^2*b^4*x^8))
```

3.378 $\int x^4 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=115

$$-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

```
[Out] (-3*a^3*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*x^3*Sqrt[a + b*x^2])/(64*b) + (
a*x^5*Sqrt[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.0423694, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (-3*a^3*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*x^3*Sqrt[a + b*x^2])/(64*b) + (
a*x^5*Sqrt[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4 \left(a+bx^2\right)^{3/2} dx = \frac{1}{8}x^5 \left(a+bx^2\right)^{3/2} + \frac{1}{8}(3a) \int x^4\sqrt{a+bx^2}\,dx$$
$$= \frac{1}{16}ax^5\sqrt{a+bx^2} + \frac{1}{8}x^5\left(a+bx^2\right)^{3/2} + \frac{1}{16}a^2 \int \frac{x^4}{\sqrt{a+bx^2}}dx$$
$$= \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{1}{16}ax^5\sqrt{a+bx^2} + \frac{1}{8}x^5\left(a+bx^2\right)^{3/2} - \frac{\left(3a^3\right)\int \frac{x^2}{\sqrt{a+bx^2}}dx}{64b}$$
$$= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2} + \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{1}{16}ax^5\sqrt{a+bx^2} + \frac{1}{8}x^5\left(a+bx^2\right)^{3/2} + \frac{\left(3a^4\right)\int \frac{1}{\sqrt{a+bx^2}}dx}{128b^2}$$
$$= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2} + \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{1}{16}ax^5\sqrt{a+bx^2} + \frac{1}{8}x^5\left(a+bx^2\right)^{3/2} + \frac{\left(3a^4\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{128b^2}$$
$$= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2} + \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{1}{16}ax^5\sqrt{a+bx^2} + \frac{1}{8}x^5\left(a+bx^2\right)^{3/2} + \frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

Mathematica [A] time = 0.130364, size = 94, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(2a^2bx^2-3a^3+24ab^2x^4+16b^3x^6\right)+\frac{3a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-3*a^3 + 2*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x
^6) + (3*a^(7/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(128*b
^(5/2))
```

Maple [A] time = 0.006, size = 95, normalized size = 0.8

$$\frac{x^3}{8b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{ax}{16b^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{a^2x}{64b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3a^3x}{128b^2}\sqrt{bx^2+a} + \frac{3a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*x^3*(b*x^2+a)^(5/2)/b-1/16/b^2*a*x*(b*x^2+a)^(5/2)+1/64/b^2*a^2*x*(b*x^
2+a)^(3/2)+3/128*a^3*x*(b*x^2+a)^(1/2)/b^2+3/128/b^(5/2)*a^4*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66426, size = 396, normalized size = 3.44

$$\left[\frac{3\,a^4\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx\right)\sqrt{bx^2+a}}{256\,b^3},-\frac{3\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{-}}{\sqrt{bx}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
16*b^4*x^7 + 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3
, -1/128*(3*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (16*b^4*x^7 +
 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 7.56633, size = 148, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - a**(5/2)*x**3/(128*b*sqrt(1 +
 b*x**2/a)) + 13*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*x**7/(
16*sqrt(1 + b*x**2/a)) + 3*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + b
**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.00105, size = 103, normalized size = 0.9

$$\frac{1}{128}\left(2\left(4\left(2\,bx^2+3\,a\right)x^2+\frac{a^2}{b}\right)x^2-\frac{3\,a^3}{b^2}\right)\sqrt{bx^2+a}x-\frac{3\,a^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(2*(4*(2*b*x^2 + 3*a)*x^2 + a^2/b)*x^2 - 3*a^3/b^2)*sqrt(b*x^2 + a)*x
 - 3/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.379 $\int x^2 \left(a + bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=91

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} + \frac{a^2 x\sqrt{a+bx^2}}{16b} + \frac{1}{8}ax^3\sqrt{a+bx^2} + \frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

```
[Out] (a^2*x*Sqrt[a + b*x^2])/(16*b) + (a*x^3*Sqrt[a + b*x^2])/8 + (x^3*(a + b*x^
2)^(3/2))/6 - (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.0306598, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} + \frac{a^2 x\sqrt{a+bx^2}}{16b} + \frac{1}{8}ax^3\sqrt{a+bx^2} + \frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^2])/(16*b) + (a*x^3*Sqrt[a + b*x^2])/8 + (x^3*(a + b*x^
2)^(3/2))/6 - (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2 \left(a+bx^2\right)^{3/2} dx = \frac{1}{6}x^3 \left(a+bx^2\right)^{3/2} + \frac{1}{2}a \int x^2\sqrt{a+bx^2}\, dx$$
$$= \frac{1}{8}ax^3\sqrt{a+bx^2} + \frac{1}{6}x^3 \left(a+bx^2\right)^{3/2} + \frac{1}{8}a^2 \int \frac{x^2}{\sqrt{a+bx^2}} dx$$
$$= \frac{a^2x\sqrt{a+bx^2}}{16b} + \frac{1}{8}ax^3\sqrt{a+bx^2} + \frac{1}{6}x^3 \left(a+bx^2\right)^{3/2} - \frac{a^3 \int \frac{1}{\sqrt{a+bx^2}} dx}{16b}$$
$$= \frac{a^2x\sqrt{a+bx^2}}{16b} + \frac{1}{8}ax^3\sqrt{a+bx^2} + \frac{1}{6}x^3 \left(a+bx^2\right)^{3/2} - \frac{a^3 \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b}$$
$$= \frac{a^2x\sqrt{a+bx^2}}{16b} + \frac{1}{8}ax^3\sqrt{a+bx^2} + \frac{1}{6}x^3 \left(a+bx^2\right)^{3/2} - \frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

Mathematica [A] time = 0.115548, size = 83, normalized size = 0.91

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(3a^2+14abx^2+8b^2x^4\right) - \frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2 + 14*a*b*x^2 + 8*b^2*x^4) - (3*a^(5/2)*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

Maple [A] time = 0.006, size = 75, normalized size = 0.8

$$\frac{x}{6\,b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{ax}{24\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{a^2x}{16\,b}\sqrt{bx^2+a} - \frac{a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)/b-1/24/b*a*x*(b*x^2+a)^(3/2)-1/16*a^2*x*(b*x^2+a)^(1/
2)/b-1/16/b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61282, size = 346, normalized size = 3.8

$$\left[\frac{3\,a^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx\right)\sqrt{bx^2+a}}{96\,b^2}, \frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(8\,b\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2*b*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2
*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 4.97693, size = 119, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{b^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*x**3/(48*sqrt(1 + b*x**2
/a)) + 11*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(3/2)) + b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.66679, size = 85, normalized size = 0.93

$$\frac{1}{48}\left(2\left(4\,bx^2+7\,a\right)x^2+\frac{3\,a^2}{b}\right)\sqrt{bx^2+a}x+\frac{a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2 + 7*a)*x^2 + 3*a^2/b)*sqrt(b*x^2 + a)*x + 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.380 $\int \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.0147713, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{3/2} dx &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2}\,dx \\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right) \\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0867275, size = 65, normalized size = 1.

$$\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+5ax+2bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/8
```

Maple [A] time = 0.001, size = 51, normalized size = 0.8

$$\frac{x}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,ax}{8}\sqrt{bx^2+a}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a*x*(b*x^2+a)^(1/2)+3/8*a^2/b^(1/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52616, size = 294, normalized size = 4.52

$$\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{16\,b},-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3+5\,ab\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 2.8023, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Giac [A] time = 2.40712, size = 66, normalized size = 1.02

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x-\frac{3\,a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

3.381 $\int \frac{(a+bx^2)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{x}+\frac{3}{2}bx\sqrt{a+bx^2}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] (3*b*x*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/x + (3*a*Sqrt[b]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi [A] time = 0.0177573, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$-\frac{(a+bx^2)^{3/2}}{x}+\frac{3}{2}bx\sqrt{a+bx^2}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] (3*b*x*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/x + (3*a*Sqrt[b]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx &= -\frac{\left(a+bx^2\right)^{3/2}}{x} + (3b)\int \sqrt{a+bx^2}\,dx \\
&= \frac{3}{2}bx\sqrt{a+bx^2} - \frac{\left(a+bx^2\right)^{3/2}}{x} + \frac{1}{2}(3ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{3}{2}bx\sqrt{a+bx^2} - \frac{\left(a+bx^2\right)^{3/2}}{x} + \frac{1}{2}(3ab)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{3}{2}bx\sqrt{a+bx^2} - \frac{\left(a+bx^2\right)^{3/2}}{x} + \frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0079266, size = 50, normalized size = 0.79

$$
-\frac{a\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(x*S
qrt[1 + (b*x^2)/a]))
```

Maple [A] time = 0.004, size = 69, normalized size = 1.1

$$
-\frac{1}{ax}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{bx}{a}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,bx}{2}\sqrt{bx^2+a} + \frac{3\,a}{2}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(5/2)+b/a*x*(b*x^2+a)^(3/2)+3/2*b*x*(b*x^2+a)^(1/2)+3/2*b^
(1/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56303, size = 271, normalized size = 4.3

$$\left[\frac{3\,a\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{4\,x},-\frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqr
t(b*x^2 + a)*(b*x^2 - 2*a))/x, -1/2*(3*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - sqrt(b*x^2 + a)*(b*x^2 - 2*a))/x]
```

Sympy [A] time = 2.29281, size = 88, normalized size = 1.4

$$-\frac{a^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}bx}{2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}+\frac{b^2x^3}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -a**(3/2)/(x*sqrt(1 + b*x**2/a)) - sqrt(a)*b*x/(2*sqrt(1 + b*x**2/a)) + 3*a
*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2 + b**2*x**3/(2*sqrt(a)*sqrt(1 + b*x**2/
a))
```

Giac [A] time = 2.47025, size = 99, normalized size = 1.57

$$\frac{1}{2}\,\sqrt{bx^2+a}bx-\frac{3}{4}\,a\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x - 3/4*a*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2
) + 2*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.382 $\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=61

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

```
[Out] -((b*Sqrt[a + b*x^2])/x) - (a + b*x^2)^(3/2)/(3*x^3) + b^(3/2)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0184478, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 206}

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -((b*Sqrt[a + b*x^2])/x) - (a + b*x^2)^(3/2)/(3*x^3) + b^(3/2)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = -\frac{(a+bx^2)^{3/2}}{3x^3} + b\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3} + b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3} + b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3} + b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.0086008, size = 52, normalized size = 0.85

$$-\frac{a\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -(a*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^2)/a)])/(3*x
^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.005, size = 92, normalized size = 1.5

$$-\frac{1}{3\,a x^3}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{2\,b}{3\,a^2 x}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{2\,b^2 x}{3\,a^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{b^2 x}{a}\sqrt{bx^2+a} + b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(5/2)-2/3*b/a^2/x*(b*x^2+a)^(5/2)+2/3*b^2/a^2*x*(b*x^2
+a)^(3/2)+b^2/a*x*(b*x^2+a)^(1/2)+b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60172, size = 279, normalized size = 4.57

$$\left[\frac{3\,b^{\frac{3}{2}}x^{3}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\,\left(4\,bx^{2}+a\right)\sqrt{bx^{2}+a}}{6\,x^{3}},-\frac{3\,\sqrt{-b}bx^{3}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)+\left(4\,bx^{2}+a\right)\sqrt{bx^{2}+a}}{3\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4*
b*x^2 + a)*sqrt(b*x^2 + a))/x^3, -1/3*(3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (4*b*x^2 + a)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] time = 1.97584, size = 78, normalized size = 1.28

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3}-\frac{b^{\frac{3}{2}}\log\left(\frac{a}{bx^{2}}\right)}{2}+b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^{2}}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 4*b**(3/2)*sqrt(a/(b*x**2) + 1)/
3 - b**(3/2)*log(a/(b*x**2))/2 + b**(3/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

Giac [B] time = 2.90562, size = 154, normalized size = 2.52

$$-\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}ab^{\frac{3}{2}}-3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{2}b^{\frac{3}{2}}+2\,a^{3}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 4/3*(3*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2) +
 2*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.383 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^2\right)^{5/2}}{5ax^5}$$

```
[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

Rubi [A] time = 0.0049624, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(a+bx^2\right)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^6,x]

[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx = -\frac{\left(a+bx^2\right)^{5/2}}{5ax^5}$$

Mathematica [A] time = 0.00613, size = 21, normalized size = 1.

$$-\frac{\left(a+bx^2\right)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^6,x]

[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$-\frac{1}{5\,ax^5}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*(b*x^2+a)^(5/2)/a/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.4898, size = 78, normalized size = 3.71

$$-\frac{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right) \sqrt{b x^{2}+a}}{5 \, a x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(a*x^5)
```

Sympy [B] time = 0.967514, size = 68, normalized size = 3.24

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{4}}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{2}}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(5*x**2) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

Giac [B] time = 2.67829, size = 116, normalized size = 5.52

$$\frac{2 \left(5 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{8} b^{\frac{5}{2}}+10 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} a^{2} b^{\frac{5}{2}}+a^{4} b^{\frac{5}{2}}\right)}{5 \left(\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2) + 10*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(5/2) + a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.384 $\int \frac{(a+bx^2)^{3/2}}{x^8} dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5} - \frac{(a+bx^2)^{5/2}}{7ax^7}$$

```
[Out] -(a + b*x^2)^(5/2)/(7*a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)
```

Rubi [A] time = 0.0124121, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5} - \frac{(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(7*a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^8} dx &= -\frac{(a+bx^2)^{5/2}}{7ax^7} - \frac{(2b)\int \frac{(a+bx^2)^{3/2}}{x^6} dx}{7a} \\
&= -\frac{(a+bx^2)^{5/2}}{7ax^7} + \frac{2b(a+bx^2)^{5/2}}{35a^2x^5}
\end{aligned}$$

Mathematica [A] time = 0.0095871, size = 31, normalized size = 0.7

$$\frac{(a+bx^2)^{5/2}(2bx^2-5a)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-5*a + 2*b*x^2))/(35*a^2*x^7)
```

Maple [A] time = 0.002, size = 28, normalized size = 0.6

$$-\frac{-2\,b x^{2}+5\,a}{35\,x^{7} a^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^8,x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-2*b*x^2+5*a)/x^7/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57507, size = 105, normalized size = 2.39

$$\frac{\left(2\,b^{3} x^{6}-a b^{2} x^{4}-8\,a^{2} b x^{2}-5\,a^{3}\right)\sqrt{b x^{2}+a}}{35\,a^{2} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^6 - a*b^2*x^4 - 8*a^2*b*x^2 - 5*a^3)*sqrt(b*x^2 + a)/(a^2*x^7
)
```

Sympy [B] time = 1.37544, size = 94, normalized size = 2.14

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35ax^2}+\frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 8*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(35*x**4) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*a*x**2) + 2*b**(7/2)*sqrt(a/(
b*x**2) + 1)/(35*a**2)
```

Giac [B] time = 2.03754, size = 224, normalized size = 5.09

$$\frac{4\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{2}b^{\frac{7}{2}}+14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{3}b^{\frac{7}{2}}+7\right.}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*a*b^(7/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2) + 14
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2) + 7*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^4*b^(7/2) - a^5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.385 $\int \frac{(a+bx^2)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=68

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

```
[Out] -(a + b*x^2)^(5/2)/(9*a*x^9) + (4*b*(a + b*x^2)^(5/2))/(63*a^2*x^7) - (8*b^
2*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rubi [A] time = 0.0207039, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(9*a*x^9) + (4*b*(a + b*x^2)^(5/2))/(63*a^2*x^7) - (8*b^
2*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx &= -\frac{(a+bx^2)^{5/2}}{9ax^9}-\frac{(4b)\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx}{9a}\\ &= -\frac{(a+bx^2)^{5/2}}{9ax^9}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}+\frac{(8b^2)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{63a^2}\\ &= -\frac{(a+bx^2)^{5/2}}{9ax^9}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}\end{aligned}$$

Mathematica [A] time = 0.0105376, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{5/2}(35a^2-20abx^2+8b^2x^4)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -((a + b*x^2)^(5/2)*(35*a^2 - 20*a*b*x^2 + 8*b^2*x^4))/(315*a^3*x^9)
```

Maple [A] time = 0.004, size = 39, normalized size = 0.6

$$-\frac{8\,b^2x^4-20\,abx^2+35\,a^2}{315\,x^9a^3}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(8*b^2*x^4-20*a*b*x^2+35*a^2)/x^9/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59179, size = 135, normalized size = 1.99

$$-\frac{\left(8\,b^4x^8-4\,ab^3x^6+3\,a^2b^2x^4+50\,a^3bx^2+35\,a^4\right)\sqrt{bx^2+a}}{315\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*b^4*x^8 - 4*a*b^3*x^6 + 3*a^2*b^2*x^4 + 50*a^3*b*x^2 + 35*a^4)*sq
rt(b*x^2 + a)/(a^3*x^9)
```

Sympy [B] time = 1.97982, size = 420, normalized size = 6.18

$$-\frac{35a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{120a^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{138a^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**10,x)
```

```
[Out] -35*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*
x**10 + 315*a**3*b**6*x**12) - 120*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)
/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 138*a**
4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x
**10 + 315*a**3*b**6*x**12) - 52*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(
315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 3*a**2*b*
*(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10
 + 315*a**3*b**6*x**12) - 12*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a*
*5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 8*b**(21/2)*x**
12*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**
3*b**6*x**12)
```

Giac [B] time = 2.51133, size = 259, normalized size = 3.81

$$\frac{16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{9}{2}}+441\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{9}{2}}+126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)\right.}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(9/2) + 315*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a*b^(9/2) + 441*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(9/2
) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(9/2) + 36*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^4*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(9/2) +
 a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.386 $\int \frac{(a+bx^2)^{3/2}}{x^{12}} dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

```
[Out] -(a + b*x^2)^(5/2)/(11*a*x^11) + (2*b*(a + b*x^2)^(5/2))/(33*a^2*x^9) - (8*
b^2*(a + b*x^2)^(5/2))/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^(5/2))/(1155*a^4
*x^5)
```

Rubi [A] time = 0.0287707, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^12,x]
```


```
[Out] -(a + b*x^2)^(5/2)/(11*a*x^11) + (2*b*(a + b*x^2)^(5/2))/(33*a^2*x^9) - (8*
b^2*(a + b*x^2)^(5/2))/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^(5/2))/(1155*a^4
*x^5)
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^{12}} dx &= -\frac{(a+bx^2)^{5/2}}{11ax^{11}}-\frac{(6b)\int \frac{(a+bx^2)^{3/2}}{x^{10}} dx}{11a}\\ &= -\frac{(a+bx^2)^{5/2}}{11ax^{11}}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}+\frac{(8b^2)\int \frac{(a+bx^2)^{3/2}}{x^8} dx}{33a^2}\\ &= -\frac{(a+bx^2)^{5/2}}{11ax^{11}}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}-\frac{(16b^3)\int \frac{(a+bx^2)^{3/2}}{x^6} dx}{231a^3}\\ &= -\frac{(a+bx^2)^{5/2}}{11ax^{11}}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}\end{aligned}$$

Mathematica [A] time = 0.0132142, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{5/2}\left(70a^2bx^2-105a^3-40ab^2x^4+16b^3x^6\right)}{1155a^4x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-105*a^3 + 70*a^2*b*x^2 - 40*a*b^2*x^4 + 16*b^3*x^6))/(
1155*a^4*x^11)
```

Maple [A] time = 0.006, size = 50, normalized size = 0.5

$$-\frac{-16\,b^3x^6+40\,ab^2x^4-70\,a^2bx^2+105\,a^3}{1155\,x^{11}a^4}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^12,x)
```

```
[Out] -1/1155*(b*x^2+a)^(5/2)*(-16*b^3*x^6+40*a*b^2*x^4-70*a^2*b*x^2+105*a^3)/x^1
1/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63865, size = 163, normalized size = 1.77

$$\frac{\left(16\,b^5x^{10}-8\,ab^4x^8+6\,a^2b^3x^6-5\,a^3b^2x^4-140\,a^4bx^2-105\,a^5\right)\sqrt{bx^2+a}}{1155\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*b^5*x^10 - 8*a*b^4*x^8 + 6*a^2*b^3*x^6 - 5*a^3*b^2*x^4 - 140*a^4
*b*x^2 - 105*a^5)*sqrt(b*x^2 + a)/(a^4*x^11)
```

Sympy [B] time = 2.66182, size = 648, normalized size = 7.04

$$-\frac{105a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{455a^7b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**12,x)
```

```
[Out] -105*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*
b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 455*a**7*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x*
*12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 740*a**6*b**(23/2)*x
**4*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 34
65*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 534*a**5*b**(25/2)*x**6*sqrt
(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*
b**11*x**14 + 1155*a**4*b**12*x**16) - 145*a**4*b**(27/2)*x**8*sqrt(a/(b*x*
*2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x*
*14 + 1155*a**4*b**12*x**16) + 5*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/
(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 115
5*a**4*b**12*x**16) + 30*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(1155*a*
*7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b
**12*x**16) + 40*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**
10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
 + 16*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**
6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
```

Giac [B] time = 1.87118, size = 297, normalized size = 3.23

$$\frac{32\left(1155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{11}{2}}+2079\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{11}{2}}+2541\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{11}{2}}+825\left(\sqrt{b}x-\sqrt{bx^2\cdots\right.\right.}{1155\left(\left(\sqrt{b}x-\sqrt{bx^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(11/2) + 2079*(sqrt(b)*x -
 sqrt(b*x^2 + a))^12*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2
*b^(11/2) + 825*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(11/2) + 165*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^4*b^(11/2) - 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a
^5*b^(11/2) + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(11/2) - a^7*b^(11/2
))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^11
```

3.387 $\int x^7 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=80

$$\frac{a^2 \left(a+bx^2\right)^{9/2}}{3b^4}-\frac{a^3 \left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{3a \left(a+bx^2\right)^{11/2}}{11b^4}$$

```
[Out] -(a^3*(a + b*x^2)^(7/2))/(7*b^4) + (a^2*(a + b*x^2)^(9/2))/(3*b^4) - (3*a*(
a + b*x^2)^(11/2))/(11*b^4) + (a + b*x^2)^(13/2)/(13*b^4)
```

Rubi [A] time = 0.0467618, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{9/2}}{3b^4}-\frac{a^3 \left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{3a \left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(7/2))/(7*b^4) + (a^2*(a + b*x^2)^(9/2))/(3*b^4) - (3*a*(
a + b*x^2)^(11/2))/(11*b^4) + (a + b*x^2)^(13/2)/(13*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7 \left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{5/2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right) dx, x, x^2\right)\\ &= -\frac{a^3 \left(a+bx^2\right)^{7/2}}{7b^4}+\frac{a^2 \left(a+bx^2\right)^{9/2}}{3b^4}-\frac{3a \left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.0267787, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{7/2} \left(56a^2bx^2-16a^3-126ab^2x^4+231b^3x^6\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))/
(3003*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-231\,b^3x^6+126\,ab^2x^4-56\,a^2bx^2+16\,a^3}{3003\,b^4}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3003*(b*x^2+a)^(7/2)*(-231*b^3*x^6+126*a*b^2*x^4-56*a^2*b*x^2+16*a^3)/b^
4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54613, size = 180, normalized size = 2.25

$$\frac{\left(231\,b^6x^{12}+567\,ab^5x^{10}+371\,a^2b^4x^8+5\,a^3b^3x^6-6\,a^4b^2x^4+8\,a^5bx^2-16\,a^6\right)\sqrt{bx^2+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(231*b^6*x^12 + 567*a*b^5*x^10 + 371*a^2*b^4*x^8 + 5*a^3*b^3*x^6 - 6
*a^4*b^2*x^4 + 8*a^5*b*x^2 - 16*a^6)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 8.7502, size = 158, normalized size = 1.98

$$\begin{cases} -\frac{16a^6\sqrt{a+bx^2}}{3003b^4}+\frac{8a^5x^2\sqrt{a+bx^2}}{3003b^3}-\frac{2a^4x^4\sqrt{a+bx^2}}{1001b^2}+\frac{5a^3x^6\sqrt{a+bx^2}}{3003b}+\frac{53a^2x^8\sqrt{a+bx^2}}{429}+\frac{27abx^{10}\sqrt{a+bx^2}}{143}+\frac{b^2x^{12}\sqrt{a+bx^2}}{13} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{5}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-16*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*a**5*x**2*sqrt(a + b*x
**2)/(3003*b**3) - 2*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*a**3*x**6*s
qrt(a + b*x**2)/(3003*b) + 53*a**2*x**8*sqrt(a + b*x**2)/429 + 27*a*b*x**10
*sqrt(a + b*x**2)/143 + b**2*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(5/2
)*x**8/8, True))
```

Giac [B] time = 1.62763, size = 301, normalized size = 3.76

$$\frac{\frac{143\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)a^2}{b^3}+\frac{26\left(315\left(bx^2+a\right)^{\frac{11}{2}}-1540\left(bx^2+a\right)^{\frac{9}{2}}a+2970\left(bx^2+a\right)^{\frac{7}{2}}a^2-2772\left(bx^2+a\right)^{\frac{5}{2}}a^3+11\right.}{b^3}}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/45045*(143*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 +
 a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*a^2/b^3 + 26*(315*(b*x^2 + a)^(1
1/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2
+ a)^(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)*a/b^3 + 5*(693*(b*x^2 + a)^(13
/2) - 4095*(b*x^2 + a)^(11/2)*a + 10010*(b*x^2 + a)^(9/2)*a^2 - 12870*(b*x^
2 + a)^(7/2)*a^3 + 9009*(b*x^2 + a)^(5/2)*a^4 - 3003*(b*x^2 + a)^(3/2)*a^5)
/b^3)/b
```

3.388 $\int x^5 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}$$

```
[Out] (a^2*(a + b*x^2)^(7/2))/(7*b^3) - (2*a*(a + b*x^2)^(9/2))/(9*b^3) + (a + b*
x^2)^(11/2)/(11*b^3)
```

Rubi [A] time = 0.0338526, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(7/2))/(7*b^3) - (2*a*(a + b*x^2)^(9/2))/(9*b^3) + (a + b*
x^2)^(11/2)/(11*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{5/2}\, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}
\end{aligned}$$

Mathematica [A] time = 0.0198446, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2-28abx^2+63b^2x^4\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(8*a^2 - 28*a*b*x^2 + 63*b^2*x^4))/(693*b^3)
```

Maple [A] time = 0.004, size = 36, normalized size = 0.6

$$\frac{63\,b^2x^4-28\,abx^2+8\,a^2}{693\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/693*(b*x^2+a)^(7/2)*(63*b^2*x^4-28*a*b*x^2+8*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5803, size = 153, normalized size = 2.59

$$\frac{\left(63\,b^5x^{10}+161\,ab^4x^8+113\,a^2b^3x^6+3\,a^3b^2x^4-4\,a^4bx^2+8\,a^5\right)\sqrt{bx^2+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^5*x^10 + 161*a*b^4*x^8 + 113*a^2*b^3*x^6 + 3*a^3*b^2*x^4 - 4*a^
4*b*x^2 + 8*a^5)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 5.51374, size = 133, normalized size = 2.25

$$\begin{cases} \frac{8a^5\sqrt{a+bx^2}}{693b^3}-\frac{4a^4x^2\sqrt{a+bx^2}}{693b^2}+\frac{a^3x^4\sqrt{a+bx^2}}{231b}+\frac{113a^2x^6\sqrt{a+bx^2}}{693}+\frac{23abx^8\sqrt{a+bx^2}}{99}+\frac{b^2x^{10}\sqrt{a+bx^2}}{11} & \text{for}\: b\neq 0 \\ \frac{a^{\frac{5}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*a**4*x**2*sqrt(a + b*x**2
)/(693*b**2) + a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*a**2*x**6*sqrt(a +
```

```
b*x**2)/693 + 23*a*b*x**8*sqrt(a + b*x**2)/99 + b**2*x**10*sqrt(a + b*x**2)
/11, Ne(b, 0)), (a**(5/2)*x**6/6, True))
```

Giac [B] time = 2.9664, size = 243, normalized size = 4.12

$$\frac{\frac{33\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)a^2}{b^2}+\frac{22\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)a}{b^2}+\frac{315\left(bx^2+a\right)^{\frac{11}{2}}-1540\left(bx^2+a\right.}{}}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^
(3/2)*a^2)*a^2/b^2 + 22*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 1
89*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*a/b^2 + (315*(b*x^2 +
 a)^(11/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(
b*x^2 + a)^(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)/b^2)/b
```

3.389 $\int x^3 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}$$

```
[Out] -(a*(a + b*x^2)^(7/2))/(7*b^2) + (a + b*x^2)^(9/2)/(9*b^2)
```

Rubi [A] time = 0.0236686, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*(a + b*x^2)^(7/2))/(7*b^2) + (a + b*x^2)^(9/2)/(9*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{5/2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right) dx, x, x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{\left(a+bx^2\right)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.0161245, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(7bx^2-2a\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-2*a + 7*b*x^2))/(63*b^2)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$-\frac{-7\,bx^2+2\,a}{63\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2),x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-7*b*x^2+2*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56116, size = 122, normalized size = 3.21

$$\frac{\left(7\,b^4x^8+19\,ab^3x^6+15\,a^2b^2x^4+a^3bx^2-2\,a^4\right)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^4*x^8 + 19*a*b^3*x^6 + 15*a^2*b^2*x^4 + a^3*b*x^2 - 2*a^4)*sqrt(b
*x^2 + a)/b^2
```

Sympy [A] time = 3.53274, size = 109, normalized size = 2.87

$$\begin{cases} -\frac{2a^4\sqrt{a+bx^2}}{63b^2}+\frac{a^3x^2\sqrt{a+bx^2}}{63b}+\frac{5a^2x^4\sqrt{a+bx^2}}{21}+\frac{19abx^6\sqrt{a+bx^2}}{63}+\frac{b^2x^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a**4*sqrt(a + b*x**2)/(63*b**2) + a**3*x**2*sqrt(a + b*x**2)/
(63*b) + 5*a**2*x**4*sqrt(a + b*x**2)/21 + 19*a*b*x**6*sqrt(a + b*x**2)/63
+ b**2*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Giac [B] time = 2.6514, size = 186, normalized size = 4.89

$$\frac{\frac{21\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)a^2}{b}+\frac{6\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)a}{b}+\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3}{b}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*a^2/b + 6*(15*(b*x^
2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*a/b + (35
*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 -
105*(b*x^2 + a)^(3/2)*a^3)/b)/b
```

3.390 $\int x\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

Rubi [A] time = 0.0034692, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Mathematica [A] time = 0.0045158, size = 18, normalized size = 1.

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{1}{7b}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*(b*x^2+a)^(7/2)/b
```

Maxima [A] time = 1.94114, size = 19, normalized size = 1.06

$$\frac{\left(bx^2 + a\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

Fricas [B] time = 1.58128, size = 90, normalized size = 5.

$$\frac{\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\sqrt{bx^2 + a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/b
```

Sympy [A] time = 1.94809, size = 85, normalized size = 4.72

$$\begin{cases} \frac{a^3\sqrt{a+bx^2}}{7b} + \frac{3a^2x^2\sqrt{a+bx^2}}{7} + \frac{3abx^4\sqrt{a+bx^2}}{7} + \frac{b^2x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((a**3*sqrt(a + b*x**2)/(7*b) + 3*a**2*x**2*sqrt(a + b*x**2)/7 + 3
*a*b*x**4*sqrt(a + b*x**2)/7 + b**2*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (a*
*(5/2)*x**2/2, True))
```

Giac [B] time = 1.67839, size = 95, normalized size = 5.28

$$\frac{15\left(bx^2 + a\right)^{\frac{7}{2}} - 42\left(bx^2 + a\right)^{\frac{5}{2}}a + 70\left(bx^2 + a\right)^{\frac{3}{2}}a^2 + 14\left(3\left(bx^2 + a\right)^{\frac{5}{2}} - 5\left(bx^2 + a\right)^{\frac{3}{2}}a\right)a}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 70*(b*x^2 + a)^(3/2)
*a^2 + 14*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*a)/b
```

3.391 $\int \frac{(a+bx^2)^{5/2}}{x} dx$

Optimal. Leaf size=72

$$a^2\sqrt{a+bx^2}+a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

```
[Out] a^2*Sqrt[a + b*x^2] + (a*(a + b*x^2)^(3/2))/3 + (a + b*x^2)^(5/2)/5 - a^(5/
2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0434574, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^2\sqrt{a+bx^2}+a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] a^2*Sqrt[a + b*x^2] + (a*(a + b*x^2)^(3/2))/3 + (a + b*x^2)^(5/2)/5 - a^(5/
2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int\frac{(a+bx^2)^{5/2}}{x}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int\frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)\\&=\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{1}{2}a\operatorname{Subst}\left(\int\frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\&=\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{1}{2}a^2\operatorname{Subst}\left(\int\frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\&=a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{1}{2}a^3\operatorname{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\&=a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{a^3\operatorname{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\&=a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.0267292, size = 62, normalized size = 0.86

$$\frac{1}{15}\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]
```

Maple [A] time = 0.005, size = 66, normalized size = 0.9

$$\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a}{3}\left(bx^2+a\right)^{\frac{3}{2}}-a^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+a^2\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)+1/3*a*(b*x^2+a)^(3/2)-a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)+a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.589, size = 311, normalized size = 4.32

$$\left[\frac{1}{2}\,a^{\frac{5}{2}}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a},\ \sqrt{-a}a^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{15}\left(3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(5/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/15*(3*
b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a), sqrt(-a)*a^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + 1/15*(3*b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a)
]
```

Sympy [A] time = 3.40694, size = 105, normalized size = 1.46

$$\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{15}+\frac{a^{\frac{5}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{11a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{15}+\frac{\sqrt{a}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x,x)
```

```
[Out] 23*a**(5/2)*sqrt(1 + b*x**2/a)/15 + a**(5/2)*log(b*x**2/a)/2 - a**(5/2)*log
(sqrt(1 + b*x**2/a) + 1) + 11*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)/15 + sqrt(
a)*b**2*x**4*sqrt(1 + b*x**2/a)/5
```

Giac [A] time = 2.772, size = 84, normalized size = 1.17

$$\frac{a^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a+\sqrt{bx^2+a}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/5*(b*x^2 + a)^(5/2) + 1/3
*(b*x^2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

3.392 $\int \frac{(a+bx^2)^{5/2}}{x^3} \, dx$

Optimal. Leaf size=80

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{5/2}}{2x^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}+\frac{5}{2}ab\sqrt{a+bx^2}$$

```
[Out] (5*a*b*Sqrt[a + b*x^2])/2 + (5*b*(a + b*x^2)^(3/2))/6 - (a + b*x^2)^(5/2)/(
2*x^2) - (5*a^(3/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.0449643, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{5/2}}{2x^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}+\frac{5}{2}ab\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (5*a*b*Sqrt[a + b*x^2])/2 + (5*b*(a + b*x^2)^(3/2))/6 - (a + b*x^2)^(5/2)/(
2*x^2) - (5*a^(3/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{4}(5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{4}(5ab)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{4}\left(5a^2b\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{2}\left(5a^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0094557, size = 37, normalized size = 0.46

$$\frac{b\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (b*x^2)/a])/(7*a^2)
```

Maple [A] time = 0.006, size = 88, normalized size = 1.1

$$-\frac{1}{2\,a x^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{b}{2\,a}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,b}{6}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,b}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+\frac{5\,ab}{2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(7/2)+1/2*b/a*(b*x^2+a)^(5/2)+5/6*b*(b*x^2+a)^(3/2)-5/
2*b*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/2*a*b*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62712, size = 342, normalized size = 4.28

$$\left[\frac{15\,a^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,b^2x^4+14\,abx^2-3\,a^2\right)\sqrt{bx^2+a}}{12\,x^2},\ \frac{15\,\sqrt{-a}abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,b^2x^4+14\,ab\right.}{6\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(2*b^2*x^4 + 14*a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a))/x^2, 1/6*(15*sqrt(-a
)*a*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*b^2*x^4 + 14*a*b*x^2 - 3*a^
2)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] time = 3.23765, size = 112, normalized size = 1.4

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{bx^2}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}+\frac{\sqrt{a}b^2x^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 7*a**(3/2)*b*sqrt(1 + b*x**2/a)/3 +
 5*a**(3/2)*b*log(b*x**2/a)/4 - 5*a**(3/2)*b*log(sqrt(1 + b*x**2/a) + 1)/2
+ sqrt(a)*b**2*x**2*sqrt(1 + b*x**2/a)/3
```

Giac [A] time = 2.0765, size = 99, normalized size = 1.24

$$\frac{1}{6}\left(\frac{15\,a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\left(bx^2+a\right)^{\frac{3}{2}}+12\,\sqrt{bx^2+a}a-\frac{3\,\sqrt{bx^2+a}a^2}{bx^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*(b*x^2 + a)^(3/2)
 + 12*sqrt(b*x^2 + a)*a - 3*sqrt(b*x^2 + a)*a^2/(b*x^2))*b
```

3.393 $\int \frac{(a+bx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=86

$$\frac{15}{8}b^2\sqrt{a+bx^2}-\frac{15}{8}\sqrt{a}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{5/2}}{4x^4}-\frac{5b(a+bx^2)^{3/2}}{8x^2}$$

```
[Out] (15*b^2*Sqrt[a + b*x^2])/8 - (5*b*(a + b*x^2)^(3/2))/(8*x^2) - (a + b*x^2)^
(5/2)/(4*x^4) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rubi [A] time = 0.0481995, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{15}{8}b^2\sqrt{a+bx^2}-\frac{15}{8}\sqrt{a}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{5/2}}{4x^4}-\frac{5b(a+bx^2)^{3/2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x^2])/8 - (5*b*(a + b*x^2)^(3/2))/(8*x^2) - (a + b*x^2)^
(5/2)/(4*x^4) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{8}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{16}\left(15b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{16}\left(15ab^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{8}(15ab)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} - \frac{15}{8}\sqrt{a}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0097769, size = 39, normalized size = 0.45

$$
-\frac{b^2\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] -(b^2*(a + b*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + (b*x^2)/a])/(7*a
^3)
```

Maple [A] time = 0.006, size = 116, normalized size = 1.4

$$
-\frac{1}{4\,a x^4}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{3\,b}{8\,a^2x^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{3\,b^2}{8\,a^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,b^2}{8\,a}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{15\,b^2}{8}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{15}{8}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(7/2)-3/8*b/a^2/x^2*(b*x^2+a)^(7/2)+3/8*b^2/a^2*(b*x^2
+a)^(5/2)+5/8*b^2/a*(b*x^2+a)^(3/2)-15/8*b^2*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)+15/8*b^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60684, size = 342, normalized size = 3.98

$$\left[\frac{15\sqrt{a}b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(8b^2x^4-9abx^2-2a^2\right)\sqrt{bx^2+a}}{16x^4}, \frac{15\sqrt{-a}b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(8b^2x^4-9\right.}{8x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(15*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4, 1/8*(15*sqrt(-
a)*b^2*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*b^2*x^4 - 9*a*b*x^2 - 2*a^
2)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A] time = 3.84351, size = 117, normalized size = 1.36

$$-\frac{15\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8}-\frac{a^3}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{11a^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**3/(4*sqrt(b)*x**5*sqrt(a
/(b*x**2) + 1)) - 11*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - a*b**(3/2
)/(8*x*sqrt(a/(b*x**2) + 1)) + b**(5/2)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A] time = 3.23042, size = 103, normalized size = 1.2

$$\frac{1}{8}\left(\frac{15a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{bx^2+a}-\frac{9\left(bx^2+a\right)^{\frac{3}{2}}a-7\sqrt{bx^2+a}a^2}{b^2x^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(15*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x^2 + a) - (
9*(b*x^2 + a)^(3/2)*a - 7*sqrt(b*x^2 + a)*a^2)/(b^2*x^4))*b^2
```

3.394 $\int \frac{(a+bx^2)^{5/2}}{x^7} dx$

Optimal. Leaf size=89

$$-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4}-\frac{\left(a+bx^2\right)^{5/2}}{6x^6}$$

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a +
 b*x^2)^(5/2)/(6*x^6) - (5*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a
])
```

Rubi [A] time = 0.0525926, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4}-\frac{\left(a+bx^2\right)^{5/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a +
 b*x^2)^(5/2)/(6*x^6) - (5*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a
])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^2\right)^{5/2}}{6x^6} + \frac{1}{12}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4} - \frac{\left(a+bx^2\right)^{5/2}}{6x^6} + \frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{24x^4} - \frac{\left(a+bx^2\right)^{5/2}}{6x^6} + \frac{1}{32}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{24x^4} - \frac{\left(a+bx^2\right)^{5/2}}{6x^6} + \frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right) \\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{24x^4} - \frac{\left(a+bx^2\right)^{5/2}}{6x^6} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0398797, size = 87, normalized size = 0.98

$$
-\frac{34a^2bx^2+8a^3+59ab^2x^4+15b^3x^6\sqrt{\frac{bx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+33b^3x^6}{48x^6\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] -(8*a^3 + 34*a^2*b*x^2 + 59*a*b^2*x^4 + 33*b^3*x^6 + 15*b^3*x^6*Sqrt[1 + (b
*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(48*x^6*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 139, normalized size = 1.6

$$
-\frac{1}{6\,ax^6}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{b}{24\,a^2x^4}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{b^2}{16\,a^3x^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{b^3}{16\,a^3}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,b^3}{48\,a^2}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5\,b^3}{16}\ln\left(\frac{1}{x}\left(2\,a\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(7/2)-1/24*b/a^2/x^4*(b*x^2+a)^(7/2)-1/16*b^2/a^3/x^2*
(b*x^2+a)^(7/2)+1/16*b^3/a^3*(b*x^2+a)^(5/2)+5/48*b^3/a^2*(b*x^2+a)^(3/2)-5
/16*b^3/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/16*b^3/a*(b*x^2+a)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.61243, size = 370, normalized size = 4.16

$$\left[\frac{15\,\sqrt{a}b^3x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(33\,ab^2x^4+26\,a^2bx^2+8\,a^3\right)\sqrt{bx^2+a}}{96\,ax^6},\ \frac{15\,\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(33\,ab^2x^4\right.}{48\,ax^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="fricas")

[Out] [1/96*(15*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(33*a*b^2*x^4 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6), 1/48*(15*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (33*a*b^2*x^4 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6)]

Sympy [A] time = 4.75573, size = 99, normalized size = 1.11

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{6x^5}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{24x^3}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{16x}-\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(5/2)/x**7,x)

[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(6*x**5) - 13*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(24*x**3) - 11*b**(5/2)*sqrt(a/(b*x**2) + 1)/(16*x) - 5*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a))

Giac [A] time = 1.72511, size = 101, normalized size = 1.13

$$\frac{1}{48}\,b^3\left(\frac{15\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33\left(bx^2+a\right)^{\frac{5}{2}}-40\left(bx^2+a\right)^{\frac{3}{2}}a+15\,\sqrt{bx^2+a}a^2}{b^3x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="giac")

[Out] 1/48*b^3*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (33*(b*x^2 + a)^(5/2) - 40*(b*x^2 + a)^(3/2)*a + 15*sqrt(b*x^2 + a)*a^2)/(b^3*x^6))

3.395 $\int \frac{(a+bx^2)^{5/2}}{x^9} dx$

Optimal. Leaf size=113

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8}$$

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(64*x^4) - (5*b^3*Sqrt[a + b*x^2])/(128*a*x^2) - (
5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rubi [A] time = 0.0672193, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(64*x^4) - (5*b^3*Sqrt[a + b*x^2])/(128*a*x^2) - (
5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^{5/2}}{8x^8} + \frac{1}{16}(5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} + \frac{1}{32}(5b^2)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} + \frac{1}{128}(5b^3)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{(5b^4)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{256a}$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{(5b^3)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+
$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} + \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

Mathematica [C] time = 0.0101187, size = 39, normalized size = 0.35

$$-\frac{b^4(a+bx^2)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] -(b^4*(a + b*x^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + (b*x^2)/a])/(7*a
^5)
```

Maple [A] time = 0.013, size = 159, normalized size = 1.4

$$-\frac{1}{8ax^8}(bx^2+a)^{\frac{7}{2}} + \frac{b}{48a^2x^6}(bx^2+a)^{\frac{7}{2}} + \frac{b^2}{192a^3x^4}(bx^2+a)^{\frac{7}{2}} + \frac{b^3}{128a^4x^2}(bx^2+a)^{\frac{7}{2}} - \frac{b^4}{128a^4}(bx^2+a)^{\frac{5}{2}} - \frac{5b^4}{384a^3}(bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(7/2)+1/48*b/a^2/x^6*(b*x^2+a)^(7/2)+1/192*b^2/a^3/x^4
*(b*x^2+a)^(7/2)+1/128*b^3/a^4/x^2*(b*x^2+a)^(7/2)-1/128*b^4/a^4*(b*x^2+a)^
(5/2)-5/384*b^4/a^3*(b*x^2+a)^(3/2)+5/128*b^4/a^(3/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)-5/128*b^4/a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6166, size = 433, normalized size = 3.83

$$\left[\frac{15\,\sqrt{a}b^4x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(15\,ab^3x^6+118\,a^2b^2x^4+136\,a^3bx^2+48\,a^4\right)\sqrt{bx^2+a}}{768\,a^2x^8},-\frac{15\,\sqrt{-a}b^4x^8\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/768*(15*sqrt(a)*b^4*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^
2 + a))/(a^2*x^8), -1/384*(15*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 +
 a)) + (15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^2
 + a))/(a^2*x^8)]
```

Sympy [A] time = 8.15153, size = 150, normalized size = 1.33

$$-\frac{a^3}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{127ab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{133b^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**9,x)
```

```
[Out] -a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*a**2*sqrt(b)/(48*x**7*sqrt
(a/(b*x**2) + 1)) - 127*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) - 133*b*
*(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(7/2)/(128*a*x*sqrt(a/(b*x**2
) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2))
```

Giac [A] time = 2.53279, size = 127, normalized size = 1.12

$$-\frac{1}{384}\,b^4\left(\frac{15\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}+73\left(bx^2+a\right)^{\frac{5}{2}}a-55\left(bx^2+a\right)^{\frac{3}{2}}a^2+15\sqrt{bx^2+a}a^3}{ab^4x^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*b^4*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x^2 +
 a)^(7/2) + 73*(b*x^2 + a)^(5/2)*a - 55*(b*x^2 + a)^(3/2)*a^2 + 15*sqrt(b*x
^2 + a)*a^3)/(a*b^4*x^8))
```

3.396 $\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=137

$$\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}$$

```
[Out] -(b^2*Sqrt[a + b*x^2])/(32*x^6) - (b^3*Sqrt[a + b*x^2])/(128*a*x^4) + (3*b^
4*Sqrt[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*
x^2)^(5/2)/(10*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2
))
```

Rubi [A] time = 0.0834827, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] -(b^2*Sqrt[a + b*x^2])/(32*x^6) - (b^3*Sqrt[a + b*x^2])/(128*a*x^4) + (3*b^
4*Sqrt[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*
x^2)^(5/2)/(10*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2
))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)^{5/2}}{10x^{10}}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}+\frac{1}{32}(3b^2)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}+\frac{1}{64}b^3\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}-\frac{(3b^4)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{256a}\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}+\frac{(3b^5)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.}{512a^2}\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}+\frac{(3b^4)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x}{b}}\right.}{256a}\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0102124, size = 39, normalized size = 0.28

$$\frac{b^5(a+bx^2)^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] (b^5*(a + b*x^2)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, 1 + (b*x^2)/a])/(7*a^
6)
```

Maple [A] time = 0.029, size = 179, normalized size = 1.3

$$-\frac{1}{10\,ax^{10}}(bx^2+a)^{\frac{7}{2}}+\frac{3\,b}{80\,a^2x^8}(bx^2+a)^{\frac{7}{2}}-\frac{b^2}{160\,a^3x^6}(bx^2+a)^{\frac{7}{2}}-\frac{b^3}{640\,a^4x^4}(bx^2+a)^{\frac{7}{2}}-\frac{3\,b^4}{1280\,a^5x^2}(bx^2+a)^{\frac{7}{2}}+\frac{3\,b^5}{1280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^11,x)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(7/2)+3/80*b/a^2/x^8*(b*x^2+a)^(7/2)-1/160*b^2/a^3/x
^6*(b*x^2+a)^(7/2)-1/640*b^3/a^4/x^4*(b*x^2+a)^(7/2)-3/1280*b^4/a^5/x^2*(b*
x^2+a)^(7/2)+3/1280*b^5/a^5*(b*x^2+a)^(5/2)+1/256*b^5/a^4*(b*x^2+a)^(3/2)-3
/256*b^5/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/256*b^5/a^3*(b*x^2
+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7218, size = 489, normalized size = 3.57

$$\left[\frac{15\,\sqrt{a}b^5x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\,a^4bx^2-128\,a^5\right)\sqrt{bx^2+a}}{2560\,a^3x^{10}},\ 15\,\sqrt{-ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x^4 - 336*a^4*b*x^2
- 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10), 1/1280*(15*sqrt(-a)*b^5*x^10*arctan
(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x
^4 - 336*a^4*b*x^2 - 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

Sympy [A] time = 11.7643, size = 175, normalized size = 1.28

$$-\frac{a^3}{10\sqrt{b}x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{29a^2\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{73ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{129b^{\frac{5}{2}}}{640x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{7}{2}}}{256ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{9}{2}}}{256a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**11,x)
```

```
[Out] -a**3/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 29*a**2*sqrt(b)/(80*x**9*sq
rt(a/(b*x**2) + 1)) - 73*a*b**(3/2)/(160*x**7*sqrt(a/(b*x**2) + 1)) - 129*b
**(5/2)/(640*x**5*sqrt(a/(b*x**2) + 1)) + b**(7/2)/(256*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*b**(9/2)/(256*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**5*asinh(sqr
```

```
t(a)/(sqrt(b)*x))/(256*a**(5/2))
```

Giac [A] time = 1.8112, size = 146, normalized size = 1.07

$$\frac{1}{1280}b^5\left(\frac{15\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{15\left(bx^2+a\right)^{\frac{9}{2}}-70\left(bx^2+a\right)^{\frac{7}{2}}a-128\left(bx^2+a\right)^{\frac{5}{2}}a^2+70\left(bx^2+a\right)^{\frac{3}{2}}a^3-15\sqrt{bx^2+a}a^4}{a^2b^5x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*b^5*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (15*(b*x^2
 + a)^(9/2) - 70*(b*x^2 + a)^(7/2)*a - 128*(b*x^2 + a)^(5/2)*a^2 + 70*(b*x^
2 + a)^(3/2)*a^3 - 15*sqrt(b*x^2 + a)*a^4)/(a^2*b^5*x^10))
```

3.397 $\int x^4 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=136

$$-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

```
[Out] (-3*a^4*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*x^3*Sqrt[a + b*x^2])/(128*b) +
(a^2*x^5*Sqrt[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x
^2)^(5/2))/10 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rubi [A] time = 0.0536311, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^4*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*x^3*Sqrt[a + b*x^2])/(128*b) +
(a^2*x^5*Sqrt[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x
^2)^(5/2))/10 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^{5/2}dx &= \frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{2}a\int x^4\left(a+bx^2\right)^{3/2}dx\\ &= \frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^4\sqrt{a+bx^2}\,dx\\ &= \frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{32}a^3\int\frac{x^4}{\sqrt{a+bx^2}}dx\\ &= \frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}-\frac{\left(3a^4\right)\int\frac{x^2}{\sqrt{a+bx^2}}dx}{128b}\\ &= -\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\\ &= -\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\\ &= -\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\end{aligned}$$

Mathematica [A] time = 0.149906, size = 105, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(248a^2b^2x^4+10a^3bx^2-15a^4+336ab^3x^6+128b^4x^8\right)+\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{1280b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-15*a^4 + 10*a^3*b*x^2 + 248*a^2*b^2*x^4 + 336
*a*b^3*x^6 + 128*b^4*x^8) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[
1 + (b*x^2)/a]))/(1280*b^(5/2))
```

Maple [A] time = 0.006, size = 113, normalized size = 0.8

$$\frac{x^3}{10\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3\,ax}{80\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{a^2x}{160\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a^3x}{128\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,a^4x}{256\,b^2}\sqrt{bx^2+a}+\frac{3\,a^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/10*x^3*(b*x^2+a)^(7/2)/b-3/80/b^2*a*x*(b*x^2+a)^(7/2)+1/160/b^2*a^2*x*(b*
x^2+a)^(5/2)+1/128/b^2*a^3*x*(b*x^2+a)^(3/2)+3/256*a^4*x*(b*x^2+a)^(1/2)/b^
2+3/256/b^(5/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71064, size = 460, normalized size = 3.38

$$\left[\frac{15\,a^{5}\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\,\left(128\,b^{5}x^{9}+336\,ab^{4}x^{7}+248\,a^{2}b^{3}x^{5}+10\,a^{3}b^{2}x^{3}-15\,a^{4}bx\right)\sqrt{bx^{2}+a}}{2560\,b^{3}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^3 - 15*a^4*b
*x)*sqrt(b*x^2 + a))/b^3, -1/1280*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^
3 - 15*a^4*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 10.927, size = 175, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}x}{256b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{a^{\frac{7}{2}}x^{3}}{256b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{129a^{\frac{5}{2}}x^{5}}{640\sqrt{1+\frac{bx^{2}}{a}}}+\frac{73a^{\frac{3}{2}}bx^{7}}{160\sqrt{1+\frac{bx^{2}}{a}}}+\frac{29\sqrt{a}b^{2}x^{9}}{80\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{5}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{b^{3}x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - a**(7/2)*x**3/(256*b*sqrt(1 +
 b*x**2/a)) + 129*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*x*
*7/(160*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a))
+ 3*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + b**3*x**11/(10*sqrt(a)*s
qrt(1 + b*x**2/a))
```

Giac [A] time = 2.34175, size = 123, normalized size = 0.9

$$-\frac{3\,a^{5}\log\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{256\,b^{\frac{5}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,b^{2}x^{2}+21\,ab\right)x^{2}+31\,a^{2}\right)x^{2}+\frac{5\,a^{3}}{b}\right)x^{2}-\frac{15\,a^{4}}{b^{2}}\right)\sqrt{bx^{2}+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/1280*(2*(4*(2
*(8*b^2*x^2 + 21*a*b)*x^2 + 31*a^2)*x^2 + 5*a^3/b)*x^2 - 15*a^4/b^2)*sqrt(b
*x^2 + a)*x
```

3.398 $\int x^2 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=112

$$-\frac{5a^4 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

```
[Out] (5*a^3*x*Sqrt[a + b*x^2])/(128*b) + (5*a^2*x^3*Sqrt[a + b*x^2])/64 + (5*a*x
^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.0423354, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{5a^4 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^3*x*Sqrt[a + b*x^2])/(128*b) + (5*a^2*x^3*Sqrt[a + b*x^2])/64 + (5*a*x
^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2 \left(a+bx^2\right)^{5/2} dx = \frac{1}{8}x^3 \left(a+bx^2\right)^{5/2} + \frac{1}{8}(5a)\int x^2 \left(a+bx^2\right)^{3/2} dx$$
$$= \frac{5}{48}ax^3 \left(a+bx^2\right)^{3/2} + \frac{1}{8}x^3 \left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^2\right)\int x^2\sqrt{a+bx^2}\,dx$$
$$= \frac{5}{64}a^2x^3\sqrt{a+bx^2} + \frac{5}{48}ax^3 \left(a+bx^2\right)^{3/2} + \frac{1}{8}x^3 \left(a+bx^2\right)^{5/2} + \frac{1}{64}\left(5a^3\right)\int \frac{x^2}{\sqrt{a+bx^2}}dx$$
$$= \frac{5a^3x\sqrt{a+bx^2}}{128b} + \frac{5}{64}a^2x^3\sqrt{a+bx^2} + \frac{5}{48}ax^3 \left(a+bx^2\right)^{3/2} + \frac{1}{8}x^3 \left(a+bx^2\right)^{5/2} - \frac{\left(5a^4\right)\int \frac{1}{\sqrt{a+bx^2}}d}{128b}$$
$$= \frac{5a^3x\sqrt{a+bx^2}}{128b} + \frac{5}{64}a^2x^3\sqrt{a+bx^2} + \frac{5}{48}ax^3 \left(a+bx^2\right)^{3/2} + \frac{1}{8}x^3 \left(a+bx^2\right)^{5/2} - \frac{\left(5a^4\right)\mathrm{Subst}\left(\int\right.}{12}$$
$$= \frac{5a^3x\sqrt{a+bx^2}}{128b} + \frac{5}{64}a^2x^3\sqrt{a+bx^2} + \frac{5}{48}ax^3 \left(a+bx^2\right)^{3/2} + \frac{1}{8}x^3 \left(a+bx^2\right)^{5/2} - \frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.}{128b^{3/2}}$$

Mathematica [A] time = 0.131013, size = 94, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(118a^2bx^2+15a^3+136ab^2x^4+48b^3x^6\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3 + 118*a^2*b*x^2 + 136*a*b^2*x^4 + 48*b^
3*x^6) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(3
84*b^(3/2))
```

Maple [A] time = 0.004, size = 93, normalized size = 0.8

$$\frac{x}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{ax}{48\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,a^2x}{192\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,a^3x}{128\,b}\sqrt{bx^2+a}-\frac{5\,a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*x*(b*x^2+a)^(7/2)/b-1/48/b*a*x*(b*x^2+a)^(5/2)-5/192/b*a^2*x*(b*x^2+a)^
(3/2)-5/128*a^3*x*(b*x^2+a)^(1/2)/b-5/128/b^(3/2)*a^4*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68193, size = 408, normalized size = 3.64

$$\left[\frac{15\,a^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(48\,b^4x^7+136\,ab^3x^5+118\,a^2b^2x^3+15\,a^3bx\right)\sqrt{bx^2+a}}{768\,b^2},\frac{15\,a^4\sqrt{-b}\,\mathrm{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(48*b^4*x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a)
)/b^2, 1/384*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*
x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 7.12454, size = 150, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{b^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*x**3/(384*sqrt(1 + b
*x**2/a)) + 127*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**2*
x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2
)) + b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.60728, size = 104, normalized size = 0.93

$$\frac{5\,a^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{384}\left(2\left(4\left(6\,b^2x^2+17\,ab\right)x^2+59\,a^2\right)x^2+\frac{15\,a^3}{b}\right)\sqrt{bx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/384*(2*(4*(6*b
^2*x^2 + 17*a*b)*x^2 + 59*a^2)*x^2 + 15*a^3/b)*sqrt(b*x^2 + a)*x
```

3.399 $\int \left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] time = 0.0216615, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2} dx = \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2} dx$$
$$= \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}}dx$$
$$= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{x}{\sqrt{a+bx^2}}\right.$$
$$= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

Mathematica [A] time = 0.106692, size = 76, normalized size = 0.9

$$\frac{1}{48}\sqrt{a+bx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} + 33a^2x + 26abx^3 + 8b^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/48
```

Maple [A] time = 0.003, size = 66, normalized size = 0.8

$$\frac{x}{6}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,ax}{24}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5\,a^2x}{16}\sqrt{bx^2+a} + \frac{5\,a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/24*a*x*(b*x^2+a)^(3/2)+5/16*a^2*x*(b*x^2+a)^(1/2)+5
/16*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.594, size = 347, normalized size = 4.13

$$\left[\frac{15\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx\right)\sqrt{bx^2+a}}{96\,b},-\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 4.23791, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

Giac [A] time = 1.71163, size = 85, normalized size = 1.01

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

3.400 $\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=83

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{x}+\frac{5}{4}bx(a+bx^2)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

```
[Out] (15*a*b*x*Sqrt[a + b*x^2])/8 + (5*b*x*(a + b*x^2)^(3/2))/4 - (a + b*x^2)^(5
/2)/x + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rubi [A] time = 0.0250642, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{x}+\frac{5}{4}bx(a+bx^2)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (15*a*b*x*Sqrt[a + b*x^2])/8 + (5*b*x*(a + b*x^2)^(3/2))/4 - (a + b*x^2)^(5
/2)/x + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = -\frac{(a+bx^2)^{5/2}}{x} + (5b)\int (a+bx^2)^{3/2}\,dx$$
$$= \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{4}(15ab)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{8}(15a^2b)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{8}(15a^2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.0075302, size = 52, normalized size = 0.63

$$-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a]))
```

Maple [A] time = 0.004, size = 85, normalized size = 1.

$$-\frac{1}{ax}(bx^2+a)^{\frac{7}{2}} + \frac{bx}{a}(bx^2+a)^{\frac{5}{2}} + \frac{5\,bx}{4}(bx^2+a)^{\frac{3}{2}} + \frac{15\,abx}{8}\sqrt{bx^2+a} + \frac{15\,a^2}{8}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(7/2)+b/a*x*(b*x^2+a)^(5/2)+5/4*b*x*(b*x^2+a)^(3/2)+15/8*a
*b*x*(b*x^2+a)^(1/2)+15/8*b^(1/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57637, size = 329, normalized size = 3.96

$$\left[\frac{15\,a^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2x^4+9\,abx^2-8\,a^2\right)\sqrt{bx^2+a}}{16\,x}, -\frac{15\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2\right.}{8\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(15*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a))/x, -1/8*(15*a^2*sqrt(-b)*
x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt
(b*x^2 + a))/x]
```

Sympy [A] time = 3.67323, size = 117, normalized size = 1.41

$$-\frac{a^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{b^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)/(x*sqrt(1 + b*x**2/a)) + a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 11
*sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*a**2*sqrt(b)*asinh(sqrt(b)*x
/sqrt(a))/8 + b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.86719, size = 117, normalized size = 1.41

$$-\frac{15}{16}\,a^2\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{8}\left(2\,b^2x^2+9\,ab\right)\sqrt{bx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*a^2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^3*sqrt(b)/((s
qrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*b^2*x^2 + 9*a*b)*sqrt(b*x^2 + a
)*x
```

3.401 $\int \frac{(a+bx^2)^{5/2}}{x^4} dx$

Optimal. Leaf size=86

$$\frac{5}{2}b^2x\sqrt{a+bx^2}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}-\frac{5b(a+bx^2)^{3/2}}{3x}$$

```
[Out] (5*b^2*x*Sqrt[a + b*x^2])/2 - (5*b*(a + b*x^2)^(3/2))/(3*x) - (a + b*x^2)^(
5/2)/(3*x^3) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi [A] time = 0.0270558, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{5}{2}b^2x\sqrt{a+bx^2}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}-\frac{5b(a+bx^2)^{3/2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (5*b^2*x*Sqrt[a + b*x^2])/2 - (5*b*(a + b*x^2)^(3/2))/(3*x) - (a + b*x^2)^(
5/2)/(3*x^3) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx = -\frac{\left(a+bx^2\right)^{5/2}}{3x^3} + \frac{1}{3}(5b)\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx$$
$$= -\frac{5b\left(a+bx^2\right)^{3/2}}{3x} - \frac{\left(a+bx^2\right)^{5/2}}{3x^3} + \left(5b^2\right)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{5}{2}b^2x\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{3x} - \frac{\left(a+bx^2\right)^{5/2}}{3x^3} + \frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{5}{2}b^2x\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{3x} - \frac{\left(a+bx^2\right)^{5/2}}{3x^3} + \frac{1}{2}\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{5}{2}b^2x\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{3x} - \frac{\left(a+bx^2\right)^{5/2}}{3x^3} + \frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.0076101, size = 54, normalized size = 0.63

$$-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] -(a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x^2)/a)])/(3
*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.006, size = 110, normalized size = 1.3

$$-\frac{1}{3\,ax^3}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{4\,b}{3\,a^2x}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{4\,b^2x}{3\,a^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,b^2x}{3\,a}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5\,b^2x}{2}\sqrt{bx^2+a} + \frac{5\,a}{2}b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(7/2)-4/3*b/a^2/x*(b*x^2+a)^(7/2)+4/3*b^2/a^2*x*(b*x^2
+a)^(5/2)+5/3*b^2/a*x*(b*x^2+a)^(3/2)+5/2*b^2*x*(b*x^2+a)^(1/2)+5/2*b^(3/2)
*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60384, size = 340, normalized size = 3.95

$$\left[\frac{15\,ab^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(3\,b^2x^4-14\,abx^2-2\,a^2\right)\sqrt{bx^2+a}}{12\,x^3},-\frac{15\,a\sqrt{-b}bx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^3, -1/6*(15*a*sqrt(-b)
*b*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*b^2*x^4 - 14*a*b*x^2 - 2*a^2
)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] time = 3.22183, size = 112, normalized size = 1.3

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{7ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{5}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 7*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 - 5*a*b**(3/2)*log(a/(b*x**2))/4 + 5*a*b**(3/2)*log(sqrt(a/(b*x**2)
+ 1) + 1)/2 + b**(5/2)*x**2*sqrt(a/(b*x**2) + 1)/2
```

Giac [A] time = 2.50965, size = 178, normalized size = 2.07

$$\frac{1}{2}\sqrt{bx^2+a}b^2x-\frac{5}{4}ab^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{3}{2}}+7\,a^4b\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x - 5/4*a*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2) - 12*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a^3*b^(3/2) + 7*a^4*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^3
```

3.402 $\int \frac{(a+bx^2)^{5/2}}{x^6} dx$

Optimal. Leaf size=82

$$-\frac{b^2\sqrt{a+bx^2}}{x}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

```
[Out] -((b^2*Sqrt[a + b*x^2])/x) - (b*(a + b*x^2)^(3/2))/(3*x^3) - (a + b*x^2)^(5
/2)/(5*x^5) + b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0280164, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 206}

$$-\frac{b^2\sqrt{a+bx^2}}{x}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -((b^2*Sqrt[a + b*x^2])/x) - (b*(a + b*x^2)^(3/2))/(3*x^3) - (a + b*x^2)^(5
/2)/(5*x^5) + b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = -\frac{(a+bx^2)^{5/2}}{5x^5} + b\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$$
$$= -\frac{b(a+bx^2)^{3/2}}{3x^3} - \frac{(a+bx^2)^{5/2}}{5x^5} + b^2\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{b^2\sqrt{a+bx^2}}{x} - \frac{b(a+bx^2)^{3/2}}{3x^3} - \frac{(a+bx^2)^{5/2}}{5x^5} + b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{b^2\sqrt{a+bx^2}}{x} - \frac{b(a+bx^2)^{3/2}}{3x^3} - \frac{(a+bx^2)^{5/2}}{5x^5} + b^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{b^2\sqrt{a+bx^2}}{x} - \frac{b(a+bx^2)^{3/2}}{3x^3} - \frac{(a+bx^2)^{5/2}}{5x^5} + b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.0093549, size = 54, normalized size = 0.66

$$-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -(a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, -((b*x^2)/a)])/(5
*x^5*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.007, size = 130, normalized size = 1.6

$$-\frac{1}{5\,ax^5}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{2\,b}{15\,a^2x^3}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{8\,b^2}{15\,a^3x}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{8\,b^3x}{15\,a^3}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{2\,b^3x}{3\,a^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{b^3x}{a}\sqrt{bx^2+a} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^6,x)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(7/2)-2/15*b/a^2/x^3*(b*x^2+a)^(7/2)-8/15*b^2/a^3/x*(b
*x^2+a)^(7/2)+8/15*b^3/a^3*x*(b*x^2+a)^(5/2)+2/3*b^3/a^2*x*(b*x^2+a)^(3/2)+
b^3/a*x*(b*x^2+a)^(1/2)+b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58854, size = 342, normalized size = 4.17

$$\left[\frac{15\,b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(23\,b^2x^4+11\,abx^2+3\,a^2\right)\sqrt{bx^2+a}}{30\,x^5},-\frac{15\,\sqrt{-b}b^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(23}{15\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(
23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5, -1/15*(15*sqrt(-b)*b
^2*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (23*b^2*x^4 + 11*a*b*x^2 + 3*a^
2)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A] time = 3.64533, size = 105, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{23b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}-\frac{b^{\frac{5}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(15*x**2) - 23*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 - b**(5/2)*log(a/(b*x
**2))/2 + b**(5/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

Giac [B] time = 2.73945, size = 227, normalized size = 2.77

$$-\frac{1}{2}\,b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{5}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a\right.}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*a*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(5/2
) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(5/2) - 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*b^(5/2) + 23*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^5
```

3.403 $\int \frac{(a+bx^2)^{5/2}}{x^8} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

```
[Out] -(a + b*x^2)^(7/2)/(7*a*x^7)
```

Rubi [A] time = 0.0046399, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(7*a*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^8} dx = -\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Mathematica [A] time = 0.0080252, size = 21, normalized size = 1.

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(7*a*x^7)
```

Maple [A] time = 0.002, size = 18, normalized size = 0.9

$$-\frac{1}{7\,ax^7}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^8,x)
```

```
[Out] -1/7*(b*x^2+a)^(7/2)/a/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58271, size = 100, normalized size = 4.76

$$-\frac{\left(b^{3} x^{6}+3 \, a b^{2} x^{4}+3 \, a^{2} b x^{2}+a^{3}\right) \sqrt{b x^{2}+a}}{7 \, a x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/(a*x^7)
```

Sympy [B] time = 1.695, size = 95, normalized size = 4.52

$$-\frac{a^{2} \sqrt{b} \sqrt{\frac{a}{b x^{2}}+1}}{7 x^{6}}-\frac{3 a b^{\frac{3}{2}} \sqrt{\frac{a}{b x^{2}}+1}}{7 x^{4}}-\frac{3 b^{\frac{5}{2}} \sqrt{\frac{a}{b x^{2}}+1}}{7 x^{2}}-\frac{b^{\frac{7}{2}} \sqrt{\frac{a}{b x^{2}}+1}}{7 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**8,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 3*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(7*x**4) - 3*b**(5/2)*sqrt(a/(b*x**2) + 1)/(7*x**2) - b**(7/2)*sqrt(a/
(b*x**2) + 1)/(7*a)
```

Giac [B] time = 2.08011, size = 153, normalized size = 7.29

$$\frac{2 \left(7 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{12} b^{\frac{7}{2}}+35 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{8} a^{2} b^{\frac{7}{2}}+21 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} a^{4} b^{\frac{7}{2}}+a^{6} b^{\frac{7}{2}}\right)}{7 \left(\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a^2*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(7/2) + a^
6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.404 $\int \frac{(a+bx^2)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

```
[Out] -(a + b*x^2)^(7/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)
```

Rubi [A] time = 0.0105772, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^{10}} dx &= -\frac{(a+bx^2)^{7/2}}{9ax^9}-\frac{(2b)\int \frac{(a+bx^2)^{5/2}}{x^8} dx}{9a}\\ &= -\frac{(a+bx^2)^{7/2}}{9ax^9}+\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A] time = 0.0106238, size = 31, normalized size = 0.7

$$\frac{(a+bx^2)^{7/2}(2bx^2-7a)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-7*a + 2*b*x^2))/(63*a^2*x^9)
```

Maple [A] time = 0.003, size = 28, normalized size = 0.6

$$-\frac{-2\,bx^{2}+7\,a}{63\,x^{9}a^{2}}\left(bx^{2}+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^10,x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-2*b*x^2+7*a)/x^9/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60237, size = 130, normalized size = 2.95

$$\frac{\left(2\,b^{4}x^{8}-ab^{3}x^{6}-15\,a^{2}b^{2}x^{4}-19\,a^{3}bx^{2}-7\,a^{4}\right)\sqrt{bx^{2}+a}}{63\,a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(2*b^4*x^8 - a*b^3*x^6 - 15*a^2*b^2*x^4 - 19*a^3*b*x^2 - 7*a^4)*sqrt(b
*x^2 + a)/(a^2*x^9)
```

Sympy [B] time = 2.16759, size = 121, normalized size = 2.75

$$-\frac{a^{2}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{9x^{8}}-\frac{19ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{63x^{6}}-\frac{5b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{21x^{4}}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{63ax^{2}}+\frac{2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{63a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**10,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 19*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(63*x**6) - 5*b**(5/2)*sqrt(a/(b*x**2) + 1)/(21*x**4) - b**(7/2)*sqrt
(a/(b*x**2) + 1)/(63*a*x**2) + 2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(63*a**2)
```

Giac [B] time = 1.69998, size = 297, normalized size = 6.75

$$\frac{4\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{9}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^3b\right.}{63\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(9/2) + 105*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)
+ 189*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2) + 189*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^4*b^(9/2) + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2) +
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2) - a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

3.405 $\int \frac{(a+bx^2)^{5/2}}{x^{12}} dx$

Optimal. Leaf size=68

$$-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

```
[Out] -(a + b*x^2)^(7/2)/(11*a*x^11) + (4*b*(a + b*x^2)^(7/2))/(99*a^2*x^9) - (8*
b^2*(a + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rubi [A] time = 0.0189424, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(11*a*x^11) + (4*b*(a + b*x^2)^(7/2))/(99*a^2*x^9) - (8*
b^2*(a + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^{12}} dx &= -\frac{(a+bx^2)^{7/2}}{11ax^{11}}-\frac{(4b)\int \frac{(a+bx^2)^{5/2}}{x^{10}} dx}{11a}\\ &= -\frac{(a+bx^2)^{7/2}}{11ax^{11}}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}+\frac{(8b^2)\int \frac{(a+bx^2)^{5/2}}{x^8} dx}{99a^2}\\ &= -\frac{(a+bx^2)^{7/2}}{11ax^{11}}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}\end{aligned}$$

Mathematica [A] time = 0.011223, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{7/2}(63a^2-28abx^2+8b^2x^4)}{693a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] -((a + b*x^2)^(7/2)*(63*a^2 - 28*a*b*x^2 + 8*b^2*x^4))/(693*a^3*x^11)
```

Maple [A] time = 0.003, size = 39, normalized size = 0.6

$$-\frac{8\,b^2x^4-28\,abx^2+63\,a^2}{693\,x^{11}a^3}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^12,x)
```

```
[Out] -1/693*(b*x^2+a)^(7/2)*(8*b^2*x^4-28*a*b*x^2+63*a^2)/x^11/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71763, size = 163, normalized size = 2.4

$$-\frac{\left(8\,b^5x^{10}-4\,ab^4x^8+3\,a^2b^3x^6+113\,a^3b^2x^4+161\,a^4bx^2+63\,a^5\right)\sqrt{bx^2+a}}{693\,a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(8*b^5*x^10 - 4*a*b^4*x^8 + 3*a^2*b^3*x^6 + 113*a^3*b^2*x^4 + 161*a^
4*b*x^2 + 63*a^5)*sqrt(b*x^2 + a)/(a^3*x^11)
```

Sympy [B] time = 3.18987, size = 481, normalized size = 7.07

$$-\frac{63a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\left(693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}\right)}-\frac{287a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{498a^5b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx^2}+}}{693a^5b^4x^8+1386a^4b^5x^{10}+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**12,x)
```

```
[Out] -63*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**2*(693*a**5*b**4*x**8 + 1386*a**
4*b**5*x**10 + 693*a**3*b**6*x**12)) - 287*a**6*b**(11/2)*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 498
*a**5*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b
**5*x**10 + 693*a**3*b**6*x**12) - 390*a**4*b**(15/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 11
5*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*
b**5*x**10 + 693*a**3*b**6*x**12) - 3*a**2*b**(19/2)*x**8*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 12*
a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5
*x**10 + 693*a**3*b**6*x**12) - 8*b**(23/2)*x**12*sqrt(a/(b*x**2) + 1)/(693
*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12)
```

Giac [B] time = 1.86646, size = 332, normalized size = 4.88

$$\frac{16\left(462\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}b^{\frac{11}{2}}+1155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}ab^{\frac{11}{2}}+2541\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^2b^{\frac{11}{2}}+2079\left(\sqrt{b}x-\sqrt{b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 16/693*(462*(sqrt(b)*x - sqrt(b*x^2 + a))^16*b^(11/2) + 1155*(sqrt(b)*x - s
qrt(b*x^2 + a))^14*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b
^(11/2) + 2079*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(11/2) + 1485*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^4*b^(11/2) + 297*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^5*b^(11/2) + 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(11/2) - 11*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^7*b^(11/2) + a^8*b^(11/2))/((sqrt(b)*x - sqrt(b
*x^2 + a))^2 - a)^11
```

3.406 $\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

```
[Out] -(a + b*x^2)^(7/2)/(13*a*x^13) + (6*b*(a + b*x^2)^(7/2))/(143*a^2*x^11) - (
8*b^2*(a + b*x^2)^(7/2))/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^(7/2))/(3003*a
^4*x^7)
```

Rubi [A] time = 0.028226, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(13*a*x^13) + (6*b*(a + b*x^2)^(7/2))/(143*a^2*x^11) - (
8*b^2*(a + b*x^2)^(7/2))/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^(7/2))/(3003*a
^4*x^7)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx &= -\frac{(a+bx^2)^{7/2}}{13ax^{13}}-\frac{(6b)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{13a}\\
&= -\frac{(a+bx^2)^{7/2}}{13ax^{13}}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}+\frac{(24b^2)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{143a^2}\\
&= -\frac{(a+bx^2)^{7/2}}{13ax^{13}}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}-\frac{(16b^3)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{429a^3}\\
&= -\frac{(a+bx^2)^{7/2}}{13ax^{13}}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}+\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}
\end{aligned}$$

Mathematica [A] time = 0.0140055, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{7/2}\left(126a^2bx^2-231a^3-56ab^2x^4+16b^3x^6\right)}{3003a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-231*a^3 + 126*a^2*b*x^2 - 56*a*b^2*x^4 + 16*b^3*x^6))/
(3003*a^4*x^13)
```

Maple [A] time = 0.004, size = 50, normalized size = 0.5

$$-\frac{-16\,b^3x^6+56\,ab^2x^4-126\,a^2bx^2+231\,a^3}{3003\,x^{13}a^4}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^14,x)
```

```
[Out] -1/3003*(b*x^2+a)^(7/2)*(-16*b^3*x^6+56*a*b^2*x^4-126*a^2*b*x^2+231*a^3)/x^
13/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86854, size = 189, normalized size = 2.05

$$\frac{\left(16\,b^6x^{12}-8\,ab^5x^{10}+6\,a^2b^4x^8-5\,a^3b^3x^6-371\,a^4b^2x^4-567\,a^5bx^2-231\,a^6\right)\sqrt{bx^2+a}}{3003\,a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/3003*(16*b^6*x^12 - 8*a*b^5*x^10 + 6*a^2*b^4*x^8 - 5*a^3*b^3*x^6 - 371*a^
4*b^2*x^4 - 567*a^5*b*x^2 - 231*a^6)*sqrt(b*x^2 + a)/(a^4*x^13)
```

Sympy [B] time = 4.4519, size = 721, normalized size = 7.84

$$-\frac{231a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{1260a^8b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+300}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**14,x)
```

```
[Out] -231*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*
b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1260*a**8*b*
*(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x
**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 2765*a**7*b**(23/2)
*x**4*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 +
9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 3050*a**6*b**(25/2)*x**6*s
qrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a*
*5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1689*a**5*b**(27/2)*x**8*sqrt(a/(
b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**1
1*x**16 + 3003*a**4*b**12*x**18) - 376*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2)
 + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16
 + 3003*a**4*b**12*x**18) + 5*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(30
03*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a
**4*b**12*x**18) + 30*a**2*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(3003*a**7*
b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**1
2*x**18) + 40*a*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12
+ 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) +
16*b**(37/2)*x**18*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b
**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18)
```

Giac [B] time = 1.88285, size = 370, normalized size = 4.02

$$\frac{32\left(3003\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}b^{\frac{13}{2}}+9009\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{13}{2}}+18018\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^2b^{\frac{13}{2}}+16302\left(\sqrt{b}x-\sqrt{b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 32/3003*(3003*(sqrt(b)*x - sqrt(b*x^2 + a))^18*b^(13/2) + 9009*(sqrt(b)*x -
 sqrt(b*x^2 + a))^16*a*b^(13/2) + 18018*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^
2*b^(13/2) + 16302*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(13/2) + 10296*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(13/2) + 2288*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^5*b^(13/2) + 286*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(13/2) - 7
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(13/2) + 13*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^8*b^(13/2) - a^9*b^(13/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^13
```

3.407 $\int \frac{(a+bx^2)^{5/2}}{x^{16}} dx$

Optimal. Leaf size=116

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}+\frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9}-\frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}}+\frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

```
[Out] -(a + b*x^2)^(7/2)/(15*a*x^15) + (8*b*(a + b*x^2)^(7/2))/(195*a^2*x^13) - (
16*b^2*(a + b*x^2)^(7/2))/(715*a^3*x^11) + (64*b^3*(a + b*x^2)^(7/2))/(6435
*a^4*x^9) - (128*b^4*(a + b*x^2)^(7/2))/(45045*a^5*x^7)
```

Rubi [A] time = 0.0422674, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}+\frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9}-\frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}}+\frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(15*a*x^15) + (8*b*(a + b*x^2)^(7/2))/(195*a^2*x^13) - (
16*b^2*(a + b*x^2)^(7/2))/(715*a^3*x^11) + (64*b^3*(a + b*x^2)^(7/2))/(6435
*a^4*x^9) - (128*b^4*(a + b*x^2)^(7/2))/(45045*a^5*x^7)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx &= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} - \frac{(8b)\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx}{15a}\\ &= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} + \frac{(16b^2)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{65a^2}\\ &= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} - \frac{(64b^3)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{715a^3}\\ &= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} + \frac{(128b^4)\int \frac{(a+bx^2)^{5/2}}{x^{8}}\,dx}{6435a^4}\\ &= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}\end{aligned}$$

Mathematica [A] time = 0.0148535, size = 64, normalized size = 0.55

$$-\frac{(a+bx^2)^{7/2}(1008a^2b^2x^4 - 1848a^3bx^2 + 3003a^4 - 448ab^3x^6 + 128b^4x^8)}{45045a^5x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -((a + b*x^2)^(7/2)*(3003*a^4 - 1848*a^3*b*x^2 + 1008*a^2*b^2*x^4 - 448*a*b
^3*x^6 + 128*b^4*x^8))/(45045*a^5*x^15)
```

Maple [A] time = 0.004, size = 61, normalized size = 0.5

$$-\frac{128\,b^4x^8 - 448\,b^3x^6a + 1008\,b^2x^4a^2 - 1848\,bx^2a^3 + 3003\,a^4}{45045\,x^{15}a^5}(bx^2+a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(b*x^2+a)^(7/2)*(128*b^4*x^8-448*a*b^3*x^6+1008*a^2*b^2*x^4-1848*a
^3*b*x^2+3003*a^4)/x^15/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05689, size = 225, normalized size = 1.94

$$-\frac{\left(128\,b^{7}x^{14}-64\,ab^{6}x^{12}+48\,a^{2}b^{5}x^{10}-40\,a^{3}b^{4}x^{8}+35\,a^{4}b^{3}x^{6}+4473\,a^{5}b^{2}x^{4}+7161\,a^{6}bx^{2}+3003\,a^{7}\right)\sqrt{bx^{2}+a}}{45045\,a^{5}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*b^7*x^14 - 64*a*b^6*x^12 + 48*a^2*b^5*x^10 - 40*a^3*b^4*x^8 +
 35*a^4*b^3*x^6 + 4473*a^5*b^2*x^4 + 7161*a^6*b*x^2 + 3003*a^7)*sqrt(b*x^2
+ a)/(a^5*x^15)
```

Sympy [B] time = 5.81163, size = 1012, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**16,x)
```

```
[Out] -3003*a**11*b**(33/2)*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180
*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 450
45*a**5*b**20*x**22) - 19173*a**10*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(450
45*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 1
80180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 51135*a**9*b**(37/2)*x**
4*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 +
270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22)
 - 72905*a**8*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 +
 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**2
0 + 45045*a**5*b**20*x**22) - 58585*a**7*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1
)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**
18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 25151*a**6*b**(43/
2)*x**10*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x
**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20
*x**22) - 4501*a**5*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*
x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**
19*x**20 + 45045*a**5*b**20*x**22) - 35*a**4*b**(47/2)*x**14*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 280*a**3*b**(
49/2)*x**16*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**1
7*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b*
*20*x**22) - 560*a**2*b**(51/2)*x**18*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**1
6*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b
**19*x**20 + 45045*a**5*b**20*x**22) - 448*a*b**(53/2)*x**20*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 128*b**(55/2)
*x**22*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**
16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x
**22)
```

Giac [B] time = 2.68954, size = 405, normalized size = 3.49

$$\frac{256\left(18018\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}b^{\frac{15}{2}}+60060\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}ab^{\frac{15}{2}}+115830\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{15}{2}}+109395\left(\sqrt{b}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(15/2) + 60060*(sqrt(b)
*x - sqrt(b*x^2 + a))^18*a*b^(15/2) + 115830*(sqrt(b)*x - sqrt(b*x^2 + a))^
16*a^2*b^(15/2) + 109395*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b^(15/2) + 65
065*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(15/2) + 15015*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*a^5*b^(15/2) + 1365*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(
15/2) - 455*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(15/2) + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^8*b^(15/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b
^(15/2) + a^10*b^(15/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^15
```

3.408 $\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=140

$$\frac{256b^5\left(a+bx^2\right)^{7/2}}{153153a^6x^7}-\frac{128b^4\left(a+bx^2\right)^{7/2}}{21879a^5x^9}+\frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}}+\frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

```
[Out] -(a + b*x^2)^(7/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(7/2))/(51*a^2*x^15) - (1
6*b^2*(a + b*x^2)^(7/2))/(663*a^3*x^13) + (32*b^3*(a + b*x^2)^(7/2))/(2431*
a^4*x^11) - (128*b^4*(a + b*x^2)^(7/2))/(21879*a^5*x^9) + (256*b^5*(a + b*x
^2)^(7/2))/(153153*a^6*x^7)
```

Rubi [A] time = 0.0557573, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{256b^5\left(a+bx^2\right)^{7/2}}{153153a^6x^7}-\frac{128b^4\left(a+bx^2\right)^{7/2}}{21879a^5x^9}+\frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}}+\frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(7/2))/(51*a^2*x^15) - (1
6*b^2*(a + b*x^2)^(7/2))/(663*a^3*x^13) + (32*b^3*(a + b*x^2)^(7/2))/(2431*
a^4*x^11) - (128*b^4*(a + b*x^2)^(7/2))/(21879*a^5*x^9) + (256*b^5*(a + b*x
^2)^(7/2))/(153153*a^6*x^7)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = -\frac{(a+bx^2)^{7/2}}{17ax^{17}} - \frac{(10b)\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx}{17a}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} + \frac{(16b^2)\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx}{51a^2}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} - \frac{(32b^3)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{221a^3}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} + \frac{(128b^4)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{2431a^4}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9} - (25$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9} + 256$$

Mathematica [A] time = 0.0177302, size = 75, normalized size = 0.54

$$\frac{(a+bx^2)^{7/2}\left(2016a^2b^3x^6 - 3696a^3b^2x^4 + 6006a^4bx^2 - 9009a^5 - 896ab^4x^8 + 256b^5x^{10}\right)}{153153a^6x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-9009*a^5 + 6006*a^4*b*x^2 - 3696*a^3*b^2*x^4 + 2016*a^
2*b^3*x^6 - 896*a*b^4*x^8 + 256*b^5*x^10))/(153153*a^6*x^17)
```

Maple [A] time = 0.005, size = 72, normalized size = 0.5

$$-\frac{-256\,b^5x^{10} + 896\,ab^4x^8 - 2016\,a^2b^3x^6 + 3696\,a^3b^2x^4 - 6006\,a^4bx^2 + 9009\,a^5}{153153\,x^{17}a^6}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(b*x^2+a)^(7/2)*(-256*b^5*x^10+896*a*b^4*x^8-2016*a^2*b^3*x^6+369
6*a^3*b^2*x^4-6006*a^4*b*x^2+9009*a^5)/x^17/a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52008, size = 254, normalized size = 1.81

$$\frac{\left(256\,b^8x^{16}-128\,ab^7x^{14}+96\,a^2b^6x^{12}-80\,a^3b^5x^{10}+70\,a^4b^4x^8-63\,a^5b^3x^6-12705\,a^6b^2x^4-21021\,a^7bx^2-9009\,a^8\right)\sqrt{b x^2+a}}{153153\,a^6x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/153153*(256*b^8*x^16 - 128*a*b^7*x^14 + 96*a^2*b^6*x^12 - 80*a^3*b^5*x^10
 + 70*a^4*b^4*x^8 - 63*a^5*b^3*x^6 - 12705*a^6*b^2*x^4 - 21021*a^7*b*x^2 -
9009*a^8)*sqrt(b*x^2 + a)/(a^6*x^17)
```

Sympy [B] time = 8.26149, size = 1346, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**18,x)
```

```
[Out] -9009*a**13*b**(51/2)*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7657
65*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22
+ 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 66066*a**12*b**(53/2
)*x**2*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*
x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b
**29*x**24 + 153153*a**6*b**30*x**26) - 207900*a**11*b**(55/2)*x**4*sqrt(a/
(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 153153
0*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 1
53153*a**6*b**30*x**26) - 363888*a**10*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/
(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x
**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**
30*x**26) - 382550*a**9*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(153153*a**11*b
**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*
a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 241
524*a**8*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7
65765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**
22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 84780*a**7*b**(63
/2)*x**12*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**
26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**
7*b**29*x**24 + 153153*a**6*b**30*x**26) - 12768*a**6*b**(65/2)*x**14*sqrt(
a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531
530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 +
 153153*a**6*b**30*x**26) + 63*a**5*b**(67/2)*x**16*sqrt(a/(b*x**2) + 1)/(1
53153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**
20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30
*x**26) + 630*a**4*b**(69/2)*x**18*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25
*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8
*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 1680*a*
*3*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*
a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 7
65765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 2016*a**2*b**(73/2)*x**
22*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**1
8 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29
*x**24 + 153153*a**6*b**30*x**26) + 1152*a*b**(75/2)*x**24*sqrt(a/(b*x**2)
```

```
+ 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b*
*27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**
6*b**30*x**26) + 256*b**(77/2)*x**26*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**
25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a*
*8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26)
```

Giac [B] time = 2.46902, size = 443, normalized size = 3.16

$$512\left(102102\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{17}{2}}+364650\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{17}{2}}+692835\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{17}{2}}+668525\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] 512/153153*(102102*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(17/2) + 364650*(sqrt
(b)*x - sqrt(b*x^2 + a))^20*a*b^(17/2) + 692835*(sqrt(b)*x - sqrt(b*x^2 + a
))^18*a^2*b^(17/2) + 668525*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(17/2) +
 384098*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*b^(17/2) + 89726*(sqrt(b)*x -
sqrt(b*x^2 + a))^12*a^5*b^(17/2) + 6188*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
6*b^(17/2) - 2380*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(17/2) + 680*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*b^(17/2) - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a^9*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(17/2) - a^11*b^
(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^17
```

3.409 $\int x^{15} \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=161

$$\frac{a^2 \left(a+bx^2\right)^{21/2}}{b^8}-\frac{35a^3 \left(a+bx^2\right)^{19/2}}{19b^8}+\frac{35a^4 \left(a+bx^2\right)^{17/2}}{17b^8}-\frac{7a^5 \left(a+bx^2\right)^{15/2}}{5b^8}+\frac{7a^6 \left(a+bx^2\right)^{13/2}}{13b^8}-\frac{a^7 \left(a+bx^2\right)^{11/2}}{11b^8}+$$

```
[Out] -(a^7*(a + b*x^2)^(11/2))/(11*b^8) + (7*a^6*(a + b*x^2)^(13/2))/(13*b^8) -
(7*a^5*(a + b*x^2)^(15/2))/(5*b^8) + (35*a^4*(a + b*x^2)^(17/2))/(17*b^8) -
 (35*a^3*(a + b*x^2)^(19/2))/(19*b^8) + (a^2*(a + b*x^2)^(21/2))/b^8 - (7*a
*(a + b*x^2)^(23/2))/(23*b^8) + (a + b*x^2)^(25/2)/(25*b^8)
```

Rubi [A] time = 0.0999879, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{21/2}}{b^8}-\frac{35a^3 \left(a+bx^2\right)^{19/2}}{19b^8}+\frac{35a^4 \left(a+bx^2\right)^{17/2}}{17b^8}-\frac{7a^5 \left(a+bx^2\right)^{15/2}}{5b^8}+\frac{7a^6 \left(a+bx^2\right)^{13/2}}{13b^8}-\frac{a^7 \left(a+bx^2\right)^{11/2}}{11b^8}+$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^7*(a + b*x^2)^(11/2))/(11*b^8) + (7*a^6*(a + b*x^2)^(13/2))/(13*b^8) -
(7*a^5*(a + b*x^2)^(15/2))/(5*b^8) + (35*a^4*(a + b*x^2)^(17/2))/(17*b^8) -
 (35*a^3*(a + b*x^2)^(19/2))/(19*b^8) + (a^2*(a + b*x^2)^(21/2))/b^8 - (7*a
*(a + b*x^2)^(23/2))/(23*b^8) + (a + b*x^2)^(25/2)/(25*b^8)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{15} \left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^7(a+bx)^{9/2} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-\frac{35a^3}{}\right.\right.\\ &= -\frac{a^7 \left(a+bx^2\right)^{11/2}}{11b^8}+\frac{7a^6 \left(a+bx^2\right)^{13/2}}{13b^8}-\frac{7a^5 \left(a+bx^2\right)^{15/2}}{5b^8}+\frac{35a^4 \left(a+bx^2\right)^{17/2}}{17b^8}-\frac{35a^3 \left(a+b\right.}{19b^8}\end{aligned}$$

Mathematica [A] time = 0.0568496, size = 94, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{11/2} \left(369512a^2b^5x^{10}-194480a^3b^4x^8+91520a^4b^3x^6-36608a^5b^2x^4+11264a^6bx^2-2048a^7-646646ab^6x^{12}\right.}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-2048*a^7 + 11264*a^6*b*x^2 - 36608*a^5*b^2*x^4 + 9152
0*a^4*b^3*x^6 - 194480*a^3*b^4*x^8 + 369512*a^2*b^5*x^10 - 646646*a*b^6*x^1
2 + 1062347*b^7*x^14))/(26558675*b^8)
```

Maple [A] time = 0.006, size = 91, normalized size = 0.6

$$-\frac{-1062347\,x^{14}b^{7}+646646\,ax^{12}b^{6}-369512\,a^{2}x^{10}b^{5}+194480\,a^{3}x^{8}b^{4}-91520\,a^{4}x^{6}b^{3}+36608\,a^{5}x^{4}b^{2}-11264\,a^{6}x^{2}b+20}{26558675\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/26558675*(b*x^2+a)^(11/2)*(-1062347*b^7*x^14+646646*a*b^6*x^12-369512*a^
2*b^5*x^10+194480*a^3*b^4*x^8-91520*a^4*b^3*x^6+36608*a^5*b^2*x^4-11264*a^6
*b*x^2+2048*a^7)/b^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83018, size = 386, normalized size = 2.4

$$\frac{\left(1062347\,b^{12}x^{24}+4665089\,ab^{11}x^{22}+7759752\,a^{2}b^{10}x^{20}+5810090\,a^{3}b^{9}x^{18}+1659515\,a^{4}b^{8}x^{16}+429\,a^{5}b^{7}x^{14}-462\,a^{6}b^{6}x\right.}{26558675\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/26558675*(1062347*b^12*x^24 + 4665089*a*b^11*x^22 + 7759752*a^2*b^10*x^20
 + 5810090*a^3*b^9*x^18 + 1659515*a^4*b^8*x^16 + 429*a^5*b^7*x^14 - 462*a^6
*b^6*x^12 + 504*a^7*b^5*x^10 - 560*a^8*b^4*x^8 + 640*a^9*b^3*x^6 - 768*a^10
*b^2*x^4 + 1024*a^11*b*x^2 - 2048*a^12)*sqrt(b*x^2 + a)/b^8
```

Sympy [A] time = 99.6531, size = 301, normalized size = 1.87

$$\begin{cases} -\frac{2048a^{12}\sqrt{a+bx^2}}{26558675b^8}+\frac{1024a^{11}x^2\sqrt{a+bx^2}}{26558675b^7}-\frac{768a^{10}x^4\sqrt{a+bx^2}}{26558675b^6}+\frac{128a^9x^6\sqrt{a+bx^2}}{5311735b^5}-\frac{112a^8x^8\sqrt{a+bx^2}}{5311735b^4}+\frac{504a^7x^{10}\sqrt{a+bx^2}}{26558675b^3}-\frac{42a^6x^{12}\sqrt{a+bx^2}}{2414425b^2}+\frac{3a^5x^{14}\sqrt{}}{18572} \\ \frac{a^{\frac{9}{2}}x^{16}}{16} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2048*a**12*sqrt(a + b*x**2)/(26558675*b**8) + 1024*a**11*x**2*s
qrt(a + b*x**2)/(26558675*b**7) - 768*a**10*x**4*sqrt(a + b*x**2)/(26558675
*b**6) + 128*a**9*x**6*sqrt(a + b*x**2)/(5311735*b**5) - 112*a**8*x**8*sqrt
(a + b*x**2)/(5311735*b**4) + 504*a**7*x**10*sqrt(a + b*x**2)/(26558675*b**
3) - 42*a**6*x**12*sqrt(a + b*x**2)/(2414425*b**2) + 3*a**5*x**14*sqrt(a +
b*x**2)/(185725*b) + 2321*a**4*x**16*sqrt(a + b*x**2)/37145 + 478*a**3*b*x*
*18*sqrt(a + b*x**2)/2185 + 168*a**2*b**2*x**20*sqrt(a + b*x**2)/575 + 101*
a*b**3*x**22*sqrt(a + b*x**2)/575 + b**4*x**24*sqrt(a + b*x**2)/25, Ne(b, 0
)), (a**(9/2)*x**16/16, True))
```

Giac [B] time = 1.74369, size = 973, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1673196525*(15295*(6435*(b*x^2 + a)^(17/2) - 51051*(b*x^2 + a)^(15/2)*a +
 176715*(b*x^2 + a)^(13/2)*a^2 - 348075*(b*x^2 + a)^(11/2)*a^3 + 425425*(b*
x^2 + a)^(9/2)*a^4 - 328185*(b*x^2 + a)^(7/2)*a^5 + 153153*(b*x^2 + a)^(5/2
)*a^6 - 36465*(b*x^2 + a)^(3/2)*a^7)*a^4/b^7 + 3220*(109395*(b*x^2 + a)^(19
/2) - 978120*(b*x^2 + a)^(17/2)*a + 3879876*(b*x^2 + a)^(15/2)*a^2 - 895356
0*(b*x^2 + a)^(13/2)*a^3 + 13226850*(b*x^2 + a)^(11/2)*a^4 - 12932920*(b*x^
2 + a)^(9/2)*a^5 + 8314020*(b*x^2 + a)^(7/2)*a^6 - 3325608*(b*x^2 + a)^(5/2
)*a^7 + 692835*(b*x^2 + a)^(3/2)*a^8)*a^3/b^7 + 2070*(230945*(b*x^2 + a)^(2
1/2) - 2297295*(b*x^2 + a)^(19/2)*a + 10270260*(b*x^2 + a)^(17/2)*a^2 - 271
59132*(b*x^2 + a)^(15/2)*a^3 + 47006190*(b*x^2 + a)^(13/2)*a^4 - 55552770*(
b*x^2 + a)^(11/2)*a^5 + 45265220*(b*x^2 + a)^(9/2)*a^6 - 24942060*(b*x^2 +
a)^(7/2)*a^7 + 8729721*(b*x^2 + a)^(5/2)*a^8 - 1616615*(b*x^2 + a)^(3/2)*a^
9)*a^2/b^7 + 300*(969969*(b*x^2 + a)^(23/2) - 10623470*(b*x^2 + a)^(21/2)*a
 + 52837785*(b*x^2 + a)^(19/2)*a^2 - 157477320*(b*x^2 + a)^(17/2)*a^3 + 312
330018*(b*x^2 + a)^(15/2)*a^4 - 432456948*(b*x^2 + a)^(13/2)*a^5 + 42590457
0*(b*x^2 + a)^(11/2)*a^6 - 297457160*(b*x^2 + a)^(9/2)*a^7 + 143416845*(b*x
^2 + a)^(7/2)*a^8 - 44618574*(b*x^2 + a)^(5/2)*a^9 + 7436429*(b*x^2 + a)^(3
/2)*a^10)*a/b^7 + 33*(2028117*(b*x^2 + a)^(25/2) - 24249225*(b*x^2 + a)^(23
/2)*a + 132793375*(b*x^2 + a)^(21/2)*a^2 - 440314875*(b*x^2 + a)^(19/2)*a^3
 + 984233250*(b*x^2 + a)^(17/2)*a^4 - 1561650090*(b*x^2 + a)^(15/2)*a^5 + 1
801903950*(b*x^2 + a)^(13/2)*a^6 - 1521087750*(b*x^2 + a)^(11/2)*a^7 + 9295
53625*(b*x^2 + a)^(9/2)*a^8 - 398380125*(b*x^2 + a)^(7/2)*a^9 + 111546435*(
b*x^2 + a)^(5/2)*a^10 - 16900975*(b*x^2 + a)^(3/2)*a^11)/b^7)/b
```

3.410 $\int x^{13} \left(a + bx^2\right)^{9/2} dx$

Optimal. Leaf size=140

$$\frac{15a^2 \left(a + bx^2\right)^{19/2}}{19b^7} - \frac{20a^3 \left(a + bx^2\right)^{17/2}}{17b^7} + \frac{a^4 \left(a + bx^2\right)^{15/2}}{b^7} - \frac{6a^5 \left(a + bx^2\right)^{13/2}}{13b^7} + \frac{a^6 \left(a + bx^2\right)^{11/2}}{11b^7} + \frac{\left(a + bx^2\right)^{23/2}}{23b^7} - \frac{2a \left(a\right.}{}$$

```
[Out] (a^6*(a + b*x^2)^(11/2))/(11*b^7) - (6*a^5*(a + b*x^2)^(13/2))/(13*b^7) + (
a^4*(a + b*x^2)^(15/2))/b^7 - (20*a^3*(a + b*x^2)^(17/2))/(17*b^7) + (15*a^
2*(a + b*x^2)^(19/2))/(19*b^7) - (2*a*(a + b*x^2)^(21/2))/(7*b^7) + (a + b*
x^2)^(23/2)/(23*b^7)
```

Rubi [A] time = 0.0797439, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{15a^2 \left(a + bx^2\right)^{19/2}}{19b^7} - \frac{20a^3 \left(a + bx^2\right)^{17/2}}{17b^7} + \frac{a^4 \left(a + bx^2\right)^{15/2}}{b^7} - \frac{6a^5 \left(a + bx^2\right)^{13/2}}{13b^7} + \frac{a^6 \left(a + bx^2\right)^{11/2}}{11b^7} + \frac{\left(a + bx^2\right)^{23/2}}{23b^7} - \frac{2a \left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^(9/2),x]
```


```
[Out] (a^6*(a + b*x^2)^(11/2))/(11*b^7) - (6*a^5*(a + b*x^2)^(13/2))/(13*b^7) + (
a^4*(a + b*x^2)^(15/2))/b^7 - (20*a^3*(a + b*x^2)^(17/2))/(17*b^7) + (15*a^
2*(a + b*x^2)^(19/2))/(19*b^7) - (2*a*(a + b*x^2)^(21/2))/(7*b^7) + (a + b*
x^2)^(23/2)/(23*b^7)
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^{13} \left(a + bx^2\right)^{9/2} dx &= \frac{1}{2} \text{Subst}\left(\int x^6 (a + bx)^{9/2} dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^6 (a + bx)^{9/2}}{b^6} - \frac{6a^5 (a + bx)^{11/2}}{b^6} + \frac{15a^4 (a + bx)^{13/2}}{b^6} - \frac{20a^3 (a + bx)^{15/2}}{b^6} + \frac{15a^2 (a + \right.\right.}{b^6} \\
&= \frac{a^6 \left(a + bx^2\right)^{11/2}}{11b^7} - \frac{6a^5 \left(a + bx^2\right)^{13/2}}{13b^7} + \frac{a^4 \left(a + bx^2\right)^{15/2}}{b^7} - \frac{20a^3 \left(a + bx^2\right)^{17/2}}{17b^7} + \frac{15a^2 \left(a + bx^2\right)^{19/2}}{19b^7}
\end{aligned}$$

Mathematica [A] time = 0.0441173, size = 83, normalized size = 0.59

$$\frac{\left(a + bx^2\right)^{11/2} \left(97240a^2b^4x^8 - 45760a^3b^3x^6 + 18304a^4b^2x^4 - 5632a^5bx^2 + 1024a^6 - 184756ab^5x^{10} + 323323b^6x^{12}\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(1024*a^6 - 5632*a^5*b*x^2 + 18304*a^4*b^2*x^4 - 45760*
a^3*b^3*x^6 + 97240*a^2*b^4*x^8 - 184756*a*b^5*x^10 + 323323*b^6*x^12))/(74
36429*b^7)
```

Maple [A] time = 0.005, size = 80, normalized size = 0.6

$$\frac{323323\,x^{12}b^{6}-184756\,a x^{10}b^{5}+97240\,a^{2}x^{8}b^{4}-45760\,a^{3}x^{6}b^{3}+18304\,a^{4}x^{4}b^{2}-5632\,a^{5}x^{2}b+1024\,a^{6}}{7436429\,b^{7}}\left(bx^{2}+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7436429*(b*x^2+a)^(11/2)*(323323*b^6*x^12-184756*a*b^5*x^10+97240*a^2*b^4
*x^8-45760*a^3*b^3*x^6+18304*a^4*b^2*x^4-5632*a^5*b*x^2+1024*a^6)/b^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72266, size = 352, normalized size = 2.51

$$\frac{\left(323323\,b^{11}x^{22}+1431859\,ab^{10}x^{20}+2406690\,a^{2}b^{9}x^{18}+1826110\,a^{3}b^{8}x^{16}+530959\,a^{4}b^{7}x^{14}+231\,a^{5}b^{6}x^{12}-252\,a^{6}b^{5}x\right.}{7436429\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7436429*(323323*b^11*x^22 + 1431859*a*b^10*x^20 + 2406690*a^2*b^9*x^18 +
1826110*a^3*b^8*x^16 + 530959*a^4*b^7*x^14 + 231*a^5*b^6*x^12 - 252*a^6*b^5
*x^10 + 280*a^7*b^4*x^8 - 320*a^8*b^3*x^6 + 384*a^9*b^2*x^4 - 512*a^10*b*x^
2 + 1024*a^11)*sqrt(b*x^2 + a)/b^7
```

Sympy [A] time = 80.1342, size = 277, normalized size = 1.98

$$\begin{cases}\frac{1024a^{11}\sqrt{a+bx^{2}}}{7436429b^{7}}-\frac{512a^{10}x^{2}\sqrt{a+bx^{2}}}{7436429b^{6}}+\frac{384a^{9}x^{4}\sqrt{a+bx^{2}}}{7436429b^{5}}-\frac{320a^{8}x^{6}\sqrt{a+bx^{2}}}{7436429b^{4}}+\frac{40a^{7}x^{8}\sqrt{a+bx^{2}}}{1062347b^{3}}-\frac{36a^{6}x^{10}\sqrt{a+bx^{2}}}{1062347b^{2}}+\frac{3a^{5}x^{12}\sqrt{a+bx^{2}}}{96577b}+\frac{3713a^{4}x^{14}\sqrt{}}{52003} \\ \frac{a^{\frac{9}{2}}x^{14}}{14}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((1024*a**11*sqrt(a + b*x**2)/(7436429*b**7) - 512*a**10*x**2*sqrt
(a + b*x**2)/(7436429*b**6) + 384*a**9*x**4*sqrt(a + b*x**2)/(7436429*b**5)
 - 320*a**8*x**6*sqrt(a + b*x**2)/(7436429*b**4) + 40*a**7*x**8*sqrt(a + b*
x**2)/(1062347*b**3) - 36*a**6*x**10*sqrt(a + b*x**2)/(1062347*b**2) + 3*a*
*5*x**12*sqrt(a + b*x**2)/(96577*b) + 3713*a**4*x**14*sqrt(a + b*x**2)/5200
3 + 12770*a**3*b*x**16*sqrt(a + b*x**2)/52003 + 990*a**2*b**2*x**18*sqrt(a
+ b*x**2)/3059 + 31*a*b**3*x**20*sqrt(a + b*x**2)/161 + b**4*x**22*sqrt(a +
 b*x**2)/23, Ne(b, 0)), (a**(9/2)*x**14/14, True))
```

Giac [B] time = 1.8674, size = 879, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/334639305*(7429*(3003*(b*x^2 + a)^(15/2) - 20790*(b*x^2 + a)^(13/2)*a + 6
1425*(b*x^2 + a)^(11/2)*a^2 - 100100*(b*x^2 + a)^(9/2)*a^3 + 96525*(b*x^2 +
 a)^(7/2)*a^4 - 54054*(b*x^2 + a)^(5/2)*a^5 + 15015*(b*x^2 + a)^(3/2)*a^6)*
a^4/b^6 + 12236*(6435*(b*x^2 + a)^(17/2) - 51051*(b*x^2 + a)^(15/2)*a + 176
715*(b*x^2 + a)^(13/2)*a^2 - 348075*(b*x^2 + a)^(11/2)*a^3 + 425425*(b*x^2
+ a)^(9/2)*a^4 - 328185*(b*x^2 + a)^(7/2)*a^5 + 153153*(b*x^2 + a)^(5/2)*a^
6 - 36465*(b*x^2 + a)^(3/2)*a^7)*a^3/b^6 + 966*(109395*(b*x^2 + a)^(19/2) -
 978120*(b*x^2 + a)^(17/2)*a + 3879876*(b*x^2 + a)^(15/2)*a^2 - 8953560*(b*
x^2 + a)^(13/2)*a^3 + 13226850*(b*x^2 + a)^(11/2)*a^4 - 12932920*(b*x^2 + a
)^(9/2)*a^5 + 8314020*(b*x^2 + a)^(7/2)*a^6 - 3325608*(b*x^2 + a)^(5/2)*a^7
 + 692835*(b*x^2 + a)^(3/2)*a^8)*a^2/b^6 + 276*(230945*(b*x^2 + a)^(21/2) -
 2297295*(b*x^2 + a)^(19/2)*a + 10270260*(b*x^2 + a)^(17/2)*a^2 - 27159132*
(b*x^2 + a)^(15/2)*a^3 + 47006190*(b*x^2 + a)^(13/2)*a^4 - 55552770*(b*x^2
+ a)^(11/2)*a^5 + 45265220*(b*x^2 + a)^(9/2)*a^6 - 24942060*(b*x^2 + a)^(7/
2)*a^7 + 8729721*(b*x^2 + a)^(5/2)*a^8 - 1616615*(b*x^2 + a)^(3/2)*a^9)*a/b
^6 + 15*(969969*(b*x^2 + a)^(23/2) - 10623470*(b*x^2 + a)^(21/2)*a + 528377
85*(b*x^2 + a)^(19/2)*a^2 - 157477320*(b*x^2 + a)^(17/2)*a^3 + 312330018*(b
*x^2 + a)^(15/2)*a^4 - 432456948*(b*x^2 + a)^(13/2)*a^5 + 425904570*(b*x^2
+ a)^(11/2)*a^6 - 297457160*(b*x^2 + a)^(9/2)*a^7 + 143416845*(b*x^2 + a)^(
7/2)*a^8 - 44618574*(b*x^2 + a)^(5/2)*a^9 + 7436429*(b*x^2 + a)^(3/2)*a^10)
/b^6)/b
```

3.411 $\int x^{11} \left(a+bx^2\right)^{9/2} \, dx$

Optimal. Leaf size=122

$$\frac{10a^2\left(a+bx^2\right)^{17/2}}{17b^6}-\frac{2a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{5a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{5a\left(a+bx^2\right)^{19/2}}{19b^6}$$

```
[Out] -(a^5*(a + b*x^2)^(11/2))/(11*b^6) + (5*a^4*(a + b*x^2)^(13/2))/(13*b^6) -
(2*a^3*(a + b*x^2)^(15/2))/(3*b^6) + (10*a^2*(a + b*x^2)^(17/2))/(17*b^6) -
 (5*a*(a + b*x^2)^(19/2))/(19*b^6) + (a + b*x^2)^(21/2)/(21*b^6)
```

Rubi [A] time = 0.0705596, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{10a^2\left(a+bx^2\right)^{17/2}}{17b^6}-\frac{2a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{5a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{5a\left(a+bx^2\right)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^5*(a + b*x^2)^(11/2))/(11*b^6) + (5*a^4*(a + b*x^2)^(13/2))/(13*b^6) -
(2*a^3*(a + b*x^2)^(15/2))/(3*b^6) + (10*a^2*(a + b*x^2)^(17/2))/(17*b^6) -
 (5*a*(a + b*x^2)^(19/2))/(19*b^6) + (a + b*x^2)^(21/2)/(21*b^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^5(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\frac{5a(a\right.\right.\\ &= -\frac{a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{5a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{2a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{10a^2\left(a+bx^2\right)^{17/2}}{17b^6}-\frac{5a\left(a+bx^2\right)}{19b^6}\end{aligned}$$

Mathematica [A] time = 0.040828, size = 72, normalized size = 0.59

$$\frac{\left(a+bx^2\right)^{11/2}\left(11440a^2b^3x^6-4576a^3b^2x^4+1408a^4bx^2-256a^5-24310ab^4x^8+46189b^5x^{10}\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*a
^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^10))/(969969*b^6)
```

Maple [A] time = 0.005, size = 69, normalized size = 0.6

$$-\frac{-46189\,b^{5}x^{10}+24310\,ab^{4}x^{8}-11440\,a^{2}b^{3}x^{6}+4576\,a^{3}b^{2}x^{4}-1408\,a^{4}bx^{2}+256\,a^{5}}{969969\,b^{6}}\left(bx^{2}+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/969969*(b*x^2+a)^(11/2)*(-46189*b^5*x^10+24310*a*b^4*x^8-11440*a^2*b^3*x
^6+4576*a^3*b^2*x^4-1408*a^4*b*x^2+256*a^5)/b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72267, size = 309, normalized size = 2.53

$$\frac{\left(46189\,b^{10}x^{20}+206635\,ab^{9}x^{18}+351780\,a^{2}b^{8}x^{16}+271414\,a^{3}b^{7}x^{14}+80773\,a^{4}b^{6}x^{12}+63\,a^{5}b^{5}x^{10}-70\,a^{6}b^{4}x^{8}+80\,a^{7}b^{3}x}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*b^10*x^20 + 206635*a*b^9*x^18 + 351780*a^2*b^8*x^16 + 27141
4*a^3*b^7*x^14 + 80773*a^4*b^6*x^12 + 63*a^5*b^5*x^10 - 70*a^6*b^4*x^8 + 80
*a^7*b^3*x^6 - 96*a^8*b^2*x^4 + 128*a^9*b*x^2 - 256*a^10)*sqrt(b*x^2 + a)/b
^6
```

Sympy [A] time = 62.4883, size = 253, normalized size = 2.07

$$\begin{cases} -\frac{256a^{10}\sqrt{a+bx^2}}{969969b^6}+\frac{128a^9x^2\sqrt{a+bx^2}}{969969b^5}-\frac{32a^8x^4\sqrt{a+bx^2}}{323323b^4}+\frac{80a^7x^6\sqrt{a+bx^2}}{969969b^3}-\frac{10a^6x^8\sqrt{a+bx^2}}{138567b^2}+\frac{3a^5x^{10}\sqrt{a+bx^2}}{46189b}+\frac{1049a^4x^{12}\sqrt{a+bx^2}}{12597}+\frac{1898a^3bx^{14}\sqrt{a+b}}{6783} \\ \frac{a^{\frac{9}{2}}x^{12}}{12} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-256*a**10*sqrt(a + b*x**2)/(969969*b**6) + 128*a**9*x**2*sqrt(a
 + b*x**2)/(969969*b**5) - 32*a**8*x**4*sqrt(a + b*x**2)/(323323*b**4) + 80
*a**7*x**6*sqrt(a + b*x**2)/(969969*b**3) - 10*a**6*x**8*sqrt(a + b*x**2)/(
138567*b**2) + 3*a**5*x**10*sqrt(a + b*x**2)/(46189*b) + 1049*a**4*x**12*sq
rt(a + b*x**2)/12597 + 1898*a**3*b*x**14*sqrt(a + b*x**2)/6783 + 820*a**2*b
**2*x**16*sqrt(a + b*x**2)/2261 + 85*a*b**3*x**18*sqrt(a + b*x**2)/399 + b*
*4*x**20*sqrt(a + b*x**2)/21, Ne(b, 0)), (a**(9/2)*x**12/12, True))
```

Giac [B] time = 2.42774, size = 784, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/14549535*(1615*(693*(b*x^2 + a)^(13/2) - 4095*(b*x^2 + a)^(11/2)*a + 1001
0*(b*x^2 + a)^(9/2)*a^2 - 12870*(b*x^2 + a)^(7/2)*a^3 + 9009*(b*x^2 + a)^(5
/2)*a^4 - 3003*(b*x^2 + a)^(3/2)*a^5)*a^4/b^5 + 1292*(3003*(b*x^2 + a)^(15/
2) - 20790*(b*x^2 + a)^(13/2)*a + 61425*(b*x^2 + a)^(11/2)*a^2 - 100100*(b*
x^2 + a)^(9/2)*a^3 + 96525*(b*x^2 + a)^(7/2)*a^4 - 54054*(b*x^2 + a)^(5/2)*
a^5 + 15015*(b*x^2 + a)^(3/2)*a^6)*a^3/b^5 + 798*(6435*(b*x^2 + a)^(17/2) -
 51051*(b*x^2 + a)^(15/2)*a + 176715*(b*x^2 + a)^(13/2)*a^2 - 348075*(b*x^2
 + a)^(11/2)*a^3 + 425425*(b*x^2 + a)^(9/2)*a^4 - 328185*(b*x^2 + a)^(7/2)*
a^5 + 153153*(b*x^2 + a)^(5/2)*a^6 - 36465*(b*x^2 + a)^(3/2)*a^7)*a^2/b^5 +
 28*(109395*(b*x^2 + a)^(19/2) - 978120*(b*x^2 + a)^(17/2)*a + 3879876*(b*x
^2 + a)^(15/2)*a^2 - 8953560*(b*x^2 + a)^(13/2)*a^3 + 13226850*(b*x^2 + a)^
(11/2)*a^4 - 12932920*(b*x^2 + a)^(9/2)*a^5 + 8314020*(b*x^2 + a)^(7/2)*a^6
 - 3325608*(b*x^2 + a)^(5/2)*a^7 + 692835*(b*x^2 + a)^(3/2)*a^8)*a/b^5 + 3*
(230945*(b*x^2 + a)^(21/2) - 2297295*(b*x^2 + a)^(19/2)*a + 10270260*(b*x^2
 + a)^(17/2)*a^2 - 27159132*(b*x^2 + a)^(15/2)*a^3 + 47006190*(b*x^2 + a)^(
13/2)*a^4 - 55552770*(b*x^2 + a)^(11/2)*a^5 + 45265220*(b*x^2 + a)^(9/2)*a^
6 - 24942060*(b*x^2 + a)^(7/2)*a^7 + 8729721*(b*x^2 + a)^(5/2)*a^8 - 161661
5*(b*x^2 + a)^(3/2)*a^9)/b^5)/b
```

3.412 $\int x^9 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=101

$$\frac{2a^2\left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{a^4\left(a+bx^2\right)^{11/2}}{11b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{4a\left(a+bx^2\right)^{17/2}}{17b^5}$$

```
[Out] (a^4*(a + b*x^2)^(11/2))/(11*b^5) - (4*a^3*(a + b*x^2)^(13/2))/(13*b^5) + (
2*a^2*(a + b*x^2)^(15/2))/(5*b^5) - (4*a*(a + b*x^2)^(17/2))/(17*b^5) + (a
+ b*x^2)^(19/2)/(19*b^5)
```

Rubi [A] time = 0.0575089, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{a^4\left(a+bx^2\right)^{11/2}}{11b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{4a\left(a+bx^2\right)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^4*(a + b*x^2)^(11/2))/(11*b^5) - (4*a^3*(a + b*x^2)^(13/2))/(13*b^5) + (
2*a^2*(a + b*x^2)^(15/2))/(5*b^5) - (4*a*(a + b*x^2)^(17/2))/(17*b^5) + (a
+ b*x^2)^(19/2)/(19*b^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^4(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a+bx)^{17/2}}{b^4}\right)\right)\\ &= \frac{a^4\left(a+bx^2\right)^{11/2}}{11b^5}-\frac{4a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{2a^2\left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a\left(a+bx^2\right)^{17/2}}{17b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}\end{aligned}$$

Mathematica [A] time = 0.0334344, size = 61, normalized size = 0.6

$$\frac{\left(a+bx^2\right)^{11/2}\left(2288a^2b^2x^4-704a^3bx^2+128a^4-5720ab^3x^6+12155b^4x^8\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(128*a^4 - 704*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 5720*a*b^
3*x^6 + 12155*b^4*x^8))/(230945*b^5)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.6

$$\frac{12155\,x^{8}b^{4}-5720\,ax^{6}b^{3}+2288\,a^{2}x^{4}b^{2}-704\,a^{3}x^{2}b+128\,a^{4}}{230945\,b^{5}}\left(bx^{2}+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/230945*(b*x^2+a)^(11/2)*(12155*b^4*x^8-5720*a*b^3*x^6+2288*a^2*b^2*x^4-70
4*a^3*b*x^2+128*a^4)/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6932, size = 277, normalized size = 2.74

$$\frac{\left(12155\,b^{9}x^{18}+55055\,ab^{8}x^{16}+95238\,a^{2}b^{7}x^{14}+75086\,a^{3}b^{6}x^{12}+23063\,a^{4}b^{5}x^{10}+35\,a^{5}b^{4}x^{8}-40\,a^{6}b^{3}x^{6}+48\,a^{7}b^{2}x^{4}-\cdots}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/230945*(12155*b^9*x^18 + 55055*a*b^8*x^16 + 95238*a^2*b^7*x^14 + 75086*a^
3*b^6*x^12 + 23063*a^4*b^5*x^10 + 35*a^5*b^4*x^8 - 40*a^6*b^3*x^6 + 48*a^7*
b^2*x^4 - 64*a^8*b*x^2 + 128*a^9)*sqrt(b*x^2 + a)/b^5
```

Sympy [A] time = 46.2677, size = 230, normalized size = 2.28

$$\begin{cases} \frac{128a^{9}\sqrt{a+bx^{2}}}{230945b^{5}}-\frac{64a^{8}x^{2}\sqrt{a+bx^{2}}}{230945b^{4}}+\frac{48a^{7}x^{4}\sqrt{a+bx^{2}}}{230945b^{3}}-\frac{8a^{6}x^{6}\sqrt{a+bx^{2}}}{46189b^{2}}+\frac{7a^{5}x^{8}\sqrt{a+bx^{2}}}{46189b}+\frac{23063a^{4}x^{10}\sqrt{a+bx^{2}}}{230945}+\frac{6826a^{3}bx^{12}\sqrt{a+bx^{2}}}{20995}+\frac{666a^{2}b^{2}x^{14}\sqrt{\cdots}}{1615}\cdots \\ \frac{a^{\frac{9}{2}}x^{10}}{10} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**9*sqrt(a + b*x**2)/(230945*b**5) - 64*a**8*x**2*sqrt(a +
b*x**2)/(230945*b**4) + 48*a**7*x**4*sqrt(a + b*x**2)/(230945*b**3) - 8*a**
6*x**6*sqrt(a + b*x**2)/(46189*b**2) + 7*a**5*x**8*sqrt(a + b*x**2)/(46189*
b) + 23063*a**4*x**10*sqrt(a + b*x**2)/230945 + 6826*a**3*b*x**12*sqrt(a +
b*x**2)/20995 + 666*a**2*b**2*x**14*sqrt(a + b*x**2)/1615 + 77*a*b**3*x**16
*sqrt(a + b*x**2)/323 + b**4*x**18*sqrt(a + b*x**2)/19, Ne(b, 0)), (a**(9/2
)*x**10/10, True))
```

Giac [B] time = 2.28212, size = 690, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/14549535*(4199*(315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*
(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)
*a^4)*a^4/b^4 + 6460*(693*(b*x^2 + a)^(13/2) - 4095*(b*x^2 + a)^(11/2)*a +
10010*(b*x^2 + a)^(9/2)*a^2 - 12870*(b*x^2 + a)^(7/2)*a^3 + 9009*(b*x^2 + a
)^(5/2)*a^4 - 3003*(b*x^2 + a)^(3/2)*a^5)*a^3/b^4 + 1938*(3003*(b*x^2 + a)^
(15/2) - 20790*(b*x^2 + a)^(13/2)*a + 61425*(b*x^2 + a)^(11/2)*a^2 - 100100
*(b*x^2 + a)^(9/2)*a^3 + 96525*(b*x^2 + a)^(7/2)*a^4 - 54054*(b*x^2 + a)^(5
/2)*a^5 + 15015*(b*x^2 + a)^(3/2)*a^6)*a^2/b^4 + 532*(6435*(b*x^2 + a)^(17/
2) - 51051*(b*x^2 + a)^(15/2)*a + 176715*(b*x^2 + a)^(13/2)*a^2 - 348075*(b
*x^2 + a)^(11/2)*a^3 + 425425*(b*x^2 + a)^(9/2)*a^4 - 328185*(b*x^2 + a)^(7
/2)*a^5 + 153153*(b*x^2 + a)^(5/2)*a^6 - 36465*(b*x^2 + a)^(3/2)*a^7)*a/b^4
 + 7*(109395*(b*x^2 + a)^(19/2) - 978120*(b*x^2 + a)^(17/2)*a + 3879876*(b*
x^2 + a)^(15/2)*a^2 - 8953560*(b*x^2 + a)^(13/2)*a^3 + 13226850*(b*x^2 + a)
^(11/2)*a^4 - 12932920*(b*x^2 + a)^(9/2)*a^5 + 8314020*(b*x^2 + a)^(7/2)*a^
6 - 3325608*(b*x^2 + a)^(5/2)*a^7 + 692835*(b*x^2 + a)^(3/2)*a^8)/b^4)/b
```

3.413 $\int x^7 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=80

$$\frac{3a^2\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{a\left(a+bx^2\right)^{15/2}}{5b^4}$$

```
[Out] -(a^3*(a + b*x^2)^(11/2))/(11*b^4) + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) -
(a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)
```

Rubi [A] time = 0.0458746, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{a\left(a+bx^2\right)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(11/2))/(11*b^4) + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) -
(a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^{9/2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{3a^2\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a\left(a+bx^2\right)^{15/2}}{5b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}
\end{aligned}$$

Mathematica [A] time = 0.0296801, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{11/2}\left(88a^2bx^2-16a^3-286ab^2x^4+715b^3x^6\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6))
/(12155*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-715\,b^3x^6+286\,ab^2x^4-88\,a^2bx^2+16\,a^3}{12155\,b^4}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/12155*(b*x^2+a)^(11/2)*(-715*b^3*x^6+286*a*b^2*x^4-88*a^2*b*x^2+16*a^3)/
b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7245, size = 238, normalized size = 2.98

$$\frac{\left(715\,b^8x^{16}+3289\,ab^7x^{14}+5808\,a^2b^6x^{12}+4714\,a^3b^5x^{10}+1515\,a^4b^4x^8+5\,a^5b^3x^6-6\,a^6b^2x^4+8\,a^7bx^2-16\,a^8\right)\sqrt{bx^2+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/12155*(715*b^8*x^16 + 3289*a*b^7*x^14 + 5808*a^2*b^6*x^12 + 4714*a^3*b^5*
x^10 + 1515*a^4*b^4*x^8 + 5*a^5*b^3*x^6 - 6*a^6*b^2*x^4 + 8*a^7*b*x^2 - 16*
a^8)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 34.4877, size = 204, normalized size = 2.55

$$\begin{cases} -\frac{16a^8\sqrt{a+bx^2}}{12155b^4}+\frac{8a^7x^2\sqrt{a+bx^2}}{12155b^3}-\frac{6a^6x^4\sqrt{a+bx^2}}{12155b^2}+\frac{a^5x^6\sqrt{a+bx^2}}{2431b}+\frac{303a^4x^8\sqrt{a+bx^2}}{2431}+\frac{4714a^3bx^{10}\sqrt{a+bx^2}}{12155}+\frac{528a^2b^2x^{12}\sqrt{a+bx^2}}{1105}+\frac{23ab^3x^{14}\sqrt{a+bx^2}}{85} \\ \frac{a^{\frac{9}{2}}x^8}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**8*sqrt(a + b*x**2)/(12155*b**4) + 8*a**7*x**2*sqrt(a + b*
x**2)/(12155*b**3) - 6*a**6*x**4*sqrt(a + b*x**2)/(12155*b**2) + a**5*x**6*
sqrt(a + b*x**2)/(2431*b) + 303*a**4*x**8*sqrt(a + b*x**2)/2431 + 4714*a**3
*b*x**10*sqrt(a + b*x**2)/12155 + 528*a**2*b**2*x**12*sqrt(a + b*x**2)/1105
 + 23*a*b**3*x**14*sqrt(a + b*x**2)/85 + b**4*x**16*sqrt(a + b*x**2)/17, Ne
(b, 0)), (a**(9/2)*x**8/8, True))
```

Giac [B] time = 2.64018, size = 595, normalized size = 7.44

$$\frac{\frac{2431\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)a^4}{b^3}+\frac{884\left(315\left(bx^2+a\right)^{\frac{11}{2}}-1540\left(bx^2+a\right)^{\frac{9}{2}}a+2970\left(bx^2+a\right)^{\frac{7}{2}}a^2-2772\left(bx^2+a\right)^{\frac{5}{2}}a^3+\right.}{b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/765765*(2431*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2
 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*a^4/b^3 + 884*(315*(b*x^2 + a)
^(11/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x
^2 + a)^(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)*a^3/b^3 + 510*(693*(b*x^2 +
 a)^(13/2) - 4095*(b*x^2 + a)^(11/2)*a + 10010*(b*x^2 + a)^(9/2)*a^2 - 1287
0*(b*x^2 + a)^(7/2)*a^3 + 9009*(b*x^2 + a)^(5/2)*a^4 - 3003*(b*x^2 + a)^(3/
2)*a^5)*a^2/b^3 + 68*(3003*(b*x^2 + a)^(15/2) - 20790*(b*x^2 + a)^(13/2)*a
+ 61425*(b*x^2 + a)^(11/2)*a^2 - 100100*(b*x^2 + a)^(9/2)*a^3 + 96525*(b*x^
2 + a)^(7/2)*a^4 - 54054*(b*x^2 + a)^(5/2)*a^5 + 15015*(b*x^2 + a)^(3/2)*a^
6)*a/b^3 + 7*(6435*(b*x^2 + a)^(17/2) - 51051*(b*x^2 + a)^(15/2)*a + 176715
*(b*x^2 + a)^(13/2)*a^2 - 348075*(b*x^2 + a)^(11/2)*a^3 + 425425*(b*x^2 + a
)^(9/2)*a^4 - 328185*(b*x^2 + a)^(7/2)*a^5 + 153153*(b*x^2 + a)^(5/2)*a^6 -
 36465*(b*x^2 + a)^(3/2)*a^7)/b^3)/b
```

3.414 $\int x^5 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=59

$$\frac{a^2 \left(a+bx^2\right)^{11/2}}{11b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}-\frac{2a \left(a+bx^2\right)^{13/2}}{13b^3}$$

```
[Out] (a^2*(a + b*x^2)^(11/2))/(11*b^3) - (2*a*(a + b*x^2)^(13/2))/(13*b^3) + (a
+ b*x^2)^(15/2)/(15*b^3)
```

Rubi [A] time = 0.0350742, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{11/2}}{11b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}-\frac{2a \left(a+bx^2\right)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(11/2))/(11*b^3) - (2*a*(a + b*x^2)^(13/2))/(13*b^3) + (a
+ b*x^2)^(15/2)/(15*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{9/2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right) dx, x, x^2\right)\\ &= \frac{a^2 \left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a \left(a+bx^2\right)^{13/2}}{13b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.0215029, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{11/2} \left(8a^2-44abx^2+143b^2x^4\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(8*a^2 - 44*a*b*x^2 + 143*b^2*x^4))/(2145*b^3)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.6

$$\frac{143\,b^2x^4-44\,abx^2+8\,a^2}{2145\,b^3}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2145*(b*x^2+a)^(11/2)*(143*b^2*x^4-44*a*b*x^2+8*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6103, size = 209, normalized size = 3.54

$$\frac{\left(143\,b^7x^{14}+671\,ab^6x^{12}+1218\,a^2b^5x^{10}+1030\,a^3b^4x^8+355\,a^4b^3x^6+3\,a^5b^2x^4-4\,a^6bx^2+8\,a^7\right)\sqrt{bx^2+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/2145*(143*b^7*x^14 + 671*a*b^6*x^12 + 1218*a^2*b^5*x^10 + 1030*a^3*b^4*x^
8 + 355*a^4*b^3*x^6 + 3*a^5*b^2*x^4 - 4*a^6*b*x^2 + 8*a^7)*sqrt(b*x^2 + a)/
b^3
```

Sympy [A] time = 25.2969, size = 180, normalized size = 3.05

$$\begin{cases}\frac{8a^7\sqrt{a+bx^2}}{2145b^3}-\frac{4a^6x^2\sqrt{a+bx^2}}{2145b^2}+\frac{a^5x^4\sqrt{a+bx^2}}{715b}+\frac{71a^4x^6\sqrt{a+bx^2}}{429}+\frac{206a^3bx^8\sqrt{a+bx^2}}{429}+\frac{406a^2b^2x^{10}\sqrt{a+bx^2}}{715}+\frac{61ab^3x^{12}\sqrt{a+bx^2}}{195}+\frac{b^4x^{14}\sqrt{a+bx^2}}{15}\\ \frac{a^{\frac{9}{2}}x^6}{6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(a + b*x**2)/(2145*b**3) - 4*a**6*x**2*sqrt(a + b*x**
2)/(2145*b**2) + a**5*x**4*sqrt(a + b*x**2)/(715*b) + 71*a**4*x**6*sqrt(a +
 b*x**2)/429 + 206*a**3*b*x**8*sqrt(a + b*x**2)/429 + 406*a**2*b**2*x**10*s
qrt(a + b*x**2)/715 + 61*a*b**3*x**12*sqrt(a + b*x**2)/195 + b**4*x**14*sqr
t(a + b*x**2)/15, Ne(b, 0)), (a**(9/2)*x**6/6, True))
```

Giac [B] time = 2.76058, size = 500, normalized size = 8.47

$$\frac{\frac{429\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)a^4}{b^2}+\frac{572\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)a^3}{b^2}+\frac{78\left(315\left(bx^2+a\right)^{\frac{11}{2}}-1540\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/45045*(429*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a
)^(3/2)*a^2)*a^4/b^2 + 572*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a
+ 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*a^3/b^2 + 78*(315*
(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2
- 2772*(b*x^2 + a)^(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)*a^2/b^2 + 20*(69
3*(b*x^2 + a)^(13/2) - 4095*(b*x^2 + a)^(11/2)*a + 10010*(b*x^2 + a)^(9/2)*
a^2 - 12870*(b*x^2 + a)^(7/2)*a^3 + 9009*(b*x^2 + a)^(5/2)*a^4 - 3003*(b*x^
2 + a)^(3/2)*a^5)*a/b^2 + (3003*(b*x^2 + a)^(15/2) - 20790*(b*x^2 + a)^(13/
2)*a + 61425*(b*x^2 + a)^(11/2)*a^2 - 100100*(b*x^2 + a)^(9/2)*a^3 + 96525*
(b*x^2 + a)^(7/2)*a^4 - 54054*(b*x^2 + a)^(5/2)*a^5 + 15015*(b*x^2 + a)^(3/
2)*a^6)/b^2)/b
```

3.415 $\int x^3 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}$$

```
[Out] -(a*(a + b*x^2)^(11/2))/(11*b^2) + (a + b*x^2)^(13/2)/(13*b^2)
```

Rubi [A] time = 0.0239998, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a*(a + b*x^2)^(11/2))/(11*b^2) + (a + b*x^2)^(13/2)/(13*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{9/2}dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}+\frac{\left(a+bx^2\right)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A] time = 0.0182053, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{11/2}\left(11bx^2-2a\right)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-2*a + 11*b*x^2))/(143*b^2)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$-\frac{-11\,bx^2+2\,a}{143\,b^2}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/143*(b*x^2+a)^(11/2)*(-11*b*x^2+2*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62493, size = 174, normalized size = 4.58

$$\frac{\left(11\,b^6x^{12}+53\,ab^5x^{10}+100\,a^2b^4x^8+90\,a^3b^3x^6+35\,a^4b^2x^4+a^5bx^2-2\,a^6\right)\sqrt{bx^2+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/143*(11*b^6*x^12 + 53*a*b^5*x^10 + 100*a^2*b^4*x^8 + 90*a^3*b^3*x^6 + 35*
a^4*b^2*x^4 + a^5*b*x^2 - 2*a^6)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 18.0402, size = 156, normalized size = 4.11

$$\begin{cases} -\frac{2a^6\sqrt{a+bx^2}}{143b^2}+\frac{a^5x^2\sqrt{a+bx^2}}{143b}+\frac{35a^4x^4\sqrt{a+bx^2}}{143}+\frac{90a^3bx^6\sqrt{a+bx^2}}{143}+\frac{100a^2b^2x^8\sqrt{a+bx^2}}{143}+\frac{53ab^3x^{10}\sqrt{a+bx^2}}{143}+\frac{b^4x^{12}\sqrt{a+bx^2}}{13} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2*a**6*sqrt(a + b*x**2)/(143*b**2) + a**5*x**2*sqrt(a + b*x**2)
/(143*b) + 35*a**4*x**4*sqrt(a + b*x**2)/143 + 90*a**3*b*x**6*sqrt(a + b*x*
*2)/143 + 100*a**2*b**2*x**8*sqrt(a + b*x**2)/143 + 53*a*b**3*x**10*sqrt(a
+ b*x**2)/143 + b**4*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(9/2)*x**4/4
```

```
, True))
```

Giac [B] time = 2.4424, size = 406, normalized size = 10.68

$$\frac{3003\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)a^4}{b}+\frac{1716\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)a^3}{b}+\frac{858\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/45045*(3003*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*a^4/b + 1716*(1
5*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*a^
3/b + 858*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)
^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*a^2/b + 52*(315*(b*x^2 + a)^(11/2)
- 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^
(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)*a/b + 5*(693*(b*x^2 + a)^(13/2) - 4
095*(b*x^2 + a)^(11/2)*a + 10010*(b*x^2 + a)^(9/2)*a^2 - 12870*(b*x^2 + a)^
(7/2)*a^3 + 9009*(b*x^2 + a)^(5/2)*a^4 - 3003*(b*x^2 + a)^(3/2)*a^5)/b)/b
```

3.416 $\int x\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

Rubi [A] time = 0.0034928, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(9/2),x]

[Out] (a + b*x^2)^(11/2)/(11*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Mathematica [A] time = 0.0055945, size = 18, normalized size = 1.

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(9/2),x]

[Out] (a + b*x^2)^(11/2)/(11*b)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{1}{11\,b}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/11*(b*x^2+a)^(11/2)/b
```

Maxima [A] time = 2.12982, size = 19, normalized size = 1.06

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

Fricas [B] time = 1.64273, size = 139, normalized size = 7.72

$$\frac{\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{bx^2+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^
2 + a^5)*sqrt(b*x^2 + a)/b
```

Sympy [A] time = 12.2733, size = 133, normalized size = 7.39

$$\begin{cases} \frac{a^5\sqrt{a+bx^2}}{11b}+\frac{5a^4x^2\sqrt{a+bx^2}}{11}+\frac{10a^3bx^4\sqrt{a+bx^2}}{11}+\frac{10a^2b^2x^6\sqrt{a+bx^2}}{11}+\frac{5ab^3x^8\sqrt{a+bx^2}}{11}+\frac{b^4x^{10}\sqrt{a+bx^2}}{11} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((a**5*sqrt(a + b*x**2)/(11*b) + 5*a**4*x**2*sqrt(a + b*x**2)/11 +
 10*a**3*b*x**4*sqrt(a + b*x**2)/11 + 10*a**2*b**2*x**6*sqrt(a + b*x**2)/11
 + 5*a*b**3*x**8*sqrt(a + b*x**2)/11 + b**4*x**10*sqrt(a + b*x**2)/11, Ne(b
, 0)), (a**(9/2)*x**2/2, True))
```

Giac [B] time = 2.59245, size = 267, normalized size = 14.83

$$315\left(bx^2+a\right)^{\frac{11}{2}}-1540\left(bx^2+a\right)^{\frac{9}{2}}a+2970\left(bx^2+a\right)^{\frac{7}{2}}a^2-2772\left(bx^2+a\right)^{\frac{5}{2}}a^3+2310\left(bx^2+a\right)^{\frac{3}{2}}a^4+924\left(3\left(bx^2+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a
)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3 + 2310*(b*x^2 + a)^(3/2)*a^4 + 924
*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*a^3 + 198*(15*(b*x^2 + a)^(7
/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*a^2 + 44*(35*(b*x^
2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b
*x^2 + a)^(3/2)*a^3)*a)/b
```

3.417 $\int \frac{(a+bx^2)^{9/2}}{x} dx$

Optimal. Leaf size=108

$$a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

```
[Out] a^4*Sqrt[a + b*x^2] + (a^3*(a + b*x^2)^(3/2))/3 + (a^2*(a + b*x^2)^(5/2))/5
 + (a*(a + b*x^2)^(7/2))/7 + (a + b*x^2)^(9/2)/9 - a^(9/2)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0702031, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] a^4*Sqrt[a + b*x^2] + (a^3*(a + b*x^2)^(3/2))/3 + (a^2*(a + b*x^2)^(5/2))/5
 + (a*(a + b*x^2)^(7/2))/7 + (a + b*x^2)^(9/2)/9 - a^(9/2)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^3\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^4\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx\right.$$
$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^5\operatorname{Subst}\Big($$
$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{a^5\operatorname{Subst}\Big(\int}{}$$
$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} - a^{9/2}\tanh^{-1}\Big($$

Mathematica [A] time = 0.0393282, size = 84, normalized size = 0.78

$$\frac{1}{315}\sqrt{a+bx^2}\left(408a^2b^2x^4 + 506a^3bx^2 + 563a^4 + 185ab^3x^6 + 35b^4x^8\right) - a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6
 + 35*b^4*x^8))/315 - a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Maple [A] time = 0.004, size = 94, normalized size = 0.9

$$\frac{1}{9}(bx^2+a)^{\frac{9}{2}} + \frac{a}{7}(bx^2+a)^{\frac{7}{2}} + \frac{a^2}{5}(bx^2+a)^{\frac{5}{2}} + \frac{a^3}{3}(bx^2+a)^{\frac{3}{2}} - a^{\frac{9}{2}}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + a^4\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x,x)
```

```
[Out] 1/9*(b*x^2+a)^(9/2)+1/7*a*(b*x^2+a)^(7/2)+1/5*a^2*(b*x^2+a)^(5/2)+1/3*a^3*(
b*x^2+a)^(3/2)-a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+a^4*(b*x^2+a)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69137, size = 419, normalized size = 3.88

$$\left[\frac{1}{2}\,a^{\frac{9}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{315}\left(35\,b^4x^8+185\,ab^3x^6+408\,a^2b^2x^4+506\,a^3bx^2+563\,a^4\right)\sqrt{bx^2+a},\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(9/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/315*(3
5*b^4*x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt
(b*x^2 + a), sqrt(-a)*a^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/315*(35*b^4*
x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt(b*x^2
 + a)]
```

Sympy [A] time = 9.37214, size = 160, normalized size = 1.48

$$\frac{563a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{315}+\frac{a^{\frac{9}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{506a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{315}+\frac{136a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{105}+\frac{37a^{\frac{3}{2}}b^3x^6\sqrt{1}}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x,x)
```

```
[Out] 563*a**(9/2)*sqrt(1 + b*x**2/a)/315 + a**(9/2)*log(b*x**2/a)/2 - a**(9/2)*l
og(sqrt(1 + b*x**2/a) + 1) + 506*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)/315 + 1
36*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)/105 + 37*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)/63 + sqrt(a)*b**4*x**8*sqrt(1 + b*x**2/a)/9
```

Giac [A] time = 1.58725, size = 122, normalized size = 1.13

$$\frac{a^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}}+\frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3+\sqrt{bx^2+a}a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] a^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/9*(b*x^2 + a)^(9/2) + 1/7
*(b*x^2 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^
3 + sqrt(b*x^2 + a)*a^4
```

3.418 $\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=118

$$\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{2}a^3b\sqrt{a+bx^2}-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}$$

```
[Out] (9*a^3*b*Sqrt[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b
*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) -
(9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.071303, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{2}a^3b\sqrt{a+bx^2}-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (9*a^3*b*Sqrt[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b
*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) -
(9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)\\
&= \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}(9ab)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}\left(9a^2b\right)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}\left(9a^3b\right)\,\text{Subst}\left(\int \sqrt{\right.\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}\left(9a\right.\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{2}\left(9a\right.\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} - \frac{9}{2}a^{7/2}
\end{aligned}$$

Mathematica [C] time = 0.0119086, size = 37, normalized size = 0.31

$$\frac{b\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x^2)/a])/(11*
a^2)
```

Maple [A] time = 0.004, size = 118, normalized size = 1.

$$-\frac{1}{2\,ax^2}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{b}{2\,a}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{9\,b}{14}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{9\,ab}{10}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{3\,a^2b}{2}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{9\,b}{2}a^{\frac{7}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(11/2)+1/2*b/a*(b*x^2+a)^(9/2)+9/14*b*(b*x^2+a)^(7/2)+
9/10*a*b*(b*x^2+a)^(5/2)+3/2*a^2*b*(b*x^2+a)^(3/2)-9/2*b*a^(7/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x)+9/2*a^3*b*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6598, size = 452, normalized size = 3.83

$$\left[\frac{315\,a^{\frac{7}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx^2-35\,a^4\right)\sqrt{bx^2+a}}{140\,x^2}, \frac{315\,\sqrt{-a}a^3bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a^(7/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*a^
4)*sqrt(b*x^2 + a))/x^2, 1/70*(315*sqrt(-a)*a^3*b*x^2*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) + (10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2
- 35*a^4)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] time = 9.35442, size = 167, normalized size = 1.42

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{194a^{\frac{7}{2}}b\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}+\frac{78a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{29a^{\frac{3}{2}}b^3x^4\sqrt{1+\frac{b}{}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**3,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 194*a**(7/2)*b*sqrt(1 + b*x**2/a)/3
5 + 9*a**(7/2)*b*log(b*x**2/a)/4 - 9*a**(7/2)*b*log(sqrt(1 + b*x**2/a) + 1)
/2 + 78*a**(5/2)*b**2*x**2*sqrt(1 + b*x**2/a)/35 + 29*a**(3/2)*b**3*x**4*sq
rt(1 + b*x**2/a)/35 + sqrt(a)*b**4*x**6*sqrt(1 + b*x**2/a)/7
```

Giac [A] time = 2.6218, size = 136, normalized size = 1.15

$$\frac{1}{70}\left(\frac{315\,a^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\left(bx^2+a\right)^{\frac{7}{2}}+28\left(bx^2+a\right)^{\frac{5}{2}}a+70\left(bx^2+a\right)^{\frac{3}{2}}a^2+280\sqrt{bx^2+a}a^3-\frac{35\sqrt{bx^2+a}a^4}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 10*(b*x^2 + a)^(7
/2) + 28*(b*x^2 + a)^(5/2)*a + 70*(b*x^2 + a)^(3/2)*a^2 + 280*sqrt(b*x^2 +
a)*a^3 - 35*sqrt(b*x^2 + a)*a^4/(b*x^2))*b
```

3.419 $\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx$

Optimal. Leaf size=126

$$\frac{63}{8}a^2b^2\sqrt{a+bx^2}-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}$$

```
[Out] (63*a^2*b^2*Sqrt[a + b*x^2])/8 + (21*a*b^2*(a + b*x^2)^(3/2))/8 + (63*b^2*(
a + b*x^2)^(5/2))/40 - (9*b*(a + b*x^2)^(7/2))/(8*x^2) - (a + b*x^2)^(9/2)/
(4*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rubi [A] time = 0.0764493, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{63}{8}a^2b^2\sqrt{a+bx^2}-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x^2])/8 + (21*a*b^2*(a + b*x^2)^(3/2))/8 + (63*b^2*(
a + b*x^2)^(5/2))/40 - (9*b*(a + b*x^2)^(7/2))/(8*x^2) - (a + b*x^2)^(9/2)/
(4*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^5}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{8}(9b)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}\left(63b^2\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)$$
$$= \frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}\left(63ab^2\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)$$
$$= \frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}\left(63a^2b^2\right)\mathrm{Subst}\left(\int$$
$$= \frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}$$
$$= \frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{8}($$
$$= \frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}-\frac{63}{8}$$

Mathematica [C] time = 0.0120723, size = 39, normalized size = 0.31

$$-\frac{b^2\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] -(b^2*(a + b*x^2)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, 1 + (b*x^2)/a])/(
11*a^3)
```

Maple [A] time = 0.007, size = 148, normalized size = 1.2

$$-\frac{1}{4\,a x^4}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{7\,b}{8\,a^2x^2}\left(bx^2+a\right)^{\frac{11}{2}}+\frac{7\,b^2}{8\,a^2}\left(bx^2+a\right)^{\frac{9}{2}}+\frac{9\,b^2}{8\,a}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{63\,b^2}{40}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{21\,ab^2}{8}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(11/2)-7/8*b/a^2/x^2*(b*x^2+a)^(11/2)+7/8*b^2/a^2*(b*x
^2+a)^(9/2)+9/8*b^2/a*(b*x^2+a)^(7/2)+63/40*b^2*(b*x^2+a)^(5/2)+21/8*a*b^2*
(b*x^2+a)^(3/2)-63/8*b^2*a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+63/8
*a^2*b^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65255, size = 451, normalized size = 3.58

$$\left[\frac{315\,a^{\frac{5}{2}}b^2x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(8\,b^4x^8+56\,ab^3x^6+288\,a^2b^2x^4-85\,a^3bx^2-10\,a^4\right)\sqrt{bx^2+a}}{80\,x^4},\ \frac{315\,\sqrt{-a}a^2b^2x^4\,\mathrm{a}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^(5/2)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 - 10*a^4
)*sqrt(b*x^2 + a))/x^4, 1/40*(315*sqrt(-a)*a^2*b^2*x^4*arctan(sqrt(-a)/sqrt
(b*x^2 + a)) + (8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A] time = 8.48189, size = 175, normalized size = 1.39

$$-\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8}-\frac{a^5}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{19a^4\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{203a^3b^{\frac{3}{2}}}{40x\sqrt{\frac{a}{bx^2}+1}}+\frac{43a^2b^{\frac{5}{2}}x}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{8ab^{\frac{7}{2}}x^3}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x^5}{5\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**5,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**5/(4*sqrt(b)*x**5*sqrt(
a/(b*x**2) + 1)) - 19*a**4*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) + 203*a**3
*b**(3/2)/(40*x*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)*x/(5*sqrt(a/(b*x**
2) + 1)) + 8*a*b**(7/2)*x**3/(5*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**5/(5*sq
rt(a/(b*x**2) + 1))
```

Giac [A] time = 2.82305, size = 143, normalized size = 1.13

$$\frac{1}{40}\left(\frac{315\,a^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\left(bx^2+a\right)^{\frac{5}{2}}+40\left(bx^2+a\right)^{\frac{3}{2}}a+240\sqrt{bx^2+a}a^2-\frac{5\left(17\left(bx^2+a\right)^{\frac{3}{2}}a^3-15\sqrt{bx^2+a}a^4\right.}{b^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*(b*x^2 + a)^(5/
2) + 40*(b*x^2 + a)^(3/2)*a + 240*sqrt(b*x^2 + a)*a^2 - 5*(17*(b*x^2 + a)^(
3/2)*a^3 - 15*sqrt(b*x^2 + a)*a^4)/(b^2*x^4))*b^2
```

3.420 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^7}\,dx$

Optimal. Leaf size=126

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-\frac{3b\left(a+bx^2\right)}{8x^4}$$

```
[Out] (105*a*b^3*Sqrt[a + b*x^2])/16 + (35*b^3*(a + b*x^2)^(3/2))/16 - (21*b^2*(a
 + b*x^2)^(5/2))/(16*x^2) - (3*b*(a + b*x^2)^(7/2))/(8*x^4) - (a + b*x^2)^(
9/2)/(6*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/16
```

Rubi [A] time = 0.0763166, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-\frac{3b\left(a+bx^2\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x^2])/16 + (35*b^3*(a + b*x^2)^(3/2))/16 - (21*b^2*(a
 + b*x^2)^(5/2))/(16*x^2) - (3*b*(a + b*x^2)^(7/2))/(8*x^4) - (a + b*x^2)^(
9/2)/(6*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/16
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{9/2}}{6x^6} + \frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4} - \frac{\left(a+bx^2\right)^{9/2}}{6x^6} + \frac{1}{16}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2} - \frac{3b\left(a+bx^2\right)^{7/2}}{8x^4} - \frac{\left(a+bx^2\right)^{9/2}}{6x^6} + \frac{1}{32}\left(105b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right.$$
$$= \frac{35}{16}b^3\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2} - \frac{3b\left(a+bx^2\right)^{7/2}}{8x^4} - \frac{\left(a+bx^2\right)^{9/2}}{6x^6} + \frac{1}{32}\left(105ab^3\right)\text{Subst}\left(\int\right.$$
$$= \frac{105}{16}ab^3\sqrt{a+bx^2} + \frac{35}{16}b^3\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2} - \frac{3b\left(a+bx^2\right)^{7/2}}{8x^4} - \frac{\left(a+bx^2\right)^{9/2}}{6x^6} + \frac{1}{32}\left(\right.$$
$$= \frac{105}{16}ab^3\sqrt{a+bx^2} + \frac{35}{16}b^3\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2} - \frac{3b\left(a+bx^2\right)^{7/2}}{8x^4} - \frac{\left(a+bx^2\right)^{9/2}}{6x^6} + \frac{1}{16}\left(\right.$$
$$= \frac{105}{16}ab^3\sqrt{a+bx^2} + \frac{35}{16}b^3\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2} - \frac{3b\left(a+bx^2\right)^{7/2}}{8x^4} - \frac{\left(a+bx^2\right)^{9/2}}{6x^6} - \frac{105}{16}$$

Mathematica [C] time = 0.0119343, size = 39, normalized size = 0.31

$$\frac{b^3\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(4,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, 1 + (b*x^2)/a])/(1
1*a^4)
```

Maple [A] time = 0.01, size = 168, normalized size = 1.3

$$-\frac{1}{6\,ax^6}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{5\,b}{24\,a^2x^4}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{35\,b^2}{48\,a^3x^2}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{35\,b^3}{48\,a^3}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{15\,b^3}{16\,a^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{21\,b^3}{16\,a}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(11/2)-5/24*b/a^2/x^4*(b*x^2+a)^(11/2)-35/48*b^2/a^3/x
^2*(b*x^2+a)^(11/2)+35/48*b^3/a^3*(b*x^2+a)^(9/2)+15/16*b^3/a^2*(b*x^2+a)^(
7/2)+21/16*b^3/a*(b*x^2+a)^(5/2)+35/16*b^3*(b*x^2+a)^(3/2)-105/16*b^3*a^(3/
2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+105/16*a*b^3*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62125, size = 451, normalized size = 3.58

$$\left[\frac{315\,a^{\frac{3}{2}}b^{3}x^{6}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(16\,b^{4}x^{8}+208\,ab^{3}x^{6}-165\,a^{2}b^{2}x^{4}-50\,a^{3}bx^{2}-8\,a^{4}\right)\sqrt{bx^{2}+a}}{96\,x^{6}}, \frac{315\,\sqrt{-a}ab^{3}x^{6}\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^(3/2)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2 - 8*a^
4)*sqrt(b*x^2 + a))/x^6, 1/48*(315*sqrt(-a)*a*b^3*x^6*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) + (16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2
- 8*a^4)*sqrt(b*x^2 + a))/x^6]
```

Sympy [A] time = 7.82761, size = 175, normalized size = 1.39

$$-\frac{105a^{\frac{3}{2}}b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{16}-\frac{a^{5}}{6\sqrt{b}x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{29a^{4}\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{215a^{3}b^{\frac{3}{2}}}{48x^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{43a^{2}b^{\frac{5}{2}}}{48x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{14ab^{\frac{7}{2}}x}{3\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{9}{2}}x^{3}}{3\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**7,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*x))/16 - a**5/(6*sqrt(b)*x**7*sqr
t(a/(b*x**2) + 1)) - 29*a**4*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) - 215*a
**3*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)/(48*x*sqrt(a
/(b*x**2) + 1)) + 14*a*b**(7/2)*x/(3*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**3/
(3*sqrt(a/(b*x**2) + 1))
```

Giac [A] time = 1.85332, size = 143, normalized size = 1.13

$$\frac{1}{48}\left(\frac{315\,a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\left(bx^2+a\right)^{\frac{3}{2}}+192\,\sqrt{bx^2+a}a-\frac{165\left(bx^2+a\right)^{\frac{5}{2}}a^2-280\left(bx^2+a\right)^{\frac{3}{2}}a^3+123\,\sqrt{bx^2+a}a\right.}{b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(315*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 16*(b*x^2 + a)^(3
/2) + 192*sqrt(b*x^2 + a)*a - (165*(b*x^2 + a)^(5/2)*a^2 - 280*(b*x^2 + a)^
(3/2)*a^3 + 123*sqrt(b*x^2 + a)*a^4)/(b^3*x^6))*b^3
```

3.421 $\int \frac{(a+bx^2)^{9/2}}{x^9} dx$

Optimal. Leaf size=128

$$-\frac{21b^2(a+bx^2)^{5/2}}{64x^4}-\frac{105b^3(a+bx^2)^{3/2}}{128x^2}+\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{9/2}}{8x^8}-\frac{3b(a+bx^2)^{7/2}}{16x^6}$$

```
[Out] (315*b^4*Sqrt[a + b*x^2])/128 - (105*b^3*(a + b*x^2)^(3/2))/(128*x^2) - (21
*b^2*(a + b*x^2)^(5/2))/(64*x^4) - (3*b*(a + b*x^2)^(7/2))/(16*x^6) - (a +
b*x^2)^(9/2)/(8*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/1
28
```

Rubi [A] time = 0.0793392, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{21b^2(a+bx^2)^{5/2}}{64x^4}-\frac{105b^3(a+bx^2)^{3/2}}{128x^2}+\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{9/2}}{8x^8}-\frac{3b(a+bx^2)^{7/2}}{16x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x^2])/128 - (105*b^3*(a + b*x^2)^(3/2))/(128*x^2) - (21
*b^2*(a + b*x^2)^(5/2))/(64*x^4) - (3*b*(a + b*x^2)^(7/2))/(16*x^6) - (a +
b*x^2)^(9/2)/(8*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/1
28
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^9}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{16}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{32}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{128}\left(105b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,x\right.\\
&= -\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{256}\left(315b^4\right)\text{Subst}\left(\right.\\
&= \frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{256}\\
&= \frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{128}\\
&= \frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}-\frac{315}{128}
\end{aligned}$$

Mathematica [C] time = 0.0123607, size = 39, normalized size = 0.3

$$-\frac{b^4\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] -(b^4*(a + b*x^2)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, 1 + (b*x^2)/a])/(
11*a^5)
```

Maple [A] time = 0.018, size = 190, normalized size = 1.5

$$-\frac{1}{8\,ax^8}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{b}{16\,a^2x^6}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{5\,b^2}{64\,a^3x^4}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{35\,b^3}{128\,a^4x^2}\left(bx^2+a\right)^{\frac{11}{2}}+\frac{35\,b^4}{128\,a^4}\left(bx^2+a\right)^{\frac{9}{2}}+\frac{45\,b^4}{128\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(11/2)-1/16*b/a^2/x^6*(b*x^2+a)^(11/2)-5/64*b^2/a^3/x^
4*(b*x^2+a)^(11/2)-35/128*b^3/a^4/x^2*(b*x^2+a)^(11/2)+35/128*b^4/a^4*(b*x^
2+a)^(9/2)+45/128*b^4/a^3*(b*x^2+a)^(7/2)+63/128*b^4/a^2*(b*x^2+a)^(5/2)+10
5/128*b^4/a*(b*x^2+a)^(3/2)-315/128*b^4*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x)+315/128*b^4*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66289, size = 456, normalized size = 3.56

$$\left[\frac{315\sqrt{a}b^4x^8\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(128b^4x^8-325ab^3x^6-210a^2b^2x^4-88a^3bx^2-16a^4\right)\sqrt{bx^2+a}}{256x^8}, \frac{315\sqrt{-a}b^4x^8\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(315*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x^2 - 16
*a^4)*sqrt(b*x^2 + a))/x^8, 1/128*(315*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqr
t(b*x^2 + a)) + (128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x
^2 - 16*a^4)*sqrt(b*x^2 + a))/x^8]
```

Sympy [A] time = 8.46165, size = 173, normalized size = 1.35

$$-\frac{315\sqrt{a}b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128}-\frac{a^5}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{13a^4\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{149a^3b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{535a^2b^{\frac{5}{2}}}{128x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{197ab^{\frac{7}{2}}}{128x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x}{\sqrt{\frac{a}{bx^2}+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**9,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*x))/128 - a**5/(8*sqrt(b)*x**9*sqr
t(a/(b*x**2) + 1)) - 13*a**4*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 149*a
**3*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) - 535*a**2*b**(5/2)/(128*x**3*s
qrt(a/(b*x**2) + 1)) - 197*a*b**(7/2)/(128*x*sqrt(a/(b*x**2) + 1)) + b**(9/
2)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A] time = 2.65703, size = 140, normalized size = 1.09

$$\frac{1}{128}\left(\frac{315\,a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\sqrt{bx^2+a}-\frac{325\left(bx^2+a\right)^{\frac{7}{2}}a-765\left(bx^2+a\right)^{\frac{5}{2}}a^2+643\left(bx^2+a\right)^{\frac{3}{2}}a^3-187\sqrt{bx^2+}}{b^4x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(315*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x^2 + a
) - (325*(b*x^2 + a)^(7/2)*a - 765*(b*x^2 + a)^(5/2)*a^2 + 643*(b*x^2 + a)^
(3/2)*a^3 - 187*sqrt(b*x^2 + a)*a^4)/(b^4*x^8))*b^4
```

3.422 $\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx$

Optimal. Leaf size=131

$$-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$

```
[Out] (-63*b^4*Sqrt[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4)
- (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) -
 (a + b*x^2)^(9/2)/(10*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
56*Sqrt[a])
```

Rubi [A] time = 0.0826313, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4)
- (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) -
 (a + b*x^2)^(9/2)/(10*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
56*Sqrt[a])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{11}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{20}(9b)\operatorname{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{160}\left(63b^2\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{64}\left(21b^3\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{256}\left(63b^4\right)\operatorname{Subst}\Bigg(\int$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{512}\Big($$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{256}\Big($$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} - \frac{63b^5}{}$$

Mathematica [A] time = 0.0479415, size = 109, normalized size = 0.83

$$-\frac{2858a^2b^3x^6+2024a^3b^2x^4+784a^4bx^2+128a^5+2455ab^4x^8+315b^5x^{10}\sqrt{\frac{bx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+965b^5x^{10}}{1280x^{10}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] -(128*a^5 + 784*a^4*b*x^2 + 2024*a^3*b^2*x^4 + 2858*a^2*b^3*x^6 + 2455*a*b^
4*x^8 + 965*b^5*x^10 + 315*b^5*x^10*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b
*x^2)/a]])/(1280*x^10*Sqrt[a + b*x^2])
```

Maple [B] time = 0.046, size = 213, normalized size = 1.6

$$-\frac{1}{10\,ax^{10}}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{b}{80\,a^2x^8}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{b^2}{160\,a^3x^6}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{b^3}{128\,a^4x^4}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{7\,b^4}{256\,a^5x^2}\left(bx^2+a\right)^{\frac{11}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^11,x)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(11/2)-1/80*b/a^2/x^8*(b*x^2+a)^(11/2)-1/160*b^2/a^3
/x^6*(b*x^2+a)^(11/2)-1/128*b^3/a^4/x^4*(b*x^2+a)^(11/2)-7/256*b^4/a^5/x^2*
```

```
(b*x^2+a)^(11/2)+7/256*b^5/a^5*(b*x^2+a)^(9/2)+9/256*b^5/a^4*(b*x^2+a)^(7/2
)+63/1280*b^5/a^3*(b*x^2+a)^(5/2)+21/256*b^5/a^2*(b*x^2+a)^(3/2)-63/256*b^5
/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+63/256*b^5/a*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72751, size = 497, normalized size = 3.79

$$\left[\frac{315\,\sqrt{a}b^5x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(965\,ab^4x^8+1490\,a^2b^3x^6+1368\,a^3b^2x^4+656\,a^4bx^2+128\,a^5\right)\sqrt{bx^2+a}}{2560\,ax^{10}},\ 315\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="fricas")

[Out] [1/2560*(315*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a^3*b^2*x^4 + 656*a^4*b
*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10), 1/1280*(315*sqrt(-a)*b^5*x^10*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) - (965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a
^3*b^2*x^4 + 656*a^4*b*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10)]
```

Sympy [A] time = 10.3829, size = 153, normalized size = 1.17

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{10x^9}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{80x^7}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{160x^5}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{128x^3}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{256x}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{256\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**11,x)

[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(10*x**9) - 41*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(80*x**7) - 171*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(160*x**5) - 1
49*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(128*x**3) - 193*b**(9/2)*sqrt(a/(b*x**2
) + 1)/(256*x) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*x))/(256*sqrt(a))
```

Giac [A] time = 1.66556, size = 139, normalized size = 1.06

$$\frac{1}{1280}\,b^5\left(\frac{315\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\left(bx^2+a\right)^{\frac{9}{2}}-2370\left(bx^2+a\right)^{\frac{7}{2}}a+2688\left(bx^2+a\right)^{\frac{5}{2}}a^2-1470\left(bx^2+a\right)^{\frac{3}{2}}a^3+315\sqrt{bx}}{b^5x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*b^5*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (965*(b*x^2 + a
)^(9/2) - 2370*(b*x^2 + a)^(7/2)*a + 2688*(b*x^2 + a)^(5/2)*a^2 - 1470*(b*x
^2 + a)^(3/2)*a^3 + 315*sqrt(b*x^2 + a)*a^4)/(b^5*x^10))
```

3.423 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^{13}}\,dx$

Optimal. Leaf size=155

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8}-\frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}}-\frac{(a+b}{12}$$

```
[Out] (-21*b^4*Sqrt[a + b*x^2])/(512*x^4) - (21*b^5*Sqrt[a + b*x^2])/(1024*a*x^2)
 - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^
8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*
b^6*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Rubi [A] time = 0.101011, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8}-\frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}}-\frac{(a+b}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x^2])/(512*x^4) - (21*b^5*Sqrt[a + b*x^2])/(1024*a*x^2)
 - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^
8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*
b^6*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^7}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^6}\,dx,x,x^2\right)$$
$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{80}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx,x,x^2\right)$$
$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{128}\left(21b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x,x^2\right.$$
$$= -\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{256}\left(21b^4\right)\text{Subst}\left(\int\right.$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \left(21b^5\right)$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{(a+}{1}$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{(a+}{1}$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{(a+}{1}$$

Mathematica [C] time = 0.0122679, size = 39, normalized size = 0.25

$$-\frac{b^6\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(\frac{11}{2},7;\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] -(b^6*(a + b*x^2)^(11/2)*Hypergeometric2F1[11/2, 7, 13/2, 1 + (b*x^2)/a])/(
11*a^7)
```

Maple [A] time = 0.113, size = 233, normalized size = 1.5

$$-\frac{1}{12\,ax^{12}}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{b}{120\,a^2x^{10}}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{b^2}{960\,a^3x^8}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{b^3}{1920\,a^4x^6}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{b^4}{1536\,a^5x^4}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^13,x)
```

```
[Out] -1/12/a/x^12*(b*x^2+a)^(11/2)+1/120*b/a^2/x^10*(b*x^2+a)^(11/2)+1/960*b^2/a
^3/x^8*(b*x^2+a)^(11/2)+1/1920*b^3/a^4/x^6*(b*x^2+a)^(11/2)+1/1536*b^4/a^5/
x^4*(b*x^2+a)^(11/2)+7/3072*b^5/a^6/x^2*(b*x^2+a)^(11/2)-7/3072*b^6/a^6*(b*
x^2+a)^(9/2)-3/1024*b^6/a^5*(b*x^2+a)^(7/2)-21/5120*b^6/a^4*(b*x^2+a)^(5/2)
-7/1024*b^6/a^3*(b*x^2+a)^(3/2)+21/1024*b^6/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^
2+a)^(1/2))/x)-21/1024*b^6/a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84903, size = 571, normalized size = 3.68

$$\left[\frac{315\,\sqrt{a}b^6x^{12}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(315\,ab^5x^{10}+4910\,a^2b^4x^8+11432\,a^3b^3x^6+12144\,a^4b^2x^4+6272\,a^5bx^2+128\right.}{30720\,a^2x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/30720*(315*sqrt(a)*b^6*x^12*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(315*a*b^5*x^10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*
a^4*b^2*x^4 + 6272*a^5*b*x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12), -1/15
360*(315*sqrt(-a)*b^6*x^12*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^5*x^
10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*a^4*b^2*x^4 + 6272*a^5*b*
x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12)]
```

Sympy [A] time = 16.2194, size = 204, normalized size = 1.32

$$-\frac{a^5}{12\sqrt{b}x^{13}\sqrt{\frac{a}{bx^2}+1}}-\frac{59a^4\sqrt{b}}{120x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{960x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{1920x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{8171ab^{\frac{7}{2}}}{7680x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{1045b^{\frac{9}{2}}}{3072x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**13,x)
```

```
[Out] -a**5/(12*sqrt(b)*x**13*sqrt(a/(b*x**2) + 1)) - 59*a**4*sqrt(b)/(120*x**11*
sqrt(a/(b*x**2) + 1)) - 1151*a**3*b**(3/2)/(960*x**9*sqrt(a/(b*x**2) + 1))
```

```
- 2947*a**2*b**(5/2)/(1920*x**7*sqrt(a/(b*x**2) + 1)) - 8171*a*b**(7/2)/(76
80*x**5*sqrt(a/(b*x**2) + 1)) - 1045*b**(9/2)/(3072*x**3*sqrt(a/(b*x**2) +
1)) - 21*b**(11/2)/(1024*a*x*sqrt(a/(b*x**2) + 1)) + 21*b**6*asinh(sqrt(a)/
(sqrt(b)*x))/(1024*a**(3/2))
```

Giac [A] time = 2.37557, size = 165, normalized size = 1.06

$$-\frac{1}{15360}\,b^6\left(\frac{315\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{315\left(bx^2+a\right)^{\frac{11}{2}}+3335\left(bx^2+a\right)^{\frac{9}{2}}a-5058\left(bx^2+a\right)^{\frac{7}{2}}a^2+4158\left(bx^2+a\right)^{\frac{5}{2}}a^3-1}{ab^6x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/15360*b^6*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x
^2 + a)^(11/2) + 3335*(b*x^2 + a)^(9/2)*a - 5058*(b*x^2 + a)^(7/2)*a^2 + 41
58*(b*x^2 + a)^(5/2)*a^3 - 1785*(b*x^2 + a)^(3/2)*a^4 + 315*sqrt(b*x^2 + a)
*a^5)/(a*b^6*x^12))
```

3.424 $\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx$

Optimal. Leaf size=179

$$\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}}-\frac{3b\left(a+bx^2\right)}{56x^{12}}$$

```
[Out] (-3*b^4*Sqrt[a + b*x^2])/(256*x^6) - (3*b^5*Sqrt[a + b*x^2])/(1024*a*x^4) +
 (9*b^6*Sqrt[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^(3/2))/(128*x^
8) - (3*b^2*(a + b*x^2)^(5/2))/(80*x^10) - (3*b*(a + b*x^2)^(7/2))/(56*x^12
) - (a + b*x^2)^(9/2)/(14*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2048*a^(5/2))
```

Rubi [A] time = 0.121738, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}}-\frac{3b\left(a+bx^2\right)}{56x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x^2])/(256*x^6) - (3*b^5*Sqrt[a + b*x^2])/(1024*a*x^4) +
 (9*b^6*Sqrt[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^(3/2))/(128*x^
8) - (3*b^2*(a + b*x^2)^(5/2))/(80*x^10) - (3*b*(a + b*x^2)^(7/2))/(56*x^12
) - (a + b*x^2)^(9/2)/(14*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2048*a^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{15}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{28}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{16}\left(3b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{32}\left(3b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{256}\left(9b^4\right)\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{512}\left(3b^5\right.$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right.}{14x^1}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+b\right.}{56x^1}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+b\right.}{56x^1}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+b\right.}{56x^1}$$

Mathematica [C] time = 0.0123766, size = 39, normalized size = 0.22

$$\frac{b^7\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(\frac{11}{2},8;\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (b^7*(a + b*x^2)^(11/2)*Hypergeometric2F1[11/2, 8, 13/2, 1 + (b*x^2)/a])/(1
1*a^8)
```

Maple [A] time = 0.29, size = 253, normalized size = 1.4

$$-\frac{1}{14\,a x^{14}}\left(b x^2+a\right)^{\frac{11}{2}}+\frac{b}{56\,a^2 x^{12}}\left(b x^2+a\right)^{\frac{11}{2}}-\frac{b^2}{560\,a^3 x^{10}}\left(b x^2+a\right)^{\frac{11}{2}}-\frac{b^3}{4480\,a^4 x^8}\left(b x^2+a\right)^{\frac{11}{2}}-\frac{b^4}{8960\,a^5 x^6}\left(b x^2+a\right)^{\frac{11}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^15,x)
```

```
[Out] -1/14/a/x^14*(b*x^2+a)^(11/2)+1/56*b/a^2/x^12*(b*x^2+a)^(11/2)-1/560*b^2/a^
3/x^10*(b*x^2+a)^(11/2)-1/4480*b^3/a^4/x^8*(b*x^2+a)^(11/2)-1/8960*b^4/a^5/
x^6*(b*x^2+a)^(11/2)-1/7168*b^5/a^6/x^4*(b*x^2+a)^(11/2)-1/2048*b^6/a^7/x^2
*(b*x^2+a)^(11/2)+1/2048*b^7/a^7*(b*x^2+a)^(9/2)+9/14336*b^7/a^6*(b*x^2+a)^
(7/2)+9/10240*b^7/a^5*(b*x^2+a)^(5/2)+3/2048*b^7/a^4*(b*x^2+a)^(3/2)-9/2048
*b^7/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+9/2048*b^7/a^3*(b*x^2+a)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1011, size = 628, normalized size = 3.51

$$\left[\frac{315\,\sqrt{a}b^7x^{14}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(315\,ab^6x^{12}-210\,a^2b^5x^{10}-14168\,a^3b^4x^8-39056\,a^4b^3x^6-44928\,a^5b^2x^4-24\right.}{143360\,a^3x^{14}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="fricas")
```

```
[Out] [1/143360*(315*sqrt(a)*b^7*x^14*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056
*a^4*b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 +
 a))/(a^3*x^14), 1/71680*(315*sqrt(-a)*b^7*x^14*arctan(sqrt(-a)/sqrt(b*x^2
+ a)) + (315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056*a^4*
b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 + a))/
(a^3*x^14)]
```

Sympy [A] time = 22.8739, size = 231, normalized size = 1.29

$$-\frac{a^5}{14\sqrt{b}x^{15}\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^4\sqrt{b}}{56x^{13}\sqrt{\frac{a}{bx^2}+1}}-\frac{541a^3b^{\frac{3}{2}}}{560x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{5249a^2b^{\frac{5}{2}}}{4480x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{6653ab^{\frac{7}{2}}}{8960x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{1027b^{\frac{9}{2}}}{5120x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**15,x)
```

```
[Out] -a**5/(14*sqrt(b)*x**15*sqrt(a/(b*x**2) + 1)) - 23*a**4*sqrt(b)/(56*x**13*s
qrt(a/(b*x**2) + 1)) - 541*a**3*b**(3/2)/(560*x**11*sqrt(a/(b*x**2) + 1)) -
 5249*a**2*b**(5/2)/(4480*x**9*sqrt(a/(b*x**2) + 1)) - 6653*a*b**(7/2)/(896
0*x**7*sqrt(a/(b*x**2) + 1)) - 1027*b**(9/2)/(5120*x**5*sqrt(a/(b*x**2) + 1
)) + 3*b**(11/2)/(2048*a*x**3*sqrt(a/(b*x**2) + 1)) + 9*b**(13/2)/(2048*a**
2*x*sqrt(a/(b*x**2) + 1)) - 9*b**7*asinh(sqrt(a)/(sqrt(b)*x))/(2048*a**(5/2
))
```

Giac [A] time = 2.35695, size = 184, normalized size = 1.03

$$\frac{1}{71680}b^7\left(\frac{315\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{315\left(bx^2+a\right)^{\frac{13}{2}}-2100\left(bx^2+a\right)^{\frac{11}{2}}a-8393\left(bx^2+a\right)^{\frac{9}{2}}a^2+9216\left(bx^2+a\right)^{\frac{7}{2}}a^3-59}{a^2b^7x^{14}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="giac")
```

```
[Out] 1/71680*b^7*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*
x^2 + a)^(13/2) - 2100*(b*x^2 + a)^(11/2)*a - 8393*(b*x^2 + a)^(9/2)*a^2 +
9216*(b*x^2 + a)^(7/2)*a^3 - 5943*(b*x^2 + a)^(5/2)*a^4 + 2100*(b*x^2 + a)^
(3/2)*a^5 - 315*sqrt(b*x^2 + a)*a^6)/(a^2*b^7*x^14))
```

3.425 $\int x^6 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=202

$$\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}$$

```
[Out] (45*a^7*x*Sqrt[a + b*x^2])/(32768*b^3) - (15*a^6*x^3*Sqrt[a + b*x^2])/(1638
4*b^2) + (3*a^5*x^5*Sqrt[a + b*x^2])/(4096*b) + (9*a^4*x^7*Sqrt[a + b*x^2])
/2048 + (3*a^3*x^7*(a + b*x^2)^(3/2))/256 + (3*a^2*x^7*(a + b*x^2)^(5/2))/1
28 + (9*a*x^7*(a + b*x^2)^(7/2))/224 + (x^7*(a + b*x^2)^(9/2))/16 - (45*a^8
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32768*b^(7/2))
```

Rubi [A] time = 0.11412, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^(9/2),x]
```


```
[Out] (45*a^7*x*Sqrt[a + b*x^2])/(32768*b^3) - (15*a^6*x^3*Sqrt[a + b*x^2])/(1638
4*b^2) + (3*a^5*x^5*Sqrt[a + b*x^2])/(4096*b) + (9*a^4*x^7*Sqrt[a + b*x^2])
/2048 + (3*a^3*x^7*(a + b*x^2)^(3/2))/256 + (3*a^2*x^7*(a + b*x^2)^(5/2))/1
28 + (9*a*x^7*(a + b*x^2)^(7/2))/224 + (x^7*(a + b*x^2)^(9/2))/16 - (45*a^8
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32768*b^(7/2))
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^6\left(a+bx^2\right)^{9/2}dx &= \frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{16}(9a)\int x^6\left(a+bx^2\right)^{7/2}dx\\
&= \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{32}\left(9a^2\right)\int x^6\left(a+bx^2\right)^{5/2}dx\\
&= \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{128}\left(15a^3\right)\int x^6\left(a+bx^2\right)^{3}\\
&= \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{256}\\
&= \frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\\
&= \frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7(a\\
&= -\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7(a+\\
&= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3}\\
&= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3}\\
&= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3}
\end{aligned}
$$

Mathematica [A] time = 0.213618, size = 138, normalized size = 0.68

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(119040a^2b^5x^{10}+98432a^3b^4x^8+32624a^4b^3x^6+168a^5b^2x^4-210a^6bx^2+315a^7+66560ab^6x^{12}+143\right.\right.}{229376b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(315*a^7 - 210*a^6*b*x^2 + 168*a^5*b^2*x^4 + 32
624*a^4*b^3*x^6 + 98432*a^3*b^4*x^8 + 119040*a^2*b^5*x^10 + 66560*a*b^6*x^1
2 + 14336*b^7*x^14) - (315*a^(15/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 +
(b*x^2)/a]))/(229376*b^(7/2))
```

Maple [A] time = 0.011, size = 169, normalized size = 0.8

$$
\frac{x^5}{16b}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{5ax^3}{224b^2}\left(bx^2+a\right)^{\frac{11}{2}}+\frac{5a^2x}{896b^3}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{a^3x}{1792b^3}\left(bx^2+a\right)^{\frac{9}{2}}-\frac{9a^4x}{14336b^3}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3a^5x}{4096b^3}(b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/16*x^5*(b*x^2+a)^(11/2)/b-5/224/b^2*a*x^3*(b*x^2+a)^(11/2)+5/896/b^3*a^2*
x*(b*x^2+a)^(11/2)-1/1792/b^3*a^3*x*(b*x^2+a)^(9/2)-9/14336/b^3*a^4*x*(b*x^
2+a)^(7/2)-3/4096/b^3*a^5*x*(b*x^2+a)^(5/2)-15/16384/b^3*a^6*x*(b*x^2+a)^(3
/2)-45/32768*a^7*x*(b*x^2+a)^(1/2)/b^3-45/32768/b^(7/2)*a^8*ln(x*b^(1/2)+(b
*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.53812, size = 656, normalized size = 3.25

$$\left[\frac{315\, a^{8} \sqrt{b} \log\left(-2\, b x^{2} + 2\, \sqrt{b x^{2} + a} \sqrt{b} x - a\right) + 2\, \left(14336\, b^{8} x^{15} + 66560\, a b^{7} x^{13} + 119040\, a^{2} b^{6} x^{11} + 98432\, a^{3} b^{5} x^{9} + 32624\right.}{458752\, b^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/458752*(315*a^8*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 98432*a^3*b^
5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 315*a^7*b*x
)*sqrt(b*x^2 + a))/b^4, 1/229376*(315*a^8*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 984
32*a^3*b^5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 31
5*a^7*b*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] time = 27.2185, size = 258, normalized size = 1.28

$$\frac{45a^{\frac{15}{2}}x}{32768b^{3}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{15a^{\frac{13}{2}}x^{3}}{32768b^{2}\sqrt{1+\frac{bx^{2}}{a}}} - \frac{3a^{\frac{11}{2}}x^{5}}{16384b\sqrt{1+\frac{bx^{2}}{a}}} + \frac{4099a^{\frac{9}{2}}x^{7}}{28672\sqrt{1+\frac{bx^{2}}{a}}} + \frac{8191a^{\frac{7}{2}}bx^{9}}{14336\sqrt{1+\frac{bx^{2}}{a}}} + \frac{1699a^{\frac{5}{2}}b^{2}x^{11}}{1792\sqrt{1+\frac{bx^{2}}{a}}} + \frac{725}{896}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(9/2),x)
```

```
[Out] 45*a**(15/2)*x/(32768*b**3*sqrt(1 + b*x**2/a)) + 15*a**(13/2)*x**3/(32768*b
**2*sqrt(1 + b*x**2/a)) - 3*a**(11/2)*x**5/(16384*b*sqrt(1 + b*x**2/a)) + 4
099*a**(9/2)*x**7/(28672*sqrt(1 + b*x**2/a)) + 8191*a**(7/2)*b*x**9/(14336*
sqrt(1 + b*x**2/a)) + 1699*a**(5/2)*b**2*x**11/(1792*sqrt(1 + b*x**2/a)) +
725*a**(3/2)*b**3*x**13/(896*sqrt(1 + b*x**2/a)) + 79*sqrt(a)*b**4*x**15/(2
24*sqrt(1 + b*x**2/a)) - 45*a**8*asinh(sqrt(b)*x/sqrt(a))/(32768*b**(7/2))
+ b**5*x**17/(16*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.54701, size = 180, normalized size = 0.89

$$\frac{45\,a^8\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{32768\,b^{\frac{7}{2}}}+\frac{1}{229376}\left(\frac{315\,a^7}{b^3}-2\left(\frac{105\,a^6}{b^2}-4\left(\frac{21\,a^5}{b}+2\left(2039\,a^4+8\left(769\,a^3b+2\left(465\,a^2b^2+4\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 45/32768*a^8*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2) + 1/229376*(315
*a^7/b^3 - 2*(105*a^6/b^2 - 4*(21*a^5/b + 2*(2039*a^4 + 8*(769*a^3*b + 2*(4
65*a^2*b^2 + 4*(14*b^4*x^2 + 65*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b
*x^2 + a)*x
```

3.426 $\int x^4 \left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=178

$$-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right.$$

```
[Out] (-9*a^6*x*Sqrt[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*Sqrt[a + b*x^2])/(1024*b
) + (3*a^4*x^5*Sqrt[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^(3/2))/128 + (
3*a^2*x^5*(a + b*x^2)^(5/2))/80 + (3*a*x^5*(a + b*x^2)^(7/2))/56 + (x^5*(a
+ b*x^2)^(9/2))/14 + (9*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2048*b^(
5/2))
```

Rubi [A] time = 0.0855914, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (-9*a^6*x*Sqrt[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*Sqrt[a + b*x^2])/(1024*b
) + (3*a^4*x^5*Sqrt[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^(3/2))/128 + (
3*a^2*x^5*(a + b*x^2)^(5/2))/80 + (3*a*x^5*(a + b*x^2)^(7/2))/56 + (x^5*(a
+ b*x^2)^(9/2))/14 + (9*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2048*b^(
5/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4 \left(a+bx^2\right)^{9/2} dx = \frac{1}{14}x^5\left(a+bx^2\right)^{9/2} + \frac{1}{14}(9a)\int x^4\left(a+bx^2\right)^{7/2} dx$$
$$= \frac{3}{56}ax^5\left(a+bx^2\right)^{7/2} + \frac{1}{14}x^5\left(a+bx^2\right)^{9/2} + \frac{1}{8}\left(3a^2\right)\int x^4\left(a+bx^2\right)^{5/2} dx$$
$$= \frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2} + \frac{3}{56}ax^5\left(a+bx^2\right)^{7/2} + \frac{1}{14}x^5\left(a+bx^2\right)^{9/2} + \frac{1}{16}\left(3a^3\right)\int x^4\left(a+bx^2\right)^{3/2} dx$$
$$= \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2} + \frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2} + \frac{3}{56}ax^5\left(a+bx^2\right)^{7/2} + \frac{1}{14}x^5\left(a+bx^2\right)^{9/2} + \frac{1}{128}(9$$
$$= \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2} + \frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2} + \frac{3}{56}ax^5\left(a+bx^2\right)^{7/2} + \frac{1}{14}x^5$$
$$= \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2} + \frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2} + \frac{3}{56}ax^5(a$$
$$= -\frac{9a^6x\sqrt{a+bx^2}}{2048b^2} + \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2} + \frac{3}{80}a^2x^5(a+$$
$$= -\frac{9a^6x\sqrt{a+bx^2}}{2048b^2} + \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2} + \frac{3}{80}a^2x^5(a+$$
$$= -\frac{9a^6x\sqrt{a+bx^2}}{2048b^2} + \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2} + \frac{3}{80}a^2x^5(a+$$

Mathematica [A] time = 0.184786, size = 127, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(44928a^2b^4x^8+39056a^3b^3x^6+14168a^4b^2x^4+210a^5bx^2-315a^6+24320ab^5x^{10}+5120b^6x^{12}\right)+\frac{315a^1}{}\right.}{71680b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-315*a^6 + 210*a^5*b*x^2 + 14168*a^4*b^2*x^4 +
 39056*a^3*b^3*x^6 + 44928*a^2*b^4*x^8 + 24320*a*b^5*x^10 + 5120*b^6*x^12)
+ (315*a^(13/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(71680*
b^(5/2))
```

Maple [A] time = 0.008, size = 149, normalized size = 0.8

$$\frac{x^3}{14b}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{ax}{56b^2}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{a^2x}{560b^2}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{9a^3x}{4480b^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{3a^4x}{1280b^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{3a^5x}{1024b^2}(bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/14*x^3*(b*x^2+a)^(11/2)/b-1/56/b^2*a*x*(b*x^2+a)^(11/2)+1/560/b^2*a^2*x*(
b*x^2+a)^(9/2)+9/4480/b^2*a^3*x*(b*x^2+a)^(7/2)+3/1280/b^2*a^4*x*(b*x^2+a)^
(5/2)+3/1024/b^2*a^5*x*(b*x^2+a)^(3/2)+9/2048*a^6*x*(b*x^2+a)^(1/2)/b^2+9/2
```

```
048/b^(5/2)*a^7*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20406, size = 599, normalized size = 3.37

$$\left[\frac{315\,a^7\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\left(5120\,b^7x^{13} + 24320\,ab^6x^{11} + 44928\,a^2b^5x^9 + 39056\,a^3b^4x^7 + 14168\,a^4b\right.}{143360\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/143360*(315*a^7*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(5120*b^7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x
^7 + 14168*a^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^
3, -1/71680*(315*a^7*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (5120*b^
7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x^7 + 14168*a
^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 19.973, size = 231, normalized size = 1.3

$$-\frac{9a^{\frac{13}{2}}x}{2048b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{11}{2}}x^3}{2048b\sqrt{1+\frac{bx^2}{a}}} + \frac{1027a^{\frac{9}{2}}x^5}{5120\sqrt{1+\frac{bx^2}{a}}} + \frac{6653a^{\frac{7}{2}}bx^7}{8960\sqrt{1+\frac{bx^2}{a}}} + \frac{5249a^{\frac{5}{2}}b^2x^9}{4480\sqrt{1+\frac{bx^2}{a}}} + \frac{541a^{\frac{3}{2}}b^3x^{11}}{560\sqrt{1+\frac{bx^2}{a}}} + \frac{23\sqrt{a}b^4x^{13}}{56\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(9/2),x)
```

```
[Out] -9*a**(13/2)*x/(2048*b**2*sqrt(1 + b*x**2/a)) - 3*a**(11/2)*x**3/(2048*b*sq
rt(1 + b*x**2/a)) + 1027*a**(9/2)*x**5/(5120*sqrt(1 + b*x**2/a)) + 6653*a**
(7/2)*b*x**7/(8960*sqrt(1 + b*x**2/a)) + 5249*a**(5/2)*b**2*x**9/(4480*sqrt
(1 + b*x**2/a)) + 541*a**(3/2)*b**3*x**11/(560*sqrt(1 + b*x**2/a)) + 23*sqr
t(a)*b**4*x**13/(56*sqrt(1 + b*x**2/a)) + 9*a**7*asinh(sqrt(b)*x/sqrt(a))/(
2048*b**(5/2)) + b**5*x**15/(14*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.98651, size = 161, normalized size = 0.9

$$-\frac{9\,a^7\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2048\,b^{\frac{5}{2}}} - \frac{1}{71680}\left(\frac{315\,a^6}{b^2} - 2\left(\frac{105\,a^5}{b} + 4\left(1771\,a^4 + 2\left(2441\,a^3b + 8\left(351\,a^2b^2 + 10\left(4\,b^4x^2 + 19\,a\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2048*a^7*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/71680*(315*a
^6/b^2 - 2*(105*a^5/b + 4*(1771*a^4 + 2*(2441*a^3*b + 8*(351*a^2*b^2 + 10*(
4*b^4*x^2 + 19*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

3.427 $\int x^2 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=154

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(\right.$$

```
[Out] (21*a^5*x*Sqrt[a + b*x^2])/(1024*b) + (21*a^4*x^3*Sqrt[a + b*x^2])/512 + (7
*a^3*x^3*(a + b*x^2)^(3/2))/128 + (21*a^2*x^3*(a + b*x^2)^(5/2))/320 + (3*a
*x^3*(a + b*x^2)^(7/2))/40 + (x^3*(a + b*x^2)^(9/2))/12 - (21*a^6*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

Rubi [A] time = 0.0702928, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (21*a^5*x*Sqrt[a + b*x^2])/(1024*b) + (21*a^4*x^3*Sqrt[a + b*x^2])/512 + (7
*a^3*x^3*(a + b*x^2)^(3/2))/128 + (21*a^2*x^3*(a + b*x^2)^(5/2))/320 + (3*a
*x^3*(a + b*x^2)^(7/2))/40 + (x^3*(a + b*x^2)^(9/2))/12 - (21*a^6*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{9/2}dx=\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{4}(3a)\int x^2\left(a+bx^2\right)^{7/2}dx$$
$$=\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{40}\left(21a^2\right)\int x^2\left(a+bx^2\right)^{5/2}dx$$
$$=\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{64}\left(21a^3\right)\int x^2\left(a+bx^2\right)^{3/2}$$
$$=\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{128}$$
$$=\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x$$
$$=\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3$$
$$=\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3$$
$$=\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3$$

Mathematica [A] time = 0.167707, size = 116, normalized size = 0.75

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(12144a^2b^3x^6+11432a^3b^2x^4+4910a^4bx^2+315a^5+6272ab^4x^8+1280b^5x^{10}\right)-\frac{315a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{15360b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(315*a^5 + 4910*a^4*b*x^2 + 11432*a^3*b^2*x^4 +
 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^10) - (315*a^(11/2)*ArcSin
h[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(15360*b^(3/2))
```

Maple [A] time = 0.007, size = 129, normalized size = 0.8

$$\frac{x}{12b}\left(bx^2+a\right)^{\frac{11}{2}}-\frac{ax}{120b}\left(bx^2+a\right)^{\frac{9}{2}}-\frac{3a^2x}{320b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{7a^3x}{640b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{7a^4x}{512b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{21a^5x}{1024b}\sqrt{bx^2+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/12*x*(b*x^2+a)^(11/2)/b-1/120/b*a*x*(b*x^2+a)^(9/2)-3/320/b*a^2*x*(b*x^2+
a)^(7/2)-7/640/b*a^3*x*(b*x^2+a)^(5/2)-7/512/b*a^4*x*(b*x^2+a)^(3/2)-21/102
4*a^5*x*(b*x^2+a)^(1/2)/b-21/1024/b^(3/2)*a^6*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11596, size = 540, normalized size = 3.51

$$\left[\frac{315\,a^6\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7+11432\,a^3b^3x^5+4910\,a^4b^2x^3\right.}{30720\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*a^6*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(1280*b^6*x^11 + 6272*a*b^5*x^9 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5
+ 4910*a^4*b^2*x^3 + 315*a^5*b*x)*sqrt(b*x^2 + a))/b^2, 1/15360*(315*a^6*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1280*b^6*x^11 + 6272*a*b^5*x^9
 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5 + 4910*a^4*b^2*x^3 + 315*a^5*b*x)*
sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 13.8789, size = 204, normalized size = 1.32

$$\frac{21a^{\frac{11}{2}}x}{1024b\sqrt{1+\frac{bx^2}{a}}}+\frac{1045a^{\frac{9}{2}}x^3}{3072\sqrt{1+\frac{bx^2}{a}}}+\frac{8171a^{\frac{7}{2}}bx^5}{7680\sqrt{1+\frac{bx^2}{a}}}+\frac{2947a^{\frac{5}{2}}b^2x^7}{1920\sqrt{1+\frac{bx^2}{a}}}+\frac{1151a^{\frac{3}{2}}b^3x^9}{960\sqrt{1+\frac{bx^2}{a}}}+\frac{59\sqrt{a}b^4x^{11}}{120\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^6\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(9/2),x)
```

```
[Out] 21*a**(11/2)*x/(1024*b*sqrt(1 + b*x**2/a)) + 1045*a**(9/2)*x**3/(3072*sqrt(
1 + b*x**2/a)) + 8171*a**(7/2)*b*x**5/(7680*sqrt(1 + b*x**2/a)) + 2947*a**(
5/2)*b**2*x**7/(1920*sqrt(1 + b*x**2/a)) + 1151*a**(3/2)*b**3*x**9/(960*sqr
t(1 + b*x**2/a)) + 59*sqrt(a)*b**4*x**11/(120*sqrt(1 + b*x**2/a)) - 21*a**6
*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(3/2)) + b**5*x**13/(12*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

Giac [A] time = 1.47423, size = 142, normalized size = 0.92

$$\frac{21\,a^6\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{3}{2}}}+\frac{1}{15360}\left(\frac{315\,a^5}{b}+2\left(2455\,a^4+4\left(1429\,a^3b+2\left(759\,a^2b^2+8\left(10\,b^4x^2+49\,ab^3\right)x^2\right)x^2\right)x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 21/1024*a^6*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/15360*(315*a
^5/b + 2*(2455*a^4 + 4*(1429*a^3*b + 2*(759*a^2*b^2 + 8*(10*b^4*x^2 + 49*a*
b^3)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

3.428 $\int \left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=122

$$\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+b\right.$$

```
[Out] (63*a^4*x*Sqrt[a + b*x^2])/256 + (21*a^3*x*(a + b*x^2)^(3/2))/128 + (21*a^2
*x*(a + b*x^2)^(5/2))/160 + (9*a*x*(a + b*x^2)^(7/2))/80 + (x*(a + b*x^2)^(
9/2))/10 + (63*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqrt[b])
```

Rubi [A] time = 0.0421447, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2),x]
```

```
[Out] (63*a^4*x*Sqrt[a + b*x^2])/256 + (21*a^3*x*(a + b*x^2)^(3/2))/128 + (21*a^2
*x*(a + b*x^2)^(5/2))/160 + (9*a*x*(a + b*x^2)^(7/2))/80 + (x*(a + b*x^2)^(
9/2))/10 + (63*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{9/2} dx = \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{10}(9a)\int \left(a+bx^2\right)^{7/2} dx$$
$$= \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{80}\left(63a^2\right)\int \left(a+bx^2\right)^{5/2} dx$$
$$= \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{32}\left(21a^3\right)\int \left(a+bx^2\right)^{3/2} dx$$
$$= \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{128}\left(63a^4\right)$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+b\right.$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+b\right.$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+b\right.$$

Mathematica [A] time = 0.131381, size = 98, normalized size = 0.8

$$\frac{\sqrt{a+bx^2}\left(1368a^2b^2x^5+1490a^3bx^3+\frac{315a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+965a^4x+656ab^3x^7+128b^4x^9\right)}{1280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(965*a^4*x + 1490*a^3*b*x^3 + 1368*a^2*b^2*x^5 + 656*a*b^3
*x^7 + 128*b^4*x^9 + (315*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sq
rt[1 + (b*x^2)/a])))/1280
```

Maple [A] time = 0.002, size = 96, normalized size = 0.8

$$\frac{x}{10}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{9\,ax}{80}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{21\,a^2x}{160}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{21\,a^3x}{128}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{63\,a^4x}{256}\sqrt{bx^2+a} + \frac{63\,a^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2),x)
```

```
[Out] 1/10*x*(b*x^2+a)^(9/2)+9/80*a*x*(b*x^2+a)^(7/2)+21/160*a^2*x*(b*x^2+a)^(5/2
)+21/128*a^3*x*(b*x^2+a)^(3/2)+63/256*a^4*x*(b*x^2+a)^(1/2)+63/256*a^5/b^(1
/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9904, size = 468, normalized size = 3.84

$$\left[\frac{315\,a^5\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) + 2\left(128\,b^5x^9 + 656\,ab^4x^7 + 1368\,a^2b^3x^5 + 1490\,a^3b^2x^3 + 965\,a^4bx\right)\sqrt{bx^2 +}}{2560\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3*b^2*x^3 + 965*
a^4*b*x)*sqrt(b*x^2 + a))/b, -1/1280*(315*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3
*b^2*x^3 + 965*a^4*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 9.49426, size = 151, normalized size = 1.24

$$\frac{193a^{\frac{9}{2}}x\sqrt{1+\frac{bx^2}{a}}}{256} + \frac{149a^{\frac{7}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{128} + \frac{171a^{\frac{5}{2}}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{160} + \frac{41a^{\frac{3}{2}}b^3x^7\sqrt{1+\frac{bx^2}{a}}}{80} + \frac{\sqrt{a}b^4x^9\sqrt{1+\frac{bx^2}{a}}}{10} + \frac{63a^5\operatorname{asinh}\left(\right.}{256\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2),x)
```

```
[Out] 193*a**(9/2)*x*sqrt(1 + b*x**2/a)/256 + 149*a**(7/2)*b*x**3*sqrt(1 + b*x**2
/a)/128 + 171*a**(5/2)*b**2*x**5*sqrt(1 + b*x**2/a)/160 + 41*a**(3/2)*b**3*
x**7*sqrt(1 + b*x**2/a)/80 + sqrt(a)*b**4*x**9*sqrt(1 + b*x**2/a)/10 + 63*a
**5*asinh(sqrt(b)*x/sqrt(a))/(256*sqrt(b))
```

Giac [A] time = 1.48273, size = 123, normalized size = 1.01

$$-\frac{63\,a^5\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{256\,\sqrt{b}} + \frac{1}{1280}\left(965\,a^4 + 2\left(745\,a^3b + 4\left(171\,a^2b^2 + 2\left(8\,b^4x^2 + 41\,ab^3\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2 + a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -63/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/1280*(965*a^
4 + 2*(745*a^3*b + 4*(171*a^2*b^2 + 2*(8*b^4*x^2 + 41*a*b^3)*x^2)*x^2)*x^2)
*sqrt(b*x^2 + a)*x
```

3.429 $\int \frac{(a+bx^2)^{9/2}}{x^2} dx$

Optimal. Leaf size=123

$$\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{315}{128}a^4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}abx$$

```
[Out] (315*a^3*b*x*Sqrt[a + b*x^2])/128 + (105*a^2*b*x*(a + b*x^2)^(3/2))/64 + (2
1*a*b*x*(a + b*x^2)^(5/2))/16 + (9*b*x*(a + b*x^2)^(7/2))/8 - (a + b*x^2)^(
9/2)/x + (315*a^4*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/128
```

Rubi [A] time = 0.0439185, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{315}{128}a^4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}abx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] (315*a^3*b*x*Sqrt[a + b*x^2])/128 + (105*a^2*b*x*(a + b*x^2)^(3/2))/64 + (2
1*a*b*x*(a + b*x^2)^(5/2))/16 + (9*b*x*(a + b*x^2)^(7/2))/8 - (a + b*x^2)^(
9/2)/x + (315*a^4*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/128
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx = -\frac{\left(a+bx^2\right)^{9/2}}{x} + (9b)\int \left(a+bx^2\right)^{7/2}\,dx$$
$$= \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} + \frac{1}{8}(63ab)\int \left(a+bx^2\right)^{5/2}\,dx$$
$$= \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} + \frac{1}{16}\left(105a^2b\right)\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} + \frac{1}{64}\left(315a^3b\right)\int \sqrt{a+$$
$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} +$$
$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} +$$
$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} +$$

Mathematica [C] time = 0.0096428, size = 52, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] -((a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a]))
```

Maple [A] time = 0.004, size = 117, normalized size = 1.

$$-\frac{1}{ax}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{bx}{a}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{9\,bx}{8}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{21\,abx}{16}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{105\,a^2bx}{64}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{315\,a^3bx}{128}\sqrt{bx^2+a} + \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(11/2)+b/a*x*(b*x^2+a)^(9/2)+9/8*b*x*(b*x^2+a)^(7/2)+21/16
*a*b*x*(b*x^2+a)^(5/2)+105/64*a^2*b*x*(b*x^2+a)^(3/2)+315/128*a^3*b*x*(b*x^
2+a)^(1/2)+315/128*b^(1/2)*a^4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67496, size = 444, normalized size = 3.61

$$\left[\frac{315\,a^4\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4\right)\sqrt{bx^2+a}}{256\,x}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*a^4*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128*a^4)*
sqrt(b*x^2 + a))/x, -1/128*(315*a^4*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128
*a^4)*sqrt(b*x^2 + a))/x]
```

Sympy [A] time = 8.28635, size = 173, normalized size = 1.41

$$-\frac{a^{\frac{9}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{197a^{\frac{7}{2}}bx}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{535a^{\frac{5}{2}}b^2x^3}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{149a^{\frac{3}{2}}b^3x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{13\sqrt{a}b^4x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{315a^4\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128}+\frac{b^5x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)/(x*sqrt(1 + b*x**2/a)) + 197*a**(7/2)*b*x/(128*sqrt(1 + b*x**2/a)
) + 535*a**(5/2)*b**2*x**3/(128*sqrt(1 + b*x**2/a)) + 149*a**(3/2)*b**3*x**
5/(64*sqrt(1 + b*x**2/a)) + 13*sqrt(a)*b**4*x**7/(16*sqrt(1 + b*x**2/a)) +
315*a**4*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/128 + b**5*x**9/(8*sqrt(a)*sqrt(1
 + b*x**2/a))
```

Giac [A] time = 1.69715, size = 155, normalized size = 1.26

$$-\frac{315}{256}\,a^4\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^5\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{128}\left(325\,a^3b+2\left(105\,a^2b^2+4\left(2\,b^4x^2+11\,ab^3\right)x^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] -315/256*a^4*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^5*sqrt(b)/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/128*(325*a^3*b + 2*(105*a^2*b^2 +
4*(2*b^4*x^2 + 11*a*b^3)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

3.430 $\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx$

Optimal. Leaf size=128

$$\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}-\frac{3b\left(a+b\cdots\right.}{x}$$

```
[Out] (105*a^2*b^2*x*Sqrt[a + b*x^2])/16 + (35*a*b^2*x*(a + b*x^2)^(3/2))/8 + (7*
b^2*x*(a + b*x^2)^(5/2))/2 - (3*b*(a + b*x^2)^(7/2))/x - (a + b*x^2)^(9/2)/
(3*x^3) + (105*a^3*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/16
```

Rubi [A] time = 0.0487097, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}-\frac{3b\left(a+b\cdots\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] (105*a^2*b^2*x*Sqrt[a + b*x^2])/16 + (35*a*b^2*x*(a + b*x^2)^(3/2))/8 + (7*
b^2*x*(a + b*x^2)^(5/2))/2 - (3*b*(a + b*x^2)^(7/2))/x - (a + b*x^2)^(9/2)/
(3*x^3) + (105*a^3*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/16
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx = -\frac{(a+bx^2)^{9/2}}{3x^3} + (3b)\int \frac{(a+bx^2)^{7/2}}{x^2}\,dx$$
$$= -\frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} + (21b^2)\int (a+bx^2)^{5/2}\,dx$$
$$= \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} + \frac{1}{2}(35ab^2)\int (a+bx^2)^{3/2}\,dx$$
$$= \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} + \frac{1}{8}(105a^2b^2)\int \sqrt{a+$$
$$= \frac{105}{16}a^2b^2x\sqrt{a+bx^2} + \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} +$$
$$= \frac{105}{16}a^2b^2x\sqrt{a+bx^2} + \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} +$$
$$= \frac{105}{16}a^2b^2x\sqrt{a+bx^2} + \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} +$$

Mathematica [C] time = 0.0101454, size = 54, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] -(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -3/2, -1/2, -((b*x^2)/a)])/(3
*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.006, size = 146, normalized size = 1.1

$$-\frac{1}{3ax^3}(bx^2+a)^{\frac{11}{2}} - \frac{8b}{3a^2x}(bx^2+a)^{\frac{11}{2}} + \frac{8b^2x}{3a^2}(bx^2+a)^{\frac{9}{2}} + 3\frac{b^2x(bx^2+a)^{7/2}}{a} + \frac{7b^2x}{2}(bx^2+a)^{\frac{5}{2}} + \frac{35ab^2x}{8}(bx^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(11/2)-8/3*b/a^2/x*(b*x^2+a)^(11/2)+8/3*b^2/a^2*x*(b*x
^2+a)^(9/2)+3*b^2/a*x*(b*x^2+a)^(7/2)+7/2*b^2*x*(b*x^2+a)^(5/2)+35/8*a*b^2*
x*(b*x^2+a)^(3/2)+105/16*a^2*b^2*x*(b*x^2+a)^(1/2)+105/16*b^(3/2)*a^3*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74687, size = 450, normalized size = 3.52

$$\left[\frac{315\,a^3b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{96\,x^3},-\frac{315\,a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 - 16*a^4)*s
qrt(b*x^2 + a))/x^3, -1/48*(315*a^3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 -
16*a^4)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] time = 7.65697, size = 175, normalized size = 1.37

$$-\frac{a^{\frac{9}{2}}}{3x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{14a^{\frac{7}{2}}b}{3x\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2x}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{215a^{\frac{3}{2}}b^3x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}b^4x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{105a^3b^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16}+\frac{b^5x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**4,x)
```

```
[Out] -a**(9/2)/(3*x**3*sqrt(1 + b*x**2/a)) - 14*a**(7/2)*b/(3*x*sqrt(1 + b*x**2/
a)) - 43*a**(5/2)*b**2*x/(48*sqrt(1 + b*x**2/a)) + 215*a**(3/2)*b**3*x**3/(
48*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**4*x**5/(24*sqrt(1 + b*x**2/a)) + 105
*a**3*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/16 + b**5*x**7/(6*sqrt(a)*sqrt(1 +
b*x**2/a))
```

Giac [A] time = 1.64402, size = 216, normalized size = 1.69

$$-\frac{105}{32}\,a^3b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{48}\left(165\,a^2b^2+2\left(4\,b^4x^2+25\,ab^3\right)x^2\right)\sqrt{bx^2+a}x+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^4b^{\frac{3}{2}}\cdots\right.}{3\left(\left(\sqrt{b}x\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] -105/32*a^3*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/48*(165*a^2*b^
2 + 2*(4*b^4*x^2 + 25*a*b^3)*x^2)*sqrt(b*x^2 + a)*x + 2/3*(15*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^4*b^(3/2) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
3/2) + 13*a^6*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.431 $\int \frac{(a+bx^2)^{9/2}}{x^6} dx$

Optimal. Leaf size=129

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+bx\right.}{5x^3}$$

```
[Out] (63*a*b^3*x*Sqrt[a + b*x^2])/8 + (21*b^3*x*(a + b*x^2)^(3/2))/4 - (21*b^2*(
a + b*x^2)^(5/2))/(5*x) - (3*b*(a + b*x^2)^(7/2))/(5*x^3) - (a + b*x^2)^(9/
2)/(5*x^5) + (63*a^2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rubi [A] time = 0.0496843, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+bx\right.}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] (63*a*b^3*x*Sqrt[a + b*x^2])/8 + (21*b^3*x*(a + b*x^2)^(3/2))/4 - (21*b^2*(
a + b*x^2)^(5/2))/(5*x) - (3*b*(a + b*x^2)^(7/2))/(5*x^3) - (a + b*x^2)^(9/
2)/(5*x^5) + (63*a^2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = -\frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{5}(9b)\int \frac{(a+bx^2)^{7/2}}{x^4}\,dx$$
$$= -\frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{5}(21b^2)\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx$$
$$= -\frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + (21b^3)\int (a+bx^2)^{3/2}\,dx$$
$$= \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{4}(63ab^3)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{8}(63$$
$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{8}(63$$
$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{63}{8}a^2b$$

Mathematica [C] time = 0.0099537, size = 54, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] -(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -5/2, -3/2, -((b*x^2)/a)])/(5
*x^5*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.008, size = 166, normalized size = 1.3

$$-\frac{1}{5ax^5}(bx^2+a)^{\frac{11}{2}} - \frac{2b}{5a^2x^3}(bx^2+a)^{\frac{11}{2}} - \frac{16b^2}{5a^3x}(bx^2+a)^{\frac{11}{2}} + \frac{16b^3x}{5a^3}(bx^2+a)^{\frac{9}{2}} + \frac{18b^3x}{5a^2}(bx^2+a)^{\frac{7}{2}} + \frac{21b^3x}{5a}(bx^2+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^6,x)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(11/2)-2/5*b/a^2/x^3*(b*x^2+a)^(11/2)-16/5*b^2/a^3/x*(
b*x^2+a)^(11/2)+16/5*b^3/a^3*x*(b*x^2+a)^(9/2)+18/5*b^3/a^2*x*(b*x^2+a)^(7/
2)+21/5*b^3/a*x*(b*x^2+a)^(5/2)+21/4*b^3*x*(b*x^2+a)^(3/2)+63/8*a*b^3*x*(b*
x^2+a)^(1/2)+63/8*b^(5/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.968, size = 450, normalized size = 3.49

$$\left[\frac{315\,a^2b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{80\,x^5},-\frac{31}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^2*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*sq
rt(b*x^2 + a))/x^5, -1/40*(315*a^2*sqrt(-b)*b^2*x^5*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - (10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 -
 8*a^4)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A] time = 8.31751, size = 175, normalized size = 1.36

$$-\frac{a^{\frac{9}{2}}}{5x^5\sqrt{1+\frac{bx^2}{a}}}-\frac{8a^{\frac{7}{2}}b}{5x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2}{5x\sqrt{1+\frac{bx^2}{a}}}-\frac{203a^{\frac{3}{2}}b^3x}{40\sqrt{1+\frac{bx^2}{a}}}+\frac{19\sqrt{a}b^4x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^2b^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{b^5x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**6,x)
```

```
[Out] -a**(9/2)/(5*x**5*sqrt(1 + b*x**2/a)) - 8*a**(7/2)*b/(5*x**3*sqrt(1 + b*x**
2/a)) - 43*a**(5/2)*b**2/(5*x*sqrt(1 + b*x**2/a)) - 203*a**(3/2)*b**3*x/(40
*sqrt(1 + b*x**2/a)) + 19*sqrt(a)*b**4*x**3/(8*sqrt(1 + b*x**2/a)) + 63*a**
2*b**(5/2)*asinh(sqrt(b)*x/sqrt(a))/8 + b**5*x**5/(4*sqrt(a)*sqrt(1 + b*x**
2/a))
```

Giac [A] time = 2.38459, size = 270, normalized size = 2.09

$$-\frac{63}{16}\,a^2b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{8}\left(2\,b^4x^2+17\,ab^3\right)\sqrt{bx^2+a}x+\frac{4\left(25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8a^3b^{\frac{5}{2}}-75\left(\sqrt{b}x-\sqrt{b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] -63/16*a^2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/8*(2*b^4*x^2 +
17*a*b^3)*sqrt(b*x^2 + a)*x + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b
^(5/2) - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(5/2) + 105*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^5*b^(5/2) - 65*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(
5/2) + 18*a^7*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.432 $\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx$

Optimal. Leaf size=126

$$-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}+\frac{9}{2}b^4x\sqrt{a+bx^2}+\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}$$

```
[Out] (9*b^4*x*Sqrt[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x
^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(
7*x^7) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi [A] time = 0.0509176, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}+\frac{9}{2}b^4x\sqrt{a+bx^2}+\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] (9*b^4*x*Sqrt[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x
^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(
7*x^7) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^8}dx = -\frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{7}(9b)\int \frac{(a+bx^2)^{7/2}}{x^6}dx$$
$$= -\frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{5}(9b^2)\int \frac{(a+bx^2)^{5/2}}{x^4}dx$$
$$= -\frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + (3b^3)\int \frac{(a+bx^2)^{3/2}}{x^2}dx$$
$$= -\frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + (9b^4)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{2}(9ab^4)$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{2}(9ab^4)$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{9}{2}ab^{7/2}\,\text{ta}$$

Mathematica [C] time = 0.0102338, size = 54, normalized size = 0.43

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{7}{2};-\frac{5}{2};-\frac{bx^2}{a}\right)}{7x^7\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] -(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -7/2, -5/2, -((b*x^2)/a)])/(7
*x^7*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.014, size = 186, normalized size = 1.5

$$-\frac{1}{7ax^7}(bx^2+a)^{\frac{11}{2}} - \frac{4b}{35a^2x^5}(bx^2+a)^{\frac{11}{2}} - \frac{8b^2}{35a^3x^3}(bx^2+a)^{\frac{11}{2}} - \frac{64b^3}{35a^4x}(bx^2+a)^{\frac{11}{2}} + \frac{64b^4x}{35a^4}(bx^2+a)^{\frac{9}{2}} + \frac{72b^4x}{35a^3}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^8,x)
```

```
[Out] -1/7/a/x^7*(b*x^2+a)^(11/2)-4/35*b/a^2/x^5*(b*x^2+a)^(11/2)-8/35*b^2/a^3/x^
3*(b*x^2+a)^(11/2)-64/35*b^3/a^4/x*(b*x^2+a)^(11/2)+64/35*b^4/a^4*x*(b*x^2+
a)^(9/2)+72/35*b^4/a^3*x*(b*x^2+a)^(7/2)+12/5*b^4/a^2*x*(b*x^2+a)^(5/2)+3*b
^4/a*x*(b*x^2+a)^(3/2)+9/2*b^4*x*(b*x^2+a)^(1/2)+9/2*b^(7/2)*a*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1316, size = 451, normalized size = 3.58

$$\left[\frac{315\,ab^{\frac{7}{2}}x^{7}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\left(35\,b^{4}x^{8}-388\,ab^{3}x^{6}-156\,a^{2}b^{2}x^{4}-58\,a^{3}bx^{2}-10\,a^{4}\right)\sqrt{bx^{2}+a}}{140\,x^{7}},-\frac{315\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a*b^(7/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)*s
qrt(b*x^2 + a))/x^7, -1/70*(315*a*sqrt(-b)*b^3*x^7*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^7]
```

Sympy [A] time = 9.03291, size = 167, normalized size = 1.33

$$-\frac{a^{4}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{7x^{6}}-\frac{29a^{3}b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35x^{4}}-\frac{78a^{2}b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35x^{2}}-\frac{194ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35}-\frac{9ab^{\frac{7}{2}}\log\left(\frac{a}{bx^{2}}\right)}{4}+\frac{9ab^{\frac{7}{2}}\log\left(\sqrt{\frac{a}{bx^{2}}+1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**8,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 29*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(35*x**4) - 78*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*x**2) - 194*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/35 - 9*a*b**(7/2)*log(a/(b*x**2))/4 + 9*a*b
**(7/2)*log(sqrt(a/(b*x**2) + 1) + 1)/2 + b**(9/2)*x**2*sqrt(a/(b*x**2) + 1
)/2
```

Giac [B] time = 1.93647, size = 324, normalized size = 2.57

$$\frac{1}{2}\sqrt{bx^{2}+a}b^{4}x-\frac{9}{4}ab^{\frac{7}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{4\left(175\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}a^{2}b^{\frac{7}{2}}-700\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}a^{3}b^{\frac{7}{2}}+15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^4*x - 9/4*a*b^(7/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 4/35*(175*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2) - 700*(sqrt(b)
```

```
*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*a^4*b^(7/2) - 1820*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2) + 1337*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a^7*b^(7/2) + 97*a^8*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.433 $\int \frac{(a+bx^2)^{9/2}}{x^{10}} dx$

Optimal. Leaf size=124

$$-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{b^2\left(a+bx^2\right)^{5/2}}{5x^5}+b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{7/2}}{7x^7}-\frac{\left(a+bx^2\right)^{9/2}}{9x^9}$$

```
[Out] -((b^4*Sqrt[a + b*x^2])/x) - (b^3*(a + b*x^2)^(3/2))/(3*x^3) - (b^2*(a + b*
x^2)^(5/2))/(5*x^5) - (b*(a + b*x^2)^(7/2))/(7*x^7) - (a + b*x^2)^(9/2)/(9*
x^9) + b^(9/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0524736, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 206}

$$-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{b^2\left(a+bx^2\right)^{5/2}}{5x^5}+b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{7/2}}{7x^7}-\frac{\left(a+bx^2\right)^{9/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -((b^4*Sqrt[a + b*x^2])/x) - (b^3*(a + b*x^2)^(3/2))/(3*x^3) - (b^2*(a + b*
x^2)^(5/2))/(5*x^5) - (b*(a + b*x^2)^(7/2))/(7*x^7) - (a + b*x^2)^(9/2)/(9*
x^9) + b^(9/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = -\frac{(a+bx^2)^{9/2}}{9x^9} + b\int \frac{(a+bx^2)^{7/2}}{x^8}\,dx$$
$$= -\frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^2\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx$$
$$= -\frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^3\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$$
$$= -\frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^4\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\int \frac{1}{\sqrt{a+bx}}$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\,\text{Subst}\Big(\int$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^{9/2}\tanh^{-1}\Big($$

Mathematica [C] time = 0.009773, size = 54, normalized size = 0.44

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{9}{2};-\frac{7}{2};-\frac{bx^2}{a}\right)}{9x^9\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -9/2, -7/2, -((b*x^2)/a)])/(9
*x^9*Sqrt[1 + (b*x^2)/a])
```

Maple [B] time = 0.03, size = 206, normalized size = 1.7

$$-\frac{1}{9\,ax^9}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{2\,b}{63\,a^2x^7}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{8\,b^2}{315\,a^3x^5}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{16\,b^3}{315\,a^4x^3}\left(bx^2+a\right)^{\frac{11}{2}} - \frac{128\,b^4}{315\,a^5x}\left(bx^2+a\right)^{\frac{11}{2}} + \frac{128}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^10,x)
```

```
[Out] -1/9/a/x^9*(b*x^2+a)^(11/2)-2/63*b/a^2/x^7*(b*x^2+a)^(11/2)-8/315*b^2/a^3/x
^5*(b*x^2+a)^(11/2)-16/315*b^3/a^4/x^3*(b*x^2+a)^(11/2)-128/315*b^4/a^5/x*(
b*x^2+a)^(11/2)+128/315*b^5/a^5*x*(b*x^2+a)^(9/2)+16/35*b^5/a^4*x*(b*x^2+a)
^(7/2)+8/15*b^5/a^3*x*(b*x^2+a)^(5/2)+2/3*b^5/a^2*x*(b*x^2+a)^(3/2)+b^5/a*x
*(b*x^2+a)^(1/2)+b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11358, size = 452, normalized size = 3.65

$$\left[\frac{315\,b^{\frac{9}{2}}x^9\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(563\,b^4x^8+506\,ab^3x^6+408\,a^2b^2x^4+185\,a^3bx^2+35\,a^4\right)\sqrt{bx^2+a}}{630\,x^9},-\frac{315}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/630*(315*b^(9/2)*x^9*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*(563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2 + 35*a^4)*s
qrt(b*x^2 + a))/x^9, -1/315*(315*sqrt(-b)*b^4*x^9*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) + (563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2
+ 35*a^4)*sqrt(b*x^2 + a))/x^9]
```

Sympy [A] time = 10.3483, size = 160, normalized size = 1.29

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8}-\frac{37a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6}-\frac{136a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{105x^4}-\frac{506ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{315x^2}-\frac{563b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315}-\frac{b^{\frac{9}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{9}{2}}\log\Big(\Big.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**10,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 37*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(63*x**6) - 136*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(105*x**4) - 50
6*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(315*x**2) - 563*b**(9/2)*sqrt(a/(b*x**2)
 + 1)/315 - b**(9/2)*log(a/(b*x**2))/2 + b**(9/2)*log(sqrt(a/(b*x**2) + 1)
+ 1)
```

Giac [B] time = 2.87017, size = 373, normalized size = 3.01

$$-\frac{1}{2}\,b^{\frac{9}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{9}{2}}-6300\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^2b^{\frac{9}{2}}+21000\left(\sqrt{b}x-\sqrt{b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(1575*(sqrt(b)*x
- sqrt(b*x^2 + a))^16*a*b^(9/2) - 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2
```

```
*b^(9/2) + 21000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(9/2) - 31500*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(9/2) + 39438*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a^5*b^(9/2) - 26292*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(9/2) + 1396
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(9/2) - 3492*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^8*b^(9/2) + 563*a^9*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)^9
```

3.434 $\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

```
[Out] -(a + b*x^2)^(11/2)/(11*a*x^11)
```

Rubi [A] time = 0.0049336, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(11*a*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx = -\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Mathematica [A] time = 0.0089415, size = 21, normalized size = 1.

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(11*a*x^11)
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$-\frac{1}{11\,ax^{11}}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^12,x)
```

```
[Out] -1/11*(b*x^2+a)^(11/2)/a/x^11
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83294, size = 150, normalized size = 7.14

$$-\frac{\left(b^{5} x^{10} + 5 \, a b^{4} x^{8} + 10 \, a^{2} b^{3} x^{6} + 10 \, a^{3} b^{2} x^{4} + 5 \, a^{4} b x^{2} + a^{5}\right) \sqrt{b x^{2} + a}}{11 \, a x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x
^2 + a^5)*sqrt(b*x^2 + a)/(a*x^11)
```

Sympy [B] time = 4.84791, size = 150, normalized size = 7.14

$$-\frac{a^{4} \sqrt{b} \sqrt{\frac{a}{b x^{2}} + 1}}{11 x^{10}} - \frac{5 a^{3} b^{\frac{3}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{11 x^{8}} - \frac{10 a^{2} b^{\frac{5}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{11 x^{6}} - \frac{10 a b^{\frac{7}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{11 x^{4}} - \frac{5 b^{\frac{9}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{11 x^{2}} - \frac{b^{\frac{11}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{11 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**12,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(11*x**10) - 5*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(11*x**8) - 10*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(11*x**6) - 10*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(11*x**4) - 5*b**(9/2)*sqrt(a/(b*x**2) + 1)
/(11*x**2) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(11*a)
```

Giac [B] time = 1.59613, size = 225, normalized size = 10.71

$$\frac{2\left(11\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{20} b^{\frac{11}{2}} + 165\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{16} a^{2} b^{\frac{11}{2}} + 462\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{12} a^{4} b^{\frac{11}{2}} + 330\left(\sqrt{b}x - \sqrt{bx^{2} + \right.}{11\left(\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{2} - a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="giac")
```

```
[Out] 2/11*(11*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(11/2) + 165*(sqrt(b)*x - sqrt(
b*x^2 + a))^16*a^2*b^(11/2) + 462*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(1
1/2) + 330*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(11/2) + 55*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*a^8*b^(11/2) + a^10*b^(11/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^11
```

3.435 $\int \frac{(a+bx^2)^{9/2}}{x^{14}} dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

```
[Out] -(a + b*x^2)^(11/2)/(13*a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)
```

Rubi [A] time = 0.0112088, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(13*a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^{14}} dx &= -\frac{(a+bx^2)^{11/2}}{13ax^{13}} - \frac{(2b)\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx}{13a}\\ &= -\frac{(a+bx^2)^{11/2}}{13ax^{13}} + \frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}\end{aligned}$$

Mathematica [A] time = 0.0133952, size = 31, normalized size = 0.7

$$\frac{(a+bx^2)^{11/2}(2bx^2-11a)}{143a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-11*a + 2*b*x^2))/(143*a^2*x^13)
```

Maple [A] time = 0.004, size = 28, normalized size = 0.6

$$-\frac{-2\,bx^{2}+11\,a}{143\,x^{13}a^{2}}\left(bx^{2}+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^14,x)
```

```
[Out] -1/143*(b*x^2+a)^(11/2)*(-2*b*x^2+11*a)/x^13/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12072, size = 184, normalized size = 4.18

$$\frac{\left(2\,b^{6}x^{12}-ab^{5}x^{10}-35\,a^{2}b^{4}x^{8}-90\,a^{3}b^{3}x^{6}-100\,a^{4}b^{2}x^{4}-53\,a^{5}bx^{2}-11\,a^{6}\right)\sqrt{bx^{2}+a}}{143\,a^{2}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/143*(2*b^6*x^12 - a*b^5*x^10 - 35*a^2*b^4*x^8 - 90*a^3*b^3*x^6 - 100*a^4*
b^2*x^4 - 53*a^5*b*x^2 - 11*a^6)*sqrt(b*x^2 + a)/(a^2*x^13)
```

Sympy [B] time = 6.51651, size = 175, normalized size = 3.98

$$-\frac{a^{4}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{13x^{12}}-\frac{53a^{3}b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{10}}-\frac{100a^{2}b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{8}}-\frac{90ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{6}}-\frac{35b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{4}}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143ax^{2}}+\frac{2b^{\frac{13}{2}}\sqrt{\frac{a}{bx^{2}}}}{143a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**14,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(13*x**12) - 53*a**3*b**(3/2)*sqrt(a/(b*
x**2) + 1)/(143*x**10) - 100*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(143*x**8)
- 90*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(143*x**6) - 35*b**(9/2)*sqrt(a/(b*x**
2) + 1)/(143*x**4) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(143*a*x**2) + 2*b**(13
```

```
/2)*sqrt(a/(b*x**2) + 1)/(143*a**2)
```

Giac [B] time = 1.68215, size = 443, normalized size = 10.07

$$\frac{4\left(143\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{13}{2}}+429\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{13}{2}}+2145\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{13}{2}}+3003\left(\sqrt{b}x-\sqrt{bx^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="giac")
```

```
[Out] 4/143*(143*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(13/2) + 429*(sqrt(b)*x - sqr
t(b*x^2 + a))^20*a*b^(13/2) + 2145*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^2*b^(
13/2) + 3003*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(13/2) + 6006*(sqrt(b)*
x - sqrt(b*x^2 + a))^14*a^4*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^1
2*a^5*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*b^(13/2) + 1430*
(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(13/2) + 715*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^8*b^(13/2) + 65*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*b^(13/2) + 13
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(13/2) - a^11*b^(13/2))/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^13
```

3.436 $\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

```
[Out] -(a + b*x^2)^(11/2)/(15*a*x^15) + (4*b*(a + b*x^2)^(11/2))/(195*a^2*x^13) -
 (8*b^2*(a + b*x^2)^(11/2))/(2145*a^3*x^11)
```

Rubi [A] time = 0.021, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^16,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(15*a*x^15) + (4*b*(a + b*x^2)^(11/2))/(195*a^2*x^13) -
 (8*b^2*(a + b*x^2)^(11/2))/(2145*a^3*x^11)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx &= -\frac{(a+bx^2)^{11/2}}{15ax^{15}}-\frac{(4b)\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx}{15a}\\
&= -\frac{(a+bx^2)^{11/2}}{15ax^{15}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}+\frac{(8b^2)\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx}{195a^2}\\
&= -\frac{(a+bx^2)^{11/2}}{15ax^{15}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.0144015, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{11/2}(143a^2-44abx^2+8b^2x^4)}{2145a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^16,x]
```

```
[Out] -((a + b*x^2)^(11/2)*(143*a^2 - 44*a*b*x^2 + 8*b^2*x^4))/(2145*a^3*x^15)
```

Maple [A] time = 0.003, size = 39, normalized size = 0.6

$$-\frac{8 b^2x^4-44 abx^2+143 a^2}{2145 x^{15}a^3}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^16,x)
```

```
[Out] -1/2145*(b*x^2+a)^(11/2)*(8*b^2*x^4-44*a*b*x^2+143*a^2)/x^15/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36999, size = 220, normalized size = 3.24

$$-\frac{\left(8 b^7x^{14}-4 ab^6x^{12}+3 a^2b^5x^{10}+355 a^3b^4x^8+1030 a^4b^3x^6+1218 a^5b^2x^4+671 a^6bx^2+143 a^7\right)\sqrt{bx^2+a}}{2145 a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/2145*(8*b^7*x^14 - 4*a*b^6*x^12 + 3*a^2*b^5*x^10 + 355*a^3*b^4*x^8 + 103
0*a^4*b^3*x^6 + 1218*a^5*b^2*x^4 + 671*a^6*b*x^2 + 143*a^7)*sqrt(b*x^2 + a)
/(a^3*x^15)
```

Sympy [B] time = 10.0584, size = 604, normalized size = 8.88

$$-\frac{143a^9b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^6\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{957a^8b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^4\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{27\cdots}{x^2\left(2145a^5b^4x^8\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**16,x)
```

```
[Out] -143*a**9*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**6*(2145*a**5*b**4*x**8 + 4290*a
**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 957*a**8*b**(11/2)*sqrt(a/(b*x**2
) + 1)/(x**4*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x
**12)) - 2703*a**7*b**(13/2)*sqrt(a/(b*x**2) + 1)/(x**2*(2145*a**5*b**4*x**
8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 4137*a**6*b**(15/2)*sqr
t(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b
**6*x**12) - 3633*a**5*b**(17/2)*x**2*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*
x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 1743*a**4*b**(19/2)*x
**4*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145
*a**3*b**6*x**12) - 357*a**3*b**(21/2)*x**6*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 3*a**2*b**(23/2
)*x**8*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2
145*a**3*b**6*x**12) - 12*a*b**(25/2)*x**10*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 8*b**(27/2)*x**
12*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*
a**3*b**6*x**12)
```

Giac [B] time = 3.06728, size = 478, normalized size = 7.03

$$16\left(1430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}b^{\frac{15}{2}}+6435\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}ab^{\frac{15}{2}}+24453\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}a^2b^{\frac{15}{2}}+45045\left(\sqrt{b}x-\sqrt{b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="giac")
```

```
[Out] 16/2145*(1430*(sqrt(b)*x - sqrt(b*x^2 + a))^24*b^(15/2) + 6435*(sqrt(b)*x -
 sqrt(b*x^2 + a))^22*a*b^(15/2) + 24453*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^
2*b^(15/2) + 45045*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^3*b^(15/2) + 70785*(s
qrt(b)*x - sqrt(b*x^2 + a))^16*a^4*b^(15/2) + 64350*(sqrt(b)*x - sqrt(b*x^2
 + a))^14*a^5*b^(15/2) + 50050*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^6*b^(15/2
) + 21450*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^7*b^(15/2) + 7800*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^8*b^(15/2) + 975*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^9*
b^(15/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^10*b^(15/2) - 15*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^11*b^(15/2) + a^12*b^(15/2))/((sqrt(b)*x - sqrt(b*
x^2 + a))^2 - a)^15
```

3.437 $\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

```
[Out] -(a + b*x^2)^(11/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(11/2))/(85*a^2*x^15) -
(8*b^2*(a + b*x^2)^(11/2))/(1105*a^3*x^13) + (16*b^3*(a + b*x^2)^(11/2))/(1
2155*a^4*x^11)
```

Rubi [A] time = 0.0296989, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^18,x]
```


```
[Out] -(a + b*x^2)^(11/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(11/2))/(85*a^2*x^15) -
(8*b^2*(a + b*x^2)^(11/2))/(1105*a^3*x^13) + (16*b^3*(a + b*x^2)^(11/2))/(1
2155*a^4*x^11)
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx &= -\frac{(a+bx^2)^{11/2}}{17ax^{17}}-\frac{(6b)\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx}{17a}\\ &= -\frac{(a+bx^2)^{11/2}}{17ax^{17}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}+\frac{(8b^2)\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx}{85a^2}\\ &= -\frac{(a+bx^2)^{11/2}}{17ax^{17}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}-\frac{(16b^3)\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx}{1105a^3}\\ &= -\frac{(a+bx^2)^{11/2}}{17ax^{17}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}\end{aligned}$$

Mathematica [A] time = 0.017672, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{11/2}(286a^2bx^2-715a^3-88ab^2x^4+16b^3x^6)}{12155a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-715*a^3 + 286*a^2*b*x^2 - 88*a*b^2*x^4 + 16*b^3*x^6))
/(12155*a^4*x^17)
```

Maple [A] time = 0.003, size = 50, normalized size = 0.5

$$-\frac{-16\,b^3x^6+88\,ab^2x^4-286\,a^2bx^2+715\,a^3}{12155\,x^{17}a^4}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^18,x)
```

```
[Out] -1/12155*(b*x^2+a)^(11/2)*(-16*b^3*x^6+88*a*b^2*x^4-286*a^2*b*x^2+715*a^3)/
x^17/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76667, size = 247, normalized size = 2.68

$$\frac{\left(16\,b^8x^{16}-8\,ab^7x^{14}+6\,a^2b^6x^{12}-5\,a^3b^5x^{10}-1515\,a^4b^4x^8-4714\,a^5b^3x^6-5808\,a^6b^2x^4-3289\,a^7bx^2-715\,a^8\right)\sqrt{bx^2+a}}{12155\,a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/12155*(16*b^8*x^16 - 8*a*b^7*x^14 + 6*a^2*b^6*x^12 - 5*a^3*b^5*x^10 - 151
5*a^4*b^4*x^8 - 4714*a^5*b^3*x^6 - 5808*a^6*b^2*x^4 - 3289*a^7*b*x^2 - 715*
a^8)*sqrt(b*x^2 + a)/(a^4*x^17)
```

Sympy [B] time = 13.0886, size = 867, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**18,x)
```

```
[Out] -715*a**11*b**(19/2)*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a*
*6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 5434*a*
*10*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6
*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 17820*a**
9*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b
**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 32720*a**8*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**
10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 36370*a**7*b*
*(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10
*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 24500*a**6*b**(
29/2)*x**10*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*
x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 9268*a**5*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x*
*18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 1520*a**4*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**1
8 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 5*a**3*b**(35/2)*x**
16*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 3
6465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 30*a**2*b**(37/2)*x**18*s
qrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465
*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 40*a*b**(39/2)*x**20*sqrt(a/(
b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b
**11*x**20 + 12155*a**4*b**12*x**22) + 16*b**(41/2)*x**22*sqrt(a/(b*x**2) +
 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**2
0 + 12155*a**4*b**12*x**22)
```

Giac [B] time = 2.85873, size = 516, normalized size = 5.61

$$\frac{32\left(12155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}b^{\frac{17}{2}}+65637\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}ab^{\frac{17}{2}}+233376\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}a^2b^{\frac{17}{2}}+466752\left(\sqrt{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="giac")
```

```
[Out] 32/12155*(12155*(sqrt(b)*x - sqrt(b*x^2 + a))^26*b^(17/2) + 65637*(sqrt(b)*
x - sqrt(b*x^2 + a))^24*a*b^(17/2) + 233376*(sqrt(b)*x - sqrt(b*x^2 + a))^2
2*a^2*b^(17/2) + 466752*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^3*b^(17/2) + 692
835*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^4*b^(17/2) + 668525*(sqrt(b)*x - sqr
t(b*x^2 + a))^16*a^5*b^(17/2) + 486200*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^6
*b^(17/2) + 221000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^7*b^(17/2) + 71825*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^8*b^(17/2) + 9775*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^9*b^(17/2) + 680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^10*b^(17/2) -
136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^11*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a^12*b^(17/2) - a^13*b^(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^17
```

3.438 $\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx$

Optimal. Leaf size=116

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

```
[Out] -(a + b*x^2)^(11/2)/(19*a*x^19) + (8*b*(a + b*x^2)^(11/2))/(323*a^2*x^17) -
 (16*b^2*(a + b*x^2)^(11/2))/(1615*a^3*x^15) + (64*b^3*(a + b*x^2)^(11/2))/
(20995*a^4*x^13) - (128*b^4*(a + b*x^2)^(11/2))/(230945*a^5*x^11)
```

Rubi [A] time = 0.0445839, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(19*a*x^19) + (8*b*(a + b*x^2)^(11/2))/(323*a^2*x^17) -
 (16*b^2*(a + b*x^2)^(11/2))/(1615*a^3*x^15) + (64*b^3*(a + b*x^2)^(11/2))/
(20995*a^4*x^13) - (128*b^4*(a + b*x^2)^(11/2))/(230945*a^5*x^11)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{20}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} - \frac{(8b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}\,dx}{19a}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} + \frac{\left(48b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx}{323a^2}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} - \frac{16b^2\left(a+bx^2\right)^{11/2}}{1615a^3x^{15}} - \frac{\left(64b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{14}}\,dx}{1615a^3}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} - \frac{16b^2\left(a+bx^2\right)^{11/2}}{1615a^3x^{15}} + \frac{64b^3\left(a+bx^2\right)^{11/2}}{20995a^4x^{13}} + \frac{\left(128b^4\right)\int \frac{\left(a+bx^2\right)^{9/}}{x^{12}}}{20995a^4}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} - \frac{16b^2\left(a+bx^2\right)^{11/2}}{1615a^3x^{15}} + \frac{64b^3\left(a+bx^2\right)^{11/2}}{20995a^4x^{13}} - \frac{128b^4\left(a+bx^2\right)^{11/}}{230945a^5x^{11}}$$

Mathematica [A] time = 0.0168273, size = 64, normalized size = 0.55

$$-\frac{\left(a+bx^2\right)^{11/2}\left(2288a^2b^2x^4-5720a^3bx^2+12155a^4-704ab^3x^6+128b^4x^8\right)}{230945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -((a + b*x^2)^(11/2)*(12155*a^4 - 5720*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 704*a
*b^3*x^6 + 128*b^4*x^8))/(230945*a^5*x^19)
```

Maple [A] time = 0.003, size = 61, normalized size = 0.5

$$-\frac{128\,b^4x^8-704\,b^3x^6a+2288\,b^2x^4a^2-5720\,bx^2a^3+12155\,a^4}{230945\,x^{19}a^5}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^20,x)
```

```
[Out] -1/230945*(b*x^2+a)^(11/2)*(128*b^4*x^8-704*a*b^3*x^6+2288*a^2*b^2*x^4-5720
*a^3*b*x^2+12155*a^4)/x^19/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.32294, size = 288, normalized size = 2.48

$$-\frac{\left(128\,b^{9}x^{18}-64\,ab^{8}x^{16}+48\,a^{2}b^{7}x^{14}-40\,a^{3}b^{6}x^{12}+35\,a^{4}b^{5}x^{10}+23063\,a^{5}b^{4}x^{8}+75086\,a^{6}b^{3}x^{6}+95238\,a^{7}b^{2}x^{4}+55055\right.}{230945\,a^{5}x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/230945*(128*b^9*x^18 - 64*a*b^8*x^16 + 48*a^2*b^7*x^14 - 40*a^3*b^6*x^12
 + 35*a^4*b^5*x^10 + 23063*a^5*b^4*x^8 + 75086*a^6*b^3*x^6 + 95238*a^7*b^2*
x^4 + 55055*a^8*b*x^2 + 12155*a^9)*sqrt(b*x^2 + a)/(a^5*x^19)
```

Sympy [B] time = 17.0971, size = 1182, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**20,x)
```

```
[Out] -12155*a**13*b**(33/2)*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 9237
80*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 +
230945*a**5*b**20*x**26) - 103675*a**12*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)
/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x*
*22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 388388*a**11*b**
(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**
17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5
*b**20*x**26) - 834988*a**10*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(230945*a*
*9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 92378
0*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 1127210*a**9*b**(41/2)*x**8
*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 +
1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**2
6) - 978810*a**8*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x*
*18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**1
9*x**24 + 230945*a**5*b**20*x**26) - 534060*a**7*b**(45/2)*x**12*sqrt(a/(b*
x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**
7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 167436
*a**6*b**(47/2)*x**14*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 92378
0*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 2
30945*a**5*b**20*x**26) - 23091*a**5*b**(49/2)*x**16*sqrt(a/(b*x**2) + 1)/(
230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**2
2 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 35*a**4*b**(51/2)*
x**18*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**
20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20
*x**26) - 280*a**3*b**(53/2)*x**20*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*
x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b*
*19*x**24 + 230945*a**5*b**20*x**26) - 560*a**2*b**(55/2)*x**22*sqrt(a/(b*x
**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7
*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 448*a*b
**(57/2)*x**24*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*
b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a
**5*b**20*x**26) - 128*b**(59/2)*x**26*sqrt(a/(b*x**2) + 1)/(230945*a**9*b*
*16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**
6*b**19*x**24 + 230945*a**5*b**20*x**26)
```

Giac [B] time = 3.05115, size = 551, normalized size = 4.75

$$256\left(92378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}b^{\frac{19}{2}}+554268\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}ab^{\frac{19}{2}}+1939938\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}a^2b^{\frac{19}{2}}+40184\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="giac")
```

```
[Out] 256/230945*(92378*(sqrt(b)*x - sqrt(b*x^2 + a))^28*b^(19/2) + 554268*(sqrt(
b)*x - sqrt(b*x^2 + a))^26*a*b^(19/2) + 1939938*(sqrt(b)*x - sqrt(b*x^2 + a
))^24*a^2*b^(19/2) + 4018443*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^3*b^(19/2)
+ 5866003*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^4*b^(19/2) + 5773625*(sqrt(b)*
x - sqrt(b*x^2 + a))^18*a^5*b^(19/2) + 4094025*(sqrt(b)*x - sqrt(b*x^2 + a)
)^16*a^6*b^(19/2) + 1889550*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^7*b^(19/2) +
 581400*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^8*b^(19/2) + 80750*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^9*b^(19/2) + 3876*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^1
0*b^(19/2) - 969*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^11*b^(19/2) + 171*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^12*b^(19/2) - 19*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^13*b^(19/2) + a^14*b^(19/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^19
```

3.439 $\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx$

Optimal. Leaf size=140

$$\frac{256b^5\left(a+bx^2\right)^{11/2}}{969969a^6x^{11}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{88179a^5x^{13}}+\frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}}-\frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}}+\frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}$$

```
[Out] -(a + b*x^2)^(11/2)/(21*a*x^21) + (10*b*(a + b*x^2)^(11/2))/(399*a^2*x^19)
- (80*b^2*(a + b*x^2)^(11/2))/(6783*a^3*x^17) + (32*b^3*(a + b*x^2)^(11/2))
/(6783*a^4*x^15) - (128*b^4*(a + b*x^2)^(11/2))/(88179*a^5*x^13) + (256*b^5
*(a + b*x^2)^(11/2))/(969969*a^6*x^11)
```

Rubi [A] time = 0.0562635, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{256b^5\left(a+bx^2\right)^{11/2}}{969969a^6x^{11}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{88179a^5x^{13}}+\frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}}-\frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}}+\frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(21*a*x^21) + (10*b*(a + b*x^2)^(11/2))/(399*a^2*x^19)
- (80*b^2*(a + b*x^2)^(11/2))/(6783*a^3*x^17) + (32*b^3*(a + b*x^2)^(11/2))
/(6783*a^4*x^15) - (128*b^4*(a + b*x^2)^(11/2))/(88179*a^5*x^13) + (256*b^5
*(a + b*x^2)^(11/2))/(969969*a^6*x^11)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{22}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} - \frac{(10b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{20}}\,dx}{21a}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} + \frac{\left(80b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}\,dx}{399a^2}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} - \frac{\left(160b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx}{2261a^3}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} + \frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}} + \frac{\left(128b^4\right)\int \frac{\left(a+bx^2\right)}{x^{14}}}{6783a^4}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} + \frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}} - \frac{128b^4\left(a+bx^2\right)^{1}}{88179a^5x^{13}}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} + \frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}} - \frac{128b^4\left(a+bx^2\right)^{1}}{88179a^5x^{13}}$$

Mathematica [A] time = 0.0204524, size = 75, normalized size = 0.54

$$\frac{\left(a+bx^2\right)^{11/2}\left(4576a^2b^3x^6-11440a^3b^2x^4+24310a^4bx^2-46189a^5-1408ab^4x^8+256b^5x^{10}\right)}{969969a^6x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-46189*a^5 + 24310*a^4*b*x^2 - 11440*a^3*b^2*x^4 + 457
6*a^2*b^3*x^6 - 1408*a*b^4*x^8 + 256*b^5*x^10))/(969969*a^6*x^21)
```

Maple [A] time = 0.005, size = 72, normalized size = 0.5

$$-\frac{-256\,b^5x^{10}+1408\,ab^4x^8-4576\,a^2b^3x^6+11440\,a^3b^2x^4-24310\,a^4bx^2+46189\,a^5}{969969\,x^{21}a^6}\left(bx^2+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^22,x)
```

```
[Out] -1/969969*(b*x^2+a)^(11/2)*(-256*b^5*x^10+1408*a*b^4*x^8-4576*a^2*b^3*x^6+1
1440*a^3*b^2*x^4-24310*a^4*b*x^2+46189*a^5)/x^21/a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.3392, size = 319, normalized size = 2.28

$$\frac{\left(256\, b^{10} x^{20} - 128\, a b^{9} x^{18} + 96\, a^{2} b^{8} x^{16} - 80\, a^{3} b^{7} x^{14} + 70\, a^{4} b^{6} x^{12} - 63\, a^{5} b^{5} x^{10} - 80773\, a^{6} b^{4} x^{8} - 271414\, a^{7} b^{3} x^{6} - 351780\, a\right.}{969969\, a^{6} x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="fricas")
```

```
[Out] 1/969969*(256*b^10*x^20 - 128*a*b^9*x^18 + 96*a^2*b^8*x^16 - 80*a^3*b^7*x^1
4 + 70*a^4*b^6*x^12 - 63*a^5*b^5*x^10 - 80773*a^6*b^4*x^8 - 271414*a^7*b^3*
x^6 - 351780*a^8*b^2*x^4 - 206635*a^9*b*x^2 - 46189*a^10)*sqrt(b*x^2 + a)/(
a^6*x^21)
```

Sympy [B] time = 18.8148, size = 1540, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**22,x)
```

```
[Out] -46189*a**15*b**(51/2)*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 484
9845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**2
6 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 437580*a**14*b**(
53/2)*x**2*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b
**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*
a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 1846845*a**13*b**(55/2)*x**4*
sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22
+ 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*
x**28 + 969969*a**6*b**30*x**30) - 4558554*a**12*b**(57/2)*x**6*sqrt(a/(b*x
**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a
**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969
969*a**6*b**30*x**30) - 7252938*a**11*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(
969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x
**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b*
*30*x**30) - 7715232*a**10*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(969969*a**
11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969
9690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30)
 - 5487650*a**9*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x*
*20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b
**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 2516940*
a**8*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 48498
45*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26
+ 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 675513*a**7*b**(67/
2)*x**16*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**
26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a*
*7*b**29*x**28 + 969969*a**6*b**30*x**30) - 80836*a**6*b**(69/2)*x**18*sqrt
(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 96
99690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**2
8 + 969969*a**6*b**30*x**30) + 63*a**5*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)
/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27
*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*
b**30*x**30) + 630*a**4*b**(73/2)*x**22*sqrt(a/(b*x**2) + 1)/(969969*a**11*
b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969969
```

```
0*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) +
1680*a**3*b**(75/2)*x**24*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 +
4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x
**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 2016*a**2*b**(
77/2)*x**26*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*
b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845
*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 1152*a*b**(79/2)*x**28*sqrt(
a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 969
9690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28
 + 969969*a**6*b**30*x**30) + 256*b**(81/2)*x**30*sqrt(a/(b*x**2) + 1)/(969
969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**2
4 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30
*x**30)
```

Giac [B] time = 2.50813, size = 589, normalized size = 4.21

$$512\left(646646\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{30}b^{\frac{21}{2}}+4157010\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}ab^{\frac{21}{2}}+14549535\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}a^2b^{\frac{21}{2}}+307\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="giac")
```

```
[Out] 512/969969*(646646*(sqrt(b)*x - sqrt(b*x^2 + a))^30*b^(21/2) + 4157010*(sqr
t(b)*x - sqrt(b*x^2 + a))^28*a*b^(21/2) + 14549535*(sqrt(b)*x - sqrt(b*x^2
+ a))^26*a^2*b^(21/2) + 30715685*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^3*b^(21
/2) + 44618574*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^4*b^(21/2) + 44265858*(sq
rt(b)*x - sqrt(b*x^2 + a))^20*a^5*b^(21/2) + 31009615*(sqrt(b)*x - sqrt(b*x
^2 + a))^18*a^6*b^(21/2) + 14346045*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^7*b^
(21/2) + 4273290*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^8*b^(21/2) + 592382*(sq
rt(b)*x - sqrt(b*x^2 + a))^12*a^9*b^(21/2) + 20349*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^10*b^(21/2) - 5985*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^11*b^(21/2)
 + 1330*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^12*b^(21/2) - 210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a^13*b^(21/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^14*b^
(21/2) - a^15*b^(21/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^21
```

3.440 $\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx$

Optimal. Leaf size=164

$$-\frac{1024b^6\left(a+bx^2\right)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5\left(a+bx^2\right)^{11/2}}{676039a^6x^{13}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{52003a^5x^{15}}+\frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}}-\frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}}+\frac{4b\left(a+\right.}{161a}$$

```
[Out] -(a + b*x^2)^(11/2)/(23*a*x^23) + (4*b*(a + b*x^2)^(11/2))/(161*a^2*x^21) -
 (40*b^2*(a + b*x^2)^(11/2))/(3059*a^3*x^19) + (320*b^3*(a + b*x^2)^(11/2))
/(52003*a^4*x^17) - (128*b^4*(a + b*x^2)^(11/2))/(52003*a^5*x^15) + (512*b^
5*(a + b*x^2)^(11/2))/(676039*a^6*x^13) - (1024*b^6*(a + b*x^2)^(11/2))/(74
36429*a^7*x^11)
```

Rubi [A] time = 0.0719655, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{1024b^6\left(a+bx^2\right)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5\left(a+bx^2\right)^{11/2}}{676039a^6x^{13}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{52003a^5x^{15}}+\frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}}-\frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}}+\frac{4b\left(a+\right.}{161a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(23*a*x^23) + (4*b*(a + b*x^2)^(11/2))/(161*a^2*x^21) -
 (40*b^2*(a + b*x^2)^(11/2))/(3059*a^3*x^19) + (320*b^3*(a + b*x^2)^(11/2))
/(52003*a^4*x^17) - (128*b^4*(a + b*x^2)^(11/2))/(52003*a^5*x^15) + (512*b^
5*(a + b*x^2)^(11/2))/(676039*a^6*x^13) - (1024*b^6*(a + b*x^2)^(11/2))/(74
36429*a^7*x^11)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}dx = -\frac{(a+bx^2)^{11/2}}{23ax^{23}} - \frac{(12b)\int \frac{(a+bx^2)^{9/2}}{x^{22}}dx}{23a}$$
$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} + \frac{(40b^2)\int \frac{(a+bx^2)^{9/2}}{x^{20}}dx}{161a^2}$$
$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} - \frac{(320b^3)\int \frac{(a+bx^2)^{9/2}}{x^{18}}dx}{3059a^3}$$
$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} + \frac{(1920b^4)\int \frac{(a+bx}{x^{1}}}{52003a^4}$$
$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a+bx^2)^{1}}{52003a^5x^{15}}$$
$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a+bx^2)^{1}}{52003a^5x^{15}}$$
$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a+bx^2)^{1}}{52003a^5x^{15}}$$

Mathematica [A] time = 0.0211539, size = 86, normalized size = 0.52

$$-\frac{(a+bx^2)^{11/2}(18304a^2b^4x^8 - 45760a^3b^3x^6 + 97240a^4b^2x^4 - 184756a^5bx^2 + 323323a^6 - 5632ab^5x^{10} + 1024b^6x^{12})}{7436429a^7x^{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -((a + b*x^2)^(11/2)*(323323*a^6 - 184756*a^5*b*x^2 + 97240*a^4*b^2*x^4 - 4
5760*a^3*b^3*x^6 + 18304*a^2*b^4*x^8 - 5632*a*b^5*x^10 + 1024*b^6*x^12))/(7
436429*a^7*x^23)
```

Maple [A] time = 0.005, size = 83, normalized size = 0.5

$$-\frac{1024\,b^6x^{12} - 5632\,b^5x^{10}a + 18304\,b^4x^8a^2 - 45760\,b^3x^6a^3 + 97240\,b^2x^4a^4 - 184756\,bx^2a^5 + 323323\,a^6}{7436429\,x^{23}a^7}(bx^2+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^24,x)
```

```
[Out] -1/7436429*(b*x^2+a)^(11/2)*(1024*b^6*x^12-5632*a*b^5*x^10+18304*a^2*b^4*x^
8-45760*a^3*b^3*x^6+97240*a^4*b^2*x^4-184756*a^5*b*x^2+323323*a^6)/x^23/a^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.68819, size = 363, normalized size = 2.21

$$-\frac{\left(1024\,b^{11}x^{22} - 512\,ab^{10}x^{20} + 384\,a^{2}b^{9}x^{18} - 320\,a^{3}b^{8}x^{16} + 280\,a^{4}b^{7}x^{14} - 252\,a^{5}b^{6}x^{12} + 231\,a^{6}b^{5}x^{10} + 530959\,a^{7}b^{4}x^{8} + 1\right.}{7436429\,a^{7}x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/7436429*(1024*b^11*x^22 - 512*a*b^10*x^20 + 384*a^2*b^9*x^18 - 320*a^3*b
^8*x^16 + 280*a^4*b^7*x^14 - 252*a^5*b^6*x^12 + 231*a^6*b^5*x^10 + 530959*a
^7*b^4*x^8 + 1826110*a^8*b^3*x^6 + 2406690*a^9*b^2*x^4 + 1431859*a^10*b*x^2
 + 323323*a^11)*sqrt(b*x^2 + a)/(a^7*x^23)
```

Sympy [B] time = 23.7845, size = 1950, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**24,x)
```

```
[Out] -323323*a**17*b**(73/2)*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 4
4618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b
**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 74364
29*a**7*b**42*x**34) - 3371797*a**16*b**(75/2)*x**2*sqrt(a/(b*x**2) + 1)/(7
436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**
38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618
574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 15847689*a**15*b**(77/2)
*x**4*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**3
7*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 11154
6435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**3
4) - 44210595*a**14*b**(79/2)*x**6*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**3
6*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728
580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x*
*32 + 7436429*a**7*b**42*x**34) - 81074994*a**13*b**(81/2)*x**8*sqrt(a/(b*x
**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115464
35*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x
**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 102129258*a*
*12*b**(83/2)*x**10*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618
574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39
*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a
**7*b**42*x**34) - 89502546*a**11*b**(85/2)*x**12*sqrt(a/(b*x**2) + 1)/(743
6429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38
*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 4461857
4*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 53885062*a**10*b**(87/2)*x
**14*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37
*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546
435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34
) - 21329935*a**9*b**(89/2)*x**16*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36
```

```
*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1487285
80*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**
32 + 7436429*a**7*b**42*x**34) - 5012953*a**8*b**(91/2)*x**18*sqrt(a/(b*x**
2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435
*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**
30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 531157*a**7*b*
*(93/2)*x**20*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a*
*12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28
 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b*
*42*x**34) - 231*a**6*b**(95/2)*x**22*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b
**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148
728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41
*x**32 + 7436429*a**7*b**42*x**34) - 2772*a**5*b**(97/2)*x**24*sqrt(a/(b*x*
*2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 11154643
5*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x*
*30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 9240*a**4*b**
(99/2)*x**26*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**
12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28
+ 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**
42*x**34) - 14784*a**3*b**(101/2)*x**28*sqrt(a/(b*x**2) + 1)/(7436429*a**13
*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1
48728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**
41*x**32 + 7436429*a**7*b**42*x**34) - 12672*a**2*b**(103/2)*x**30*sqrt(a/(
b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115
46435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**4
0*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 5632*a*b*
*(105/2)*x**32*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a
**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**2
8 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b
**42*x**34) - 1024*b**(107/2)*x**34*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**
36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 14872
8580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x
**32 + 7436429*a**7*b**42*x**34)
```

Giac [B] time = 3.05804, size = 624, normalized size = 3.8

$$\frac{2048\left(4249388\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{32}b^{\frac{23}{2}}+28683369\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{30}ab^{\frac{23}{2}}+100922965\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}a^2b^{\frac{23}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="giac")
```

```
[Out] 2048/7436429*(4249388*(sqrt(b)*x - sqrt(b*x^2 + a))^32*b^(23/2) + 28683369*
(sqrt(b)*x - sqrt(b*x^2 + a))^30*a*b^(23/2) + 100922965*(sqrt(b)*x - sqrt(b
*x^2 + a))^28*a^2*b^(23/2) + 215656441*(sqrt(b)*x - sqrt(b*x^2 + a))^26*a^3
*b^(23/2) + 313006057*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^4*b^(23/2) + 31165
3979*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^5*b^(23/2) + 216800507*(sqrt(b)*x -
 sqrt(b*x^2 + a))^20*a^6*b^(23/2) + 100105775*(sqrt(b)*x - sqrt(b*x^2 + a))
^18*a^7*b^(23/2) + 29173683*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^8*b^(23/2) +
 4004231*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^9*b^(23/2) + 100947*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*a^10*b^(23/2) - 33649*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*a^11*b^(23/2) + 8855*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^12*b^(23/2) - 1771
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^13*b^(23/2) + 253*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^14*b^(23/2) - 23*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^15*b^(23/2)
```

```
+ a^16*b^(23/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^23
```

3.441 $\int x^5\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

```
[Out] (27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/4
48
```

Rubi [A] time = 0.0195838, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 + 4*x^2],x]
```

```
[Out] (27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/4
48
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{9+4x}-\frac{9}{8}(9+4x)^{3/2}+\frac{1}{16}(9+4x)^{5/2}\right)dx,x,x^2\right)\\ &= \frac{27}{64}\left(9+4x^2\right)^{3/2}-\frac{9}{160}\left(9+4x^2\right)^{5/2}+\frac{1}{448}\left(9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0100116, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2+9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 + 4*x^2],x]
```

```
[Out] ((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

Maple [A] time = 0.003, size = 24, normalized size = 0.5

$$\frac{10x^4 - 18x^2 + 27}{280}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/280*(4*x^2+9)^(3/2)*(10*x^4-18*x^2+27)
```

Maxima [A] time = 1.7444, size = 54, normalized size = 1.17

$$\frac{1}{28}\left(4x^2+9\right)^{\frac{3}{2}}x^4 - \frac{9}{140}\left(4x^2+9\right)^{\frac{3}{2}}x^2 + \frac{27}{280}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 + 9)^(3/2)*x^4 - 9/140*(4*x^2 + 9)^(3/2)*x^2 + 27/280*(4*x^2 +
9)^(3/2)
```

Fricas [A] time = 1.64842, size = 76, normalized size = 1.65

$$\frac{1}{280}\left(40x^6 + 18x^4 - 54x^2 + 243\right)\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(4*x^2 + 9)
```

Sympy [A] time = 1.88655, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2+9}}{7} + \frac{9x^4\sqrt{4x^2+9}}{140} - \frac{27x^2\sqrt{4x^2+9}}{140} + \frac{243\sqrt{4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 + 9)/7 + 9*x**4*sqrt(4*x**2 + 9)/140 - 27*x**2*sqrt(4*x**2
 + 9)/140 + 243*sqrt(4*x**2 + 9)/280
```

Giac [A] time = 1.41474, size = 46, normalized size = 1.

$$\frac{1}{448}\left(4x^2+9\right)^{\frac{7}{2}}-\frac{9}{160}\left(4x^2+9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 + 9)^(7/2) - 9/160*(4*x^2 + 9)^(5/2) + 27/64*(4*x^2 + 9)^(3/2)
```

3.442 $\int x^4\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=63

$$\frac{1}{6}\sqrt{4x^2+9}x^5+\frac{3}{32}\sqrt{4x^2+9}x^3-\frac{81}{256}\sqrt{4x^2+9}x+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (-81*x*Sqrt[9 + 4*x^2])/256 + (3*x^3*Sqrt[9 + 4*x^2])/32 + (x^5*Sqrt[9 + 4*
x^2])/6 + (729*ArcSinh[(2*x)/3])/512
```

Rubi [A] time = 0.0155562, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 215}

$$\frac{1}{6}\sqrt{4x^2+9}x^5+\frac{3}{32}\sqrt{4x^2+9}x^3-\frac{81}{256}\sqrt{4x^2+9}x+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 + 4*x^2])/256 + (3*x^3*Sqrt[9 + 4*x^2])/32 + (x^5*Sqrt[9 + 4*
x^2])/6 + (729*ArcSinh[(2*x)/3])/512
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{9+4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9+4x^2}}\,dx\\
&= \frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}-\frac{81}{32}\int\frac{x^2}{\sqrt{9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.0117959, size = 39, normalized size = 0.62

$$\frac{1}{768}x\sqrt{4x^2+9}\left(128x^4+72x^2-243\right)+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 + (729*ArcSinh[(2*x)/3])/
512
```

Maple [A] time = 0.005, size = 46, normalized size = 0.7

$$\frac{x^3}{24}\left(4x^2+9\right)^{\frac{3}{2}}-\frac{9x}{128}\left(4x^2+9\right)^{\frac{3}{2}}+\frac{81x}{256}\sqrt{4x^2+9}+\frac{729}{512}\mathrm{Arcsinh}\left(\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*x^3*(4*x^2+9)^(3/2)-9/128*x*(4*x^2+9)^(3/2)+81/256*x*(4*x^2+9)^(1/2)+7
29/512*arcsinh(2/3*x)
```

Maxima [A] time = 3.11349, size = 61, normalized size = 0.97

$$\frac{1}{24}\left(4x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(4x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{4x^2+9}x+\frac{729}{512}\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 + 9)^(3/2)*x^3 - 9/128*(4*x^2 + 9)^(3/2)*x + 81/256*sqrt(4*x^2
+ 9)*x + 729/512*arcsinh(2/3*x)
```

Fricas [A] time = 1.70871, size = 119, normalized size = 1.89

$$\frac{1}{768}\left(128x^5+72x^3-243x\right)\sqrt{4x^2+9}-\frac{729}{512}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(4*x^2 + 9) - 729/512*log(-2*x + sqrt(
4*x^2 + 9))
```

Sympy [A] time = 4.55124, size = 75, normalized size = 1.19

$$\frac{2x^7}{3\sqrt{4x^2+9}}+\frac{15x^5}{8\sqrt{4x^2+9}}-\frac{27x^3}{64\sqrt{4x^2+9}}-\frac{729x}{256\sqrt{4x^2+9}}+\frac{729\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2+9)**(1/2),x)
```

```
[Out] 2*x**7/(3*sqrt(4*x**2 + 9)) + 15*x**5/(8*sqrt(4*x**2 + 9)) - 27*x**3/(64*sq
rt(4*x**2 + 9)) - 729*x/(256*sqrt(4*x**2 + 9)) + 729*asinh(2*x/3)/512
```

Giac [A] time = 1.48252, size = 58, normalized size = 0.92

$$\frac{1}{768}\left(8\left(16x^2+9\right)x^2-243\right)\sqrt{4x^2+9}x-\frac{729}{512}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(4*x^2 + 9)*x - 729/512*log(-2*x + sqr
t(4*x^2 + 9))
```

3.443 $\int x^3\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

```
[Out] (-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80
```

Rubi [A] time = 0.014846, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 + 4*x^2],x]

[Out] (-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9+4x^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{9}{4}\sqrt{9+4x}+\frac{1}{4}(9+4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{3}{16}\left(9+4x^2\right)^{3/2}+\frac{1}{80}\left(9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.006506, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2-3\right)\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 + 4*x^2],x]

[Out] ((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40
```

Maple [A] time = 0.003, size = 19, normalized size = 0.6

$$\frac{2x^2-3}{40}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(4*x^2+9)^(3/2)*(2*x^2-3)
```

Maxima [A] time = 3.28448, size = 35, normalized size = 1.13

$$\frac{1}{20}\left(4x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(4*x^2 + 9)^(3/2)*x^2 - 3/40*(4*x^2 + 9)^(3/2)
```

Fricas [A] time = 1.66133, size = 58, normalized size = 1.87

$$\frac{1}{40}\left(8x^4+6x^2-27\right)\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(4*x^2 + 9)
```

Sympy [A] time = 0.584487, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2+9}}{5}+\frac{3x^2\sqrt{4x^2+9}}{20}-\frac{27\sqrt{4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 + 9)/5 + 3*x**2*sqrt(4*x**2 + 9)/20 - 27*sqrt(4*x**2 + 9)/
40
```

Giac [A] time = 2.61329, size = 31, normalized size = 1.

$$\frac{1}{80}\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{16}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 + 9)^(5/2) - 3/16*(4*x^2 + 9)^(3/2)
```

3.444 $\int x^2\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\sqrt{4x^2+9}x^3+\frac{9}{32}\sqrt{4x^2+9}x-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (9*x*Sqrt[9 + 4*x^2])/32 + (x^3*Sqrt[9 + 4*x^2])/4 - (81*ArcSinh[(2*x)/3])/
64
```

Rubi [A] time = 0.0095313, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 215}

$$\frac{1}{4}\sqrt{4x^2+9}x^3+\frac{9}{32}\sqrt{4x^2+9}x-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9*x*Sqrt[9 + 4*x^2])/32 + (x^3*Sqrt[9 + 4*x^2])/4 - (81*ArcSinh[(2*x)/3])/
64
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{9+4x^2}\,dx &= \frac{1}{4}x^3\sqrt{9+4x^2}+\frac{9}{4}\int\frac{x^2}{\sqrt{9+4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{9+4x^2}+\frac{1}{4}x^3\sqrt{9+4x^2}-\frac{81}{32}\int\frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{9+4x^2}+\frac{1}{4}x^3\sqrt{9+4x^2}-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0143258, size = 36, normalized size = 0.8

$$\sqrt{4x^2+9}\left(\frac{x^3}{4}+\frac{9x}{32}\right)-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]*((9*x)/32 + x^3/4) - (81*ArcSinh[(2*x)/3])/64
```

Maple [A] time = 0.004, size = 32, normalized size = 0.7

$$\frac{x}{16}\left(4x^2+9\right)^{\frac{3}{2}}-\frac{9x}{32}\sqrt{4x^2+9}-\frac{81}{64}\text{Arcsinh}\left(\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/16*x*(4*x^2+9)^(3/2)-9/32*x*(4*x^2+9)^(1/2)-81/64*arcsinh(2/3*x)
```

Maxima [A] time = 2.7876, size = 42, normalized size = 0.93

$$\frac{1}{16}\left(4x^2+9\right)^{\frac{3}{2}}x-\frac{9}{32}\sqrt{4x^2+9}x-\frac{81}{64}\,\text{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 + 9)^(3/2)*x - 9/32*sqrt(4*x^2 + 9)*x - 81/64*arcsinh(2/3*x)
```

Fricas [A] time = 1.44884, size = 97, normalized size = 2.16

$$\frac{1}{32}\left(8x^3+9x\right)\sqrt{4x^2+9}+\frac{81}{64}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 + 9*x)*sqrt(4*x^2 + 9) + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 2.66339, size = 54, normalized size = 1.2

$$\frac{x^5}{\sqrt{4x^2+9}}+\frac{27x^3}{8\sqrt{4x^2+9}}+\frac{81x}{32\sqrt{4x^2+9}}-\frac{81\,\text{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2+9)**(1/2),x)
```

```
[Out] x**5/sqrt(4*x**2 + 9) + 27*x**3/(8*sqrt(4*x**2 + 9)) + 81*x/(32*sqrt(4*x**2
 + 9)) - 81*asinh(2*x/3)/64
```

Giac [A] time = 2.48487, size = 49, normalized size = 1.09

$$\frac{1}{32}\left(8x^2+9\right)\sqrt{4x^2+9}x+\frac{81}{64}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(8*x^2 + 9)*sqrt(4*x^2 + 9)*x + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

3.445 $\int x\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

```
[Out] (9 + 4*x^2)^(3/2)/12
```

Rubi [A] time = 0.0021563, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 + 4*x^2],x]

[Out] (9 + 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

Mathematica [A] time = 0.0018369, size = 15, normalized size = 1.

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 + 4*x^2],x]

[Out] (9 + 4*x^2)^(3/2)/12
```

Maple [A] time = 0.001, size = 12, normalized size = 0.8

$$\frac{1}{12}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/12*(4*x^2+9)^(3/2)
```

Maxima [A] time = 1.22952, size = 15, normalized size = 1.

$$\frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

Fricas [A] time = 1.49535, size = 32, normalized size = 2.13

$$\frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

Sympy [B] time = 0.184337, size = 27, normalized size = 1.8

$$\frac{x^2\sqrt{4x^2+9}}{3}+\frac{3\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 + 9)/3 + 3*sqrt(4*x**2 + 9)/4
```

Giac [A] time = 2.55571, size = 15, normalized size = 1.

$$\frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

3.446 $\int \sqrt{9+4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{4x^2+9}x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Rubi [A] time = 0.0037743, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{4x^2+9}x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9+4x^2}\,dx&=\frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\&=\frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0060767, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{4x^2+9}x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Maple [A] time = 0.002, size = 20, normalized size = 0.7

$$\frac{9}{4}\text{Arcsinh}\left(\frac{2\,x}{3}\right)+\frac{x}{2}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2),x)
```

```
[Out] 9/4*arcsinh(2/3*x)+1/2*x*(4*x^2+9)^(1/2)
```

Maxima [A] time = 2.2904, size = 26, normalized size = 0.96

$$\frac{1}{2}\,\sqrt{4\,x^2+9}x+\frac{9}{4}\,\text{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x + 9/4*arcsinh(2/3*x)
```

Fricas [A] time = 1.44501, size = 77, normalized size = 2.85

$$\frac{1}{2}\,\sqrt{4\,x^2+9}x-\frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 0.193213, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\text{asinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 + 9)/2 + 9*asinh(2*x/3)/4
```

Giac [A] time = 1.33756, size = 39, normalized size = 1.44

$$\frac{1}{2}\,\sqrt{4\,x^2+9}x-\frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

3.447 $\int \frac{\sqrt{9+4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{4x^2+9} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rubi [A] time = 0.0162719, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 207}

$$\sqrt{4x^2+9} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{9+4x}}{x}\,dx,x,x^2\right)\\ &= \sqrt{9+4x^2}+\frac{9}{2}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= \sqrt{9+4x^2}+\frac{9}{4}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{9+4x^2}\right)\\ &= \sqrt{9+4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0040806, size = 30, normalized size = 1.

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$\sqrt{4\,x^2+9}-3\,\mathrm{Artanh}\left(3\,\frac{1}{\sqrt{4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x,x)
```

```
[Out] (4*x^2+9)^(1/2)-3*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A] time = 3.55119, size = 26, normalized size = 0.87

$$\sqrt{4\,x^2+9}-3\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 + 9) - 3*arcsinh(3/2/abs(x))
```

Fricas [A] time = 1.44154, size = 120, normalized size = 4.

$$\sqrt{4\,x^2+9}-3\,\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)+3\,\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(4*x^2 + 9) - 3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 3*log(-2*x + sqrt(4*x
^2 + 9) - 3)
```

Sympy [A] time = 1.22376, size = 39, normalized size = 1.3

$$\frac{2x}{\sqrt{1+\frac{9}{4x^2}}} - 3\operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x,x)
```

```
[Out] 2*x/sqrt(1 + 9/(4*x**2)) - 3*asinh(3/(2*x)) + 9/(2*x*sqrt(1 + 9/(4*x**2)))
```

Giac [A] time = 2.47241, size = 51, normalized size = 1.7

$$\sqrt{4x^2+9} - \frac{3}{2}\log\left(\sqrt{4x^2+9}+3\right) + \frac{3}{2}\log\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(4*x^2 + 9) - 3/2*log(sqrt(4*x^2 + 9) + 3) + 3/2*log(sqrt(4*x^2 + 9) -
3)
```

3.448 $\int \frac{\sqrt{9+4x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Rubi [A] time = 0.0048838, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 215}

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^2}\,dx &= -\frac{\sqrt{9+4x^2}}{x}+4\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{x}+2\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0057339, size = 25, normalized size = 1.

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Maple [A] time = 0.002, size = 34, normalized size = 1.4

$$-\frac{1}{9x}\left(4x^2+9\right)^{\frac{3}{2}}+\frac{4x}{9}\sqrt{4x^2+9}+2\,\mathrm{Arcsinh}\,(2/3\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(4*x^2+9)^(3/2)+4/9*x*(4*x^2+9)^(1/2)+2*arcsinh(2/3*x)
```

Maxima [A] time = 3.04903, size = 28, normalized size = 1.12

$$-\frac{\sqrt{4x^2+9}}{x}+2\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(4*x^2 + 9)/x + 2*arcsinh(2/3*x)
```

Fricas [A] time = 1.45516, size = 84, normalized size = 3.36

$$-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2+9}\right)+2\,x+\sqrt{4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*x*log(-2*x + sqrt(4*x^2 + 9)) + 2*x + sqrt(4*x^2 + 9))/x
```

Sympy [A] time = 0.23143, size = 19, normalized size = 0.76

$$2\,\mathrm{asinh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**2,x)
```

```
[Out] 2*asinh(2*x/3) - sqrt(4*x**2 + 9)/x
```

Giac [A] time = 1.96634, size = 54, normalized size = 2.16

$$\frac{36}{\left(2x - \sqrt{4x^2 + 9}\right)^2 - 9} - 2\log\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 36/((2*x - sqrt(4*x^2 + 9))^2 - 9) - 2*log(-2*x + sqrt(4*x^2 + 9))
```

3.449 $\int \frac{\sqrt{9+4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -Sqrt[9 + 4*x^2]/(2*x^2) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Rubi [A] time = 0.0164569, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 207}

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(2*x^2) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} + \text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0176511, size = 37, normalized size = 0.95

$$-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(2*x^2) - (2*ArcTanh[Sqrt[1 + (4*x^2)/9]])/3
```

Maple [A] time = 0.004, size = 41, normalized size = 1.1

$$-\frac{1}{18\,x^2}\left(4\,x^2+9\right)^{\frac{3}{2}} + \frac{2}{9}\sqrt{4\,x^2+9} - \frac{2}{3}\text{Artanh}\left(3\,\frac{1}{\sqrt{4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^3,x)
```

```
[Out] -1/18/x^2*(4*x^2+9)^(3/2)+2/9*(4*x^2+9)^(1/2)-2/3*arctanh(3/(4*x^2+9)^(1/2)
)
```

Maxima [A] time = 3.54023, size = 47, normalized size = 1.21

$$\frac{2}{9}\sqrt{4\,x^2+9} - \frac{\left(4\,x^2+9\right)^{\frac{3}{2}}}{18\,x^2} - \frac{2}{3}\,\text{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(4*x^2 + 9) - 1/18*(4*x^2 + 9)^(3/2)/x^2 - 2/3*arcsinh(3/2/abs(x))
```

Fricas [A] time = 1.55267, size = 149, normalized size = 3.82

$$-\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right) - 4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right) + 3\,\sqrt{4\,x^2+9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) + 3*sqrt(4*x^2 + 9))/x^2
```

Sympy [A] time = 1.71282, size = 24, normalized size = 0.62

$$-\frac{2\,\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] -2*asinh(3/(2*x))/3 - sqrt(1 + 9/(4*x**2))/x
```

Giac [A] time = 2.23136, size = 58, normalized size = 1.49

$$-\frac{\sqrt{4\,x^2+9}}{2\,x^2}-\frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(4*x^2 + 9)/x^2 - 1/3*log(sqrt(4*x^2 + 9) + 3) + 1/3*log(sqrt(4*x^
2 + 9) - 3)
```

3.450 $\int \frac{\sqrt{9+4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

```
[Out] -(9 + 4*x^2)^(3/2)/(27*x^3)
```

Rubi [A] time = 0.002985, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^4,x]
```

```
[Out] -(9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{(9+4x^2)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.0034218, size = 18, normalized size = 1.

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^4,x]
```

```
[Out] -(9 + 4*x^2)^(3/2)/(27*x^3)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$-\frac{1}{27\,x^3}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^4,x)
```

```
[Out] -1/27*(4*x^2+9)^(3/2)/x^3
```

Maxima [A] time = 3.61069, size = 19, normalized size = 1.06

$$-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(4*x^2 + 9)^(3/2)/x^3
```

Fricas [A] time = 1.51408, size = 53, normalized size = 2.94

$$-\frac{8\,x^3+\left(4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(8*x^3 + (4*x^2 + 9)^(3/2))/x^3
```

Sympy [B] time = 1.00305, size = 34, normalized size = 1.89

$$-\frac{8\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] -8*sqrt(1 + 9/(4*x**2))/27 - 2*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

Giac [B] time = 2.45481, size = 57, normalized size = 3.17

$$\frac{16\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^4+27\right)}{\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 16*((2*x - sqrt(4*x^2 + 9))^4 + 27)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

3.451 $\int \frac{\sqrt{9+4x^2}}{x^5} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{4x^2+9}}{18x^2}-\frac{\sqrt{4x^2+9}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -Sqrt[9 + 4*x^2]/(4*x^4) - Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4
*x^2]/3])/27
```

Rubi [A] time = 0.0216427, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 207}

$$-\frac{\sqrt{4x^2+9}}{18x^2}-\frac{\sqrt{4x^2+9}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(4*x^4) - Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4
*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9+4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{9+4x}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{9+4x^2}}{4x^4} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right) \\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.0051901, size = 32, normalized size = 0.56

$$-\frac{16\left(4x^2+9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4x^2}{9}+1\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] (-16*(9 + 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (4*x^2)/9])/2187
```

Maple [A] time = 0.006, size = 55, normalized size = 1.

$$-\frac{1}{36\,x^4}\left(4\,x^2+9\right)^{\frac{3}{2}}+\frac{1}{162\,x^2}\left(4\,x^2+9\right)^{\frac{3}{2}}-\frac{2}{81}\sqrt{4\,x^2+9}+\frac{2}{27}\operatorname{Artanh}\left(3\,\frac{1}{\sqrt{4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^5,x)
```

```
[Out] -1/36/x^4*(4*x^2+9)^(3/2)+1/162/x^2*(4*x^2+9)^(3/2)-2/81*(4*x^2+9)^(1/2)+2/
27*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A] time = 3.95155, size = 66, normalized size = 1.16

$$-\frac{2}{81}\sqrt{4\,x^2+9}+\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}}{162\,x^2}-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}}{36\,x^4}+\frac{2}{27}\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 + 9) + 1/162*(4*x^2 + 9)^(3/2)/x^2 - 1/36*(4*x^2 + 9)^(3/2
)/x^4 + 2/27*arcsinh(3/2/abs(x))
```

Fricas [A] time = 1.51197, size = 166, normalized size = 2.91

$$\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\left(2\,x^2+9\right)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
+ 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 + 9))/x^4
```

Sympy [A] time = 3.47304, size = 63, normalized size = 1.11

$$\frac{2\,\text{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 3/(4*x**3*sqrt(1 + 9/(
4*x**2))) - 9/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [A] time = 2.68474, size = 74, normalized size = 1.3

$$-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}+9\,\sqrt{4\,x^2+9}}{72\,x^4}+\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((4*x^2 + 9)^(3/2) + 9*sqrt(4*x^2 + 9))/x^4 + 1/27*log(sqrt(4*x^2 + 9
) + 3) - 1/27*log(sqrt(4*x^2 + 9) - 3)
```

3.452 $\int x^5\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

```
[Out] (-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/
448
```

Rubi [A] time = 0.0205247, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/
448
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9-4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{9-4x}x^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{9-4x}-\frac{9}{8}(9-4x)^{3/2}+\frac{1}{16}(9-4x)^{5/2}\right)dx,x,x^2\right)\\&=-\frac{27}{64}\left(9-4x^2\right)^{3/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{1}{448}\left(9-4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0120286, size = 27, normalized size = 0.59

$$-\frac{1}{280}\left(9-4x^2\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] -((9 - 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

Maple [A] time = 0.002, size = 34, normalized size = 0.7

$$\frac{(-3+2x)(3+2x)\left(10x^4+18x^2+27\right)\sqrt{-4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/280*(-3+2*x)*(3+2*x)*(10*x^4+18*x^2+27)*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.88775, size = 54, normalized size = 1.17

$$-\frac{1}{28}\left(-4x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(-4x^2+9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 + 9)^(3/2)*x^4 - 9/140*(-4*x^2 + 9)^(3/2)*x^2 - 27/280*(-4*x^
2 + 9)^(3/2)
```

Fricas [A] time = 1.49467, size = 77, normalized size = 1.67

$$\frac{1}{280}\left(40x^6-18x^4-54x^2-243\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(-4*x^2 + 9)
```

Sympy [A] time = 1.92776, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{9-4x^2}}{7}-\frac{9x^4\sqrt{9-4x^2}}{140}-\frac{27x^2\sqrt{9-4x^2}}{140}-\frac{243\sqrt{9-4x^2}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(9 - 4*x**2)/7 - 9*x**4*sqrt(9 - 4*x**2)/140 - 27*x**2*sqrt(9 - 4*
x**2)/140 - 243*sqrt(9 - 4*x**2)/280
```

Giac [A] time = 2.33858, size = 70, normalized size = 1.52

$$\frac{1}{448}\left(4\,x^2-9\right)^3\sqrt{-4\,x^2+9}+\frac{9}{160}\left(4\,x^2-9\right)^2\sqrt{-4\,x^2+9}-\frac{27}{64}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 - 9)^3*sqrt(-4*x^2 + 9) + 9/160*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9)
 - 27/64*(-4*x^2 + 9)^(3/2)
```

3.453 $\int x^4\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=63

$$\frac{1}{6}\sqrt{9-4x^2}x^5-\frac{3}{32}\sqrt{9-4x^2}x^3-\frac{81}{256}\sqrt{9-4x^2}x+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (-81*x*Sqrt[9 - 4*x^2])/256 - (3*x^3*Sqrt[9 - 4*x^2])/32 + (x^5*Sqrt[9 - 4*
x^2])/6 + (729*ArcSin[(2*x)/3])/512
```

Rubi [A] time = 0.0161792, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 216}

$$\frac{1}{6}\sqrt{9-4x^2}x^5-\frac{3}{32}\sqrt{9-4x^2}x^3-\frac{81}{256}\sqrt{9-4x^2}x+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 - 4*x^2])/256 - (3*x^3*Sqrt[9 - 4*x^2])/32 + (x^5*Sqrt[9 - 4*
x^2])/6 + (729*ArcSin[(2*x)/3])/512
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{9-4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{81}{32}\int\frac{x^2}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.0117474, size = 39, normalized size = 0.62

$$\frac{1}{768}x\sqrt{9-4x^2}\left(128x^4-72x^2-243\right)+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*ArcSin[(2*x)/3])/5
12
```

Maple [A] time = 0.006, size = 46, normalized size = 0.7

$$-\frac{x^3}{24}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{9x}{128}\left(-4x^2+9\right)^{\frac{3}{2}}+\frac{81x}{256}\sqrt{-4x^2+9}+\frac{729}{512}\arcsin\left(\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/24*x^3*(-4*x^2+9)^(3/2)-9/128*x*(-4*x^2+9)^(3/2)+81/256*x*(-4*x^2+9)^(1/
2)+729/512*arcsin(2/3*x)
```

Maxima [A] time = 4.07798, size = 61, normalized size = 0.97

$$-\frac{1}{24}\left(-4x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(-4x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4x^2+9}x+\frac{729}{512}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 + 9)^(3/2)*x^3 - 9/128*(-4*x^2 + 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

Fricas [A] time = 1.54162, size = 132, normalized size = 2.1

$$\frac{1}{768}\left(128x^5-72x^3-243x\right)\sqrt{-4x^2+9}-\frac{729}{256}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(-4*x^2 + 9) - 729/256*arctan(1/2*(sqr
t(-4*x^2 + 9) - 3)/x)
```

Sympy [A] time = 4.51964, size = 167, normalized size = 2.65

$$\begin{cases} \frac{2ix^7}{3\sqrt{4x^2-9}} - \frac{15ix^5}{8\sqrt{4x^2-9}} - \frac{27ix^3}{64\sqrt{4x^2-9}} + \frac{729ix}{256\sqrt{4x^2-9}} - \frac{729i\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{2x^7}{3\sqrt{9-4x^2}} + \frac{15x^5}{8\sqrt{9-4x^2}} + \frac{27x^3}{64\sqrt{9-4x^2}} - \frac{729x}{256\sqrt{9-4x^2}} + \frac{729\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((2*I*x**7/(3*sqrt(4*x**2 - 9)) - 15*I*x**5/(8*sqrt(4*x**2 - 9)) -
 27*I*x**3/(64*sqrt(4*x**2 - 9)) + 729*I*x/(256*sqrt(4*x**2 - 9)) - 729*I*a
cosh(2*x/3)/512, 4*Abs(x**2)/9 > 1), (-2*x**7/(3*sqrt(9 - 4*x**2)) + 15*x**
5/(8*sqrt(9 - 4*x**2)) + 27*x**3/(64*sqrt(9 - 4*x**2)) - 729*x/(256*sqrt(9
- 4*x**2)) + 729*asin(2*x/3)/512, True))
```

Giac [A] time = 2.5577, size = 45, normalized size = 0.71

$$\frac{1}{768}\left(8\left(16x^2-9\right)x^2-243\right)\sqrt{-4x^2+9}x+\frac{729}{512}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(-4*x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

3.454 $\int x^3\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

```
[Out] (-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80
```

Rubi [A] time = 0.0151302, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{9-4x}x\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{9-4x}-\frac{1}{4}(9-4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{3}{16}\left(9-4x^2\right)^{3/2}+\frac{1}{80}\left(9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0085122, size = 22, normalized size = 0.71

$$-\frac{1}{40}\left(9-4x^2\right)^{3/2}\left(2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 - 4*x^2],x]
```

```
[Out] -((9 - 4*x^2)^(3/2)*(3 + 2*x^2))/40
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{(-3+2x)(3+2x)\left(2x^2+3\right)}{40}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(-3+2*x)*(3+2*x)*(2*x^2+3)*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.10749, size = 35, normalized size = 1.13

$$-\frac{1}{20}\left(-4x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(-4*x^2 + 9)^(3/2)*x^2 - 3/40*(-4*x^2 + 9)^(3/2)
```

Fricas [A] time = 1.49148, size = 59, normalized size = 1.9

$$\frac{1}{40}\left(8x^4-6x^2-27\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(-4*x^2 + 9)
```

Sympy [A] time = 0.591265, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{9-4x^2}}{5}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(9 - 4*x**2)/5 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2)/
40
```

Giac [A] time = 2.693, size = 43, normalized size = 1.39

$$\frac{1}{80}\left(4x^2-9\right)^2\sqrt{-4x^2+9}-\frac{3}{16}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) - 3/16*(-4*x^2 + 9)^(3/2)
```

3.455 $\int x^2\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\sqrt{9-4x^2}x^3-\frac{9}{32}\sqrt{9-4x^2}x+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (-9*x*Sqrt[9 - 4*x^2])/32 + (x^3*Sqrt[9 - 4*x^2])/4 + (81*ArcSin[(2*x)/3])/
64
```

Rubi [A] time = 0.0104155, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{9-4x^2}x^3-\frac{9}{32}\sqrt{9-4x^2}x+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - 4*x^2])/32 + (x^3*Sqrt[9 - 4*x^2])/4 + (81*ArcSin[(2*x)/3])/
64
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{9-4x^2}\,dx &= \frac{1}{4}x^3\sqrt{9-4x^2}+\frac{9}{4}\int\frac{x^2}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{9-4x^2}+\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{81}{32}\int\frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{9-4x^2}+\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0155067, size = 36, normalized size = 0.8

$$\sqrt{9-4x^2}\left(\frac{x^3}{4}-\frac{9x}{32}\right)+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] Sqrt[9 - 4*x^2]*((-9*x)/32 + x^3/4) + (81*ArcSin[(2*x)/3])/64
```

Maple [A] time = 0.003, size = 32, normalized size = 0.7

$$-\frac{x}{16}\left(-4x^2+9\right)^{\frac{3}{2}}+\frac{9x}{32}\sqrt{-4x^2+9}+\frac{81}{64}\arcsin\left(\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/16*x*(-4*x^2+9)^(3/2)+9/32*x*(-4*x^2+9)^(1/2)+81/64*arcsin(2/3*x)
```

Maxima [A] time = 3.07358, size = 42, normalized size = 0.93

$$-\frac{1}{16}\left(-4x^2+9\right)^{\frac{3}{2}}x+\frac{9}{32}\sqrt{-4x^2+9}x+\frac{81}{64}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(-4*x^2 + 9)^(3/2)*x + 9/32*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

Fricas [A] time = 1.54845, size = 111, normalized size = 2.47

$$\frac{1}{32}\left(8x^3-9x\right)\sqrt{-4x^2+9}-\frac{81}{32}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 9*x)*sqrt(-4*x^2 + 9) - 81/32*arctan(1/2*(sqrt(-4*x^2 + 9) -
3)/x)
```

Sympy [A] time = 2.65184, size = 124, normalized size = 2.76

$$\begin{cases} \frac{ix^5}{\sqrt{4x^2-9}}-\frac{27ix^3}{8\sqrt{4x^2-9}}+\frac{81ix}{32\sqrt{4x^2-9}}-\frac{81i\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9}>1 \\ -\frac{x^5}{\sqrt{9-4x^2}}+\frac{27x^3}{8\sqrt{9-4x^2}}-\frac{81x}{32\sqrt{9-4x^2}}+\frac{81\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/sqrt(4*x**2 - 9) - 27*I*x**3/(8*sqrt(4*x**2 - 9)) + 81*I*
x/(32*sqrt(4*x**2 - 9)) - 81*I*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-x**5/
sqrt(9 - 4*x**2) + 27*x**3/(8*sqrt(9 - 4*x**2)) - 81*x/(32*sqrt(9 - 4*x**2)
) + 81*asin(2*x/3)/64, True))
```

Giac [A] time = 2.25919, size = 35, normalized size = 0.78

$$\frac{1}{32}\left(8\,x^2-9\right)\sqrt{-4\,x^2+9}x+\frac{81}{64}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(8*x^2 - 9)*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

3.456 $\int x\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

```
[Out] -(9 - 4*x^2)^(3/2)/12
```

Rubi [A] time = 0.0023749, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 - 4*x^2],x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9-4x^2}\,dx=-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Mathematica [A] time = 0.0017305, size = 15, normalized size = 1.

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 - 4*x^2],x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/12
```

Maple [A] time = 0.001, size = 22, normalized size = 1.5

$$\frac{(-3+2x)(3+2x)}{12}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/12*(-3+2*x)*(3+2*x)*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 2.80043, size = 15, normalized size = 1.

$$-\frac{1}{12}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

Fricas [A] time = 1.49731, size = 47, normalized size = 3.13

$$\frac{1}{12}\left(4x^2-9\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 - 9)*sqrt(-4*x^2 + 9)
```

Sympy [B] time = 0.187359, size = 27, normalized size = 1.8

$$\frac{x^2\sqrt{9-4x^2}}{3}-\frac{3\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(9 - 4*x**2)/3 - 3*sqrt(9 - 4*x**2)/4
```

Giac [A] time = 2.69025, size = 15, normalized size = 1.

$$-\frac{1}{12}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

3.457 $\int \sqrt{9-4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{9-4x^2}x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Rubi [A] time = 0.0032839, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{9-4x^2}x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9-4x^2}\,dx &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0059365, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{9-4x^2}x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Maple [A] time = 0.002, size = 20, normalized size = 0.7

$$\frac{9}{4}\arcsin\left(\frac{2x}{3}\right)+\frac{x}{2}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2),x)
```

```
[Out] 9/4*arcsin(2/3*x)+1/2*x*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.62502, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-4x^2+9}x+\frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Fricas [A] time = 1.53536, size = 90, normalized size = 3.33

$$\frac{1}{2}\sqrt{-4x^2+9}x-\frac{9}{2}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x - 9/2*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] time = 0.193277, size = 22, normalized size = 0.81

$$\frac{x\sqrt{9-4x^2}}{2}+\frac{9\operatorname{asin}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(9 - 4*x**2)/2 + 9*asin(2*x/3)/4
```

Giac [A] time = 2.56643, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-4x^2+9}x+\frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

3.458 $\int \frac{\sqrt{9-4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{9-4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rubi [A] time = 0.0152687, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{9-4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x}\,dx,x,x^2\right)$$
$$= \sqrt{9-4x^2} + \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x}\,dx,x,x^2\right)$$
$$= \sqrt{9-4x^2} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)$$
$$= \sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Mathematica [A] time = 0.0042257, size = 30, normalized size = 1.

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$\sqrt{-4x^2+9} - 3\,\text{Artanh}\left(3\frac{1}{\sqrt{-4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x,x)
```

```
[Out] (-4*x^2+9)^(1/2)-3*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A] time = 2.27514, size = 47, normalized size = 1.57

$$\sqrt{-4x^2+9} - 3\log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 + 9) - 3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 1.55815, size = 70, normalized size = 2.33

$$\sqrt{-4x^2+9} + 3\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 + 9) + 3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] time = 1.30778, size = 76, normalized size = 2.53

$$\begin{cases} i\sqrt{4x^2-9} - 3\log(x) + \frac{3\log(x^2)}{2} + 3i\operatorname{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ \sqrt{9-4x^2} + \frac{3\log(x^2)}{2} - 3\log\left(\sqrt{1-\frac{4x^2}{9}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(4*x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin(3/(2*x)
), 4*Abs(x**2)/9 > 1), (sqrt(9 - 4*x**2) + 3*log(x**2)/2 - 3*log(sqrt(1 - 4
*x**2/9) + 1), True))
```

Giac [A] time = 2.41552, size = 54, normalized size = 1.8

$$\sqrt{-4\,x^2+9} - \frac{3}{2}\,\log\left(\sqrt{-4\,x^2+9}+3\right) + \frac{3}{2}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-4*x^2 + 9) - 3/2*log(sqrt(-4*x^2 + 9) + 3) + 3/2*log(-sqrt(-4*x^2 + 9
) + 3)
```

3.459 $\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Rubi [A] time = 0.0050271, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 216}

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-4x^2}}{x^2}\,dx &= -\frac{\sqrt{9-4x^2}}{x}-4\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0064594, size = 25, normalized size = 1.

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Maple [A] time = 0.002, size = 34, normalized size = 1.4

$$-\frac{1}{9x}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{4x}{9}\sqrt{-4x^2+9}-2\arcsin\left(2/3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(-4*x^2+9)^(3/2)-4/9*x*(-4*x^2+9)^(1/2)-2*arcsin(2/3*x)
```

Maxima [A] time = 3.75001, size = 28, normalized size = 1.12

$$-\frac{\sqrt{-4x^2+9}}{x}-2\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-4*x^2 + 9)/x - 2*arcsin(2/3*x)
```

Fricas [A] time = 1.5143, size = 88, normalized size = 3.52

$$\frac{4x\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)-\sqrt{-4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (4*x*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x) - sqrt(-4*x^2 + 9))/x
```

Sympy [A] time = 0.231266, size = 20, normalized size = 0.8

$$-2\operatorname{asin}\left(\frac{2x}{3}\right)-\frac{\sqrt{9-4x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**2,x)
```

```
[Out] -2*asin(2*x/3) - sqrt(9 - 4*x**2)/x
```

Giac [A] time = 2.33949, size = 53, normalized size = 2.12

$$\frac{2x}{\sqrt{-4x^2+9}-3} - \frac{\sqrt{-4x^2+9}-3}{2x} - 2\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*x/(sqrt(-4*x^2 + 9) - 3) - 1/2*(sqrt(-4*x^2 + 9) - 3)/x - 2*arcsin(2/3*x)
```

3.460 $\int \frac{\sqrt{9-4x^2}}{x^3} \, dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

```
[Out] -Sqrt[9 - 4*x^2]/(2*x^2) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

Rubi [A] time = 0.016247, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(2*x^2) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{9-4x^2}}{2x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)$$
$$= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Mathematica [A] time = 0.0251954, size = 37, normalized size = 0.95

$$\frac{2}{3}\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right) - \frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(2*x^2) + (2*ArcTanh[Sqrt[1 - (4*x^2)/9]])/3
```

Maple [A] time = 0.004, size = 41, normalized size = 1.1

$$-\frac{1}{18\,x^2}\left(-4\,x^2+9\right)^{\frac{3}{2}} - \frac{2}{9}\sqrt{-4\,x^2+9} + \frac{2}{3}\text{Artanh}\left(3\,\frac{1}{\sqrt{-4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^3,x)
```

```
[Out] -1/18/x^2*(-4*x^2+9)^(3/2)-2/9*(-4*x^2+9)^(1/2)+2/3*arctanh(3/(-4*x^2+9)^(1
/2))
```

Maxima [A] time = 3.90331, size = 69, normalized size = 1.77

$$-\frac{2}{9}\sqrt{-4\,x^2+9} - \frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{18\,x^2} + \frac{2}{3}\log\left(\frac{6\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-4*x^2 + 9) - 1/18*(-4*x^2 + 9)^(3/2)/x^2 + 2/3*log(6*sqrt(-4*x^2
 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 1.50812, size = 93, normalized size = 2.38

$$-\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)+3\,\sqrt{-4\,x^2+9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) + 3*sqrt(-4*x^2 + 9))/x^2
```

Sympy [A] time = 1.80749, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{1}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{3}-\frac{i}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/3 + 1/(x*sqrt(-1 + 9/(4*x**2))) - 9/(4*x**3*sqr
t(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*I*asin(3/(2*x))/3 - I/(x*sqr
t(1 - 9/(4*x**2))) + 9*I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 2.5649, size = 61, normalized size = 1.56

$$-\frac{\sqrt{-4\,x^2+9}}{2\,x^2}+\frac{1}{3}\,\log\left(\sqrt{-4\,x^2+9}+3\right)-\frac{1}{3}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*x^2 + 9)/x^2 + 1/3*log(sqrt(-4*x^2 + 9) + 3) - 1/3*log(-sqrt(-
4*x^2 + 9) + 3)
```

3.461 $\int \frac{\sqrt{9-4x^2}}{x^4} \, dx$

Optimal. Leaf size=18

$$-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

```
[Out] -(9 - 4*x^2)^(3/2)/(27*x^3)
```

Rubi [A] time = 0.0033959, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x^4} \, dx = -\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.0029203, size = 18, normalized size = 1.

$$-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/(27*x^3)
```

Maple [A] time = 0.003, size = 25, normalized size = 1.4

$$\frac{(-3+2x)(3+2x)}{27x^3}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^4,x)
```

```
[Out] 1/27/x^3*(-3+2*x)*(3+2*x)*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.65625, size = 19, normalized size = 1.06

$$-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(-4*x^2 + 9)^(3/2)/x^3
```

Fricas [A] time = 1.45615, size = 53, normalized size = 2.94

$$\frac{\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(4*x^2 - 9)*sqrt(-4*x^2 + 9)/x^3
```

Sympy [B] time = 1.03708, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{8i\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*sqrt(-1 + 9/(4*x**2))/27 - 2*sqrt(-1 + 9/(4*x**2))/(3*x**2), 9
/(4*Abs(x**2)) > 1), (8*I*sqrt(1 - 9/(4*x**2))/27 - 2*I*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

Giac [B] time = 2.00095, size = 99, normalized size = 5.5

$$-\frac{2\,x^3\left(\frac{3\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}-4\right)}{27\left(\sqrt{-4\,x^2+9}-3\right)^3}+\frac{\sqrt{-4\,x^2+9}-3}{18\,x}-\frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{216\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/27*x^3*(3*(sqrt(-4*x^2 + 9) - 3)^2/x^2 - 4)/(sqrt(-4*x^2 + 9) - 3)^3 + 1
/18*(sqrt(-4*x^2 + 9) - 3)/x - 1/216*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

3.462 $\int \frac{\sqrt{9-4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{9-4x^2}}{18x^2}-\frac{\sqrt{9-4x^2}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -Sqrt[9 - 4*x^2]/(4*x^4) + Sqrt[9 - 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 - 4
*x^2]/3])/27
```

Rubi [A] time = 0.0219855, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 206}

$$\frac{\sqrt{9-4x^2}}{18x^2}-\frac{\sqrt{9-4x^2}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(4*x^4) + Sqrt[9 - 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 - 4
*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{9-4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} - \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.005111, size = 32, normalized size = 0.56

$$
-\frac{16\left(9-4x^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{4x^2}{9}\right)}{2187}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^5,x]
```

```
[Out] (-16*(9 - 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (4*x^2)/9])/2187
```

Maple [A] time = 0.004, size = 55, normalized size = 1.

$$
-\frac{1}{36\,x^4}\left(-4\,x^2+9\right)^{\frac{3}{2}} - \frac{1}{162\,x^2}\left(-4\,x^2+9\right)^{\frac{3}{2}} - \frac{2}{81}\sqrt{-4\,x^2+9} + \frac{2}{27}\operatorname{Artanh}\left(3\,\frac{1}{\sqrt{-4\,x^2+9}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^5,x)
```

```
[Out] -1/36/x^4*(-4*x^2+9)^(3/2)-1/162/x^2*(-4*x^2+9)^(3/2)-2/81*(-4*x^2+9)^(1/2)
+2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A] time = 2.87239, size = 88, normalized size = 1.54

$$
-\frac{2}{81}\sqrt{-4\,x^2+9} - \frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{162\,x^2} - \frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{36\,x^4} + \frac{2}{27}\log\left(\frac{6\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 + 9) - 1/162*(-4*x^2 + 9)^(3/2)/x^2 - 1/36*(-4*x^2 + 9)^(
3/2)/x^4 + 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 1.53264, size = 112, normalized size = 1.96

$$-\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\left(2\,x^2-9\right)\sqrt{-4\,x^2+9}}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/108*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 - 9)*sqrt(-4*x^2 + 9
))/x^4
```

Sympy [A] time = 3.52182, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{3}{4x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/27 - 1/(9*x*sqrt(-1 + 9/(4*x**2))) + 3/(4*x**3*
sqrt(-1 + 9/(4*x**2))) - 9/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2))
> 1), (-2*I*asin(3/(2*x))/27 + I/(9*x*sqrt(1 - 9/(4*x**2))) - 3*I/(4*x**3*s
qrt(1 - 9/(4*x**2))) + 9*I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 1.98292, size = 77, normalized size = 1.35

$$-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}+9\,\sqrt{-4\,x^2+9}}{72\,x^4}+\frac{1}{27}\,\log\left(\sqrt{-4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((-4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 + 9))/x^4 + 1/27*log(sqrt(-4*x^2
+ 9) + 3) - 1/27*log(-sqrt(-4*x^2 + 9) + 3)
```

3.463 $\int x^5 \sqrt{-9+4x^2}\, dx$

Optimal. Leaf size=46

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

```
[Out] (27*(-9 + 4*x^2)^(3/2))/64 + (9*(-9 + 4*x^2)^(5/2))/160 + (-9 + 4*x^2)^(7/2
)/448
```

Rubi [A] time = 0.019893, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*(-9 + 4*x^2)^(3/2))/64 + (9*(-9 + 4*x^2)^(5/2))/160 + (-9 + 4*x^2)^(7/2
)/448
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{-9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{-9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{-9+4x}+\frac{9}{8}(-9+4x)^{3/2}+\frac{1}{16}(-9+4x)^{5/2}\right)dx,x,x^2\right)\\ &= \frac{27}{64}\left(-9+4x^2\right)^{3/2}+\frac{9}{160}\left(-9+4x^2\right)^{5/2}+\frac{1}{448}\left(-9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0090673, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2-9\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((-9 + 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$\frac{(-3+2x)(3+2x)\left(10x^4+18x^2+27\right)}{280}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/280*(-3+2*x)*(3+2*x)*(10*x^4+18*x^2+27)*(4*x^2-9)^(1/2)
```

Maxima [A] time = 3.40864, size = 54, normalized size = 1.17

$$\frac{1}{28}\left(4x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(4x^2-9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 - 9)^(3/2)*x^4 + 9/140*(4*x^2 - 9)^(3/2)*x^2 + 27/280*(4*x^2 -
9)^(3/2)
```

Fricas [A] time = 1.43098, size = 76, normalized size = 1.65

$$\frac{1}{280}\left(40x^6-18x^4-54x^2-243\right)\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(4*x^2 - 9)
```

Sympy [A] time = 1.91672, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2-9}}{7}-\frac{9x^4\sqrt{4x^2-9}}{140}-\frac{27x^2\sqrt{4x^2-9}}{140}-\frac{243\sqrt{4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 - 9)/7 - 9*x**4*sqrt(4*x**2 - 9)/140 - 27*x**2*sqrt(4*x**2
 - 9)/140 - 243*sqrt(4*x**2 - 9)/280
```

Giac [A] time = 2.21624, size = 46, normalized size = 1.

$$\frac{1}{448}\left(4\,x^2-9\right)^{\frac{7}{2}}+\frac{9}{160}\left(4\,x^2-9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 - 9)^(7/2) + 9/160*(4*x^2 - 9)^(5/2) + 27/64*(4*x^2 - 9)^(3/2)
```

3.464 $\int x^4\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{6}\sqrt{4x^2-9}x^5-\frac{3}{32}\sqrt{4x^2-9}x^3-\frac{81}{256}\sqrt{4x^2-9}x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] (-81*x*Sqrt[-9 + 4*x^2])/256 - (3*x^3*Sqrt[-9 + 4*x^2])/32 + (x^5*Sqrt[-9 +
 4*x^2])/6 - (729*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/512
```

Rubi [A] time = 0.0188935, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{1}{6}\sqrt{4x^2-9}x^5-\frac{3}{32}\sqrt{4x^2-9}x^3-\frac{81}{256}\sqrt{4x^2-9}x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 + 4*x^2])/256 - (3*x^3*Sqrt[-9 + 4*x^2])/32 + (x^5*Sqrt[-9 +
 4*x^2])/6 - (729*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/512
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{-9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{3}{2}\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{81}{32}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\
&= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0110108, size = 49, normalized size = 0.68

$$\frac{1}{768}x\sqrt{4x^2-9}\left(128x^4-72x^2-243\right)-\frac{729}{512}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 - (729*Log[2*x + Sqrt[-9
 + 4*x^2]])/512
```

Maple [A] time = 0.006, size = 61, normalized size = 0.9

$$\frac{x^3}{24}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{9x}{128}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{81x}{256}\sqrt{4x^2-9}-\frac{729\sqrt{4}}{1024}\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/24*x^3*(4*x^2-9)^(3/2)+9/128*x*(4*x^2-9)^(3/2)+81/256*x*(4*x^2-9)^(1/2)-7
29/1024*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A] time = 2.72062, size = 77, normalized size = 1.07

$$\frac{1}{24}\left(4x^2-9\right)^{\frac{3}{2}}x^3+\frac{9}{128}\left(4x^2-9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{4x^2-9}x-\frac{729}{512}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 - 9)^(3/2)*x^3 + 9/128*(4*x^2 - 9)^(3/2)*x + 81/256*sqrt(4*x^2
- 9)*x - 729/512*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A] time = 1.47389, size = 119, normalized size = 1.65

$$\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{4\,x^2-9}+\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(4*x^2 - 9) + 729/512*log(-2*x + sqrt(
4*x^2 - 9))
```

Sympy [A] time = 4.58974, size = 167, normalized size = 2.32

$$\begin{cases} \frac{2x^7}{3\sqrt{4x^2-9}}-\frac{15x^5}{8\sqrt{4x^2-9}}-\frac{27x^3}{64\sqrt{4x^2-9}}+\frac{729x}{256\sqrt{4x^2-9}}-\frac{729\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for}\ \frac{4|x^2|}{9}>1 \\ -\frac{2ix^7}{3\sqrt{9-4x^2}}+\frac{15ix^5}{8\sqrt{9-4x^2}}+\frac{27ix^3}{64\sqrt{9-4x^2}}-\frac{729ix}{256\sqrt{9-4x^2}}+\frac{729i\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*x**7/(3*sqrt(4*x**2 - 9)) - 15*x**5/(8*sqrt(4*x**2 - 9)) - 27*
x**3/(64*sqrt(4*x**2 - 9)) + 729*x/(256*sqrt(4*x**2 - 9)) - 729*acosh(2*x/3
)/512, 4*Abs(x**2)/9 > 1), (-2*I*x**7/(3*sqrt(9 - 4*x**2)) + 15*I*x**5/(8*s
qrt(9 - 4*x**2)) + 27*I*x**3/(64*sqrt(9 - 4*x**2)) - 729*I*x/(256*sqrt(9 -
4*x**2)) + 729*I*asin(2*x/3)/512, True))
```

Giac [A] time = 2.5384, size = 59, normalized size = 0.82

$$\frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{4\,x^2-9}x+\frac{729}{512}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(4*x^2 - 9)*x + 729/512*log(abs(-2*x +
 sqrt(4*x^2 - 9)))
```

3.465 $\int x^3\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

```
[Out] (3*(-9 + 4*x^2)^(3/2))/16 + (-9 + 4*x^2)^(5/2)/80
```

Rubi [A] time = 0.0136232, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (3*(-9 + 4*x^2)^(3/2))/16 + (-9 + 4*x^2)^(5/2)/80
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9+4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x\sqrt{-9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{-9+4x}+\frac{1}{4}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\&=\frac{3}{16}\left(-9+4x^2\right)^{3/2}+\frac{1}{80}\left(-9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0063563, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2+3\right)\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((3 + 2*x^2)*(-9 + 4*x^2)^(3/2))/40
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{(-3+2x)(3+2x)(2x^2+3)}{40}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/40*(-3+2*x)*(3+2*x)*(2*x^2+3)*(4*x^2-9)^(1/2)
```

Maxima [A] time = 2.41976, size = 35, normalized size = 1.13

$$\frac{1}{20}\left(4x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(4*x^2 - 9)^(3/2)*x^2 + 3/40*(4*x^2 - 9)^(3/2)
```

Fricas [A] time = 1.49394, size = 58, normalized size = 1.87

$$\frac{1}{40}\left(8x^4-6x^2-27\right)\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(4*x^2 - 9)
```

Sympy [A] time = 0.589467, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2-9}}{5}-\frac{3x^2\sqrt{4x^2-9}}{20}-\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/5 - 3*x**2*sqrt(4*x**2 - 9)/20 - 27*sqrt(4*x**2 - 9)/
40
```

Giac [A] time = 1.91932, size = 31, normalized size = 1.

$$\frac{1}{80}\left(4x^2-9\right)^{\frac{5}{2}}+\frac{3}{16}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^(5/2) + 3/16*(4*x^2 - 9)^(3/2)
```

3.466 $\int x^2\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4}\sqrt{4x^2-9}x^3-\frac{9}{32}\sqrt{4x^2-9}x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] (-9*x*Sqrt[-9 + 4*x^2])/32 + (x^3*Sqrt[-9 + 4*x^2])/4 - (81*ArcTanh[(2*x)/S
qrt[-9 + 4*x^2]])/64
```

Rubi [A] time = 0.0132831, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{1}{4}\sqrt{4x^2-9}x^3-\frac{9}{32}\sqrt{4x^2-9}x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[-9 + 4*x^2])/32 + (x^3*Sqrt[-9 + 4*x^2])/4 - (81*ArcTanh[(2*x)/S
qrt[-9 + 4*x^2]])/64
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{-9+4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{9}{4}\int\frac{x^2}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\int\frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\,\text{Subst}\left(\int\frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0133994, size = 46, normalized size = 0.85

$$\sqrt{4x^2-9}\left(\frac{x^3}{4}-\frac{9x}{32}\right)-\frac{81}{64}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 + 4*x^2],x]
```

```
[Out] Sqrt[-9 + 4*x^2]*((-9*x)/32 + x^3/4) - (81*Log[2*x + Sqrt[-9 + 4*x^2]])/64
```

Maple [A] time = 0.004, size = 47, normalized size = 0.9

$$\frac{x}{16}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{9x}{32}\sqrt{4x^2-9}-\frac{81\sqrt{4}}{128}\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/16*x*(4*x^2-9)^(3/2)+9/32*x*(4*x^2-9)^(1/2)-81/128*ln(x*4^(1/2)+(4*x^2-9)
^(1/2))*4^(1/2)
```

Maxima [A] time = 3.99713, size = 58, normalized size = 1.07

$$\frac{1}{16}\left(4x^2-9\right)^{\frac{3}{2}}x+\frac{9}{32}\sqrt{4x^2-9}x-\frac{81}{64}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 - 9)^(3/2)*x + 9/32*sqrt(4*x^2 - 9)*x - 81/64*log(8*x + 4*sqrt(
4*x^2 - 9))
```

Fricas [A] time = 1.46524, size = 97, normalized size = 1.8

$$\frac{1}{32}\left(8x^3-9x\right)\sqrt{4x^2-9}+\frac{81}{64}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 9*x)*sqrt(4*x^2 - 9) + 81/64*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] time = 2.72616, size = 124, normalized size = 2.3

$$\begin{cases} \frac{x^5}{\sqrt{4x^2-9}} - \frac{27x^3}{8\sqrt{4x^2-9}} + \frac{81x}{32\sqrt{4x^2-9}} - \frac{81\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{ix^5}{\sqrt{9-4x^2}} + \frac{27ix^3}{8\sqrt{9-4x^2}} - \frac{81ix}{32\sqrt{9-4x^2}} + \frac{81i\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((x**5/sqrt(4*x**2 - 9) - 27*x**3/(8*sqrt(4*x**2 - 9)) + 81*x/(32*
sqrt(4*x**2 - 9)) - 81*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-I*x**5/sqrt(9
 - 4*x**2) + 27*I*x**3/(8*sqrt(9 - 4*x**2)) - 81*I*x/(32*sqrt(9 - 4*x**2))
+ 81*I*asin(2*x/3)/64, True))
```

Giac [A] time = 2.21839, size = 50, normalized size = 0.93

$$\frac{1}{32}\left(8x^2-9\right)\sqrt{4x^2-9}x + \frac{81}{64}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(8*x^2 - 9)*sqrt(4*x^2 - 9)*x + 81/64*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

3.467 $\int x\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

```
[Out] (-9 + 4*x^2)^(3/2)/12
```

Rubi [A] time = 0.0023235, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 + 4*x^2],x]

[Out] (-9 + 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9+4x^2}\,dx=\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

Mathematica [A] time = 0.0018217, size = 15, normalized size = 1.

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 + 4*x^2],x]

[Out] (-9 + 4*x^2)^(3/2)/12
```

Maple [A] time = 0.003, size = 22, normalized size = 1.5

$$\frac{(-3+2x)(3+2x)}{12}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/12*(-3+2*x)*(3+2*x)*(4*x^2-9)^(1/2)
```

Maxima [A] time = 2.64925, size = 15, normalized size = 1.

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

Fricas [A] time = 1.43061, size = 32, normalized size = 2.13

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

Sympy [B] time = 0.186326, size = 27, normalized size = 1.8

$$\frac{x^2\sqrt{4x^2-9}}{3}-\frac{3\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 - 9)/3 - 3*sqrt(4*x**2 - 9)/4
```

Giac [A] time = 1.39767, size = 15, normalized size = 1.

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

3.468 $\int \sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4
```

Rubi [A] time = 0.0056792, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-9+4x^2}\,dx &= \frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{2}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{2}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0059698, size = 37, normalized size = 1.03

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*Log[2*x + Sqrt[-9 + 4*x^2]])/4
```

Maple [A] time = 0.003, size = 35, normalized size = 1.

$$\frac{x}{2}\sqrt{4\,x^{2}-9}-\frac{9\,\sqrt{4}}{8}\ln\left(x\sqrt{4}+\sqrt{4\,x^{2}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2),x)
```

```
[Out] 1/2*x*(4*x^2-9)^(1/2)-9/8*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A] time = 3.19076, size = 42, normalized size = 1.17

$$\frac{1}{2}\,\sqrt{4\,x^{2}-9}x-\frac{9}{4}\,\log\left(8\,x+4\,\sqrt{4\,x^{2}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 - 9)*x - 9/4*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A] time = 1.48715, size = 77, normalized size = 2.14

$$\frac{1}{2}\,\sqrt{4\,x^{2}-9}x+\frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^{2}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] time = 0.195789, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^{2}-9}}{2}-\frac{9\operatorname{acosh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 - 9)/2 - 9*acosh(2*x/3)/4
```

Giac [A] time = 1.58807, size = 41, normalized size = 1.14

$$\frac{1}{2}\sqrt{4x^2-9}x+\frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

3.469 $\int \frac{\sqrt{-9+4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Rubi [A] time = 0.0165918, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 203}

$$\sqrt{4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x}\,dx, x, x^2\right)$$
$$= \sqrt{-9+4x^2} - \frac{9}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)$$
$$= \sqrt{-9+4x^2} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)$$
$$= \sqrt{-9+4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Mathematica [A] time = 0.0042368, size = 30, normalized size = 1.

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$\sqrt{4x^2-9} + 3\arctan\left(3\frac{1}{\sqrt{4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x,x)
```

```
[Out] (4*x^2-9)^(1/2)+3*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A] time = 4.40072, size = 26, normalized size = 0.87

$$\sqrt{4x^2-9} + 3\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 - 9) + 3*arcsin(3/2/abs(x))
```

Fricas [A] time = 1.49895, size = 78, normalized size = 2.6

$$\sqrt{4x^2-9} - 6\arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(4*x^2 - 9) - 6*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

Sympy [C] time = 1.33413, size = 82, normalized size = 2.73

$$\begin{cases} \sqrt{4x^2-9} - 3i\log(x) + \frac{3i\log(x^2)}{2} + 3\operatorname{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ i\sqrt{9-4x^2} + \frac{3i\log(x^2)}{2} - 3i\log\left(\sqrt{1-\frac{4x^2}{9}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(4*x**2 - 9) - 3*I*log(x) + 3*I*log(x**2)/2 + 3*asin(3/(2*x)
), 4*Abs(x**2)/9 > 1), (I*sqrt(9 - 4*x**2) + 3*I*log(x**2)/2 - 3*I*log(sqrt
(1 - 4*x**2/9) + 1), True))
```

Giac [A] time = 2.69534, size = 32, normalized size = 1.07

$$\sqrt{4x^2-9} - 3\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(4*x^2 - 9) - 3*arctan(1/3*sqrt(4*x^2 - 9))
```

3.470 $\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

```
[Out] -(Sqrt[-9 + 4*x^2]/x) + 2*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]
```

Rubi [A] time = 0.0079967, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 206}

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 + 4*x^2]/x) + 2*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx &= -\frac{\sqrt{-9+4x^2}}{x}+4\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{\sqrt{-9+4x^2}}{x}+4\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{x}+2\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.009073, size = 48, normalized size = 1.41

$$-\frac{\sqrt{4x^2-9}+\frac{2x\sqrt{4x^2-9}\sin^{-1}\left(\frac{2x}{3}\right)}{\sqrt{9-4x^2}}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -((Sqrt[-9 + 4*x^2] + (2*x*Sqrt[-9 + 4*x^2]*ArcSin[(2*x)/3])/Sqrt[9 - 4*x^2
])/x)
```

Maple [A] time = 0.003, size = 48, normalized size = 1.4

$$\frac{1}{9x}\left(4x^2-9\right)^{\frac{3}{2}}-\frac{4x}{9}\sqrt{4x^2-9}+\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^2,x)
```

```
[Out] 1/9/x*(4*x^2-9)^(3/2)-4/9*x*(4*x^2-9)^(1/2)+ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4
^(1/2)
```

Maxima [A] time = 2.81638, size = 45, normalized size = 1.32

$$-\frac{\sqrt{4x^2-9}}{x}+2\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(4*x^2 - 9)/x + 2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A] time = 1.49452, size = 84, normalized size = 2.47

$$-\frac{2x\log\left(-2x+\sqrt{4x^2-9}\right)+2x+\sqrt{4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*x*log(-2*x + sqrt(4*x^2 - 9)) + 2*x + sqrt(4*x^2 - 9))/x
```

Sympy [A] time = 0.232093, size = 19, normalized size = 0.56

$$2\operatorname{acosh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] 2*acosh(2*x/3) - sqrt(4*x**2 - 9)/x
```

Giac [A] time = 2.20901, size = 59, normalized size = 1.74

$$-\frac{36}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}-\log\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -36/((2*x - sqrt(4*x^2 - 9))^2 + 9) - log((2*x - sqrt(4*x^2 - 9))^2)
```

3.471 $\int \frac{\sqrt{-9+4x^2}}{x^3} \, dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

```
[Out] -Sqrt[-9 + 4*x^2]/(2*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

Rubi [A] time = 0.016156, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[-9 + 4*x^2]/(2*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{-9+4x^2}}{2x^2} + \text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0074068, size = 55, normalized size = 1.41

$$-\frac{12x^2+4\sqrt{9-4x^2}x^2\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)-27}{6x^2\sqrt{4x^2-9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^3,x]
```

```
[Out] -(-27 + 12*x^2 + 4*x^2*Sqrt[9 - 4*x^2]*ArcTanh[Sqrt[1 - (4*x^2)/9]])/(6*x^2
*Sqrt[-9 + 4*x^2])
```

Maple [A] time = 0.003, size = 41, normalized size = 1.1

$$\frac{1}{18\,x^2}\left(4\,x^2-9\right)^{\frac{3}{2}}-\frac{2}{9}\sqrt{4\,x^2-9}-\frac{2}{3}\arctan\left(3\,\frac{1}{\sqrt{4\,x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^3,x)
```

```
[Out] 1/18/x^2*(4*x^2-9)^(3/2)-2/9*(4*x^2-9)^(1/2)-2/3*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A] time = 3.93929, size = 47, normalized size = 1.21

$$-\frac{2}{9}\sqrt{4\,x^2-9}+\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}}{18\,x^2}-\frac{2}{3}\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(4*x^2 - 9) + 1/18*(4*x^2 - 9)^(3/2)/x^2 - 2/3*arcsin(3/2/abs(x))
```

Fricas [A] time = 1.56207, size = 100, normalized size = 2.56

$$\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)-3\,\sqrt{4\,x^2-9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) - 3*sqrt(4*x^2 - 9))/x^2
```

Sympy [A] time = 1.78121, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{i}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9}{4|x^2|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{3}-\frac{1}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/3 + I/(x*sqrt(-1 + 9/(4*x**2))) - 9*I/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*asin(3/(2*x))/3 - 1/(x*s
qrt(1 - 9/(4*x**2))) + 9/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 2.56137, size = 39, normalized size = 1.

$$-\frac{\sqrt{4\,x^2-9}}{2\,x^2}+\frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(4*x^2 - 9)/x^2 + 2/3*arctan(1/3*sqrt(4*x^2 - 9))
```

3.472 $\int \frac{\sqrt{-9+4x^2}}{x^4} dx$

Optimal. Leaf size=18

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Rubi [A] time = 0.0030039, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x^4} dx = \frac{(-9+4x^2)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.0024081, size = 18, normalized size = 1.

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Maple [A] time = 0.002, size = 25, normalized size = 1.4

$$\frac{(-3+2x)(3+2x)}{27x^3}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^4,x)
```

```
[Out] 1/27/x^3*(-3+2*x)*(3+2*x)*(4*x^2-9)^(1/2)
```

Maxima [A] time = 3.60186, size = 19, normalized size = 1.06

$$\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*(4*x^2 - 9)^(3/2)/x^3
```

Fricas [A] time = 1.45217, size = 51, normalized size = 2.83

$$\frac{8x^3+\left(4x^2-9\right)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(8*x^3 + (4*x^2 - 9)^(3/2))/x^3
```

Sympy [B] time = 1.05356, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8i\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for } \frac{9}{4|x^2|}>1 \\ \frac{8\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*I*sqrt(-1 + 9/(4*x**2))/27 - 2*I*sqrt(-1 + 9/(4*x**2))/(3*x**2
), 9/(4*Abs(x**2)) > 1), (8*sqrt(1 - 9/(4*x**2))/27 - 2*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

Giac [B] time = 2.18852, size = 57, normalized size = 3.17

$$\frac{16\left(\left(2x-\sqrt{4x^2-9}\right)^4+27\right)}{\left(\left(2x-\sqrt{4x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 16*((2*x - sqrt(4*x^2 - 9))^4 + 27)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

3.473 $\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{\sqrt{4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] -Sqrt[-9 + 4*x^2]/(4*x^4) + Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/27
```

Rubi [A] time = 0.0210753, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{\sqrt{4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[-9 + 4*x^2]/(4*x^4) + Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{-9+4x^2}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.0042657, size = 32, normalized size = 0.56

$$\frac{16\left(4x^2-9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{4x^2}{9}\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^5,x]

[Out] (16*(-9 + 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (4*x^2)/9])/2187
```

Maple [A] time = 0.003, size = 55, normalized size = 1.

$$\frac{1}{36\,x^4}\left(4\,x^2-9\right)^{\frac{3}{2}}+\frac{1}{162\,x^2}\left(4\,x^2-9\right)^{\frac{3}{2}}-\frac{2}{81}\sqrt{4\,x^2-9}-\frac{2}{27}\arctan\left(3\,\frac{1}{\sqrt{4\,x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^5,x)

[Out] 1/36/x^4*(4*x^2-9)^(3/2)+1/162/x^2*(4*x^2-9)^(3/2)-2/81*(4*x^2-9)^(1/2)-2/2
7*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A] time = 3.50003, size = 66, normalized size = 1.16

$$-\frac{2}{81}\sqrt{4\,x^2-9}+\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}}{162\,x^2}+\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}}{36\,x^4}-\frac{2}{27}\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 - 9) + 1/162*(4*x^2 - 9)^(3/2)/x^2 + 1/36*(4*x^2 - 9)^(3/2
)/x^4 - 2/27*arcsin(3/2/abs(x))
```

Fricas [A] time = 1.43126, size = 120, normalized size = 2.11

$$\frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\left(2\,x^2-9\right)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 - 9))/x^4
```

Sympy [A] time = 3.51579, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{3i}{4x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + 3*I/(4*x
**3*sqrt(-1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x
**2)) > 1), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 3/(4*x**3
*sqrt(1 - 9/(4*x**2))) + 9/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 2.12905, size = 55, normalized size = 0.96

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}-9\,\sqrt{4\,x^2-9}}{72\,x^4}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/72*((4*x^2 - 9)^(3/2) - 9*sqrt(4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt(4*x
^2 - 9))
```

3.474 $\int x^5\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

```
[Out] (-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/
2)/448
```

Rubi [A] time = 0.0220785, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/
2)/448
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{-9-4x^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sqrt{-9-4x}x^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{81}{16}\sqrt{-9-4x}+\frac{9}{8}(-9-4x)^{3/2}+\frac{1}{16}(-9-4x)^{5/2}\right)dx,x,x^2\right)\\
&= -\frac{27}{64}\left(-9-4x^2\right)^{3/2}-\frac{9}{160}\left(-9-4x^2\right)^{5/2}-\frac{1}{448}\left(-9-4x^2\right)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.0123819, size = 27, normalized size = 0.59

$$-\frac{1}{280}\left(-4x^2-9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -((-9 - 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

Maple [A] time = 0.003, size = 24, normalized size = 0.5

$$-\frac{10\,x^{4}-18\,x^{2}+27}{280}\left(-4\,x^{2}-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/280*(10*x^4-18*x^2+27)*(-4*x^2-9)^(3/2)
```

Maxima [A] time = 3.08869, size = 54, normalized size = 1.17

$$-\frac{1}{28}\left(-4\,x^{2}-9\right)^{\frac{3}{2}}x^{4}+\frac{9}{140}\left(-4\,x^{2}-9\right)^{\frac{3}{2}}x^{2}-\frac{27}{280}\left(-4\,x^{2}-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 - 9)^(3/2)*x^4 + 9/140*(-4*x^2 - 9)^(3/2)*x^2 - 27/280*(-4*x^
2 - 9)^(3/2)
```

Fricas [A] time = 1.27527, size = 77, normalized size = 1.67

$$\frac{1}{280}\left(40\,x^{6}+18\,x^{4}-54\,x^{2}+243\right)\sqrt{-4\,x^{2}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(-4*x^2 - 9)
```

Sympy [A] time = 1.91907, size = 68, normalized size = 1.48

$$\frac{x^{6}\sqrt{-4x^{2}-9}}{7}+\frac{9x^{4}\sqrt{-4x^{2}-9}}{140}-\frac{27x^{2}\sqrt{-4x^{2}-9}}{140}+\frac{243\sqrt{-4x^{2}-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(-4*x**2 - 9)/7 + 9*x**4*sqrt(-4*x**2 - 9)/140 - 27*x**2*sqrt(-4*x
**2 - 9)/140 + 243*sqrt(-4*x**2 - 9)/280
```

Giac [C] time = 2.18085, size = 46, normalized size = 1.

$$\frac{1}{448}i\left(4x^2+9\right)^{\frac{7}{2}}-\frac{9}{160}i\left(4x^2+9\right)^{\frac{5}{2}}+\frac{27}{64}i\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*I*(4*x^2 + 9)^(7/2) - 9/160*I*(4*x^2 + 9)^(5/2) + 27/64*I*(4*x^2 + 9)
^(3/2)
```

3.475 $\int x^4\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{6}\sqrt{-4x^2-9}x^5+\frac{3}{32}\sqrt{-4x^2-9}x^3-\frac{81}{256}\sqrt{-4x^2-9}x-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] (-81*x*Sqrt[-9 - 4*x^2])/256 + (3*x^3*Sqrt[-9 - 4*x^2])/32 + (x^5*Sqrt[-9 -
 4*x^2])/6 - (729*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/512
```

Rubi [A] time = 0.0208599, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 203}

$$\frac{1}{6}\sqrt{-4x^2-9}x^5+\frac{3}{32}\sqrt{-4x^2-9}x^3-\frac{81}{256}\sqrt{-4x^2-9}x-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 - 4*x^2])/256 + (3*x^3*Sqrt[-9 - 4*x^2])/32 + (x^5*Sqrt[-9 -
 4*x^2])/6 - (729*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/512
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{-9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{3}{2}\int\frac{x^4}{\sqrt{-9-4x^2}}\,dx\\
&= \frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}+\frac{81}{32}\int\frac{x^2}{\sqrt{-9-4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{256}\int\frac{1}{\sqrt{-9-4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{256}\,\text{Subst}\left(\int\frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\
&= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0154593, size = 48, normalized size = 0.67

$$\frac{1}{768}x\sqrt{-4x^2-9}\left(128x^4+72x^2-243\right)-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*ArcTan[(2*x)/Sqrt
[-9 - 4*x^2]])/512
```

Maple [A] time = 0.007, size = 55, normalized size = 0.8

$$-\frac{x^3}{24}\left(-4x^2-9\right)^{\frac{3}{2}}+\frac{9x}{128}\left(-4x^2-9\right)^{\frac{3}{2}}+\frac{81x}{256}\sqrt{-4x^2-9}-\frac{729}{512}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/24*x^3*(-4*x^2-9)^(3/2)+9/128*x*(-4*x^2-9)^(3/2)+81/256*x*(-4*x^2-9)^(1/
2)-729/512*arctan(2*x/(-4*x^2-9)^(1/2))
```

Maxima [C] time = 3.45487, size = 61, normalized size = 0.85

$$-\frac{1}{24}\left(-4x^2-9\right)^{\frac{3}{2}}x^3+\frac{9}{128}\left(-4x^2-9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4x^2-9}x+\frac{729}{512}i\,\text{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 - 9)^(3/2)*x^3 + 9/128*(-4*x^2 - 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 - 9)*x + 729/512*I*arcsinh(2/3*x)
```

Fricas [C] time = 1.33584, size = 204, normalized size = 2.83

$$\frac{1}{768}\left(128\,x^5+72\,x^3-243\,x\right)\sqrt{-4\,x^2-9}-\frac{729}{1024}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)+\frac{729}{1024}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(-4*x^2 - 9) - 729/1024*I*log(-(8*x +
4*I*sqrt(-4*x^2 - 9))/x) + 729/1024*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

Sympy [C] time = 4.55362, size = 83, normalized size = 1.15

$$\frac{2ix^7}{3\sqrt{4x^2+9}}+\frac{15ix^5}{8\sqrt{4x^2+9}}-\frac{27ix^3}{64\sqrt{4x^2+9}}-\frac{729ix}{256\sqrt{4x^2+9}}+\frac{729i\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x**7/(3*sqrt(4*x**2 + 9)) + 15*I*x**5/(8*sqrt(4*x**2 + 9)) - 27*I*x**3/
(64*sqrt(4*x**2 + 9)) - 729*I*x/(256*sqrt(4*x**2 + 9)) + 729*I*asinh(2*x/3)
/512
```

Giac [C] time = 2.39498, size = 45, normalized size = 0.62

$$\frac{1}{768}\left(8\left(16\,x^2+9\right)x^2-243\right)\sqrt{-4\,x^2-9}x+\frac{729}{512}i\,\arcsin\left(\frac{2}{3}i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(-4*x^2 - 9)*x + 729/512*I*arcsin(2/3*
I*x)
```

3.476 $\int x^3\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

```
[Out] (3*(-9 - 4*x^2)^(3/2))/16 + (-9 - 4*x^2)^(5/2)/80
```

Rubi [A] time = 0.0139497, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (3*(-9 - 4*x^2)^(3/2))/16 + (-9 - 4*x^2)^(5/2)/80
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{-9-4x^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int\sqrt{-9-4x}x\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{9}{4}\sqrt{-9-4x}-\frac{1}{4}(-9-4x)^{3/2}\right)dx,x,x^2\right)\\
&= \frac{3}{16}\left(-9-4x^2\right)^{3/2}+\frac{1}{80}\left(-9-4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0095639, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(-4x^2-9\right)^{3/2}\left(3-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] ((-9 - 4*x^2)^(3/2)*(3 - 2*x^2))/40
```

Maple [A] time = 0.002, size = 19, normalized size = 0.6

$$-\frac{2x^2-3}{40}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/40*(2*x^2-3)*(-4*x^2-9)^(3/2)
```

Maxima [A] time = 4.20774, size = 35, normalized size = 1.13

$$-\frac{1}{20}\left(-4x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(-4*x^2 - 9)^(3/2)*x^2 + 3/40*(-4*x^2 - 9)^(3/2)
```

Fricas [A] time = 1.27106, size = 59, normalized size = 1.9

$$\frac{1}{40}\left(8x^4+6x^2-27\right)\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(-4*x^2 - 9)
```

Sympy [A] time = 0.603163, size = 49, normalized size = 1.58

$$\frac{x^4\sqrt{-4x^2-9}}{5}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(-4*x**2 - 9)/5 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2 -
9)/40
```

Giac [C] time = 2.76936, size = 31, normalized size = 1.

$$\frac{1}{80}i\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{16}i\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*I*(4*x^2 + 9)^(5/2) - 3/16*I*(4*x^2 + 9)^(3/2)
```

3.477 $\int x^2\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4}\sqrt{-4x^2-9}x^3+\frac{9}{32}\sqrt{-4x^2-9}x+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] (9*x*Sqrt[-9 - 4*x^2])/32 + (x^3*Sqrt[-9 - 4*x^2])/4 + (81*ArcTan[(2*x)/Sqr
t[-9 - 4*x^2]])/64
```

Rubi [A] time = 0.0136314, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 203}

$$\frac{1}{4}\sqrt{-4x^2-9}x^3+\frac{9}{32}\sqrt{-4x^2-9}x+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*x*Sqrt[-9 - 4*x^2])/32 + (x^3*Sqrt[-9 - 4*x^2])/4 + (81*ArcTan[(2*x)/Sqr
t[-9 - 4*x^2]])/64
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{-9-4x^2}\,dx = \frac{1}{4}x^3\sqrt{-9-4x^2} - \frac{9}{4}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$$
$$= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{32}\int \frac{1}{\sqrt{-9-4x^2}}\,dx$$
$$= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{32}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)$$
$$= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

Mathematica [A] time = 0.0156697, size = 43, normalized size = 0.8

$$\frac{1}{64}\left(2x\sqrt{-4x^2-9}\left(8x^2+9\right)+81\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 - 4*x^2]*(9 + 8*x^2) + 81*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/64
```

Maple [A] time = 0.003, size = 41, normalized size = 0.8

$$-\frac{x}{16}\left(-4x^2-9\right)^{\frac{3}{2}} - \frac{9x}{32}\sqrt{-4x^2-9} + \frac{81}{64}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/16*x*(-4*x^2-9)^(3/2)-9/32*x*(-4*x^2-9)^(1/2)+81/64*arctan(2*x/(-4*x^2-9
)^(1/2))
```

Maxima [C] time = 3.42131, size = 42, normalized size = 0.78

$$-\frac{1}{16}\left(-4x^2-9\right)^{\frac{3}{2}}x - \frac{9}{32}\sqrt{-4x^2-9}x - \frac{81}{64}i\,\text{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(-4*x^2 - 9)^(3/2)*x - 9/32*sqrt(-4*x^2 - 9)*x - 81/64*I*arcsinh(2/3*
x)
```

Fricas [C] time = 1.25942, size = 180, normalized size = 3.33

$$\frac{1}{32}\left(8x^3+9x\right)\sqrt{-4x^2-9} + \frac{81}{128}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right) - \frac{81}{128}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 + 9*x)*sqrt(-4*x^2 - 9) + 81/128*I*log(-(8*x + 4*I*sqrt(-4*x^2
- 9))/x) - 81/128*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

Sympy [C] time = 2.70224, size = 61, normalized size = 1.13

$$\frac{ix^5}{\sqrt{4x^2+9}}+\frac{27ix^3}{8\sqrt{4x^2+9}}+\frac{81ix}{32\sqrt{4x^2+9}}-\frac{81i\operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2-9)**(1/2),x)
```

```
[Out] I*x**5/sqrt(4*x**2 + 9) + 27*I*x**3/(8*sqrt(4*x**2 + 9)) + 81*I*x/(32*sqrt(
4*x**2 + 9)) - 81*I*asinh(2*x/3)/64
```

Giac [C] time = 1.54434, size = 35, normalized size = 0.65

$$\frac{1}{32}\left(8x^2+9\right)\sqrt{-4x^2-9}x-\frac{81}{64}i\arcsin\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(8*x^2 + 9)*sqrt(-4*x^2 - 9)*x - 81/64*I*arcsin(2/3*I*x)
```

3.478 $\int x\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

```
[Out] -(-9 - 4*x^2)^(3/2)/12
```

Rubi [A] time = 0.0024286, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 - 4*x^2],x]

[Out] -(-9 - 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9-4x^2}\,dx=-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

Mathematica [A] time = 0.0016278, size = 15, normalized size = 1.

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 - 4*x^2],x]

[Out] -(-9 - 4*x^2)^(3/2)/12
```

Maple [A] time = 0.002, size = 12, normalized size = 0.8

$$-\frac{1}{12}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/12*(-4*x^2-9)^(3/2)
```

Maxima [A] time = 2.09672, size = 15, normalized size = 1.

$$-\frac{1}{12}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 - 9)^(3/2)
```

Fricas [A] time = 1.26114, size = 47, normalized size = 3.13

$$\frac{1}{12}\left(4x^2+9\right)\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)*sqrt(-4*x^2 - 9)
```

Sympy [B] time = 0.203126, size = 31, normalized size = 2.07

$$\frac{x^2\sqrt{-4x^2-9}}{3}+\frac{3\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(-4*x**2 - 9)/3 + 3*sqrt(-4*x**2 - 9)/4
```

Giac [C] time = 1.58329, size = 15, normalized size = 1.

$$\frac{1}{12}i\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*I*(4*x^2 + 9)^(3/2)
```

3.479 $\int \sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] (x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Rubi [A] time = 0.0057094, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 203}

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-9-4x^2}\,dx &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{2}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0075753, size = 36, normalized size = 1.

$$\frac{1}{4}\left(2x\sqrt{-4x^2-9}-9\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 - 4*x^2] - 9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Maple [A] time = 0.002, size = 29, normalized size = 0.8

$$-\frac{9}{4}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)+\frac{x}{2}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2),x)
```

```
[Out] -9/4*arctan(2*x/(-4*x^2-9)^(1/2))+1/2*x*(-4*x^2-9)^(1/2)
```

Maxima [C] time = 4.25974, size = 26, normalized size = 0.72

$$\frac{1}{2}\sqrt{-4x^2-9}x+\frac{9}{4}i\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsinh(2/3*x)
```

Fricas [C] time = 1.26692, size = 154, normalized size = 4.28

$$\frac{1}{2}\sqrt{-4x^2-9}x-\frac{9}{8}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)+\frac{9}{8}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x - 9/8*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + 9/8*I
*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] time = 0.379129, size = 34, normalized size = 0.94

$$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2),x)
```

```
[Out] x*sqrt(-4*x**2 - 9)/2 - 9*atan(2*x/sqrt(-4*x**2 - 9))/4
```

Giac [C] time = 2.36686, size = 26, normalized size = 0.72

$$\frac{1}{2}\sqrt{-4x^2-9}x+\frac{9}{4}i\arcsin\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsin(2/3*I*x)
```

3.480 $\int \frac{\sqrt{-9-4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{-4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Rubi [A] time = 0.0151617, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 204}

$$\sqrt{-4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x,x]

[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9-4x}}{x}\,dx, x, x^2\right)$$
$$= \sqrt{-9-4x^2} - \frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x}\,dx, x, x^2\right)$$
$$= \sqrt{-9-4x^2} + \frac{9}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)$$
$$= \sqrt{-9-4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Mathematica [A] time = 0.0043035, size = 30, normalized size = 1.

$$\sqrt{-4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$\sqrt{-4x^2-9} + 3\arctan\left(3\frac{1}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x*(-4*x^2-9)^(1/2),x)
```

```
[Out] (-4*x^2-9)^(1/2)+3*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] time = 3.30422, size = 47, normalized size = 1.57

$$\sqrt{-4x^2-9} + 3i\log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 - 9) + 3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] time = 1.289, size = 142, normalized size = 4.73

$$\sqrt{-4x^2-9} - \frac{3}{2}i\log\left(-\frac{6\left(i\sqrt{-4x^2-9}-3\right)}{x}\right) + \frac{3}{2}i\log\left(-\frac{6\left(-i\sqrt{-4x^2-9}-3\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 - 9) - 3/2*I*log(-6*(I*sqrt(-4*x^2 - 9) - 3)/x) + 3/2*I*log(-6*
(-I*sqrt(-4*x^2 - 9) - 3)/x)
```

Sympy [C] time = 1.24054, size = 44, normalized size = 1.47

$$\frac{2ix}{\sqrt{1+\frac{9}{4x^2}}} - 3i\operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9i}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x/sqrt(1 + 9/(4*x**2)) - 3*I*asinh(3/(2*x)) + 9*I/(2*x*sqrt(1 + 9/(4*x*
*2)))
```

Giac [C] time = 2.42821, size = 35, normalized size = 1.17

$$i\sqrt{4x^2+9} - 3\arctan\left(\frac{1}{3}i\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] I*sqrt(4*x^2 + 9) - 3*arctan(1/3*I*sqrt(4*x^2 + 9))
```

3.481 $\int \frac{\sqrt{-9-4x^2}}{x^2} \, dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{-4x^2-9}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - 2*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]
```

Rubi [A] time = 0.0076738, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 203}

$$-\frac{\sqrt{-4x^2-9}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - 2*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^2} \, dx &= -\frac{\sqrt{-9-4x^2}}{x}-4\int \frac{1}{\sqrt{-9-4x^2}} \, dx\\
&= -\frac{\sqrt{-9-4x^2}}{x}-4\,\text{Subst}\left(\int \frac{1}{1+4x^2} \, dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0069708, size = 49, normalized size = 1.44

$$\frac{\sqrt{-4x^2-9}\left(2x\sinh^{-1}\left(\frac{2x}{3}\right)-\sqrt{4x^2+9}\right)}{x\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(-Sqrt[9 + 4*x^2] + 2*x*ArcSinh[(2*x)/3]))/(x*Sqrt[9 + 4*
x^2])
```

Maple [A] time = 0.003, size = 43, normalized size = 1.3

$$\frac{1}{9x}\left(-4x^2-9\right)^{\frac{3}{2}}+\frac{4x}{9}\sqrt{-4x^2-9}-2\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^2,x)
```

```
[Out] 1/9/x*(-4*x^2-9)^(3/2)+4/9*x*(-4*x^2-9)^(1/2)-2*arctan(2*x/(-4*x^2-9)^(1/2)
)
```

Maxima [C] time = 3.47615, size = 28, normalized size = 0.82

$$-\frac{\sqrt{-4x^2-9}}{x}+2i\,\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-4*x^2 - 9)/x + 2*I*arcsinh(2/3*x)
```

Fricas [C] time = 1.34476, size = 147, normalized size = 4.32

$$\frac{-ix\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)+ix\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)-\sqrt{-4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (-I*x*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + I*x*log(-(8*x - 4*I*sqrt(-4*x^
2 - 9))/x) - sqrt(-4*x^2 - 9))/x
```

Sympy [A] time = 0.420157, size = 32, normalized size = 0.94

$$-2\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{\sqrt{-4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] -2*atan(2*x/sqrt(-4*x**2 - 9)) - sqrt(-4*x**2 - 9)/x
```

Giac [C] time = 1.33912, size = 58, normalized size = 1.71

$$-\frac{i\sqrt{4x^2+9}+3i}{2x}-\frac{8x}{-4i\sqrt{4x^2+9}-12i}+2i\arcsin\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(I*sqrt(4*x^2 + 9) + 3*I)/x - 8*x/(-4*I*sqrt(4*x^2 + 9) - 12*I) + 2*I*
arcsin(2/3*I*x)
```

3.482 $\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] -Sqrt[-9 - 4*x^2]/(2*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Rubi [A] time = 0.0161465, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^3,x]

[Out] -Sqrt[-9 - 4*x^2]/(2*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{-9-4x^2}}{2x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{-9-4x^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)$$
$$= -\frac{\sqrt{-9-4x^2}}{2x^2} - \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Mathematica [A] time = 0.0136417, size = 55, normalized size = 1.41

$$\frac{12x^2 + 4\sqrt{4x^2+9}x^2\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right) + 27}{6x^2\sqrt{-4x^2-9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^3,x]
```

```
[Out] (27 + 12*x^2 + 4*x^2*Sqrt[9 + 4*x^2]*ArcTanh[Sqrt[1 + (4*x^2)/9]])/(6*x^2*S
qrt[-9 - 4*x^2])
```

Maple [A] time = 0.003, size = 41, normalized size = 1.1

$$\frac{1}{18\,x^2}\left(-4\,x^2-9\right)^{\frac{3}{2}} + \frac{2}{9}\sqrt{-4\,x^2-9} + \frac{2}{3}\arctan\left(3\,\frac{1}{\sqrt{-4\,x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^3,x)
```

```
[Out] 1/18/x^2*(-4*x^2-9)^(3/2)+2/9*(-4*x^2-9)^(1/2)+2/3*arctan(3/(-4*x^2-9)^(1/2
))
```

Maxima [C] time = 3.57064, size = 69, normalized size = 1.77

$$\frac{2}{9}\sqrt{-4\,x^2-9} + \frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{18\,x^2} + \frac{2}{3}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(-4*x^2 - 9) + 1/18*(-4*x^2 - 9)^(3/2)/x^2 + 2/3*I*log(6*sqrt(4*x^2
 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] time = 1.25728, size = 170, normalized size = 4.36

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\,\sqrt{-4\,x^2-9}-3\right)}{3\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\,\sqrt{-4\,x^2-9}-3\right)}{3\,x}\right)-3\,\sqrt{-4\,x^2-9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(-2*I*x^2*log(-4/3*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/3*(-I*s
qrt(-4*x^2 - 9) - 3)/x) - 3*sqrt(-4*x^2 - 9))/x^2
```

Sympy [C] time = 1.74531, size = 27, normalized size = 0.69

$$-\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{i\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**3,x)
```

```
[Out] -2*I*asinh(3/(2*x))/3 - I*sqrt(1 + 9/(4*x**2))/x
```

Giac [C] time = 2.00082, size = 39, normalized size = 1.

$$-\frac{i\,\sqrt{4\,x^2+9}}{2\,x^2}-\frac{2}{3}\,\arctan\left(\frac{1}{3}i\,\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(4*x^2 + 9)/x^2 - 2/3*arctan(1/3*I*sqrt(4*x^2 + 9))
```

3.483 $\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Rubi [A] time = 0.003333, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^4,x]
```

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = \frac{\left(-9-4x^2\right)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.0024752, size = 18, normalized size = 1.

$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^4,x]
```

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{1}{27\,x^3}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^4,x)
```

```
[Out] 1/27*(-4*x^2-9)^(3/2)/x^3
```

Maxima [A] time = 3.31359, size = 19, normalized size = 1.06

$$\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

Fricas [A] time = 1.23472, size = 39, normalized size = 2.17

$$\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

Sympy [C] time = 1.0267, size = 37, normalized size = 2.06

$$-\frac{8i\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**4,x)
```

```
[Out] -8*I*sqrt(1 + 9/(4*x**2))/27 - 2*I*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

Giac [C] time = 1.89349, size = 109, normalized size = 6.06

$$-\frac{2\,x^3\left(\frac{3\left(-i\,\sqrt{4\,x^2+9}-3i\right)^2}{x^2}-4\right)}{27\left(-i\,\sqrt{4\,x^2+9}-3i\right)^3}-\frac{81i\,\sqrt{4\,x^2+9}+243i}{1458\,x}-\frac{\left(-i\,\sqrt{4\,x^2+9}-3i\right)^3}{216\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/27*x^3*(3*(-I*sqrt(4*x^2 + 9) - 3*I)^2/x^2 - 4)/(-I*sqrt(4*x^2 + 9) - 3*
I)^3 - 1/1458*(81*I*sqrt(4*x^2 + 9) + 243*I)/x - 1/216*(-I*sqrt(4*x^2 + 9)
- 3*I)^3/x^3
```

3.484 $\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{\sqrt{-4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] -Sqrt[-9 - 4*x^2]/(4*x^4) - Sqrt[-9 - 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/27
```

Rubi [A] time = 0.0211147, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{\sqrt{-4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/(4*x^4) - Sqrt[-9 - 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0049775, size = 32, normalized size = 0.56

$$
\frac{16\left(-4x^2-9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4x^2}{9}+1\right)}{2187}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] (16*(-9 - 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (4*x^2)/9])/2187
```

Maple [A] time = 0.004, size = 55, normalized size = 1.

$$
\frac{1}{36x^4}\left(-4x^2-9\right)^{\frac{3}{2}} - \frac{1}{162x^2}\left(-4x^2-9\right)^{\frac{3}{2}} - \frac{2}{81}\sqrt{-4x^2-9} - \frac{2}{27}\arctan\left(3\frac{1}{\sqrt{-4x^2-9}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^5,x)
```

```
[Out] 1/36/x^4*(-4*x^2-9)^(3/2)-1/162/x^2*(-4*x^2-9)^(3/2)-2/81*(-4*x^2-9)^(1/2)-
2/27*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] time = 4.01728, size = 88, normalized size = 1.54

$$
-\frac{2}{81}\sqrt{-4x^2-9} - \frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{162x^2} + \frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{36x^4} - \frac{2}{27}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 - 9) - 1/162*(-4*x^2 - 9)^(3/2)/x^2 + 1/36*(-4*x^2 - 9)^(
3/2)/x^4 - 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] time = 1.319, size = 192, normalized size = 3.37

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}+3\right)}{27\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}+3\right)}{27\,x}\right)-3\left(2\,x^2+9\right)\sqrt{-4\,x^2-9}}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(-4*I*x^4*log(-4/27*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/27*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 + 9)*sqrt(-4*x^2 - 9))/x^4
```

Sympy [C] time = 3.53655, size = 68, normalized size = 1.19

$$\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3i}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] 2*I*asinh(3/(2*x))/27 - I/(9*x*sqrt(1 + 9/(4*x**2))) - 3*I/(4*x**3*sqrt(1 +
 9/(4*x**2))) - 9*I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [C] time = 2.92472, size = 58, normalized size = 1.02

$$-\frac{i\left(4\,x^2+9\right)^{\frac{3}{2}}+9i\,\sqrt{4\,x^2+9}}{72\,x^4}+\frac{2}{27}\,\arctan\left(\frac{1}{3}i\,\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*(I*(4*x^2 + 9)^(3/2) + 9*I*sqrt(4*x^2 + 9))/x^4 + 2/27*arctan(1/3*I*s
qrt(4*x^2 + 9))
```

3.485 $\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}$$

```
[Out] (a^2*Sqrt[a + b*x^2])/b^3 - (2*a*(a + b*x^2)^(3/2))/(3*b^3) + (a + b*x^2)^(
5/2)/(5*b^3)
```

Rubi [A] time = 0.0307414, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*Sqrt[a + b*x^2])/b^3 - (2*a*(a + b*x^2)^(3/2))/(3*b^3) + (a + b*x^2)^(
5/2)/(5*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a^2\sqrt{a+bx^2}}{b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0182842, size = 39, normalized size = 0.7

$$\frac{\sqrt{a+bx^2}\left(8a^2-4abx^2+3b^2x^4\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*x^2 + 3*b^2*x^4))/(15*b^3)
```

Maple [A] time = 0.004, size = 36, normalized size = 0.6

$$\frac{3\,b^{2}x^{4}-4\,abx^{2}+8\,a^{2}}{15\,b^{3}}\sqrt{bx^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(3*b^2*x^4-4*a*b*x^2+8*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27504, size = 78, normalized size = 1.39

$$\frac{\left(3\,b^{2}x^{4}-4\,abx^{2}+8\,a^{2}\right)\sqrt{bx^{2}+a}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^4 - 4*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 0.781171, size = 68, normalized size = 1.21

$$\begin{cases} \frac{8a^{2}\sqrt{a+bx^{2}}}{15b^{3}}-\frac{4ax^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{x^{4}\sqrt{a+bx^{2}}}{5b} & \text{for}\ b\neq 0\\ \frac{x^{6}}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*a*x**2*sqrt(a + b*x**2)/(1
5*b**2) + x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True))
```

Giac [A] time = 2.49782, size = 58, normalized size = 1.04

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}-10\left(bx^2+a\right)^{\frac{3}{2}}a+15\sqrt{bx^2+a}a^2}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2) - 10*(b*x^2 + a)^(3/2)*a + 15*sqrt(b*x^2 + a)*a^2
)/b^3
```

3.486 $\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=73

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.0195522, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2}}dx = \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}dx}{4b}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{(3a^2)\int \frac{1}{\sqrt{a+bx^2}}dx}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{(3a^2)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.0227279, size = 62, normalized size = 0.85

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \sqrt{b}x\sqrt{a+bx^2}\left(2bx^2-3a\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a + 2*b*x^2) + 3*a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] time = 0.005, size = 59, normalized size = 0.8

$$\frac{x^3}{4\,b}\sqrt{bx^2+a} - \frac{3\,ax}{8\,b^2}\sqrt{bx^2+a} + \frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8/b^(5/2)*a^2*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32124, size = 300, normalized size = 4.11

$$\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2x^3-3\,abx\right)\sqrt{bx^2+a}}{16\,b^3}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3-3\,abx\right)\sqrt{}}{8\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 3.70894, size = 95, normalized size = 1.3

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Giac [A] time = 2.79865, size = 73, normalized size = 1.

$$\frac{1}{8}\sqrt{bx^2+a}x\left(\frac{2x^2}{b}-\frac{3a}{b^2}\right)-\frac{3a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

3.487 $\int \frac{x^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rubi [A] time = 0.022349, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{(a+bx^2)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, x^2\right)\\ &= -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{(a+bx^2)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.0121982, size = 27, normalized size = 0.75

$$\frac{(bx^2-2a)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2],x]
```

```
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.7

$$-\frac{-bx^2+2a}{3b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27851, size = 53, normalized size = 1.47

$$\frac{\sqrt{bx^2+a}\left(bx^2-2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A] time = 0.466704, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A] time = 1.43237, size = 36, normalized size = 1.

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}-3\sqrt{bx^2+a}\,a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2) - 3*sqrt(b*x^2 + a)*a)/b^2
```

3.488 $\int \frac{x^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rubi [A] time = 0.0124689, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 217, 206}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2}}\,dx &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0172061, size = 49, normalized size = 1.

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.8

$$\frac{x}{2b}\sqrt{bx^2+a}-\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32988, size = 238, normalized size = 4.86

$$\left[\frac{2\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2\,bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}bx+a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] time = 2.08583, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

Giac [A] time = 2.85334, size = 54, normalized size = 1.1

$$\frac{\sqrt{bx^2+a}x}{2b}+\frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

3.489 $\int \frac{x}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

[Out] Sqrt[a + b*x^2]/b

Rubi [A] time = 0.0033863, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b*x^2],x]

[Out] Sqrt[a + b*x^2]/b

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2}} dx = \frac{\sqrt{a+bx^2}}{b}$$

Mathematica [A] time = 0.0017451, size = 15, normalized size = 1.

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a + b*x^2],x]

[Out] Sqrt[a + b*x^2]/b

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{1}{b}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)/b
```

Maxima [A] time = 2.23283, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Fricas [A] time = 1.18847, size = 26, normalized size = 1.73

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Sympy [A] time = 0.37488, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

Giac [A] time = 1.4133, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b
```

3.490 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0055332, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0039217, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38498, size = 153, normalized size = 6.12

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 1.00954, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A] time = 2.33426, size = 31, normalized size = 1.24

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.491 $\int \frac{1}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rubi [A] time = 0.0172745, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0045837, size = 25, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]),x]

[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Maple [A] time = 0.004, size = 29, normalized size = 1.2

$$-\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x)

[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33357, size = 154, normalized size = 6.16

$$\left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

Sympy [A] time = 1.05324, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A] time = 2.32046, size = 30, normalized size = 1.2

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

3.492 $\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rubi [A] time = 0.0045501, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{ax}$$

Mathematica [A] time = 0.0033399, size = 19, normalized size = 1.

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Maple [A] time = 0.005, size = 18, normalized size = 1.

$$-\frac{1}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -(b*x^2+a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27823, size = 32, normalized size = 1.68

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(a*x)
```

Sympy [A] time = 0.567775, size = 19, normalized size = 1.

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [A] time = 2.40265, size = 41, normalized size = 2.16

$$\frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.493 $\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rubi [A] time = 0.0261624, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a}$$
$$= -\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.0474586, size = 61, normalized size = 1.22

$$\frac{b\sqrt{a+bx^2}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\sqrt{\frac{bx^2}{a}+1}} - \frac{a}{2bx^2}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-a/(2*b*x^2) + ArcTanh[Sqrt[1 + (b*x^2)/a]]/(2*Sqrt[1 +
 (b*x^2)/a])))/a^2
```

Maple [A] time = 0.005, size = 48, normalized size = 1.

$$-\frac{1}{2\,ax^2}\sqrt{bx^2+a} + \frac{b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35134, size = 263, normalized size = 5.26

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}a}{4a^2x^2},-\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}a}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

Sympy [A] time = 2.26539, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

Giac [A] time = 2.88439, size = 65, normalized size = 1.3

$$-\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^2+a}}{abx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)/(a*
b*x^2))
```

3.494 $\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rubi [A] time = 0.0103251, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{\sqrt{a+bx^2}}{3ax^3}-\frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\ &= -\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A] time = 0.0057551, size = 29, normalized size = 0.66

$$-\frac{\left(a-2bx^2\right)\sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -((a - 2*b*x^2)*Sqrt[a + b*x^2])/(3*a^2*x^3)
```

Maple [A] time = 0.003, size = 26, normalized size = 0.6

$$-\frac{-2\,b x^{2}+a}{3\,a^{2}x^{3}}\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33137, size = 61, normalized size = 1.39

$$\frac{\left(2\,b x^{2}-a\right)\sqrt{b x^{2}+a}}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] time = 0.791755, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

Giac [A] time = 1.67347, size = 74, normalized size = 1.68

$$\frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

3.495 $\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

```
[Out] -Sqrt[a + b*x^2]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2])/(8*a^2*x^2) - (3*b^2*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] time = 0.0376781, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]),x]
```



```
[Out] -Sqrt[a + b*x^2]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2])/(8*a^2*x^2) - (3*b^2*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{8a}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{8a^2}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0068974, size = 37, normalized size = 0.5

$$-\frac{b^2\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -((b^2*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^2)/a])/a^3)
```

Maple [A] time = 0.005, size = 68, normalized size = 0.9

$$-\frac{1}{4\,ax^4}\sqrt{bx^2+a}+\frac{3\,b}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,b^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)^(1/2)/a/x^4+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2-3/8*b^2/a^(5/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3125, size = 323, normalized size = 4.36

$$\left[\frac{3\,\sqrt{a}b^{2}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(3\,abx^{2}-2\,a^{2}\right)\sqrt{bx^{2}+a}}{16\,a^{3}x^{4}},\ \frac{3\,\sqrt{-a}b^{2}x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(3\,abx^{2}-2\,a^{2}\right)\sqrt{bx^{2}+a}}{8\,a^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/(a^3*x^4), 1/8*(3*sqrt(-a)*b^2*x
^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/
(a^3*x^4)]
```

Sympy [A] time = 4.09866, size = 97, normalized size = 1.31

$$-\frac{1}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -1/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)/(8*a*x**3*sqrt(a/(b*x**2
) + 1)) + 3*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**2*asinh(sqrt(a)
/(sqrt(b)*x))/(8*a**(5/2))
```

Giac [A] time = 2.35898, size = 89, normalized size = 1.2

$$\frac{1}{8}\,b^{2}\left(\frac{3\,\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}-5\,\sqrt{bx^{2}+a}a}{a^{2}b^{2}x^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b^2*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)
^(3/2) - 5*sqrt(b*x^2 + a)*a)/(a^2*b^2*x^4))
```

3.496 $\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{3/2}}{3b^3}$$

```
[Out] -(a^2/(b^3*Sqrt[a + b*x^2])) - (2*a*Sqrt[a + b*x^2])/b^3 + (a + b*x^2)^(3/2
)/(3*b^3)
```

Rubi [A] time = 0.0327409, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2/(b^3*Sqrt[a + b*x^2])) - (2*a*Sqrt[a + b*x^2])/b^3 + (a + b*x^2)^(3/2
)/(3*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, x^2\right)\\ &= -\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.0159502, size = 38, normalized size = 0.69

$$\frac{-8a^2-4abx^2+b^2x^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$-\frac{-b^2x^4+4\,abx^2+8\,a^2}{3\,b^3}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-b^2*x^4+4*a*b*x^2+8*a^2)/(b*x^2+a)^(1/2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29551, size = 93, normalized size = 1.69

$$\frac{\left(b^2x^4-4\,abx^2-8\,a^2\right)\sqrt{bx^2+a}}{3\,\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 - 4*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a)/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 0.943373, size = 68, normalized size = 1.24

$$\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b*
x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True))
```

Giac [A] time = 1.95599, size = 55, normalized size = 1.

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}-6\sqrt{bx^2+a}a-\frac{3a^2}{\sqrt{bx^2+a}}}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2) - 6*sqrt(b*x^2 + a)*a - 3*a^2/sqrt(b*x^2 + a))/b^3
```

3.497 $\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x^3}{b\sqrt{a+bx^2}}$$

```
[Out] -(x^3/(b*Sqrt[a + b*x^2])) + (3*x*Sqrt[a + b*x^2])/(2*b^2) - (3*a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] time = 0.0225984, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x^3}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a + b*x^2])) + (3*x*Sqrt[a + b*x^2])/(2*b^2) - (3*a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{(3a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}$$
$$= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}$$
$$= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.0325632, size = 71, normalized size = 1.04

$$\frac{\sqrt{b}x\left(3a+bx^2\right)-3a^{3/2}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(3*a + b*x^2) - 3*a^(3/2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x
)/Sqrt[a]])/(2*b^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 57, normalized size = 0.8

$$\frac{x^3}{2\,b}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,ax}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^3/b/(b*x^2+a)^(1/2)+3/2/b^2*a*x/(b*x^2+a)^(1/2)-3/2/b^(5/2)*a*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34425, size = 362, normalized size = 5.32

$$\left[\frac{3\left(abx^2+a^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(b^2x^3+3\,abx\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)},\frac{3\left(abx^2+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(b}{2\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*(3*(a*
b*x^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b^2*x^3 + 3*a*b
*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 3.08214, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b
**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.68139, size = 69, normalized size = 1.01

$$\frac{x\left(\frac{x^2}{b}+\frac{3\,a}{b^2}\right)}{2\,\sqrt{bx^2+a}}+\frac{3\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(x^2/b + 3*a/b^2)/sqrt(b*x^2 + a) + 3/2*a*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

3.498 $\int \frac{x^3}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=32

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

```
[Out] a/(b^2*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b^2
```

Rubi [A] time = 0.0223081, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(3/2),x]

[Out] a/(b^2*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b^2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx, x, x^2\right)\\
&= \frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0108361, size = 24, normalized size = 0.75

$$\frac{2a+bx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(3/2),x]

[Out] (2*a + b*x^2)/(b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.003, size = 23, normalized size = 0.7

$$\frac{bx^2+2a}{b^2}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2),x)

[Out] (b*x^2+2*a)/(b*x^2+a)^(1/2)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29934, size = 66, normalized size = 2.06

$$\frac{\left(bx^2+2a\right)\sqrt{bx^2+a}}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] (b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.558642, size = 41, normalized size = 1.28

$$\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2),x)

[Out] Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0
)), (x**4/(4*a**(3/2)), True))
```

Giac [A] time = 2.11577, size = 34, normalized size = 1.06

$$\frac{\sqrt{bx^2 + a} + \frac{a}{\sqrt{bx^2+a}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(b*x^2 + a) + a/sqrt(b*x^2 + a))/b^2
```

3.499 $\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

```
[Out] -(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)
```

Rubi [A] time = 0.0131784, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {288, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x}{b\sqrt{a+bx^2}}+\frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{x}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{x}{b\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0512126, size = 59, normalized size = 1.37

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{b}x}{b^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*x) + Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(
b^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.9

$$-\frac{x}{b}\frac{1}{\sqrt{bx^2+a}}+\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32611, size = 300, normalized size = 6.98

$$\left[-\frac{2\sqrt{bx^2+a}bx-\left(bx^2+a\right)\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\left(b^3x^2+ab^2\right)},-\frac{\sqrt{bx^2+a}bx+\left(bx^2+a\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b^3x^2+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b*x^2 + a)*b*x - (b*x^2 + a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a))/(b^3*x^2 + a*b^2), -(sqrt(b*x^2 + a)*b*x + (b*x^2
+ a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(b^3*x^2 + a*b^2)]
```

Sympy [A] time = 1.6524, size = 37, normalized size = 0.86

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.86196, size = 53, normalized size = 1.23

$$-\frac{x}{\sqrt{bx^2 + a}b} - \frac{\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -x/(sqrt(b*x^2 + a)*b) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.500 $\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{b\sqrt{a+bx^2}}$$

```
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Rubi [A] time = 0.0033578, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{1}{b\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0025923, size = 16, normalized size = 1.

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$-\frac{1}{b}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)
```

Maxima [A] time = 2.86063, size = 19, normalized size = 1.19

$$-\frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

Fricas [A] time = 1.30934, size = 46, normalized size = 2.88

$$-\frac{\sqrt{bx^2+a}}{b^2x^2+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(b^2*x^2 + a*b)
```

Sympy [A] time = 0.540283, size = 24, normalized size = 1.5

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

Giac [A] time = 1.99789, size = 19, normalized size = 1.19

$$-\frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

3.501 $\int \frac{1}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

[Out] x/(a*Sqrt[a + b*x^2])

Rubi [A] time = 0.0018622, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0033032, size = 16, normalized size = 1.

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$\frac{x}{a}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x)
```

```
[Out] x/a/(b*x^2+a)^(1/2)
```

Maxima [A] time = 3.00858, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Fricas [A] time = 1.17569, size = 47, normalized size = 2.94

$$\frac{\sqrt{bx^2 + a}x}{abx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A] time = 0.55124, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 3.08707, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

3.502 $\int \frac{1}{x(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Rubi [A] time = 0.027081, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{a\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$
$$= \frac{1}{a\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}$$
$$= \frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.0058366, size = 33, normalized size = 0.8

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a]/(a*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 43, normalized size = 1.1

$$\frac{1}{a}\frac{1}{\sqrt{bx^2+a}} - \ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37665, size = 294, normalized size = 7.17

$$\left[\frac{\left(bx^2+a\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}a}{2\left(a^2bx^2+a^3\right)},\ \frac{\left(bx^2+a\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}a}{a^2bx^2+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x^2 + a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3), ((b*x^2 + a)*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3)]
```

Sympy [B] time = 1.69477, size = 184, normalized size = 4.49

$$\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/
(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) +
2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
+ 2*a**(7/2)*b*x**2)
```

Giac [A] time = 1.56115, size = 53, normalized size = 1.29

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{1}{\sqrt{bx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(b*x^2 + a)*a)
```

3.503 $\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

```
[Out] -(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0079569, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 191}

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{(2b)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{2bx}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0055127, size = 27, normalized size = 0.71

$$-\frac{a+2bx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((a + 2*b*x^2)/(a^2*x*Sqrt[a + b*x^2]))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$-\frac{2\,b x^{2}+a}{a^{2} x}\frac{1}{\sqrt{b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -(2*b*x^2+a)/x/(b*x^2+a)^(1/2)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27512, size = 70, normalized size = 1.84

$$-\frac{\left(2\,b x^{2}+a\right)\sqrt{b x^{2}+a}}{a^{2} b x^{3}+a^{3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*b*x^2 + a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

Sympy [A] time = 0.788244, size = 46, normalized size = 1.21

$$-\frac{1}{a\sqrt{b}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{2\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2)
+ 1))
```

Giac [A] time = 2.61432, size = 68, normalized size = 1.79

$$-\frac{bx}{\sqrt{bx^2+a}a^2}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -b*x/(sqrt(b*x^2 + a)*a^2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)*a)
```

3.504 $\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{3b}{2a^2\sqrt{a+bx^2}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{1}{2ax^2\sqrt{a+bx^2}}$$

```
[Out] (-3*b)/(2*a^2*Sqrt[a + b*x^2]) - 1/(2*a*x^2*Sqrt[a + b*x^2]) + (3*b*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.0390216, antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{3\sqrt{a+bx^2}}{2a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(2*a^2*x^2) + (3*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)$$
$$=\frac{1}{ax^2\sqrt{a+bx^2}}+\frac{3\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}$$
$$=\frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}-\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^2}$$
$$=\frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}-\frac{3\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^2}$$
$$=\frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [C] time = 0.007585, size = 35, normalized size = 0.51

$$-\frac{b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/(a^2*Sqrt[a + b*x^2]))
```

Maple [A] time = 0.004, size = 63, normalized size = 0.9

$$-\frac{1}{2\,ax^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,b}{2\,a^2}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a^2/(b*x^2+a)^(1/2)+3/2*b/a^(5/2)*ln((2*a+
2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35542, size = 382, normalized size = 5.54

$$\left[\frac{3\left(b^2x^4+abx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(3abx^2+a^2\right)\sqrt{bx^2+a}}{4\left(a^3bx^4+a^4x^2\right)}, -\frac{3\left(b^2x^4+abx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3a\right.}{2\left(a^3bx^4+a^4x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^2*x^4 + a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) - 2*(3*a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2),
-1/2*(3*(b^2*x^4 + a*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*
a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [A] time = 3.19302, size = 73, normalized size = 1.06

$$-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*
x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))
```

Giac [A] time = 1.72575, size = 89, normalized size = 1.29

$$-\frac{1}{2}b\left(\frac{3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3bx^2+a}{\left(\left(bx^2+a\right)^{\frac{3}{2}}-\sqrt{bx^2+a}a\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*b*x^2 + a)/(
((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2))
```

3.505 $\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

```
[Out] -1/(3*a*x^3*Sqrt[a + b*x^2]) + (4*b)/(3*a^2*x*Sqrt[a + b*x^2]) + (8*b^2*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0157467, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 191}

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/(3*a*x^3*Sqrt[a + b*x^2]) + (4*b)/(3*a^2*x*Sqrt[a + b*x^2]) + (8*b^2*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{3ax^3\sqrt{a+bx^2}}-\frac{(4b)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\
&= -\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{\left(8b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}\\
&= -\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0080129, size = 40, normalized size = 0.61

$$-\frac{a^2-4abx^2-8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(a^2 - 4*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x^3*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 37, normalized size = 0.6

$$-\frac{-8\,b^2x^4-4\,abx^2+a^2}{3\,a^3x^3}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-8*b^2*x^4-4*a*b*x^2+a^2)/x^3/(b*x^2+a)^(1/2)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21503, size = 99, normalized size = 1.5

$$\frac{\left(8\,b^2x^4+4\,abx^2-a^2\right)\sqrt{bx^2+a}}{3\,\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*b^2*x^4 + 4*a*b*x^2 - a^2)*sqrt(b*x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [B] time = 1.19645, size = 233, normalized size = 3.53

$$-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}}}{3a^5b^4x^2+6a^4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] -a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 +
3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b
**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3
*a**3*b**6*x**6)
```

Giac [A] time = 2.16911, size = 143, normalized size = 2.17

$$\frac{b^2x}{\sqrt{bx^2+a}a^3}-\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+5a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] b^2*x/(sqrt(b*x^2 + a)*a^3) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2
) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2))/(((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

3.506 $\int \frac{x^6}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=91

$$-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -x^5/(3*b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*Sqrt[a + b*x^2]) + (5*x*Sqrt[
a + b*x^2])/(2*b^3) - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2)
)
```

Rubi [A] time = 0.0291006, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] -x^5/(3*b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*Sqrt[a + b*x^2]) + (5*x*Sqrt[
a + b*x^2])/(2*b^3) - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2)
)
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^2}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^3}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.129976, size = 90, normalized size = 0.99

$$\frac{\sqrt{b}x\left(15a^2+20abx^2+3b^2x^4\right)-15a^{3/2}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6b^{7/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(15*a^2 + 20*a*b*x^2 + 3*b^2*x^4) - 15*a^(3/2)*(a + b*x^2)*Sqrt[
1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(6*b^(7/2)*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.007, size = 75, normalized size = 0.8

$$\frac{x^5}{2b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5ax^3}{6b^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5ax}{2b^3}\frac{1}{\sqrt{bx^2+a}}-\frac{5a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*x^5/b/(b*x^2+a)^(3/2)+5/6/b^2*a*x^3/(b*x^2+a)^(3/2)+5/2/b^3*a*x/(b*x^2+
a)^(1/2)-5/2/b^(7/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39588, size = 506, normalized size = 5.56

$$\left[\frac{15\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)\sqrt{b}\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\left(3\,b^{3}x^{5}+20\,ab^{2}x^{3}+15\,a^{2}bx\right)\sqrt{bx^{2}+a}}{12\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)},\;15\left(ab^{2}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(3*b^3*x^5 + 20*a*b^2*x^3 + 15*a^2*b*x)*sqrt(b*x
^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(15*(a*b^2*x^4 + 2*a^2*b*x^
2 + a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*b^3*x^5 + 20*a*b^
2*x^3 + 15*a^2*b*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [B] time = 4.71714, size = 367, normalized size = 4.03

$$-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2
)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2
/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt
(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x*
*2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)
*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**
2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.70859, size = 88, normalized size = 0.97

$$\frac{\left(x^{2}\left(\frac{3\,x^{2}}{b}+\frac{20\,a}{b^{2}}\right)+\frac{15\,a^{2}}{b^{3}}\right)x}{6\left(bx^{2}+a\right)^{\frac{3}{2}}}+\frac{5\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*(3*x^2/b + 20*a/b^2) + 15*a^2/b^3)*x/(b*x^2 + a)^(3/2) + 5/2*a*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.507 $\int \frac{x^5}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

```
[Out] -a^2/(3*b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*Sqrt[a + b*x^2]) + Sqrt[a + b*x
^2]/b^3
```

Rubi [A] time = 0.0314975, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(5/2),x]
```

```
[Out] -a^2/(3*b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*Sqrt[a + b*x^2]) + Sqrt[a + b*x
^2]/b^3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx,x,x^2\right)\\
&= -\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0173704, size = 39, normalized size = 0.72

$$\frac{8a^2+12abx^2+3b^2x^4}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(5/2),x]
```

```
[Out] (8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.7

$$\frac{3\,b^2x^4+12\,abx^2+8\,a^2}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(3*b^2*x^4+12*a*b*x^2+8*a^2)/(b*x^2+a)^(3/2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27047, size = 119, normalized size = 2.2

$$\frac{\left(3\,b^2x^4+12\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{3\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 + 12*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^2
 + a^2*b^3)
```

Sympy [A] time = 1.07568, size = 138, normalized size = 2.56

$$\begin{cases}\frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)
) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
+ 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)), N
e(b, 0)), (x**6/(6*a**(5/2)), True))
```

Giac [A] time = 2.35319, size = 58, normalized size = 1.07

$$\frac{3\sqrt{bx^2+a}+\frac{6\left(bx^2+a\right)a-a^2}{\left(bx^2+a\right)^{\frac{3}{2}}}}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(b*x^2 + a) + (6*(b*x^2 + a)*a - a^2)/(b*x^2 + a)^(3/2))/b^3
```

3.508 $\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -x^3/(3*b*(a + b*x^2)^(3/2)) - x/(b^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]]/b^(5/2)
```

Rubi [A] time = 0.0204295, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {288, 217, 206}

$$-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -x^3/(3*b*(a + b*x^2)^(3/2)) - x/(b^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]]/b^(5/2)
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = -\frac{x^3}{3b(a+bx^2)^{3/2}} + \frac{\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx}{b}$$

$$= -\frac{x^3}{3b(a+bx^2)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}$$

$$= -\frac{x^3}{3b(a+bx^2)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2}$$

$$= -\frac{x^3}{3b(a+bx^2)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.110533, size = 80, normalized size = 1.25

$$\frac{3\sqrt{a}\,(a+bx^2)\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \sqrt{b}x\,(3a+4bx^2)}{3b^{5/2}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(Sqrt[b]*x*(3*a + 4*b*x^2)) + 3*Sqrt[a]*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(5/2)*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.006, size = 54, normalized size = 0.8

$$-\frac{x^3}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{x}{b^2}\frac{1}{\sqrt{bx^2+a}} + \ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*x^3/b/(b*x^2+a)^(3/2)-x/b^2/(b*x^2+a)^(1/2)+1/b^(5/2)*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32188, size = 444, normalized size = 6.94

$$\left[\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(4\,b^2x^3+3\,abx\right)\sqrt{bx^2+a}}{6\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)},-\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), -1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4
+ 2*a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] time = 2.77414, size = 303, normalized size = 4.73

$$\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*
b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a
)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*
a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1
+ b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a
) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a))
```

Giac [A] time = 2.56272, size = 69, normalized size = 1.08

$$-\frac{x\left(\frac{4\,x^2}{b}+\frac{3\,a}{b^2}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(4*x^2/b + 3*a/b^2)/(b*x^2 + a)^(3/2) - log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(5/2)
```

3.509 $\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

```
[Out] a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0215647, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(5/2),x]

[Out] a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx,x,x^2\right)\\
&= \frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0109335, size = 28, normalized size = 0.78

$$\frac{-2a-3bx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a - 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 25, normalized size = 0.7

$$-\frac{3\,b x^{2}+2\,a}{3\,b^{2}}\left(b x^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(3*b*x^2+2*a)/(b*x^2+a)^(3/2)/b^2
```

Maxima [A] time = 2.36117, size = 45, normalized size = 1.25

$$-\frac{x^{2}}{\left(b x^{2}+a\right)^{\frac{3}{2}} b}-\frac{2\,a}{3\,\left(b x^{2}+a\right)^{\frac{3}{2}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -x^2/((b*x^2 + a)^(3/2)*b) - 2/3*a/((b*x^2 + a)^(3/2)*b^2)
```

Fricas [A] time = 1.3013, size = 97, normalized size = 2.69

$$-\frac{\left(3\,b x^{2}+2\,a\right)\sqrt{b x^{2}+a}}{3\,\left(b^{4} x^{4}+2\,a b^{3} x^{2}+a^{2} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [A] time = 1.01676, size = 92, normalized size = 2.56

$$\begin{cases} -\frac{2a}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{3bx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**4/(4*a**(5/2)), True))
```

Giac [A] time = 2.55065, size = 32, normalized size = 0.89

$$-\frac{3\,bx^2 + 2\,a}{3\left(bx^2 + a\right)^{\frac{3}{2}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2 + 2*a)/((b*x^2 + a)^(3/2)*b^2)
```

3.510 $\int \frac{x^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.0045606, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/2),x]

[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0049984, size = 21, normalized size = 1.

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/2),x]

[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$\frac{x^3}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x^3/a/(b*x^2+a)^(3/2)
```

Maxima [A] time = 3.02234, size = 46, normalized size = 2.19

$$-\frac{x}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{x}{3\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/((b*x^2 + a)^(3/2)*b) + 1/3*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [B] time = 1.2413, size = 77, normalized size = 3.67

$$\frac{\sqrt{bx^2+a}x^3}{3\left(ab^2x^4+2\,a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*x^3/(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)
```

Sympy [B] time = 0.707849, size = 44, normalized size = 2.1

$$\frac{x^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Giac [A] time = 2.04626, size = 23, normalized size = 1.1

$$\frac{x^3}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3/((b*x^2 + a)^(3/2)*a)
```

3.511 $\int \frac{x}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=18

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.0034343, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}} dx = -\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0032977, size = 18, normalized size = 1.

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$-\frac{1}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)
```

Maxima [A] time = 3.31017, size = 19, normalized size = 1.06

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

Fricas [B] time = 1.34873, size = 73, normalized size = 4.06

$$-\frac{\sqrt{bx^2+a}}{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(b*x^2 + a)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [A] time = 0.969165, size = 46, normalized size = 2.56

$$\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**2/(2*a**(5/2)), True))
```

Giac [A] time = 2.80529, size = 19, normalized size = 1.06

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

3.512 $\int \frac{1}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}$$

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0051256, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/2}} dx &= \frac{x}{3a(a+bx^2)^{3/2}}+\frac{2\int \frac{1}{(a+bx^2)^{3/2}} dx}{3a}\\ &= \frac{x}{3a(a+bx^2)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0072204, size = 29, normalized size = 0.74

$$\frac{x(3a+2bx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$\frac{x\left(2\,bx^{2}+3\,a\right)}{3\,a^{2}}\left(bx^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 2.98605, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{bx^{2}+a}a^{2}}+\frac{x}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 1.29612, size = 99, normalized size = 2.54

$$\frac{\left(2\,bx^{3}+3\,ax\right)\sqrt{bx^{2}+a}}{3\,\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] time = 0.777108, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

Giac [A] time = 2.3006, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

3.513 $\int \frac{1}{x(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=59

$$\frac{1}{a^2\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{1}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/(3*a*(a + b*x^2)^(3/2)) + 1/(a^2*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]/a^(5/2)
```

Rubi [A] time = 0.0349385, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a^2\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{1}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*(a + b*x^2)^(3/2)) + 1/(a^2*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]/a^(5/2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0062367, size = 36, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^2)/a]/(3*a*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 57, normalized size = 1.

$$\frac{1}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{1}{a^2}\frac{1}{\sqrt{bx^2+a}}-\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3/a/(b*x^2+a)^(3/2)+1/a^2/(b*x^2+a)^(1/2)-1/a^(5/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34842, size = 440, normalized size = 7.46

$$\left[\frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{a}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(3\,abx^{2}+4\,a^{2}\right)\sqrt{bx^{2}+a}}{6\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)},\ \frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{-a}\arctan\left(\right.}{3\left(a^{3}b^{2}x^{4}+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
+ 2*a^4*b*x^2 + a^5), 1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
 + 2*a^4*b*x^2 + a^5)]
```

Sympy [B] time = 2.73289, size = 740, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)
*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a*
*7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**
2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sq
rt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**
2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a*
*(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x
**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqr
t(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2
*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) -
 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b
*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
```

Giac [A] time = 1.55533, size = 68, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{3\,bx^{2}+4\,a}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*b*x^2 + 4*a)/((b*x
^2 + a)^(3/2)*a^2)
```

3.514 $\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a + b*x^2)^(3/2)) - (8*b*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0135271, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {271, 192, 191}

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a + b*x^2)^(3/2)) - (8*b*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx = -\frac{1}{ax(a+bx^2)^{3/2}} - \frac{(4b)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{a}$$
$$= -\frac{1}{ax(a+bx^2)^{3/2}} - \frac{4bx}{3a^2(a+bx^2)^{3/2}} - \frac{(8b)\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a^2}$$
$$= -\frac{1}{ax(a+bx^2)^{3/2}} - \frac{4bx}{3a^2(a+bx^2)^{3/2}} - \frac{8bx}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0084805, size = 42, normalized size = 0.7

$$\frac{-3a^2-12abx^2-8b^2x^4}{3a^3x(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2 - 12*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 39, normalized size = 0.7

$$-\frac{8\,b^2x^4+12\,abx^2+3\,a^2}{3\,a^3x}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(8*b^2*x^4+12*a*b*x^2+3*a^2)/x/(b*x^2+a)^(3/2)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34013, size = 123, normalized size = 2.05

$$-\frac{\left(8\,b^2x^4+12\,abx^2+3\,a^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*b^2*x^4 + 12*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4*
b*x^3 + a^5*x)
```

Sympy [B] time = 1.31081, size = 165, normalized size = 2.75

$$-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a
**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/
(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)
```

Giac [A] time = 1.88141, size = 86, normalized size = 1.43

$$-\frac{x\left(\frac{5b^2x^2}{a^3}+\frac{6b}{a^2}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(5*b^2*x^2/a^3 + 6*b/a^2)/(b*x^2 + a)^(3/2) + 2*sqrt(b)/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)*a^2)
```

3.515 $\int \frac{1}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{5b}{2a^3\sqrt{a+bx^2}}-\frac{5b}{6a^2\left(a+bx^2\right)^{3/2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] (-5*b)/(6*a^2*(a + b*x^2)^(3/2)) - 1/(2*a*x^2*(a + b*x^2)^(3/2)) - (5*b)/(2
*a^3*Sqrt[a + b*x^2]) + (5*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.0495339, antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{5\sqrt{a+bx^2}}{2a^3x^2}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*x^2*(a + b*x^2)^(3/2)) + 5/(3*a^2*x^2*Sqrt[a + b*x^2]) - (5*Sqrt[a +
 b*x^2])/(2*a^3*x^2) + (5*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)}{6a}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}+\frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}-\frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^3}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}-\frac{5\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^3}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0071367, size = 37, normalized size = 0.42

$$
-\frac{b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a^2\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(b*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x^2)/a])/(3*a^2*(a + b*x^2)^(3/
2))
```

Maple [A] time = 0.005, size = 78, normalized size = 0.9

$$
-\frac{1}{2\,ax^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{5\,b}{6\,a^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{5\,b}{2\,a^3}\frac{1}{\sqrt{bx^2+a}}+\frac{5\,b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(3/2)-5/6*b/a^2/(b*x^2+a)^(3/2)-5/2*b/a^3/(b*x^2+a)^(1
/2)+5/2*b/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42624, size = 527, normalized size = 5.99

$$\left[\frac{15\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(15\,ab^2x^4+20\,a^2bx^2+3\,a^3\right)\sqrt{bx^2+a}}{12\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}, -\frac{15\left(b^3x^6+2\,ab^2x^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*a*b^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sq
rt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b
^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4
+ a^6*x^2)]
```

Sympy [B] time = 4.96205, size = 864, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x
**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 +
12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) +
30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/
2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**
2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**
(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/
a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a*
*(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a*
*14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(
1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*
b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*
a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2
)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
```

```
*8)
```

Giac [A] time = 2.13894, size = 100, normalized size = 1.14

$$-\frac{1}{6}\,b\left(\frac{15\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{2\left(6\,bx^2+7\,a\right)}{\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{3\,\sqrt{bx^2+a}}{a^3bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*b*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*(6*b*x^2 + 7
*a)/((b*x^2 + a)^(3/2)*a^3) + 3*sqrt(b*x^2 + a)/(a^3*b*x^2))
```

3.516 $\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*b)/(a^2*x*(a + b*x^2)^(3/2)) + (8*b^2*x
)/(3*a^3*(a + b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0224556, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {271, 192, 191}

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*b)/(a^2*x*(a + b*x^2)^(3/2)) + (8*b^2*x
)/(3*a^3*(a + b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{5/2}}\,dx = -\frac{1}{3ax^3 \left(a+bx^2\right)^{3/2}} - \frac{(2b)\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{a}$$
$$= -\frac{1}{3ax^3 \left(a+bx^2\right)^{3/2}} + \frac{2b}{a^2x\left(a+bx^2\right)^{3/2}} + \frac{\left(8b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a^2}$$
$$= -\frac{1}{3ax^3 \left(a+bx^2\right)^{3/2}} + \frac{2b}{a^2x\left(a+bx^2\right)^{3/2}} + \frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{\left(16b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^3}$$
$$= -\frac{1}{3ax^3 \left(a+bx^2\right)^{3/2}} + \frac{2b}{a^2x\left(a+bx^2\right)^{3/2}} + \frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{16b^2x}{3a^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0098215, size = 53, normalized size = 0.62

$$\frac{6a^2bx^2-a^3+24ab^2x^4+16b^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a + b*x^2)^(3/
2))
```

Maple [A] time = 0.005, size = 48, normalized size = 0.6

$$-\frac{-16\,b^3x^6-24\,ab^2x^4-6\,a^2bx^2+a^3}{3\,x^3a^4}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-16*b^3*x^6-24*a*b^2*x^4-6*a^2*b*x^2+a^3)/x^3/(b*x^2+a)^(3/2)/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31881, size = 144, normalized size = 1.67

$$\frac{\left(16\,b^{3}x^{6} + 24\,ab^{2}x^{4} + 6\,a^{2}bx^{2} - a^{3}\right)\sqrt{bx^{2} + a}}{3\left(a^{4}b^{2}x^{7} + 2\,a^{5}bx^{5} + a^{6}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(16*b^3*x^6 + 24*a*b^2*x^4 + 6*a^2*b*x^2 - a^3)*sqrt(b*x^2 + a)/(a^4*b^
2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [B] time = 1.88013, size = 354, normalized size = 4.12

$$-\frac{a^{4}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{7}b^{9}x^{2}+9a^{6}b^{10}x^{4}+9a^{5}b^{11}x^{6}+3a^{4}b^{12}x^{8}}+\frac{5a^{3}b^{\frac{21}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{7}b^{9}x^{2}+9a^{6}b^{10}x^{4}+9a^{5}b^{11}x^{6}+3a^{4}b^{12}x^{8}}+\frac{30a^{2}b^{\frac{23}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}}}{3a^{7}b^{9}x^{2}+9a^{6}b^{10}x^{4}+9a^{5}b^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] -a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4
+ 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/(b*
x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a*
*4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x
**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*b**
(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*
a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2) +
1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12
*x**8)
```

Giac [A] time = 2.75468, size = 163, normalized size = 1.9

$$\frac{x\left(\frac{8\,b^{3}x^{2}}{a^{4}} + \frac{9\,b^{2}}{a^{3}}\right)}{3\left(bx^{2} + a\right)^{\frac{3}{2}}} - \frac{4\left(3\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{4}b^{\frac{3}{2}} - 9\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{2}ab^{\frac{3}{2}} + 4\,a^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{2} - a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(8*b^3*x^2/a^4 + 9*b^2/a^3)/(b*x^2 + a)^(3/2) - 4/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*b^(3/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 4
*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^3)
```

3.517 $\int \frac{x^{10}}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

```
[Out] -x^9/(7*b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^5)
/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*Sqrt[a + b*x^2]) + (9*x*Sqrt[a +
b*x^2])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rubi [A] time = 0.0535724, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] -x^9/(7*b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^5)
/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*Sqrt[a + b*x^2]) + (9*x*Sqrt[a +
b*x^2])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} + \frac{9\int \frac{x^8}{\left(a+bx^2\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} + \frac{9\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} + \frac{3\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{(9a)\int \frac{1}{\sqrt{a+b}}}{2b^5}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{(9a)\,\text{Subst}}{}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{9a\tanh^{-1}\left(\right.}{2b^{11/}}$$

Mathematica [A] time = 0.188903, size = 114, normalized size = 0.87

$$\frac{\sqrt{b}x\left(1218a^2b^2x^4+1050a^3bx^2+315a^4+528ab^3x^6+35b^4x^8\right)-315a^{3/2}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{70b^{11/2}\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^3*x^6 + 3
5*b^4*x^8) - 315*a^(3/2)*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]
*x)/Sqrt[a]])/(70*b^(11/2)*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.029, size = 111, normalized size = 0.9

$$\frac{x^9}{2b}\left(bx^2+a\right)^{-\frac{7}{2}} + \frac{9\,ax^7}{14\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}} + \frac{9\,ax^5}{10\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}} + \frac{3\,ax^3}{2\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}} + \frac{9\,ax}{2\,b^5}\frac{1}{\sqrt{bx^2+a}} - \frac{9\,a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2*x^9/b/(b*x^2+a)^(7/2)+9/14/b^2*a*x^7/(b*x^2+a)^(7/2)+9/10/b^3*a*x^5/(b*
x^2+a)^(5/2)+3/2/b^4*a*x^3/(b*x^2+a)^(3/2)+9/2/b^5*a*x/(b*x^2+a)^(1/2)-9/2/
```

```
b^(11/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43223, size = 798, normalized size = 6.09

$$\left[\frac{315\left(ab^{4}x^{8}+4\,a^{2}b^{3}x^{6}+6\,a^{3}b^{2}x^{4}+4\,a^{4}bx^{2}+a^{5}\right)\sqrt{b}\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\left(35\,b^{5}x^{9}+528\,ab^{4}x^{7}+12\right.}{140\left(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a^{3}b^{7}x^{2}+a^{4}b^{6}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(35*b^5*x^9 +
528*a*b^4*x^7 + 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x
^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
, 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (35*b^5*x^9 + 528*a*b^4*x^7
+ 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x^2 + a))/(b^10
*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

Sympy [B] time = 11.9994, size = 3181, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] -315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(3
09/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a*
*(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x*
*8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
+ 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/2)*b**6
7*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2
)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(14
9/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x
**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)
*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*x**4*sqrt(1 +
```

b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*x**6*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(303/2)*b**70*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(301/2)*b**71*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 315*a**(299/2)*b**72*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 315*a**155*b**(133/2)*x/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 1995*a**154*b**(135/2)*x**3/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 5313*a**153*b**(137/2)*x**5/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 7647*a**152*b**(139/2)*x**7/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 6323*a**151*b**(141/2)*x**9/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 2907*a**150*b**(143/2)*x**11/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 +

```
b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 633*a**149
*b**(145/2)*x**13/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(30
7/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*s
qrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 10
50*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2
)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2
/a)) + 35*a**148*b**(147/2)*x**15/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2
/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b
**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1
+ b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(
299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12
*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.88799, size = 123, normalized size = 0.94

$$\frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\,\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{9\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2)
```

3.518 $\int \frac{x^9}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

```
[Out] -a^4/(7*b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a^2
)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b
^5
```

Rubi [A] time = 0.0531119, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^(9/2),x]
```


```
[Out] -a^4/(7*b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a^2
)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b
^5
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{9/2}}-\frac{4a^3}{b^4(a+bx)^{7/2}}+\frac{6a^2}{b^4(a+bx)^{5/2}}-\frac{4a}{b^4(a+bx)^{3/2}}+\frac{1}{b^4\sqrt{a+bx}}\right)dx,x,x^2\right)\\
&= -\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.0281393, size = 61, normalized size = 0.65

$$\frac{560a^2b^2x^4+448a^3bx^2+128a^4+280ab^3x^6+35b^4x^8}{35b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] (128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(3
5*b^5*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.004, size = 58, normalized size = 0.6

$$\frac{35\,x^8b^4+280\,ax^6b^3+560\,a^2x^4b^2+448\,a^3x^2b+128\,a^4}{35\,b^5}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*a^4)/(b*x^
2+a)^(7/2)/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34429, size = 217, normalized size = 2.31

$$\frac{\left(35\,b^4x^8+280\,ab^3x^6+560\,a^2b^2x^4+448\,a^3bx^2+128\,a^4\right)\sqrt{bx^2+a}}{35\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^2 + a)/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 +
 a^4*b^5)
```

Sympy [A] time = 5.56295, size = 454, normalized size = 4.83

$$\begin{cases} \frac{128a^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{448a^3bx^2}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{}{35a^3b^5\sqrt{}} \\ \frac{x^{10}}{10a^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqr
t(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*
x**2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**
2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a
 + b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*
b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**
6*sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2) + 105*
a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**
8*x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x**2) + 10
5*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b
**8*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**10/(10*a**(9/2)), True))
```

Giac [A] time = 1.72733, size = 96, normalized size = 1.02

$$\frac{35\sqrt{bx^2+a} + \frac{140\left(bx^2+a\right)^3a - 70\left(bx^2+a\right)^2a^2 + 28\left(bx^2+a\right)a^3 - 5\,a^4}{\left(bx^2+a\right)^{\frac{7}{2}}}}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(35*sqrt(b*x^2 + a) + (140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 + 28
*(b*x^2 + a)*a^3 - 5*a^4)/(b*x^2 + a)^(7/2))/b^5
```

3.519 $\int \frac{x^8}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x}{b^4\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

```
[Out] -x^7/(7*b*(a + b*x^2)^(7/2)) - x^5/(5*b^2*(a + b*x^2)^(5/2)) - x^3/(3*b^3*(
a + b*x^2)^(3/2)) - x/(b^4*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]/b^(9/2)
```

Rubi [A] time = 0.0419398, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {288, 217, 206}

$$-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x}{b^4\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^(9/2),x]
```



```
[Out] -x^7/(7*b*(a + b*x^2)^(7/2)) - x^5/(5*b^2*(a + b*x^2)^(5/2)) - x^3/(3*b^3*(
a + b*x^2)^(3/2)) - x/(b^4*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]/b^(9/2)
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^6}{\left(a+bx^2\right)^{7/2}}\,dx}{b}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx}{b^2}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^4}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] time = 0.126931, size = 101, normalized size = 0.95

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}} - \frac{x\left(350a^2bx^2+105a^3+406ab^2x^4+176b^3x^6\right)}{105b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^6))/(105*b^4*(a +
b*x^2)^(7/2)) + (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/
(b^(9/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.015, size = 88, normalized size = 0.8

$$-\frac{x^7}{7b}\left(bx^2+a\right)^{-\frac{7}{2}} - \frac{x^5}{5b^2}\left(bx^2+a\right)^{-\frac{5}{2}} - \frac{x^3}{3b^3}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{x}{b^4}\frac{1}{\sqrt{bx^2+a}} + \ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*x^7/b/(b*x^2+a)^(7/2)-1/5*x^5/b^2/(b*x^2+a)^(5/2)-1/3*x^3/b^3/(b*x^2+a
)^(3/2)-x/b^4/(b*x^2+a)^(1/2)+1/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46109, size = 736, normalized size = 6.94

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(176\,b^4x^7+406\,ab^3x^5+350\right.}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(176*b^4*x^7 + 406
*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) + (176*b^4*x^7 + 406*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a
^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6
*x^2 + a^4*b^5)]
```

Sympy [B] time = 8.09565, size = 2980, normalized size = 28.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] 105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(2
05/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1
+ b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**
(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**
8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) +
 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46
*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)
*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1
575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105
/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x*
*2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)
*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x*
*2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)
*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(
1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a*
*(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x*
*12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asi
nh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a*
*(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x*
*4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a)
```

```
+ 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(1
09/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b
*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/s
qrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(1
01/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*
x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197
/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sq
rt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 63
0*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b
**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(
99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b*
*(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
 b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(1
93/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a*
*(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt
(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*
a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*
x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a
) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93
/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(
101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b
*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*s
qrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1
771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1
+ b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(
201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6
*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111
/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/2)*b
**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**
2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)
*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(
1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a*
*(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*b**(101/2)*x**11/
(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x*
*2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
+ 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(
107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b
*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 176*a**96*
b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203
/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sq
rt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 157
5*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)
*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2
/a))
```

Giac [A] time = 2.36694, size = 105, normalized size = 0.99

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.520 $\int \frac{x^7}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=75

$$\frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

```
[Out] a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*
(a + b*x^2)^(3/2)) - 1/(b^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0447213, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*
(a + b*x^2)^(3/2)) - 1/(b^4*Sqrt[a + b*x^2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{9/2}}+\frac{3a^2}{b^3(a+bx)^{7/2}}-\frac{3a}{b^3(a+bx)^{5/2}}+\frac{1}{b^3(a+bx)^{3/2}}\right)dx,x,x^2\right)\\
&= \frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0236671, size = 50, normalized size = 0.67

$$\frac{-56a^2bx^2-16a^3-70ab^2x^4-35b^3x^6}{35b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] (-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(7
/2))
```

Maple [A] time = 0.004, size = 47, normalized size = 0.6

$$-\frac{35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3}{35\,b^4}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)/(b*x^2+a)^(7/2)/b^4
```

Maxima [A] time = 1.08614, size = 99, normalized size = 1.32

$$-\frac{x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{2\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{16\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -x^6/((b*x^2 + a)^(7/2)*b) - 2*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 8/5*a^2*x^2/
((b*x^2 + a)^(7/2)*b^3) - 16/35*a^3/((b*x^2 + a)^(7/2)*b^4)
```

Fricas [A] time = 1.31997, size = 190, normalized size = 2.53

$$-\frac{\left(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3\right)\sqrt{bx^2+a}}{35\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^2 + a)/(
b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [A] time = 5.45232, size = 364, normalized size = 4.85

$$\begin{cases}-\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}}-\frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}}-\frac{}{35} \\ \frac{x^8}{8a^{\frac{9}{2}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqr
t(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*
x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2
*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
+ b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*
x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqr
t(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**
5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

Giac [A] time = 2.27929, size = 74, normalized size = 0.99

$$-\frac{35\left(bx^2+a\right)^3-35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2-5\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)/
((b*x^2 + a)^(7/2)*b^4)
```

3.521 $\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.0045666, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(9/2),x]
```

```
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx=\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] time = 0.0059893, size = 21, normalized size = 1.

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(9/2),x]
```

```
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$\frac{x^7}{7\,a}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*x^7/a/(b*x^2+a)^(7/2)
```

Maxima [B] time = 1.31785, size = 139, normalized size = 6.62

$$-\frac{x^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,ax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{x}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{x}{7\sqrt{bx^2+a}ab^3}+\frac{3\,ax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15\,a^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 1/14*x
/((b*x^2 + a)^(3/2)*b^3) + 1/7*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*a^2*x/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [B] time = 1.3269, size = 120, normalized size = 5.71

$$\frac{\sqrt{bx^2+a}x^7}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*x^7/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*
b*x^2 + a^5)
```

Sympy [B] time = 2.00362, size = 95, normalized size = 4.52

$$\frac{x^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))
```

Giac [A] time = 3.00424, size = 23, normalized size = 1.1

$$\frac{x^7}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

3.522 $\int \frac{x^5}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] -a^2/(7*b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b^3
*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.033334, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] -a^2/(7*b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b^3
*(a + b*x^2)^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{9/2}}-\frac{2a}{b^2(a+bx)^{7/2}}+\frac{1}{b^2(a+bx)^{5/2}}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0179678, size = 39, normalized size = 0.66

$$\frac{-8a^2-28abx^2-35b^2x^4}{105b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 36, normalized size = 0.6

$$-\frac{35\,b^2x^4+28\,abx^2+8\,a^2}{105\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(35*b^2*x^4+28*a*b*x^2+8*a^2)/(b*x^2+a)^(7/2)/b^3
```

Maxima [A] time = 1.4914, size = 72, normalized size = 1.22

$$-\frac{x^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,ax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^4/((b*x^2 + a)^(7/2)*b) - 4/15*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 8/105
*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [A] time = 1.31278, size = 167, normalized size = 2.83

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{105\left(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^7*x^8 + 4*a*b^6
*x^6 + 6*a^2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [A] time = 5.39027, size = 272, normalized size = 4.61

$$\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqr
t(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b
*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*
sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a
+ b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x
**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqr
t(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

Giac [A] time = 2.4579, size = 55, normalized size = 0.93

$$-\frac{35\left(bx^2+a\right)^2-42\left(bx^2+a\right)a+15\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b^
3)
```

3.523 $\int \frac{x^4}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=44

$$\frac{2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}+\frac{x^5}{5a\left(a+bx^2\right)^{7/2}}$$

```
[Out] x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.0106348, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}+\frac{x^5}{5a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{x^5}{5a\left(a+bx^2\right)^{7/2}}+\frac{(2b)\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{5a}\\ &= \frac{x^5}{5a\left(a+bx^2\right)^{7/2}}+\frac{2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0115358, size = 31, normalized size = 0.7

$$\frac{7ax^5+2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] (7*a*x^5 + 2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 28, normalized size = 0.6

$$\frac{x^5\left(2\,bx^2+7\,a\right)}{35\,a^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x^5*(2*b*x^2+7*a)/(b*x^2+a)^(7/2)/a^2
```

Maxima [B] time = 1.85136, size = 115, normalized size = 2.61

$$-\frac{x^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,x}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,x}{35\,\sqrt{bx^2+a}a^2b^2}+\frac{x}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,ax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/((b*x^2 + a)^(7/2)*b) + 3/140*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*x/(
sqrt(b*x^2 + a)*a^2*b^2) + 1/35*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*a*x/((b*
x^2 + a)^(7/2)*b^2)
```

Fricas [A] time = 1.27116, size = 146, normalized size = 3.32

$$\frac{\left(2\,bx^7+7\,ax^5\right)\sqrt{bx^2+a}}{35\left(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b*x^7 + 7*a*x^5)*sqrt(b*x^2 + a)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*a
^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

Sympy [B] time = 2.04143, size = 199, normalized size = 4.52

$$\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}+\frac{}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] 7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*
x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(
9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

Giac [A] time = 3.01034, size = 39, normalized size = 0.89

$$\frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

3.524 $\int \frac{x^3}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=38

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

```
[Out] a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))
```

Rubi [A] time = 0.0232085, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(9/2),x]
```

```
[Out] a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{9/2}}+\frac{1}{b(a+bx)^{7/2}}\right)dx,x,x^2\right)\\
&= \frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0138211, size = 28, normalized size = 0.74

$$\frac{-2a-7bx^2}{35b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(9/2),x]

[Out] (-2*a - 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 25, normalized size = 0.7

$$-\frac{7\,bx^2+2\,a}{35\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(9/2),x)

[Out] -1/35*(7*b*x^2+2*a)/(b*x^2+a)^(7/2)/b^2
```

Maxima [A] time = 2.01263, size = 45, normalized size = 1.18

$$-\frac{x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{2\,a}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")

[Out] -1/5*x^2/((b*x^2 + a)^(7/2)*b) - 2/35*a/((b*x^2 + a)^(7/2)*b^2)
```

Fricas [B] time = 1.33584, size = 142, normalized size = 3.74

$$-\frac{\left(7\,bx^2+2\,a\right)\sqrt{bx^2+a}}{35\left(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")

[Out] -1/35*(7*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^
4 + 4*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [A] time = 5.25198, size = 180, normalized size = 4.74

$$\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} & \text{fo} \\ \frac{x^4}{4a^{\frac{9}{2}}} & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a +
b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2))
, Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

Giac [A] time = 2.3914, size = 32, normalized size = 0.84

$$-\frac{7\,bx^2 + 2\,a}{35\,\left(bx^2 + a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2)
```

3.525 $\int \frac{x^2}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=68

$$\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{7/2}}$$

```
[Out] x^3/(3*a*(a + b*x^2)^(7/2)) + (4*b*x^5)/(15*a^2*(a + b*x^2)^(7/2)) + (8*b^2
*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.0205032, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(9/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(7/2)) + (4*b*x^5)/(15*a^2*(a + b*x^2)^(7/2)) + (8*b^2
*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}}+\frac{(4b)\int \frac{x^4}{\left(a+bx^2\right)^{9/2}}\,dx}{3a}\\
&= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(8b^2\right)\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^2}\\
&= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0131602, size = 42, normalized size = 0.62

$$\frac{x^3\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(9/2),x]
```

```
[Out] (x^3*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 39, normalized size = 0.6

$$\frac{x^3\left(8\,b^2x^4+28\,abx^2+35\,a^2\right)}{105\,a^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x^3*(8*b^2*x^4+28*a*b*x^2+35*a^2)/(b*x^2+a)^(7/2)/a^3
```

Maxima [A] time = 2.70413, size = 95, normalized size = 1.4

$$-\frac{x}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,x}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,x}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{x}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*x/((b*x^2 + a)^(7/2)*b) + 8/105*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*x/((
b*x^2 + a)^(3/2)*a^2*b) + 1/35*x/((b*x^2 + a)^(5/2)*a*b)
```

Fricas [A] time = 1.3595, size = 171, normalized size = 2.51

$$\frac{\left(8\,b^2x^7+28\,abx^5+35\,a^2x^3\right)\sqrt{bx^2+a}}{105\left(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^7 + 28*a*b*x^5 + 35*a^2*x^3)*sqrt(b*x^2 + a)/(a^3*b^4*x^8 +
4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

Sympy [B] time = 2.31883, size = 517, normalized size = 7.6

$$\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{}{105a^{\frac{1}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(
1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))
+ 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13
/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/
a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420
*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(1
7/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
 + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqr
t(1 + b*x**2/a))
```

Giac [A] time = 2.5777, size = 58, normalized size = 0.85

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

3.526 $\int \frac{x}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.0037261, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{9/2}}\,dx=-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] time = 0.0044308, size = 18, normalized size = 1.

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$-\frac{1}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7/b/(b*x^2+a)^(7/2)
```

Maxima [A] time = 2.29914, size = 19, normalized size = 1.06

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

Fricas [B] time = 1.37224, size = 116, normalized size = 6.44

$$-\frac{\sqrt{bx^2+a}}{7\left(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^2 + a)/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2
 + a^4*b)
```

Sympy [A] time = 5.12714, size = 90, normalized size = 5.

$$\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

Giac [A] time = 1.51784, size = 19, normalized size = 1.06

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

3.527 $\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] x/(7*a*(a + b*x^2)^(7/2)) + (6*x)/(35*a^2*(a + b*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + b*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0160159, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + b*x^2)^(7/2)) + (6*x)/(35*a^2*(a + b*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + b*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + b*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}\\
&= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{24\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{16\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{35a^3}\\
&= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{16x}{35a^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0115378, size = 51, normalized size = 0.66

$$\frac{x\left(70a^2bx^2+35a^3+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^(7/2))
```

Maple [A] time = 0.003, size = 48, normalized size = 0.6

$$\frac{x\left(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3\right)}{35\,a^4}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/(b*x^2+a)^(7/2)/a^4
```

Maxima [A] time = 2.45681, size = 82, normalized size = 1.06

$$\frac{16\,x}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,x}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,x}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{x}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(b*x^2 + a)*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((b
*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)
```

Fricas [A] time = 1.31958, size = 192, normalized size = 2.49

$$\frac{\left(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x\right)\sqrt{bx^2+a}}{35\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*sqrt(b*x^2 + a)/
(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] time = 3.04892, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**
3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 21
0*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
+ b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*
x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/
2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(
1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*
b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 +
 b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x
**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a
**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b
*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13
/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2
)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a
))
```

Giac [A] time = 1.42683, size = 74, normalized size = 0.96

$$\frac{\left(2\left(4x^2\left(\frac{2b^3x^2}{a^4} + \frac{7b^2}{a^3}\right) + \frac{35b}{a^2}\right)x^2 + \frac{35}{a}\right)x}{35\left(bx^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

3.528 $\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=95

$$\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/(7*a*(a + b*x^2)^(7/2)) + 1/(5*a^2*(a + b*x^2)^(5/2)) + 1/(3*a^3*(a + b*x
^2)^(3/2)) + 1/(a^4*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(
9/2)
```

Rubi [A] time = 0.0585164, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*(a + b*x^2)^(7/2)) + 1/(5*a^2*(a + b*x^2)^(5/2)) + 1/(3*a^3*(a + b*x
^2)^(3/2)) + 1/(a^4*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(
9/2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{7/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a^3}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^4}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^4b}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0057869, size = 36, normalized size = 0.38

$$
\frac{{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a\left(a+bx^2\right)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] Hypergeometric2F1[-7/2, 1, -5/2, 1 + (b*x^2)/a]/(7*a*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 85, normalized size = 0.9

$$
\frac{1}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{1}{5a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{1}{3a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{1}{a^4}\frac{1}{\sqrt{bx^2+a}}-\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7/a/(b*x^2+a)^(7/2)+1/5/a^2/(b*x^2+a)^(5/2)+1/3/a^3/(b*x^2+a)^(3/2)+1/a^4
/(b*x^2+a)^(1/2)-1/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.48295, size = 732, normalized size = 7.71

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+1\right.}{210\left(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9), 1/105*(105*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a)*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) + (105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*
a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*
b*x^2 + a^9)]
```

Sympy [B] time = 7.72519, size = 5250, normalized size = 55.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(9/2),x)
```

```
[Out] 352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(
71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100
*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x*
*12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/
2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/
a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b*
*5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a
**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20
) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*
```

x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 22050*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a

```
**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10
 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)
*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 5292
0*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71
/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a
**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**1
2 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)
*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1 +
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1 +
b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2
*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(6
3/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 +
 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**1
0*x**20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*log(
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2
100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6
 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)
*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100
*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**16*
sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*
b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a
**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**
14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*
b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log(sq
rt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/
2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*
x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/
2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x**18
*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2
*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(6
3/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 +
 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**1
0*x**20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
```

```
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10*x*
*20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73
/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b*
*3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a*
*(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
```

Giac [A] time = 3.13541, size = 109, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2+15\,a^3}{105\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + 1/105*(105*(b*x^2 + a)^3
+ 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2)*a^4)
```

3.529 $\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a + b*x^2)^(7/2)) - (48*b*x)
/(35*a^3*(a + b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (128*b*
x)/(35*a^5*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0263844, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {271, 192, 191}

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a + b*x^2)^(7/2)) - (48*b*x)
/(35*a^3*(a + b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (128*b*
x)/(35*a^5*Sqrt[a + b*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{(8b)\int \frac{1}{(a+bx^2)^{9/2}}\,dx}{a}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{(48b)\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7a^2}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{(192b)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35a^3}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{(128b)\int \frac{1}{(a+bx^2)}}{35a^4}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{128bx}{35a^5\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0114345, size = 64, normalized size = 0.64

$$\frac{-560a^2b^2x^4 - 280a^3bx^2 - 35a^4 - 448ab^3x^6 - 128b^4x^8}{35a^5x(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b^4*x^8)/(
35*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.004, size = 61, normalized size = 0.6

$$-\frac{128\,b^4x^8 + 448\,b^3x^6a + 560\,b^2x^4a^2 + 280\,bx^2a^3 + 35\,a^4}{35\,a^5x}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*b*x^2+35*a^4)/x/(b
*x^2+a)^(7/2)/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38824, size = 221, normalized size = 2.21

$$-\frac{\left(128\,b^{4}x^{8}+448\,ab^{3}x^{6}+560\,a^{2}b^{2}x^{4}+280\,a^{3}bx^{2}+35\,a^{4}\right)\sqrt{bx^{2}+a}}{35\,\left(a^{5}b^{4}x^{9}+4\,a^{6}b^{3}x^{7}+6\,a^{7}b^{2}x^{5}+4\,a^{8}bx^{3}+a^{9}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*
x^3 + a^9*x)
```

Sympy [B] time = 3.85121, size = 400, normalized size = 4.

$$-\frac{35a^{4}b^{\frac{33}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}+140a^{8}b^{17}x^{2}+210a^{7}b^{18}x^{4}+140a^{6}b^{19}x^{6}+35a^{5}b^{20}x^{8}}-\frac{280a^{3}b^{\frac{35}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}+140a^{8}b^{17}x^{2}+210a^{7}b^{18}x^{4}+140a^{6}b^{19}x^{6}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] -35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a
**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a
**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a
*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 +
 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(
41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*
a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
```

Giac [A] time = 2.09056, size = 122, normalized size = 1.22

$$-\frac{\left(\left(x^{2}\left(\frac{93\,b^{4}x^{2}}{a^{5}}+\frac{308\,b^{3}}{a^{4}}\right)+\frac{350\,b^{2}}{a^{3}}\right)x^{2}+\frac{140\,b}{a^{2}}\right)x}{35\,\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

3.530 $\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{9b}{2a^5\sqrt{a+bx^2}}-\frac{3b}{2a^4\left(a+bx^2\right)^{3/2}}-\frac{9b}{10a^3\left(a+bx^2\right)^{5/2}}-\frac{9b}{14a^2\left(a+bx^2\right)^{7/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{7/2}}$$

```
[Out] (-9*b)/(14*a^2*(a + b*x^2)^(7/2)) - 1/(2*a*x^2*(a + b*x^2)^(7/2)) - (9*b)/(
10*a^3*(a + b*x^2)^(5/2)) - (3*b)/(2*a^4*(a + b*x^2)^(3/2)) - (9*b)/(2*a^5*
Sqrt[a + b*x^2]) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rubi [A] time = 0.0780217, antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{9\sqrt{a+bx^2}}{2a^5x^2}+\frac{3}{a^4x^2\sqrt{a+bx^2}}+\frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}}+\frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{1}{7ax^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*x^2*(a + b*x^2)^(7/2)) + 9/(35*a^2*x^2*(a + b*x^2)^(5/2)) + 3/(5*a^3
*x^2*(a + b*x^2)^(3/2)) + 3/(a^4*x^2*Sqrt[a + b*x^2]) - (9*Sqrt[a + b*x^2])
/(2*a^5*x^2) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{9/2}}\,dx,x,x^2\right)$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{7/2}}\,dx,x,x^2\right)}{14a}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx,x,x^2\right)}{10a^2}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a^3}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a \cdots}}\right.}{2a^4 \cdots}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+bx^2}}{2a^5x^2} - \frac{(9 \cdots}{\cdots}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+bx^2}}{2a^5x^2} - \frac{9 \cdots}{\cdots}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+bx^2}}{2a^5x^2} + \frac{9b \cdots}{\cdots}$$

Mathematica [C] time = 0.0087415, size = 37, normalized size = 0.29

$$-\frac{b\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(b*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(7*a^2*(a + b*x^2)^(7/
2))
```

Maple [A] time = 0.006, size = 108, normalized size = 0.9

$$-\frac{1}{2\,ax^2}\left(bx^2+a\right)^{-\frac{7}{2}} - \frac{9\,b}{14\,a^2}\left(bx^2+a\right)^{-\frac{7}{2}} - \frac{9\,b}{10\,a^3}\left(bx^2+a\right)^{-\frac{5}{2}} - \frac{3\,b}{2\,a^4}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{9\,b}{2\,a^5}\frac{1}{\sqrt{bx^2+a}} + \frac{9\,b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a \cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(7/2)-9/14*b/a^2/(b*x^2+a)^(7/2)-9/10*b/a^3/(b*x^2+a)^
(5/2)-3/2*b/a^4/(b*x^2+a)^(3/2)-9/2*b/a^5/(b*x^2+a)^(1/2)+9/2*b/a^(11/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55049, size = 824, normalized size = 6.54

$$\left[\frac{315\left(b^{5}x^{10}+4\,ab^{4}x^{8}+6\,a^{2}b^{3}x^{6}+4\,a^{3}b^{2}x^{4}+a^{4}bx^{2}\right)\sqrt{a}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-2\left(315\,ab^{4}x^{8}+1050\,a^{2}b^{3}x^{6}+1218\right.}{140\left(a^{6}b^{4}x^{10}+4\,a^{7}b^{3}x^{8}+6\,a^{8}b^{2}x^{6}+4\,a^{9}bx^{4}+a^{10}x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b
*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(315*
a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*s
qrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4
 + a^10*x^2), -1/70*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^
2*x^4 + a^4*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^4*x
^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x
^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10
*x^2)]
```

Sympy [B] time = 12.3913, size = 5540, normalized size = 43.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] -70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4
+ 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*
b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1680
0*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x*
*20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a)/(14
0*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16
```

800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**47*b**2*x**4*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 6300*a**47*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 33956*a**46*b**3*x**6*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**46*b**3*x**6*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 28350*a**46*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 71940*a**45*b**4*x**8*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**45*b**4*x**8*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 75600*a**45*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 100260*a**44*b**5*x**10*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**44*b**5*x**10*log(b

```
*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b*
*2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a
**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**
16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*
b**10*x**22) + 132300*a**44*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(140*a**
(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a
**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**
12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/
2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 94
396*a**43*b**6*x**12*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105
/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400
*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x
**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89
/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 79380*a**43*b**6*x**12*log(b*
x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**
2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a*
*(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**1
6 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b
**10*x**22) + 158760*a**43*b**6*x**12*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(
107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a*
*(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**1
2 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2
)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 597
72*a**42*b**7*x**14*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/
2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*
a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x*
*14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/
2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**42*b**7*x**14*log(b*x
**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2
*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**
(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b*
*10*x**22) + 132300*a**42*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(1
07/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**
(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12
 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)
*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 2448
6*a**41*b**8*x**16*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2
)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a
**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**
14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2
)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**41*b**8*x**16*log(b*x*
*2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*
x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(
97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
+ 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**
10*x**22) + 75600*a**41*b**8*x**16*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107
/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(1
01/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 +
 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b
**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 5880*a
**40*b**9*x**18*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b
*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(
99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14
+ 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b
**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**40*b**9*x**18*log(b*x**2/
a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**
6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/
2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6
```

```
300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*
x**22) + 28350*a**40*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)
*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/
2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29
400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8
*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 630*a**39
*b**10*x**20*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x*
*4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/
2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1
6800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9
*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**39*b**10*x**20*log(b*x**2/a)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) + 6300*a**39*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x*
*2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*
b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400
*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x*
*18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 315*a**38*b*
*11*x**22*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 630
0*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*
x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**
(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 +
 140*a**(87/2)*b**10*x**22) + 630*a**38*b**11*x**22*log(sqrt(1 + b*x**2/a)
+ 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x
**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(9
7/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 +
 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**1
0*x**22)
```

Giac [A] time = 1.82489, size = 142, normalized size = 1.13

$$-\frac{1}{70}\,b\left(\frac{315\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5}+\frac{2\left(140\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+14\left(bx^2+a\right)a^2+5\,a^3\right)}{\left(bx^2+a\right)^{\frac{7}{2}}a^5}+\frac{35\,\sqrt{bx^2+a}}{a^5bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/70*b*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) + 2*(140*(b*x^
2 + a)^3 + 35*(b*x^2 + a)^2*a + 14*(b*x^2 + a)*a^2 + 5*a^3)/((b*x^2 + a)^(7
/2)*a^5) + 35*sqrt(b*x^2 + a)/(a^5*b*x^2))
```

3.531 $\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=132

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/2}}+\frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}}+\frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}}+\frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a + b*x^2)^(7/2)) + (80*b
^2*x)/(21*a^3*(a + b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a + b*x^2)^(5/2)) + (
128*b^2*x)/(21*a^5*(a + b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*Sqrt[a + b*x^2]
)
```

Rubi [A] time = 0.0406277, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {271, 192, 191}

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/2}}+\frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}}+\frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}}+\frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a + b*x^2)^(7/2)) + (80*b
^2*x)/(21*a^3*(a + b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a + b*x^2)^(5/2)) + (
128*b^2*x)/(21*a^5*(a + b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*Sqrt[a + b*x^2]
)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx = -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{(10b)\int \frac{1}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80b^2\right)\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a^3}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{\left(128b^2\right)\int \frac{1}{(a+bx^2}}{7a^4}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/}}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/}}$$

Mathematica [A] time = 0.0130469, size = 75, normalized size = 0.57

$$\frac{1120a^2b^3x^6 + 560a^3b^2x^4 + 70a^4bx^2 - 7a^5 + 896ab^4x^8 + 256b^5x^{10}}{21a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8
 + 256*b^5*x^10)/(21*a^6*x^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.004, size = 72, normalized size = 0.6

$$-\frac{-256\,b^5x^{10} - 896\,ab^4x^8 - 1120\,a^2b^3x^6 - 560\,a^3b^2x^4 - 70\,a^4bx^2 + 7\,a^5}{21\,x^3a^6}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/21*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a^3*b^2*x^4-70*a^4*
b*x^2+7*a^5)/x^3/(b*x^2+a)^(7/2)/a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7756, size = 250, normalized size = 1.89

$$\frac{\left(256\, b^{5} x^{10} + 896\, a b^{4} x^{8} + 1120\, a^{2} b^{3} x^{6} + 560\, a^{3} b^{2} x^{4} + 70\, a^{4} b x^{2} - 7\, a^{5}\right) \sqrt{b x^{2} + a}}{21 \, \left(a^{6} b^{4} x^{11} + 4\, a^{7} b^{3} x^{9} + 6\, a^{8} b^{2} x^{7} + 4\, a^{9} b x^{5} + a^{10} x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

Sympy [B] time = 5.32823, size = 668, normalized size = 5.06

$$-\frac{7a^{6}b^{\frac{51}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{21a^{11}b^{25}x^{2}+105a^{10}b^{26}x^{4}+210a^{9}b^{27}x^{6}+210a^{8}b^{28}x^{8}+105a^{7}b^{29}x^{10}+21a^{6}b^{30}x^{12}}+\frac{}{21a^{11}b^{25}x^{2}+105a^{10}b^{26}x^{4}+21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] -7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**
26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10
+ 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21*a*
*11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**2
8*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2)*x
**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*
a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**3
0*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x*
*2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105
*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt(a/
(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*
x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) +
1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**1
0*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x
**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(21*
a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b*
*28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
```

Giac [A] time = 2.37613, size = 198, normalized size = 1.5

$$\frac{\left(\left(x^{2}\left(\frac{158\, b^{5} x^{2}}{a^{6}} + \frac{511\, b^{4}}{a^{5}}\right) + \frac{560\, b^{3}}{a^{4}}\right)x^{2} + \frac{210\, b^{2}}{a^{3}}\right)x}{21\, \left(b x^{2} + a\right)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}x - \sqrt{b x^{2} + a}\right)^{4} b^{\frac{3}{2}} - 15\left(\sqrt{b}x - \sqrt{b x^{2} + a}\right)^{2} a b^{\frac{3}{2}} + 7\, a^{2} b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x - \sqrt{b x^{2} + a}\right)^{2} - a\right)^{3} a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

3.532 $\int \frac{x^5}{\sqrt{9+4x^2}} \, dx$

Optimal. Leaf size=46

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

```
[Out] (81*Sqrt[9 + 4*x^2])/64 - (3*(9 + 4*x^2)^(3/2))/32 + (9 + 4*x^2)^(5/2)/320
```

Rubi [A] time = 0.0194254, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[9 + 4*x^2])/64 - (3*(9 + 4*x^2)^(3/2))/32 + (9 + 4*x^2)^(5/2)/320
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{9+4x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9+4x}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9+4x}}-\frac{9}{8}\sqrt{9+4x}+\frac{1}{16}(9+4x)^{3/2}\right) dx, x, x^2\right)\\
&= \frac{81}{64}\sqrt{9+4x^2}-\frac{3}{32}\left(9+4x^2\right)^{3/2}+\frac{1}{320}\left(9+4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0081212, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2+9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 + 4*x^2],x]
```

```
[Out] (Sqrt[9 + 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

Maple [A] time = 0.003, size = 24, normalized size = 0.5

$$\frac{2 x^4 - 6 x^2 + 27}{40}\sqrt{4 x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(4*x^2+9)^(1/2)*(2*x^4-6*x^2+27)
```

Maxima [A] time = 3.06038, size = 54, normalized size = 1.17

$$\frac{1}{20}\sqrt{4 x^2 + 9}x^4 - \frac{3}{20}\sqrt{4 x^2 + 9}x^2 + \frac{27}{40}\sqrt{4 x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(4*x^2 + 9)*x^4 - 3/20*sqrt(4*x^2 + 9)*x^2 + 27/40*sqrt(4*x^2 + 9)
```

Fricas [A] time = 1.54204, size = 58, normalized size = 1.26

$$\frac{1}{40}\left(2 x^4 - 6 x^2 + 27\right)\sqrt{4 x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(2*x^4 - 6*x^2 + 27)*sqrt(4*x^2 + 9)
```

Sympy [A] time = 1.13166, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2 + 9}}{20} - \frac{3x^2\sqrt{4x^2 + 9}}{20} + \frac{27\sqrt{4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 + 9)/20 - 3*x**2*sqrt(4*x**2 + 9)/20 + 27*sqrt(4*x**2 + 9)
/40
```

Giac [A] time = 2.67957, size = 46, normalized size = 1.

$$\frac{1}{320}\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{32}\left(4x^2+9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*(4*x^2 + 9)^(5/2) - 3/32*(4*x^2 + 9)^(3/2) + 81/64*sqrt(4*x^2 + 9)
```

3.533 $\int \frac{x^4}{\sqrt{9+4x^2}} \, dx$

Optimal. Leaf size=45

$$\frac{1}{16}\sqrt{4x^2+9}x^3 - \frac{27}{128}\sqrt{4x^2+9}x + \frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (-27*x*Sqrt[9 + 4*x^2])/128 + (x^3*Sqrt[9 + 4*x^2])/16 + (243*ArcSinh[(2*x)
/3])/256
```

Rubi [A] time = 0.0093216, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 215}

$$\frac{1}{16}\sqrt{4x^2+9}x^3 - \frac{27}{128}\sqrt{4x^2+9}x + \frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 + 4*x^2],x]
```


```
[Out] (-27*x*Sqrt[9 + 4*x^2])/128 + (x^3*Sqrt[9 + 4*x^2])/16 + (243*ArcSinh[(2*x)
/3])/256
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{9+4x^2}} \, dx &= \frac{1}{16}x^3\sqrt{9+4x^2} - \frac{27}{16}\int \frac{x^2}{\sqrt{9+4x^2}} \, dx \\
&= -\frac{27}{128}x\sqrt{9+4x^2} + \frac{1}{16}x^3\sqrt{9+4x^2} + \frac{243}{128}\int \frac{1}{\sqrt{9+4x^2}} \, dx \\
&= -\frac{27}{128}x\sqrt{9+4x^2} + \frac{1}{16}x^3\sqrt{9+4x^2} + \frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.0106941, size = 34, normalized size = 0.76

$$\frac{1}{256}\left(2x\sqrt{4x^2+9}\left(8x^2-27\right)+243\sinh^{-1}\left(\frac{2x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 + 4*x^2],x]
```

```
[Out] (2*x*Sqrt[9 + 4*x^2]*(-27 + 8*x^2) + 243*ArcSinh[(2*x)/3])/256
```

Maple [A] time = 0.004, size = 34, normalized size = 0.8

$$\frac{243}{256}\mathrm{Arcsinh}\left(\frac{2\,x}{3}\right)-\frac{27\,x}{128}\sqrt{4\,x^2+9}+\frac{x^3}{16}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2+9)^(1/2),x)
```

```
[Out] 243/256*arcsinh(2/3*x)-27/128*x*(4*x^2+9)^(1/2)+1/16*x^3*(4*x^2+9)^(1/2)
```

Maxima [A] time = 3.57066, size = 45, normalized size = 1.

$$\frac{1}{16}\,\sqrt{4\,x^2+9}x^3-\frac{27}{128}\,\sqrt{4\,x^2+9}x+\frac{243}{256}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(4*x^2 + 9)*x^3 - 27/128*sqrt(4*x^2 + 9)*x + 243/256*arcsinh(2/3*x
)
```

Fricas [A] time = 1.44861, size = 103, normalized size = 2.29

$$\frac{1}{128}\,\left(8\,x^3-27\,x\right)\sqrt{4\,x^2+9}-\frac{243}{256}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*x^3 - 27*x)*sqrt(4*x^2 + 9) - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 0.671648, size = 39, normalized size = 0.87

$$\frac{x^3\sqrt{4x^2+9}}{16}-\frac{27x\sqrt{4x^2+9}}{128}+\frac{243\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2+9)**(1/2),x)
```

```
[Out] x**3*sqrt(4*x**2 + 9)/16 - 27*x*sqrt(4*x**2 + 9)/128 + 243*asinh(2*x/3)/256
```

Giac [A] time = 3.02374, size = 49, normalized size = 1.09

$$\frac{1}{128}\left(8\,x^2-27\right)\sqrt{4\,x^2+9}x-\frac{243}{256}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*x^2 - 27)*sqrt(4*x^2 + 9)*x - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

3.534 $\int \frac{x^3}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

```
[Out] (-9*Sqrt[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48
```

Rubi [A] time = 0.0142129, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[9 + 4*x^2],x]
```

```
[Out] (-9*Sqrt[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{9+4x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{9}{4\sqrt{9+4x}}+\frac{1}{4}\sqrt{9+4x}\right)dx,x,x^2\right)\\
&= -\frac{9}{16}\sqrt{9+4x^2}+\frac{1}{48}\left(9+4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0055979, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2-9\right)\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 + 4*x^2],x]
```

[Out] ((-9 + 2*x^2)*Sqrt[9 + 4*x^2])/24

Maple [A] time = 0.003, size = 19, normalized size = 0.6

$$\frac{2x^2-9}{24}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(4*x^2+9)^(1/2),x)

[Out] 1/24*(4*x^2+9)^(1/2)*(2*x^2-9)

Maxima [A] time = 2.96505, size = 35, normalized size = 1.13

$$\frac{1}{12}\sqrt{4x^2+9}x^2-\frac{3}{8}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")

[Out] 1/12*sqrt(4*x^2 + 9)*x^2 - 3/8*sqrt(4*x^2 + 9)

Fricas [A] time = 1.29076, size = 46, normalized size = 1.48

$$\frac{1}{24}\sqrt{4x^2+9}\left(2x^2-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")

[Out] 1/24*sqrt(4*x^2 + 9)*(2*x^2 - 9)

Sympy [A] time = 0.339444, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2+9}}{12}-\frac{3\sqrt{4x^2+9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(4*x**2+9)**(1/2),x)

[Out] x**2*sqrt(4*x**2 + 9)/12 - 3*sqrt(4*x**2 + 9)/8

Giac [A] time = 2.1769, size = 31, normalized size = 1.

$$\frac{1}{48}\left(4x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(4*x^2 + 9)^(3/2) - 9/16*sqrt(4*x^2 + 9)
```

3.535 $\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Rubi [A] time = 0.0060955, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 215}

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0049179, size = 27, normalized size = 1.

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Maple [A] time = 0.005, size = 20, normalized size = 0.7

$$-\frac{9}{16}\text{Arcsinh}\left(\frac{2\,x}{3}\right)+\frac{x}{8}\sqrt{4\,x^{2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2+9)^(1/2),x)
```

```
[Out] -9/16*arcsinh(2/3*x)+1/8*x*(4*x^2+9)^(1/2)
```

Maxima [A] time = 3.10261, size = 26, normalized size = 0.96

$$\frac{1}{8}\sqrt{4\,x^{2}+9}x-\frac{9}{16}\,\text{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x - 9/16*arcsinh(2/3*x)
```

Fricas [A] time = 1.26852, size = 78, normalized size = 2.89

$$\frac{1}{8}\sqrt{4\,x^{2}+9}x+\frac{9}{16}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 0.203203, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^{2}+9}}{8}-\frac{9\,\text{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 + 9)/8 - 9*asinh(2*x/3)/16
```

Giac [A] time = 2.97447, size = 39, normalized size = 1.44

$$\frac{1}{8}\sqrt{4\,x^{2}+9}x+\frac{9}{16}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

3.536 $\int \frac{x}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{4x^2+9}$$

```
[Out] Sqrt[9 + 4*x^2]/4
```

Rubi [A] time = 0.0023737, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\sqrt{9+4x^2}$$

Mathematica [A] time = 0.0012551, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

Maple [A] time = 0.002, size = 12, normalized size = 0.8

$$\frac{1}{4}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/4*(4*x^2+9)^(1/2)
```

Maxima [A] time = 2.95053, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

Fricas [A] time = 1.29, size = 28, normalized size = 1.87

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

Sympy [A] time = 0.133348, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+9)**(1/2),x)
```

```
[Out] sqrt(4*x**2 + 9)/4
```

Giac [A] time = 2.46151, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

3.537 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] ArcSinh[(2*x)/3]/2
```

Rubi [A] time = 0.001411, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.0033149, size = 10, normalized size = 1.

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Maple [A] time = 0.002, size = 7, normalized size = 0.7

$$\frac{1}{2}\mathrm{Arcsinh}\left(\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Maxima [A] time = 1.91969, size = 8, normalized size = 0.8

$$\frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Fricas [B] time = 1.2582, size = 46, normalized size = 4.6

$$-\frac{1}{2}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 0.132782, size = 7, normalized size = 0.7

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2+9)**(1/2),x)
```

```
[Out] asinh(2*x/3)/2
```

Giac [B] time = 2.54792, size = 22, normalized size = 2.2

$$-\frac{1}{2}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

3.538 $\int \frac{1}{x\sqrt{9+4x^2}} \, dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -ArcTanh[Sqrt[9 + 4*x^2]/3]/3
```

Rubi [A] time = 0.0100864, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {266, 63, 207}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 + 4*x^2]),x]

[Out] -ArcTanh[Sqrt[9 + 4*x^2]/3]/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{9+4x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}} \, dx, x, x^2\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}} \, dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0024164, size = 20, normalized size = 1.

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[9 + 4*x^2]/3]/3
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$-\frac{1}{3}\mathrm{Artanh}\left(3\,\frac{1}{\sqrt{4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/3*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A] time = 2.99021, size = 12, normalized size = 0.6

$$-\frac{1}{3}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsinh(3/2/abs(x))
```

Fricas [B] time = 1.24206, size = 103, normalized size = 5.15

$$-\frac{1}{3}\,\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)+\frac{1}{3}\,\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 1/3*log(-2*x + sqrt(4*x^2 + 9) - 3)
```

Sympy [A] time = 0.997542, size = 8, normalized size = 0.4

$$-\frac{\mathrm{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2+9)**(1/2),x)
```

```
[Out] -asinh(3/(2*x))/3
```

Giac [B] time = 2.20068, size = 39, normalized size = 1.95

$$-\frac{1}{6}\log\left(\sqrt{4x^2+9}+3\right)+\frac{1}{6}\log\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*log(sqrt(4*x^2 + 9) + 3) + 1/6*log(sqrt(4*x^2 + 9) - 3)
```

3.539 $\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{4x^2+9}}{9x}$$

```
[Out] -Sqrt[9 + 4*x^2]/(9*x)
```

Rubi [A] time = 0.003062, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[9 + 4*x^2]),x]

[Out] -Sqrt[9 + 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{9x}$$

Mathematica [A] time = 0.0023532, size = 18, normalized size = 1.

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[9 + 4*x^2]),x]

[Out] -Sqrt[9 + 4*x^2]/(9*x)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$-\frac{1}{9\,x}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2+9)^(1/2),x)

[Out] -1/9*(4*x^2+9)^(1/2)/x
```

Maxima [A] time = 2.12417, size = 19, normalized size = 1.06

$$-\frac{\sqrt{4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")

[Out] -1/9*sqrt(4*x^2 + 9)/x
```

Fricas [A] time = 1.21919, size = 43, normalized size = 2.39

$$-\frac{2\,x+\sqrt{4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")

[Out] -1/9*(2*x + sqrt(4*x^2 + 9))/x
```

Sympy [A] time = 0.706813, size = 15, normalized size = 0.83

$$-\frac{2\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2+9)**(1/2),x)

[Out] -2*sqrt(1 + 9/(4*x**2))/9
```

Giac [A] time = 2.26969, size = 31, normalized size = 1.72

$$\frac{4}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="giac")

[Out] 4/((2*x - sqrt(4*x^2 + 9))^2 - 9)
```

3.540 $\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

```
[Out] -Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Rubi [A] time = 0.0165838, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 207}

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)$$
$$= -\frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Mathematica [A] time = 0.010559, size = 37, normalized size = 0.95

$$\frac{1}{54}\left(4\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right) - \frac{3\sqrt{4x^2+9}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

```
[Out] ((-3*Sqrt[9 + 4*x^2])/x^2 + 4*ArcTanh[Sqrt[1 + (4*x^2)/9]])/54
```

Maple [A] time = 0.005, size = 30, normalized size = 0.8

$$-\frac{1}{18x^2}\sqrt{4x^2+9} + \frac{2}{27}\operatorname{Artanh}\left(3\frac{1}{\sqrt{4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A] time = 1.90796, size = 32, normalized size = 0.82

$$-\frac{\sqrt{4x^2+9}}{18x^2} + \frac{2}{27}\operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 2/27*arcsinh(3/2/abs(x))
```

Fricas [A] time = 1.23201, size = 149, normalized size = 3.82

$$\frac{4x^2\log\left(-2x+\sqrt{4x^2+9}+3\right) - 4x^2\log\left(-2x+\sqrt{4x^2+9}-3\right) - 3\sqrt{4x^2+9}}{54x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) - 3*sqrt(4*x^2 + 9))/x^2
```

Sympy [A] time = 2.17704, size = 44, normalized size = 1.13

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{1 + \frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{1 + \frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2+9)**(1/2),x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 1/(4*x**3*sqrt(1 + 9/(
4*x**2)))
```

Giac [A] time = 1.72825, size = 58, normalized size = 1.49

$$-\frac{\sqrt{4\,x^2+9}}{18\,x^2} + \frac{1}{27}\log\left(\sqrt{4\,x^2+9}+3\right) - \frac{1}{27}\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 1/27*log(sqrt(4*x^2 + 9) + 3) - 1/27*log(sqrt(4
*x^2 + 9) - 3)
```

3.541 $\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

```
[Out] -Sqrt[9 + 4*x^2]/(27*x^3) + (8*Sqrt[9 + 4*x^2])/(243*x)
```

Rubi [A] time = 0.0071159, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(27*x^3) + (8*Sqrt[9 + 4*x^2])/(243*x)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx &= -\frac{\sqrt{9+4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{27x^3}+\frac{8\sqrt{9+4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.0041132, size = 27, normalized size = 0.73

$$-\frac{\left(9-8x^2\right)\sqrt{\frac{4x^2}{9}+1}}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -((9 - 8*x^2)*Sqrt[1 + (4*x^2)/9])/(81*x^3)
```

Maple [A] time = 0.002, size = 22, normalized size = 0.6

$$\frac{8x^2-9}{243x^3}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/243*(4*x^2+9)^(1/2)*(8*x^2-9)/x^3
```

Maxima [A] time = 3.56123, size = 39, normalized size = 1.05

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/243*sqrt(4*x^2 + 9)/x - 1/27*sqrt(4*x^2 + 9)/x^3
```

Fricas [A] time = 1.25628, size = 68, normalized size = 1.84

$$\frac{16x^3+\left(8x^2-9\right)\sqrt{4x^2+9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/243*(16*x^3 + (8*x^2 - 9)*sqrt(4*x^2 + 9))/x^3
```

Sympy [A] time = 1.45535, size = 32, normalized size = 0.86

$$\frac{16\sqrt{1+\frac{9}{4x^2}}}{243}-\frac{2\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2+9)**(1/2),x)
```

```
[Out] 16*sqrt(1 + 9/(4*x**2))/243 - 2*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

Giac [A] time = 1.93105, size = 57, normalized size = 1.54

$$\frac{32\left(\left(2x-\sqrt{4x^2+9}\right)^2-3\right)}{\left(\left(2x-\sqrt{4x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 32*((2*x - sqrt(4*x^2 + 9))^2 - 3)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

3.542 $\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2+9}}{54x^2}-\frac{\sqrt{4x^2+9}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -Sqrt[9 + 4*x^2]/(36*x^4) + Sqrt[9 + 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 +
4*x^2]/3])/81
```

Rubi [A] time = 0.0229079, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 207}

$$\frac{\sqrt{4x^2+9}}{54x^2}-\frac{\sqrt{4x^2+9}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(36*x^4) + Sqrt[9 + 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 +
4*x^2]/3])/81
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}+\frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}+\frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.004676, size = 32, normalized size = 0.56

$$-\frac{16}{729}\sqrt{4x^2+9}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{4x^2}{9}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] (-16*Sqrt[9 + 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (4*x^2)/9])/729
```

Maple [A] time = 0.003, size = 44, normalized size = 0.8

$$-\frac{1}{36\,x^4}\sqrt{4\,x^2+9}+\frac{1}{54\,x^2}\sqrt{4\,x^2+9}-\frac{2}{81}\operatorname{Artanh}\left(3\,\frac{1}{\sqrt{4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(4*x^2+9)
^(1/2))
```

Maxima [A] time = 2.97595, size = 51, normalized size = 0.89

$$\frac{\sqrt{4\,x^2+9}}{54\,x^2}-\frac{\sqrt{4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 + 9)/x^2 - 1/36*sqrt(4*x^2 + 9)/x^4 - 2/81*arcsinh(3/2/abs(
x))
```

Fricas [A] time = 1.36904, size = 167, normalized size = 2.93

$$-\frac{8 x^4 \log\left(-2 x + \sqrt{4 x^2 + 9} + 3\right) - 8 x^4 \log\left(-2 x + \sqrt{4 x^2 + 9} - 3\right) - 3 \sqrt{4 x^2 + 9}\left(2 x^2 - 3\right)}{324 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/324*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
 + 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 - 3))/x^4
```

Sympy [A] time = 4.45956, size = 63, normalized size = 1.11

$$-\frac{2 \operatorname{asinh}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{1 + \frac{9}{4x^2}}} + \frac{1}{36x^3\sqrt{1 + \frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{1 + \frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*asinh(3/(2*x))/81 + 1/(27*x*sqrt(1 + 9/(4*x**2))) + 1/(36*x**3*sqrt(1 +
9/(4*x**2))) - 1/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [A] time = 3.02201, size = 74, normalized size = 1.3

$$\frac{\left(4 x^2 + 9\right)^{\frac{3}{2}} - 15 \sqrt{4 x^2 + 9}}{216 x^4} - \frac{1}{81} \log\left(\sqrt{4 x^2 + 9} + 3\right) + \frac{1}{81} \log\left(\sqrt{4 x^2 + 9} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 + 9)^(3/2) - 15*sqrt(4*x^2 + 9))/x^4 - 1/81*log(sqrt(4*x^2 +
9) + 3) + 1/81*log(sqrt(4*x^2 + 9) - 3)
```

3.543 $\int \frac{x^5}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

```
[Out] (-81*Sqrt[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320
```

Rubi [A] time = 0.0188897, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{9-4x}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{81}{16\sqrt{9-4x}}-\frac{9}{8}\sqrt{9-4x}+\frac{1}{16}(9-4x)^{3/2}\right)dx, x, x^2\right)\\
&= -\frac{81}{64}\sqrt{9-4x^2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{1}{320}\left(9-4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0082703, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{9-4x^2}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 - 4*x^2],x]
```

```
[Out] -(Sqrt[9 - 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$\frac{(-3+2x)(3+2x)\left(2x^4+6x^2+27\right)}{40}\frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(-3+2*x)*(3+2*x)*(2*x^4+6*x^2+27)/(-4*x^2+9)^(1/2)
```

Maxima [A] time = 1.80632, size = 54, normalized size = 1.17

$$-\frac{1}{20}\sqrt{-4x^2+9}x^4-\frac{3}{20}\sqrt{-4x^2+9}x^2-\frac{27}{40}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-4*x^2 + 9)*x^4 - 3/20*sqrt(-4*x^2 + 9)*x^2 - 27/40*sqrt(-4*x^2
+ 9)
```

Fricas [A] time = 1.27043, size = 61, normalized size = 1.33

$$-\frac{1}{40}\left(2x^4+6x^2+27\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^4 + 6*x^2 + 27)*sqrt(-4*x^2 + 9)
```

Sympy [A] time = 1.15974, size = 46, normalized size = 1.

$$-\frac{x^4\sqrt{9-4x^2}}{20}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**4*sqrt(9 - 4*x**2)/20 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2
)/40
```

Giac [A] time = 2.93596, size = 58, normalized size = 1.26

$$-\frac{1}{320}\left(4x^2-9\right)^2\sqrt{-4x^2+9}+\frac{3}{32}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/320*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) + 3/32*(-4*x^2 + 9)^(3/2) - 81/64*sqr
t(-4*x^2 + 9)
```

3.544 $\int \frac{x^4}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{16}\sqrt{9-4x^2}x^3-\frac{27}{128}\sqrt{9-4x^2}x+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] (-27*x*Sqrt[9 - 4*x^2])/128 - (x^3*Sqrt[9 - 4*x^2])/16 + (243*ArcSin[(2*x)/
3])/256
```

Rubi [A] time = 0.0096936, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$-\frac{1}{16}\sqrt{9-4x^2}x^3-\frac{27}{128}\sqrt{9-4x^2}x+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*x*Sqrt[9 - 4*x^2])/128 - (x^3*Sqrt[9 - 4*x^2])/16 + (243*ArcSin[(2*x)/
3])/256
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{27}{16}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0115058, size = 34, normalized size = 0.76

$$\frac{1}{256}\left(243\sin^{-1}\left(\frac{2x}{3}\right)-2x\sqrt{9-4x^2}\left(8x^2+27\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 - 4*x^2],x]

[Out] (-2*x*Sqrt[9 - 4*x^2]*(27 + 8*x^2) + 243*ArcSin[(2*x)/3])/256
```

Maple [A] time = 0.004, size = 34, normalized size = 0.8

$$\frac{243}{256}\arcsin\left(\frac{2x}{3}\right)-\frac{27x}{128}\sqrt{-4x^2+9}-\frac{x^3}{16}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2+9)^(1/2),x)

[Out] 243/256*arcsin(2/3*x)-27/128*x*(-4*x^2+9)^(1/2)-1/16*x^3*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.61109, size = 45, normalized size = 1.

$$-\frac{1}{16}\sqrt{-4x^2+9}x^3-\frac{27}{128}\sqrt{-4x^2+9}x+\frac{243}{256}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")

[Out] -1/16*sqrt(-4*x^2 + 9)*x^3 - 27/128*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3
*x)
```

Fricas [A] time = 1.28691, size = 117, normalized size = 2.6

$$-\frac{1}{128}\left(8x^3+27x\right)\sqrt{-4x^2+9}-\frac{243}{128}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")

[Out] -1/128*(8*x^3 + 27*x)*sqrt(-4*x^2 + 9) - 243/128*arctan(1/2*(sqrt(-4*x^2 +
9) - 3)/x)
```

Sympy [A] time = 0.670848, size = 39, normalized size = 0.87

$$-\frac{x^3\sqrt{9-4x^2}}{16}-\frac{27x\sqrt{9-4x^2}}{128}+\frac{243\operatorname{asin}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**3*sqrt(9 - 4*x**2)/16 - 27*x*sqrt(9 - 4*x**2)/128 + 243*asin(2*x/3)/256
```

Giac [A] time = 2.51963, size = 35, normalized size = 0.78

$$-\frac{1}{128}\left(8\,x^2+27\right)\sqrt{-4\,x^2+9}x+\frac{243}{256}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(8*x^2 + 27)*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3*x)
```

3.545 $\int \frac{x^3}{\sqrt{9-4x^2}} \, dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

```
[Out] (-9*Sqrt[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48
```

Rubi [A] time = 0.0135806, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-9*Sqrt[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{9-4x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{9-4x}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{9}{4\sqrt{9-4x}}-\frac{1}{4}\sqrt{9-4x}\right) dx, x, x^2\right)\\
&= -\frac{9}{16}\sqrt{9-4x^2}+\frac{1}{48}\left(9-4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0058884, size = 22, normalized size = 0.71

$$-\frac{1}{24}\sqrt{9-4x^2}\left(2x^2+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 - 4*x^2],x]
```

```
[Out] -(Sqrt[9 - 4*x^2]*(9 + 2*x^2))/24
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{(-3+2x)(3+2x)(2x^2+9)}{24}\frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*(-3+2*x)*(3+2*x)*(2*x^2+9)/(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.20442, size = 35, normalized size = 1.13

$$-\frac{1}{12}\sqrt{-4x^2+9}x^2-\frac{3}{8}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-4*x^2 + 9)*x^2 - 3/8*sqrt(-4*x^2 + 9)
```

Fricas [A] time = 1.25076, size = 49, normalized size = 1.58

$$-\frac{1}{24}(2x^2+9)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*x^2 + 9)*sqrt(-4*x^2 + 9)
```

Sympy [A] time = 0.344189, size = 29, normalized size = 0.94

$$-\frac{x^2\sqrt{9-4x^2}}{12}-\frac{3\sqrt{9-4x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**2*sqrt(9 - 4*x**2)/12 - 3*sqrt(9 - 4*x**2)/8
```

Giac [A] time = 2.56902, size = 31, normalized size = 1.

$$\frac{1}{48}\left(-4\,x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\,\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(-4*x^2 + 9)^(3/2) - 9/16*sqrt(-4*x^2 + 9)
```

3.546 $\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

```
[Out] -(x*Sqrt[9 - 4*x^2])/8 + (9*ArcSin[(2*x)/3])/16
```

Rubi [A] time = 0.0050559, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[9 - 4*x^2])/8 + (9*ArcSin[(2*x)/3])/16
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx&=-\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\&=-\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0052729, size = 27, normalized size = 1.

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[9 - 4*x^2])/8 + (9*ArcSin[(2*x)/3])/16
```

Maple [A] time = 0.003, size = 20, normalized size = 0.7

$$\frac{9}{16}\arcsin\left(\frac{2\,x}{3}\right)-\frac{x}{8}\sqrt{-4\,x^{2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+9)^(1/2),x)
```

```
[Out] 9/16*arcsin(2/3*x)-1/8*x*(-4*x^2+9)^(1/2)
```

Maxima [A] time = 2.57791, size = 26, normalized size = 0.96

$$-\frac{1}{8}\sqrt{-4\,x^{2}+9}x+\frac{9}{16}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

Fricas [A] time = 1.2887, size = 92, normalized size = 3.41

$$-\frac{1}{8}\sqrt{-4\,x^{2}+9}x-\frac{9}{8}\arctan\left(\frac{\sqrt{-4\,x^{2}+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x - 9/8*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] time = 0.202437, size = 22, normalized size = 0.81

$$-\frac{x\sqrt{9-4x^{2}}}{8}+\frac{9\operatorname{asin}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x*sqrt(9 - 4*x**2)/8 + 9*asin(2*x/3)/16
```

Giac [A] time = 2.96208, size = 26, normalized size = 0.96

$$-\frac{1}{8}\sqrt{-4\,x^{2}+9}x+\frac{9}{16}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

3.547 $\int \frac{x}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{9-4x^2}$$

```
[Out] -Sqrt[9 - 4*x^2]/4
```

Rubi [A] time = 0.002276, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 - 4*x^2],x]

[Out] -Sqrt[9 - 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{9-4x^2}$$

Mathematica [A] time = 0.0013072, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 - 4*x^2],x]

[Out] -Sqrt[9 - 4*x^2]/4
```

Maple [A] time = 0.002, size = 22, normalized size = 1.5

$$\frac{(-3+2\,x)\,(3+2\,x)}{4}\,\frac{1}{\sqrt{-4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/4*(-3+2*x)*(3+2*x)/(-4*x^2+9)^(1/2)
```

Maxima [A] time = 2.50057, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

Fricas [A] time = 1.19692, size = 31, normalized size = 2.07

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

Sympy [A] time = 0.135019, size = 12, normalized size = 0.8

$$-\frac{\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2+9)**(1/2),x)
```

```
[Out] -sqrt(9 - 4*x**2)/4
```

Giac [A] time = 2.81202, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

3.548 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] ArcSin[(2*x)/3]/2
```

Rubi [A] time = 0.0014207, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - 4*x^2],x]
```

```
[Out] ArcSin[(2*x)/3]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.0036364, size = 10, normalized size = 1.

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - 4*x^2],x]
```

```
[Out] ArcSin[(2*x)/3]/2
```

Maple [A] time = 0.003, size = 7, normalized size = 0.7

$$\frac{1}{2}\arcsin\left(\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+9)^(1/2),x)

[Out] 1/2*arcsin(2/3*x)
```

Maxima [A] time = 3.71767, size = 8, normalized size = 0.8

$$\frac{1}{2}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="maxima")

[Out] 1/2*arcsin(2/3*x)
```

Fricas [B] time = 1.26488, size = 53, normalized size = 5.3

$$-\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="fricas")

[Out] -arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] time = 0.134488, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+9)**(1/2),x)

[Out] asin(2*x/3)/2
```

Giac [A] time = 2.09543, size = 8, normalized size = 0.8

$$\frac{1}{2}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="giac")

[Out] 1/2*arcsin(2/3*x)
```

3.549 $\int \frac{1}{x\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -ArcTanh[Sqrt[9 - 4*x^2]/3]/3
```

Rubi [A] time = 0.0101745, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {266, 63, 206}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 - 4*x^2]),x]

[Out] -ArcTanh[Sqrt[9 - 4*x^2]/3]/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\right)\\
&= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0027887, size = 20, normalized size = 1.

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[9 - 4*x^2]/3]/3
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$-\frac{1}{3}\text{Artanh}\left(3\frac{1}{\sqrt{-4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/3*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A] time = 2.48907, size = 34, normalized size = 1.7

$$-\frac{1}{3}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 1.27376, size = 47, normalized size = 2.35

$$\frac{1}{3}\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] time = 1.05755, size = 26, normalized size = 1.3

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-acosh(3/(2*x))/3, 9/(4*Abs(x**2)) > 1), (I*asin(3/(2*x))/3, Tru
e))
```

Giac [B] time = 2.64758, size = 42, normalized size = 2.1

$$-\frac{1}{6}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{6}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*log(sqrt(-4*x^2 + 9) + 3) + 1/6*log(-sqrt(-4*x^2 + 9) + 3)
```

3.550 $\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{9-4x^2}}{9x}$$

```
[Out] -Sqrt[9 - 4*x^2]/(9*x)
```

Rubi [A] time = 0.0030861, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[9 - 4*x^2]),x]

[Out] -Sqrt[9 - 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{9-4x^2}}{9x}$$

Mathematica [A] time = 0.0024548, size = 18, normalized size = 1.

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[9 - 4*x^2]),x]

[Out] -Sqrt[9 - 4*x^2]/(9*x)
```

Maple [A] time = 0.002, size = 25, normalized size = 1.4

$$\frac{(-3+2x)(3+2x)}{9x}\frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/9/x*(-3+2*x)*(3+2*x)/(-4*x^2+9)^(1/2)
```

Maxima [A] time = 1.77081, size = 19, normalized size = 1.06

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

Fricas [A] time = 1.27145, size = 34, normalized size = 1.89

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

Sympy [A] time = 0.738667, size = 41, normalized size = 2.28

$$\begin{cases} -\frac{2\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{2i\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 9/(4*x**2))/9, 9/(4*Abs(x**2)) > 1), (-2*I*sqrt(1 -
 9/(4*x**2))/9, True))
```

Giac [B] time = 2.47954, size = 45, normalized size = 2.5

$$\frac{2\,x}{9\left(\sqrt{-4\,x^2+9}-3\right)}-\frac{\sqrt{-4\,x^2+9}-3}{18\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*x/(sqrt(-4*x^2 + 9) - 3) - 1/18*(sqrt(-4*x^2 + 9) - 3)/x
```

3.551 $\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -Sqrt[9 - 4*x^2]/(18*x^2) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Rubi [A] time = 0.0159081, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(18*x^2) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x^2}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{18x^2}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{18x^2}-\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\\ &= -\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0097624, size = 37, normalized size = 0.95

$$\frac{1}{54}\left(-\frac{3\sqrt{9-4x^2}}{x^2}-4\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

```
[Out] ((-3*Sqrt[9 - 4*x^2])/x^2 - 4*ArcTanh[Sqrt[1 - (4*x^2)/9]])/54
```

Maple [A] time = 0.004, size = 30, normalized size = 0.8

$$-\frac{1}{18\,x^2}\sqrt{-4\,x^2+9}-\frac{2}{27}\text{Artanh}\left(3\,\frac{1}{\sqrt{-4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/18*(-4*x^2+9)^(1/2)/x^2-2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A] time = 3.38938, size = 54, normalized size = 1.38

$$-\frac{\sqrt{-4\,x^2+9}}{18\,x^2}-\frac{2}{27}\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

Fricas [A] time = 1.27832, size = 93, normalized size = 2.38

$$\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,\sqrt{-4\,x^2+9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*sqrt(-4*x^2 + 9))/x^2
```

Sympy [A] time = 2.24583, size = 99, normalized size = 2.54

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/27 + 1/(9*x*sqrt(-1 + 9/(4*x**2))) - 1/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (2*I*asin(3/(2*x))/27 - I/(9
*x*sqrt(1 - 9/(4*x**2))) + I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 1.71776, size = 61, normalized size = 1.56

$$-\frac{\sqrt{-4x^2+9}}{18x^2} - \frac{1}{27}\log\left(\sqrt{-4x^2+9}+3\right) + \frac{1}{27}\log\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 1/27*log(sqrt(-4*x^2 + 9) + 3) + 1/27*log(-sqr
t(-4*x^2 + 9) + 3)
```

3.552 $\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=37

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

```
[Out] -Sqrt[9 - 4*x^2]/(27*x^3) - (8*Sqrt[9 - 4*x^2])/(243*x)
```

Rubi [A] time = 0.0070717, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(27*x^3) - (8*Sqrt[9 - 4*x^2])/(243*x)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx &= -\frac{\sqrt{9-4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx\\ &= -\frac{\sqrt{9-4x^2}}{27x^3}-\frac{8\sqrt{9-4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.003646, size = 27, normalized size = 0.73

$$-\frac{\sqrt{1-\frac{4x^2}{9}}\left(8x^2+9\right)}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -(Sqrt[1 - (4*x^2)/9]*(9 + 8*x^2))/(81*x^3)
```

Maple [A] time = 0.002, size = 32, normalized size = 0.9

$$\frac{(-3+2x)(3+2x)\left(8x^2+9\right)}{243x^3}\frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/243*(-3+2*x)*(3+2*x)*(8*x^2+9)/x^3/(-4*x^2+9)^(1/2)
```

Maxima [A] time = 3.45287, size = 39, normalized size = 1.05

$$-\frac{8\sqrt{-4x^2+9}}{243x}-\frac{\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 + 9)/x - 1/27*sqrt(-4*x^2 + 9)/x^3
```

Fricas [A] time = 1.21108, size = 55, normalized size = 1.49

$$-\frac{\left(8x^2+9\right)\sqrt{-4x^2+9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 + 9)*sqrt(-4*x^2 + 9)/x^3
```

Sympy [A] time = 1.49926, size = 80, normalized size = 2.16

$$\begin{cases} -\frac{16\sqrt{-1+\frac{9}{4x^2}}}{243}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{16i\sqrt{1-\frac{9}{4x^2}}}{243}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-16*sqrt(-1 + 9/(4*x**2))/243 - 2*sqrt(-1 + 9/(4*x**2))/(27*x**2
), 9/(4*Abs(x**2)) > 1), (-16*I*sqrt(1 - 9/(4*x**2))/243 - 2*I*sqrt(1 - 9/(
```

```
4*x**2))/(27*x**2), True))
```

Giac [B] time = 2.57631, size = 99, normalized size = 2.68

$$\frac{2x^3\left(\frac{9\left(\sqrt{-4x^2+9}-3\right)^2}{x^2}+4\right)}{243\left(\sqrt{-4x^2+9}-3\right)^3}-\frac{\sqrt{-4x^2+9}-3}{54x}-\frac{\left(\sqrt{-4x^2+9}-3\right)^3}{1944x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/243*x^3*(9*(sqrt(-4*x^2 + 9) - 3)^2/x^2 + 4)/(sqrt(-4*x^2 + 9) - 3)^3 - 1
/54*(sqrt(-4*x^2 + 9) - 3)/x - 1/1944*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

3.553 $\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{\sqrt{9-4x^2}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -Sqrt[9 - 4*x^2]/(36*x^4) - Sqrt[9 - 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 -
4*x^2]/3])/81
```

Rubi [A] time = 0.0235052, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{\sqrt{9-4x^2}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(36*x^4) - Sqrt[9 - 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 -
4*x^2]/3])/81
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x^3}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x^2}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}+\frac{1}{27}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}x}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.0048208, size = 32, normalized size = 0.56

$$-\frac{16}{729}\sqrt{9-4x^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{4x^2}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

```
[Out] (-16*Sqrt[9 - 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (4*x^2)/9])/729
```

Maple [A] time = 0.004, size = 44, normalized size = 0.8

$$-\frac{1}{36\,x^4}\sqrt{-4\,x^2+9}-\frac{1}{54\,x^2}\sqrt{-4\,x^2+9}-\frac{2}{81}\text{Artanh}\left(3\,\frac{1}{\sqrt{-4\,x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(-4*x^2
+9)^(1/2))
```

Maxima [A] time = 2.64565, size = 73, normalized size = 1.28

$$-\frac{\sqrt{-4\,x^2+9}}{54\,x^2}-\frac{\sqrt{-4\,x^2+9}}{36\,x^4}-\frac{2}{81}\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 + 9)/x^2 - 1/36*sqrt(-4*x^2 + 9)/x^4 - 2/81*log(6*sqrt(-4
*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 1.3713, size = 111, normalized size = 1.95

$$\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\left(2\,x^2+3\right)\sqrt{-4\,x^2+9}}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 + 3)*sqrt(-4*x^2 + 9)
)/x^4
```

Sympy [A] time = 4.44485, size = 136, normalized size = 2.39

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/81 + 1/(27*x*sqrt(-1 + 9/(4*x**2))) - 1/(36*x*
*3*sqrt(-1 + 9/(4*x**2))) - 1/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2
)) > 1), (2*I*asin(3/(2*x))/81 - I/(27*x*sqrt(1 - 9/(4*x**2))) + I/(36*x**3
*sqrt(1 - 9/(4*x**2))) + I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 2.40034, size = 77, normalized size = 1.35

$$\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}-15\,\sqrt{-4\,x^2+9}}{216\,x^4}-\frac{1}{81}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{81}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((-4*x^2 + 9)^(3/2) - 15*sqrt(-4*x^2 + 9))/x^4 - 1/81*log(sqrt(-4*x^2
 + 9) + 3) + 1/81*log(-sqrt(-4*x^2 + 9) + 3)
```

3.554 $\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

```
[Out] (81*Sqrt[-9 + 4*x^2])/64 + (3*(-9 + 4*x^2)^(3/2))/32 + (-9 + 4*x^2)^(5/2)/3
20
```

Rubi [A] time = 0.0187101, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[-9 + 4*x^2])/64 + (3*(-9 + 4*x^2)^(3/2))/32 + (-9 + 4*x^2)^(5/2)/3
20
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{81}{16\sqrt{-9+4x}}+\frac{9}{8}\sqrt{-9+4x}+\frac{1}{16}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\
&= \frac{81}{64}\sqrt{-9+4x^2}+\frac{3}{32}\left(-9+4x^2\right)^{3/2}+\frac{1}{320}\left(-9+4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0085672, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2-9}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$\frac{(-3+2x)(3+2x)\left(2x^{4}+6x^{2}+27\right)}{40}\frac{1}{\sqrt{4x^{2}-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/40*(-3+2*x)*(3+2*x)*(2*x^4+6*x^2+27)/(4*x^2-9)^(1/2)
```

Maxima [A] time = 2.52361, size = 54, normalized size = 1.17

$$\frac{1}{20}\sqrt{4x^{2}-9}x^{4}+\frac{3}{20}\sqrt{4x^{2}-9}x^{2}+\frac{27}{40}\sqrt{4x^{2}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(4*x^2 - 9)*x^4 + 3/20*sqrt(4*x^2 - 9)*x^2 + 27/40*sqrt(4*x^2 - 9)
```

Fricas [A] time = 1.32353, size = 58, normalized size = 1.26

$$\frac{1}{40}\left(2x^{4}+6x^{2}+27\right)\sqrt{4x^{2}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(2*x^4 + 6*x^2 + 27)*sqrt(4*x^2 - 9)
```

Sympy [A] time = 1.12322, size = 44, normalized size = 0.96

$$\frac{x^{4}\sqrt{4x^{2}-9}}{20}+\frac{3x^{2}\sqrt{4x^{2}-9}}{20}+\frac{27\sqrt{4x^{2}-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/20 + 3*x**2*sqrt(4*x**2 - 9)/20 + 27*sqrt(4*x**2 - 9)
/40
```

Giac [A] time = 1.41269, size = 46, normalized size = 1.

$$\frac{1}{320}\left(4x^2-9\right)^{\frac{5}{2}}+\frac{3}{32}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*(4*x^2 - 9)^(5/2) + 3/32*(4*x^2 - 9)^(3/2) + 81/64*sqrt(4*x^2 - 9)
```

3.555 $\int \frac{x^4}{\sqrt{-9+4x^2}} \, dx$

Optimal. Leaf size=54

$$\frac{1}{16}\sqrt{4x^2-9}x^3+\frac{27}{128}\sqrt{4x^2-9}x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] (27*x*Sqrt[-9 + 4*x^2])/128 + (x^3*Sqrt[-9 + 4*x^2])/16 + (243*ArcTanh[(2*x
)/Sqrt[-9 + 4*x^2]])/256
```

Rubi [A] time = 0.0125138, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 217, 206}

$$\frac{1}{16}\sqrt{4x^2-9}x^3+\frac{27}{128}\sqrt{4x^2-9}x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 + 4*x^2])/128 + (x^3*Sqrt[-9 + 4*x^2])/16 + (243*ArcTanh[(2*x
)/Sqrt[-9 + 4*x^2]])/256
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{-9+4x^2}} \, dx &= \frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{27}{16}\int \frac{x^2}{\sqrt{-9+4x^2}} \, dx\\
&= \frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{-9+4x^2}} \, dx\\
&= \frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{128}\text{Subst}\left(\int \frac{1}{1-4x^2} \, dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)\\
&= \frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0129595, size = 43, normalized size = 0.8

$$\frac{1}{256}\left(2x\sqrt{4x^2-9}\left(8x^2+27\right)+243\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 + 4*x^2]*(27 + 8*x^2) + 243*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/2
56
```

Maple [A] time = 0.005, size = 49, normalized size = 0.9

$$\frac{x^3}{16}\sqrt{4x^2-9}+\frac{27x}{128}\sqrt{4x^2-9}+\frac{243\sqrt{4}}{512}\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/16*x^3*(4*x^2-9)^(1/2)+27/128*x*(4*x^2-9)^(1/2)+243/512*ln(x*4^(1/2)+(4*x
^2-9)^(1/2))*4^(1/2)
```

Maxima [A] time = 3.97886, size = 61, normalized size = 1.13

$$\frac{1}{16}\sqrt{4x^2-9}x^3+\frac{27}{128}\sqrt{4x^2-9}x+\frac{243}{256}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(4*x^2 - 9)*x^3 + 27/128*sqrt(4*x^2 - 9)*x + 243/256*log(8*x + 4*s
qrt(4*x^2 - 9))
```

Fricas [A] time = 1.31189, size = 103, normalized size = 1.91

$$\frac{1}{128}\left(8x^3+27x\right)\sqrt{4x^2-9}-\frac{243}{256}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*x^3 + 27*x)*sqrt(4*x^2 - 9) - 243/256*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] time = 0.685653, size = 39, normalized size = 0.72

$$\frac{x^3\sqrt{4x^2-9}}{16}+\frac{27x\sqrt{4x^2-9}}{128}+\frac{243\operatorname{acosh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] x**3*sqrt(4*x**2 - 9)/16 + 27*x*sqrt(4*x**2 - 9)/128 + 243*acosh(2*x/3)/256
```

Giac [A] time = 1.90084, size = 50, normalized size = 0.93

$$\frac{1}{128}\left(8\,x^2+27\right)\sqrt{4\,x^2-9}x-\frac{243}{256}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*x^2 + 27)*sqrt(4*x^2 - 9)*x - 243/256*log(abs(-2*x + sqrt(4*x^2 -
9)))
```

3.556 $\int \frac{x^3}{\sqrt{-9+4x^2}} dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

```
[Out] (9*Sqrt[-9 + 4*x^2])/16 + (-9 + 4*x^2)^(3/2)/48
```

Rubi [A] time = 0.0130457, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (9*Sqrt[-9 + 4*x^2])/16 + (-9 + 4*x^2)^(3/2)/48
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{9}{4\sqrt{-9+4x}}+\frac{1}{4}\sqrt{-9+4x}\right)dx,x,x^2\right)\\
&= \frac{9}{16}\sqrt{-9+4x^2}+\frac{1}{48}\left(-9+4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0060144, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2+9\right)\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((9 + 2*x^2)*Sqrt[-9 + 4*x^2])/24
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{(-3+2x)(3+2x)\left(2x^2+9\right)}{24}\frac{1}{\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/24*(-3+2*x)*(3+2*x)*(2*x^2+9)/(4*x^2-9)^(1/2)
```

Maxima [A] time = 4.58126, size = 35, normalized size = 1.13

$$\frac{1}{12}\sqrt{4x^2-9}x^2+\frac{3}{8}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(4*x^2 - 9)*x^2 + 3/8*sqrt(4*x^2 - 9)
```

Fricas [A] time = 1.20504, size = 46, normalized size = 1.48

$$\frac{1}{24}\sqrt{4x^2-9}\left(2x^2+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(4*x^2 - 9)*(2*x^2 + 9)
```

Sympy [A] time = 0.339632, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2-9}}{12}+\frac{3\sqrt{4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 - 9)/12 + 3*sqrt(4*x**2 - 9)/8
```

Giac [A] time = 2.70622, size = 31, normalized size = 1.

$$\frac{1}{48}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{9}{16}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(4*x^2 - 9)^(3/2) + 9/16*sqrt(4*x^2 - 9)
```

3.557 $\int \frac{x^2}{\sqrt{-9+4x^2}} dx$

Optimal. Leaf size=36

$$\frac{1}{8}\sqrt{4x^2-9}x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Rubi [A] time = 0.007637, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 217, 206}

$$\frac{1}{8}\sqrt{4x^2-9}x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-9+4x^2}} dx &= \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{8}\int \frac{1}{\sqrt{-9+4x^2}} dx \\
&= \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{8}\text{Subst}\left(\int \frac{1}{1-4x^2} dx, x, \frac{x}{\sqrt{-9+4x^2}}\right) \\
&= \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.005273, size = 36, normalized size = 1.

$$\frac{1}{8}\sqrt{4x^2-9}x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Maple [A] time = 0.005, size = 35, normalized size = 1.

$$\frac{x}{8}\sqrt{4x^2-9}+\frac{9\sqrt{4}}{32}\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/8*x*(4*x^2-9)^(1/2)+9/32*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A] time = 1.84864, size = 42, normalized size = 1.17

$$\frac{1}{8}\sqrt{4x^2-9}x+\frac{9}{16}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^2 - 9)*x + 9/16*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A] time = 1.28349, size = 78, normalized size = 2.17

$$\frac{1}{8}\sqrt{4x^2-9}x-\frac{9}{16}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] time = 0.204969, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\operatorname{acosh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2-9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 - 9)/8 + 9*acosh(2*x/3)/16
```

Giac [A] time = 2.15916, size = 41, normalized size = 1.14

$$\frac{1}{8}\sqrt{4x^2-9}x-\frac{9}{16}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

3.558 $\int \frac{x}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{4x^2-9}$$

```
[Out] Sqrt[-9 + 4*x^2]/4
```

Rubi [A] time = 0.0020956, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 + 4*x^2],x]

[Out] Sqrt[-9 + 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{1}{4}\sqrt{-9+4x^2}$$

Mathematica [A] time = 0.0012402, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 + 4*x^2],x]

[Out] Sqrt[-9 + 4*x^2]/4
```

Maple [A] time = 0.002, size = 22, normalized size = 1.5

$$\frac{(-3+2x)(3+2x)}{4}\frac{1}{\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/4*(-3+2*x)*(3+2*x)/(4*x^2-9)^(1/2)
```

Maxima [A] time = 2.956, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

Fricas [A] time = 1.18828, size = 28, normalized size = 1.87

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

Sympy [A] time = 0.135896, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2-9)**(1/2),x)
```

```
[Out] sqrt(4*x**2 - 9)/4
```

Giac [A] time = 2.67957, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

3.559 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Rubi [A] time = 0.0031291, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + 4*x^2],x]
```

```
[Out] ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0025181, size = 43, normalized size = 2.26

$$\frac{1}{4}\log\left(\frac{2x}{\sqrt{4x^2-9}}+1\right)-\frac{1}{4}\log\left(1-\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + 4*x^2],x]
```

```
[Out] -Log[1 - (2*x)/Sqrt[-9 + 4*x^2]]/4 + Log[1 + (2*x)/Sqrt[-9 + 4*x^2]]/4
```

Maple [A] time = 0.002, size = 22, normalized size = 1.2

$$\frac{\sqrt{4}}{4}\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/4*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A] time = 3.60055, size = 24, normalized size = 1.26

$$\frac{1}{2}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A] time = 1.2857, size = 46, normalized size = 2.42

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] time = 0.133074, size = 7, normalized size = 0.37

$$\frac{\operatorname{acosh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-9)**(1/2),x)
```

```
[Out] acosh(2*x/3)/2
```

Giac [A] time = 2.60687, size = 23, normalized size = 1.21

$$-\frac{1}{2}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="giac")

[Out] -1/2*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

3.560 $\int \frac{1}{x\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Rubi [A] time = 0.0096452, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-9 + 4*x^2]),x]

[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0026158, size = 20, normalized size = 1.

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$-\frac{1}{3}\arctan\left(3\frac{1}{\sqrt{4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2-9)^(1/2),x)
```

```
[Out] -1/3*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A] time = 1.98175, size = 12, normalized size = 0.6

$$-\frac{1}{3}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsin(3/2/abs(x))
```

Fricas [A] time = 1.30312, size = 57, normalized size = 2.85

$$\frac{2}{3}\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

Sympy [A] time = 1.05396, size = 26, normalized size = 1.3

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(3/(2*x))/3, 9/(4*Abs(x**2)) > 1), (-asin(3/(2*x))/3, Tru
e))
```

Giac [A] time = 2.14256, size = 19, normalized size = 0.95

$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(1/3*sqrt(4*x^2 - 9))
```

3.561 $\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{4x^2-9}}{9x}$$

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Rubi [A] time = 0.0029692, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 + 4*x^2]),x]

[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{9x}$$

Mathematica [A] time = 0.0023989, size = 18, normalized size = 1.

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 + 4*x^2]),x]

[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Maple [A] time = 0.002, size = 25, normalized size = 1.4

$$\frac{(-3+2x)(3+2x)}{9x}\frac{1}{\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/9/x*(-3+2*x)*(3+2*x)/(4*x^2-9)^(1/2)
```

Maxima [A] time = 3.09762, size = 19, normalized size = 1.06

$$\frac{\sqrt{4x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(4*x^2 - 9)/x
```

Fricas [A] time = 1.28741, size = 42, normalized size = 2.33

$$\frac{2x+\sqrt{4x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*x + sqrt(4*x^2 - 9))/x
```

Sympy [A] time = 0.745963, size = 37, normalized size = 2.06

$$\begin{cases} \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for } \frac{9}{4|x^2|} > 1 \\ \frac{2\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*sqrt(-1 + 9/(4*x**2))/9, 9/(4*Abs(x**2)) > 1), (2*sqrt(1 - 9
/(4*x**2))/9, True))
```

Giac [A] time = 1.83537, size = 31, normalized size = 1.72

$$\frac{4}{\left(2x-\sqrt{4x^2-9}\right)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 4/((2*x - sqrt(4*x^2 - 9))^2 + 9)
```

3.562 $\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Rubi [A] time = 0.0158369, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)$$
$$= \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Mathematica [A] time = 0.0207119, size = 54, normalized size = 1.38

$$\frac{4}{81}\sqrt{4x^2-9}\left(\frac{9}{8x^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)}{2\sqrt{1-\frac{4x^2}{9}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (4*Sqrt[-9 + 4*x^2]*(9/(8*x^2) + ArcTanh[Sqrt[1 - (4*x^2)/9]]/(2*Sqrt[1 - (
4*x^2)/9])))/81
```

Maple [A] time = 0.003, size = 30, normalized size = 0.8

$$\frac{1}{18\,x^2}\sqrt{4\,x^2-9}-\frac{2}{27}\arctan\left(3\,\frac{1}{\sqrt{4\,x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/18*(4*x^2-9)^(1/2)/x^2-2/27*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A] time = 1.93946, size = 32, normalized size = 0.82

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}-\frac{2}{27}\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 - 2/27*arcsin(3/2/abs(x))
```

Fricas [A] time = 1.34103, size = 101, normalized size = 2.59

$$\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9))/x^2
```

Sympy [A] time = 2.23335, size = 99, normalized size = 2.54

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + I/(4*x**
3*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*asin(3/(2*x))/27 + 1/(9
*x*sqrt(1 - 9/(4*x**2))) - 1/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 2.46387, size = 39, normalized size = 1.

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 + 2/27*arctan(1/3*sqrt(4*x^2 - 9))
```

3.563 $\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

```
[Out] Sqrt[-9 + 4*x^2]/(27*x^3) + (8*Sqrt[-9 + 4*x^2])/(243*x)
```

Rubi [A] time = 0.0073577, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(27*x^3) + (8*Sqrt[-9 + 4*x^2])/(243*x)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx &= \frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx\\ &= \frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8\sqrt{-9+4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.0038396, size = 25, normalized size = 0.68

$$\frac{\sqrt{4x^2-9}\left(8x^2+9\right)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(9 + 8*x^2))/(243*x^3)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$\frac{(-3+2x)(3+2x)\left(8x^2+9\right)}{243x^3}\frac{1}{\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/243*(-3+2*x)*(3+2*x)*(8*x^2+9)/x^3/(4*x^2-9)^(1/2)
```

Maxima [A] time = 3.32919, size = 39, normalized size = 1.05

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/243*sqrt(4*x^2 - 9)/x + 1/27*sqrt(4*x^2 - 9)/x^3
```

Fricas [A] time = 1.2183, size = 68, normalized size = 1.84

$$\frac{16x^3+\left(8x^2+9\right)\sqrt{4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/243*(16*x^3 + (8*x^2 + 9)*sqrt(4*x^2 - 9))/x^3
```

Sympy [A] time = 1.5154, size = 76, normalized size = 2.05

$$\begin{cases} \frac{16i\sqrt{-1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for } \frac{9}{4|x^2|}>1 \\ \frac{16\sqrt{1-\frac{9}{4x^2}}}{243}+\frac{2\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((16*I*sqrt(-1 + 9/(4*x**2))/243 + 2*I*sqrt(-1 + 9/(4*x**2))/(27*x
**2), 9/(4*Abs(x**2)) > 1), (16*sqrt(1 - 9/(4*x**2))/243 + 2*sqrt(1 - 9/(4*
x**2))/(27*x**2), True))
```

Giac [A] time = 2.17388, size = 57, normalized size = 1.54

$$\frac{32\left(\left(2x-\sqrt{4x^2-9}\right)^2+3\right)}{\left(\left(2x-\sqrt{4x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 32*((2*x - sqrt(4*x^2 - 9))^2 + 3)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

3.564 $\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] Sqrt[-9 + 4*x^2]/(36*x^4) + Sqrt[-9 + 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/81
```

Rubi [A] time = 0.0232267, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(36*x^4) + Sqrt[-9 + 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/81
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{1}{27}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{-9+4x^2}\right)\\ &= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.0047386, size = 32, normalized size = 0.56

$$\frac{16}{729}\sqrt{4x^2-9}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{4x^2}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (16*Sqrt[-9 + 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (4*x^2)/9])/729
```

Maple [A] time = 0.004, size = 44, normalized size = 0.8

$$\frac{1}{36\,x^4}\sqrt{4\,x^2-9}+\frac{1}{54\,x^2}\sqrt{4\,x^2-9}-\frac{2}{81}\arctan\left(3\,\frac{1}{\sqrt{4\,x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(4*x^2-9)^(
1/2))
```

Maxima [A] time = 3.16137, size = 51, normalized size = 0.89

$$\frac{\sqrt{4\,x^2-9}}{54\,x^2}+\frac{\sqrt{4\,x^2-9}}{36\,x^4}-\frac{2}{81}\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 - 9)/x^2 + 1/36*sqrt(4*x^2 - 9)/x^4 - 2/81*arcsin(3/2/abs(x
))
```

Fricas [A] time = 1.25057, size = 120, normalized size = 2.11

$$\frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\left(2\,x^2+3\right)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 + 3))/x^4
```

Sympy [A] time = 4.39706, size = 136, normalized size = 2.39

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/81 - I/(27*x*sqrt(-1 + 9/(4*x**2))) + I/(36*x
**3*sqrt(-1 + 9/(4*x**2))) + I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**
2)) > 1), (-2*asin(3/(2*x))/81 + 1/(27*x*sqrt(1 - 9/(4*x**2))) - 1/(36*x**3
*sqrt(1 - 9/(4*x**2))) - 1/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A] time = 2.49245, size = 55, normalized size = 0.96

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}+15\,\sqrt{4\,x^2-9}}{216\,x^4}+\frac{2}{81}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 - 9)^(3/2) + 15*sqrt(4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sqrt(4
*x^2 - 9))
```

3.565 $\int \frac{x^5}{\sqrt{-9-4x^2}} \, dx$

Optimal. Leaf size=46

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

```
[Out] (-81*Sqrt[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/
320
```

Rubi [A] time = 0.0194125, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/
320
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{-9-4x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{-9-4x}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{-9-4x}}+\frac{9}{8}\sqrt{-9-4x}+\frac{1}{16}(-9-4x)^{3/2}\right) dx, x, x^2\right)\\
&= -\frac{81}{64}\sqrt{-9-4x^2}-\frac{3}{32}\left(-9-4x^2\right)^{3/2}-\frac{1}{320}\left(-9-4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0084343, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{-4x^2-9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

Maple [A] time = 0.003, size = 24, normalized size = 0.5

$$-\frac{2x^4-6x^2+27}{40}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/40*(2*x^4-6*x^2+27)*(-4*x^2-9)^(1/2)
```

Maxima [A] time = 3.95294, size = 54, normalized size = 1.17

$$-\frac{1}{20}\sqrt{-4x^2-9}x^4+\frac{3}{20}\sqrt{-4x^2-9}x^2-\frac{27}{40}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-4*x^2 - 9)*x^4 + 3/20*sqrt(-4*x^2 - 9)*x^2 - 27/40*sqrt(-4*x^2
- 9)
```

Fricas [A] time = 1.29725, size = 61, normalized size = 1.33

$$-\frac{1}{40}\left(2x^4-6x^2+27\right)\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^4 - 6*x^2 + 27)*sqrt(-4*x^2 - 9)
```

Sympy [A] time = 1.16989, size = 49, normalized size = 1.07

$$-\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**4*sqrt(-4*x**2 - 9)/20 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2
- 9)/40
```

Giac [C] time = 2.09956, size = 46, normalized size = 1.

$$-\frac{1}{320}i\left(4x^2+9\right)^{\frac{5}{2}}+\frac{3}{32}i\left(4x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}i\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/320*I*(4*x^2 + 9)^(5/2) + 3/32*I*(4*x^2 + 9)^(3/2) - 81/64*I*sqrt(4*x^2
+ 9)
```

3.566 $\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{16}\sqrt{-4x^2-9}x^3+\frac{27}{128}\sqrt{-4x^2-9}x+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] (27*x*Sqrt[-9 - 4*x^2])/128 - (x^3*Sqrt[-9 - 4*x^2])/16 + (243*ArcTan[(2*x)
/Sqrt[-9 - 4*x^2]])/256
```

Rubi [A] time = 0.0129458, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 217, 203}

$$-\frac{1}{16}\sqrt{-4x^2-9}x^3+\frac{27}{128}\sqrt{-4x^2-9}x+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 - 4*x^2])/128 - (x^3*Sqrt[-9 - 4*x^2])/16 + (243*ArcTan[(2*x)
/Sqrt[-9 - 4*x^2]])/256
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{-9-4x^2}-\frac{27}{16}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx\\
&= \frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\
&= \frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{128}\operatorname{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\
&= \frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0137112, size = 43, normalized size = 0.8

$$\frac{1}{256}\left(2x\sqrt{-4x^2-9}\left(27-8x^2\right)+243\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*(27 - 8*x^2)*Sqrt[-9 - 4*x^2] + 243*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/25
6
```

Maple [A] time = 0.005, size = 43, normalized size = 0.8

$$\frac{243}{256}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)+\frac{27x}{128}\sqrt{-4x^2-9}-\frac{x^3}{16}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2-9)^(1/2),x)
```

```
[Out] 243/256*arctan(2*x/(-4*x^2-9)^(1/2))+27/128*x*(-4*x^2-9)^(1/2)-1/16*x^3*(-4
*x^2-9)^(1/2)
```

Maxima [C] time = 1.87877, size = 45, normalized size = 0.83

$$-\frac{1}{16}\sqrt{-4x^2-9}x^3+\frac{27}{128}\sqrt{-4x^2-9}x-\frac{243}{256}i\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(-4*x^2 - 9)*x^3 + 27/128*sqrt(-4*x^2 - 9)*x - 243/256*I*arcsinh(
2/3*x)
```

Fricas [C] time = 1.36508, size = 186, normalized size = 3.44

$$-\frac{1}{128}\left(8x^3-27x\right)\sqrt{-4x^2-9}+\frac{243}{512}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)-\frac{243}{512}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(8*x^3 - 27*x)*sqrt(-4*x^2 - 9) + 243/512*I*log(-(8*x + 4*I*sqrt(-4*
x^2 - 9))/x) - 243/512*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] time = 0.854609, size = 53, normalized size = 0.98

$$-\frac{x^3\sqrt{-4x^2-9}}{16}+\frac{27x\sqrt{-4x^2-9}}{128}+\frac{243\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**3*sqrt(-4*x**2 - 9)/16 + 27*x*sqrt(-4*x**2 - 9)/128 + 243*atan(2*x/sqrt
(-4*x**2 - 9))/256
```

Giac [C] time = 2.25413, size = 35, normalized size = 0.65

$$-\frac{1}{128}\left(8\,x^2-27\right)\sqrt{-4\,x^2-9}x-\frac{243}{256}i\,\arcsin\left(\frac{2}{3}i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(8*x^2 - 27)*sqrt(-4*x^2 - 9)*x - 243/256*I*arcsin(2/3*I*x)
```

3.567 $\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

```
[Out] (9*Sqrt[-9 - 4*x^2])/16 + (-9 - 4*x^2)^(3/2)/48
```

Rubi [A] time = 0.013987, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*Sqrt[-9 - 4*x^2])/16 + (-9 - 4*x^2)^(3/2)/48
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{-9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{9}{4\sqrt{-9-4x}}-\frac{1}{4}\sqrt{-9-4x}\right)dx,x,x^2\right)\\ &= \frac{9}{16}\sqrt{-9-4x^2}+\frac{1}{48}\left(-9-4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0075584, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{-4x^2-9}\left(9-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(9 - 2*x^2))/24
```

Maple [A] time = 0.001, size = 19, normalized size = 0.6

$$-\frac{2x^2-9}{24}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/24*(2*x^2-9)*(-4*x^2-9)^(1/2)
```

Maxima [A] time = 3.19461, size = 35, normalized size = 1.13

$$-\frac{1}{12}\sqrt{-4x^2-9}x^2+\frac{3}{8}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-4*x^2 - 9)*x^2 + 3/8*sqrt(-4*x^2 - 9)
```

Fricas [A] time = 1.22553, size = 49, normalized size = 1.58

$$-\frac{1}{24}\left(2x^2-9\right)\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*x^2 - 9)*sqrt(-4*x^2 - 9)
```

Sympy [A] time = 0.35704, size = 31, normalized size = 1.

$$-\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**2*sqrt(-4*x**2 - 9)/12 + 3*sqrt(-4*x**2 - 9)/8
```

Giac [C] time = 1.68378, size = 31, normalized size = 1.

$$-\frac{1}{48}i\left(4x^2+9\right)^{\frac{3}{2}}+\frac{9}{16}i\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*I*(4*x^2 + 9)^(3/2) + 9/16*I*sqrt(4*x^2 + 9)
```

3.568 $\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{8}\sqrt{-4x^2-9}x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] -(x*Sqrt[-9 - 4*x^2])/8 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Rubi [A] time = 0.0072962, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 217, 203}

$$-\frac{1}{8}\sqrt{-4x^2-9}x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[-9 - 4*x^2])/8 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{8}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\ &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0059576, size = 36, normalized size = 1.

$$-\frac{1}{8}\sqrt{-4x^2-9}x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[-9 - 4*x^2])/8 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Maple [A] time = 0.003, size = 29, normalized size = 0.8

$$-\frac{9}{16}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)-\frac{x}{8}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2-9)^(1/2),x)
```

```
[Out] -9/16*arctan(2*x/(-4*x^2-9)^(1/2))-1/8*x*(-4*x^2-9)^(1/2)
```

Maxima [C] time = 3.83237, size = 26, normalized size = 0.72

$$-\frac{1}{8}\sqrt{-4x^2-9}x+\frac{9}{16}i\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^2 - 9)*x + 9/16*I*arcsinh(2/3*x)
```

Fricas [C] time = 1.2637, size = 158, normalized size = 4.39

$$-\frac{1}{8}\sqrt{-4x^2-9}x-\frac{9}{32}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)+\frac{9}{32}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 9)*x - 9/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + 9/3
2*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] time = 0.392234, size = 36, normalized size = 1.

$$-\frac{x\sqrt{-4x^2-9}}{8}-\frac{9\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x*sqrt(-4*x**2 - 9)/8 - 9*atan(2*x/sqrt(-4*x**2 - 9))/16
```

Giac [C] time = 1.53594, size = 26, normalized size = 0.72

$$-\frac{1}{8}\sqrt{-4x^2-9}x+\frac{9}{16}i\arcsin\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 - 9)*x + 9/16*I*arcsin(2/3*I*x)
```

3.569 $\int \frac{x}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

```
[Out] -Sqrt[-9 - 4*x^2]/4
```

Rubi [A] time = 0.0022589, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-9-4x^2}$$

Mathematica [A] time = 0.0014836, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/4
```

Maple [A] time = 0.002, size = 12, normalized size = 0.8

$$-\frac{1}{4}\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/4*(-4*x^2-9)^(1/2)
```

Maxima [A] time = 1.89684, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

Fricas [A] time = 1.20037, size = 31, normalized size = 2.07

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

Sympy [A] time = 0.14674, size = 14, normalized size = 0.93

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2-9)**(1/2),x)
```

```
[Out] -sqrt(-4*x**2 - 9)/4
```

Giac [C] time = 2.40221, size = 15, normalized size = 1.

$$-\frac{1}{4}i\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(4*x^2 + 9)
```

3.570 $\int \frac{1}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Rubi [A] time = 0.0032778, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - 4*x^2],x]

[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0031859, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 - 4*x^2],x]

[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{1}{2}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/2*arctan(2*x/(-4*x^2-9)^(1/2))
```

Maxima [C] time = 2.55582, size = 8, normalized size = 0.42

$$-\frac{1}{2}i\,\mathrm{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*arcsinh(2/3*x)
```

Fricas [C] time = 1.26577, size = 120, normalized size = 6.32

$$\frac{1}{4}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)-\frac{1}{4}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) - 1/4*I*log(-(8*x - 4*I*sqrt(-4*
x^2 - 9))/x)
```

Sympy [A] time = 0.318525, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2-9)**(1/2),x)
```

```
[Out] atan(2*x/sqrt(-4*x**2 - 9))/2
```

Giac [C] time = 2.44089, size = 8, normalized size = 0.42

$$-\frac{1}{2}i\arcsin\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*arcsin(2/3*I*x)
```

3.571 $\int \frac{1}{x\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Rubi [A] time = 0.0101194, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {266, 63, 204}

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4xx}}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\right)\\
&= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0023916, size = 20, normalized size = 1.

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$-\frac{1}{3}\arctan\left(3\frac{1}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/3*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] time = 1.89725, size = 34, normalized size = 1.7

$$-\frac{1}{3}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] time = 1.27353, size = 123, normalized size = 6.15

$$-\frac{1}{6}i\log\left(-\frac{2\left(i\sqrt{-4x^2-9}+3\right)}{3x}\right)+\frac{1}{6}i\log\left(-\frac{2\left(-i\sqrt{-4x^2-9}+3\right)}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*I*log(-2/3*(I*sqrt(-4*x^2 - 9) + 3)/x) + 1/6*I*log(-2/3*(-I*sqrt(-4*x^
2 - 9) + 3)/x)
```

Sympy [C] time = 1.02936, size = 8, normalized size = 0.4

$$\frac{i\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2-9)**(1/2),x)
```

```
[Out] I*asinh(3/(2*x))/3
```

Giac [C] time = 2.67627, size = 19, normalized size = 0.95

$$\frac{1}{3} \arctan\left(\frac{1}{3} i \sqrt{4 x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(1/3*I*sqrt(4*x^2 + 9))
```

3.572 $\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{-4x^2-9}}{9x}$$

```
[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

Rubi [A] time = 0.0030486, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 - 4*x^2]),x]

[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{9x}$$

Mathematica [A] time = 0.0021481, size = 18, normalized size = 1.

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 - 4*x^2]),x]

[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{1}{9x}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/9/x*(-4*x^2-9)^(1/2)
```

Maxima [A] time = 3.66322, size = 19, normalized size = 1.06

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

Fricas [A] time = 1.18863, size = 32, normalized size = 1.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

Sympy [C] time = 0.7266, size = 15, normalized size = 0.83

$$\frac{2i\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*sqrt(1 + 9/(4*x**2))/9
```

Giac [C] time = 2.56933, size = 50, normalized size = 2.78

$$\frac{i\,\sqrt{4\,x^2+9}+3i}{18\,x}+\frac{8\,x}{9\left(-4i\,\sqrt{4\,x^2+9}-12i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*(I*sqrt(4*x^2 + 9) + 3*I)/x + 8/9*x/(-4*I*sqrt(4*x^2 + 9) - 12*I)
```

3.573 $\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Rubi [A] time = 0.0160702, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 204}

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 - 4*x^2]),x]

[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{1}{18}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0215214, size = 54, normalized size = 1.38

$$
-\frac{4}{81}\sqrt{-4x^2-9}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)}{2\sqrt{\frac{4x^2}{9}+1}}-\frac{9}{8x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] (-4*Sqrt[-9 - 4*x^2]*(-9/(8*x^2) + ArcTanh[Sqrt[1 + (4*x^2)/9]]/(2*Sqrt[1 +
 (4*x^2)/9])))/81
```

Maple [A] time = 0.004, size = 30, normalized size = 0.8

$$
\frac{1}{18\,x^2}\sqrt{-4\,x^2-9}+\frac{2}{27}\arctan\left(3\,\frac{1}{\sqrt{-4\,x^2-9}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/18*(-4*x^2-9)^(1/2)/x^2+2/27*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] time = 2.95667, size = 54, normalized size = 1.38

$$
\frac{\sqrt{-4\,x^2-9}}{18\,x^2}+\frac{2}{27}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 + 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

Fricas [C] time = 1.19176, size = 174, normalized size = 4.46

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\,\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\,\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+3\,\sqrt{-4\,x^2-9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(-2*I*x^2*log(-4/27*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/27*(-
I*sqrt(-4*x^2 - 9) - 3)/x) + 3*sqrt(-4*x^2 - 9))/x^2
```

Sympy [C] time = 2.20793, size = 46, normalized size = 1.18

$$-\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}+\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -2*I*asinh(3/(2*x))/27 + I/(9*x*sqrt(1 + 9/(4*x**2))) + I/(4*x**3*sqrt(1 +
9/(4*x**2)))
```

Giac [C] time = 2.67715, size = 39, normalized size = 1.

$$\frac{i\,\sqrt{4\,x^2+9}}{18\,x^2}-\frac{2}{27}\,\arctan\left(\frac{1}{3}i\,\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*I*sqrt(4*x^2 + 9)/x^2 - 2/27*arctan(1/3*I*sqrt(4*x^2 + 9))
```

3.574 $\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

```
[Out] Sqrt[-9 - 4*x^2]/(27*x^3) - (8*Sqrt[-9 - 4*x^2])/(243*x)
```

Rubi [A] time = 0.0072743, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 - 4*x^2]),x]

[Out] Sqrt[-9 - 4*x^2]/(27*x^3) - (8*Sqrt[-9 - 4*x^2])/(243*x)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx &= \frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx\\ &= \frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8\sqrt{-9-4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.0055563, size = 25, normalized size = 0.68

$$\frac{\left(9-8x^2\right)\sqrt{-4x^2-9}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 - 4*x^2]),x]

[Out] ((9 - 8*x^2)*Sqrt[-9 - 4*x^2])/(243*x^3)
```

Maple [A] time = 0.003, size = 22, normalized size = 0.6

$$-\frac{8x^2-9}{243x^3}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/243*(8*x^2-9)/x^3*(-4*x^2-9)^(1/2)
```

Maxima [A] time = 3.60023, size = 39, normalized size = 1.05

$$-\frac{8\sqrt{-4x^2-9}}{243x}+\frac{\sqrt{-4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 - 9)/x + 1/27*sqrt(-4*x^2 - 9)/x^3
```

Fricas [A] time = 1.26705, size = 55, normalized size = 1.49

$$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 - 9)*sqrt(-4*x^2 - 9)/x^3
```

Sympy [C] time = 1.47664, size = 36, normalized size = 0.97

$$-\frac{16i\sqrt{1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -16*I*sqrt(1 + 9/(4*x**2))/243 + 2*I*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

Giac [C] time = 2.5751, size = 109, normalized size = 2.95

$$-\frac{2x^3\left(\frac{9\left(-i\sqrt{4x^2+9}-3i\right)^2}{x^2}+4\right)}{243\left(-i\sqrt{4x^2+9}-3i\right)^3}-\frac{19683i\sqrt{4x^2+9}+59049i}{1062882x}+\frac{\left(-i\sqrt{4x^2+9}-3i\right)^3}{1944x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -2/243*x^3*(9*(-I*sqrt(4*x^2 + 9) - 3*I)^2/x^2 + 4)/(-I*sqrt(4*x^2 + 9) - 3
*I)^3 - 1/1062882*(19683*I*sqrt(4*x^2 + 9) + 59049*I)/x + 1/1944*(-I*sqrt(4
*x^2 + 9) - 3*I)^3/x^3
```

3.575 $\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] Sqrt[-9 - 4*x^2]/(36*x^4) - Sqrt[-9 - 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/81
```

Rubi [A] time = 0.0232741, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(36*x^4) - Sqrt[-9 - 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/81
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x^3}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{1}{27}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}x}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}-\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{-9-4x^2}\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.0047161, size = 32, normalized size = 0.56

$$\frac{16}{729}\sqrt{-4x^2-9}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{4x^2}{9}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] (16*Sqrt[-9 - 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (4*x^2)/9])/729
```

Maple [A] time = 0.004, size = 44, normalized size = 0.8

$$\frac{1}{36\,x^4}\sqrt{-4\,x^2-9}-\frac{1}{54\,x^2}\sqrt{-4\,x^2-9}-\frac{2}{81}\arctan\left(3\,\frac{1}{\sqrt{-4\,x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(-4*x^2-9
)^(1/2))
```

Maxima [C] time = 3.63359, size = 73, normalized size = 1.28

$$-\frac{\sqrt{-4\,x^2-9}}{54\,x^2}+\frac{\sqrt{-4\,x^2-9}}{36\,x^4}-\frac{2}{81}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 - 9)/x^2 + 1/36*sqrt(-4*x^2 - 9)/x^4 - 2/81*I*log(6*sqrt(
4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] time = 1.32893, size = 192, normalized size = 3.37

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)-3\left(2\,x^2-3\right)\sqrt{-4\,x^2-9}}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(-4*I*x^4*log(-4/81*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/81*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 - 3)*sqrt(-4*x^2 - 9))/x^4
```

Sympy [C] time = 4.46695, size = 65, normalized size = 1.14

$$\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1+\frac{9}{4x^2}}}-\frac{i}{36x^3\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*asinh(3/(2*x))/81 - I/(27*x*sqrt(1 + 9/(4*x**2))) - I/(36*x**3*sqrt(1 +
 9/(4*x**2))) + I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [C] time = 2.87814, size = 58, normalized size = 1.02

$$-\frac{i\left(4\,x^2+9\right)^{\frac{3}{2}}-15i\sqrt{4\,x^2+9}}{216\,x^4}+\frac{2}{81}\arctan\left(\frac{1}{3}i\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/216*(I*(4*x^2 + 9)^(3/2) - 15*I*sqrt(4*x^2 + 9))/x^4 + 2/81*arctan(1/3*I
*sqrt(4*x^2 + 9))
```

3.576 $\int \frac{1}{\sqrt{9+bx^2}} dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rubi [A] time = 0.0021277, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+bx^2}} dx = \frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.007021, size = 17, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Maple [A] time = 0.002, size = 21, normalized size = 1.2

$$\ln\left(x\sqrt{b}+\sqrt{bx^2+9}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+9)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+9)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.26105, size = 157, normalized size = 9.24

$$\left[\frac{\log\left(-\sqrt{b}x-\sqrt{bx^2+9}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+9}\sqrt{-b}-3\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(-sqrt(b)*x - sqrt(b*x^2 + 9))/sqrt(b), -2*sqrt(-b)*arctan((sqrt(b*x^2
+ 9)*sqrt(-b) - 3*sqrt(-b))/(b*x))/b]
```

Sympy [A] time = 0.943127, size = 14, normalized size = 0.82

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+9)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/3)/sqrt(b)
```

Giac [A] time = 2.49941, size = 30, normalized size = 1.76

$$-\frac{\log\left(-\sqrt{b}x+\sqrt{bx^2+9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(b)*x + sqrt(b*x^2 + 9))/sqrt(b)
```

3.577 $\int \frac{1}{\sqrt{9-bx^2}} dx$

Optimal. Leaf size=17

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rubi [A] time = 0.0025019, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-bx^2}} \, dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0072171, size = 17, normalized size = 1.

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Maple [A] time = 0.003, size = 21, normalized size = 1.2

$$\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+9}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+9)^(1/2),x)
```

```
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+9)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.27607, size = 146, normalized size = 8.59

$$\left[-\frac{\sqrt{-b}\log\left(-\sqrt{-b}x-\sqrt{-bx^2+9}\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx^2+9}-3}{\sqrt{b}x}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-sqrt(-b)*x - sqrt(-b*x^2 + 9))/b, -2*arctan((sqrt(-b*x^2 +
9) - 3)/(sqrt(b)*x))/sqrt(b)]
```

Sympy [A] time = 1.00037, size = 39, normalized size = 2.29

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{9}>1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (asin(sqrt(b)
*x/3)/sqrt(b), True))
```

Giac [B] time = 2.40067, size = 36, normalized size = 2.12

$$-\frac{\log\left(-\sqrt{-b}x+\sqrt{-bx^2+9}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(-b)*x + sqrt(-b*x^2 + 9))/sqrt(-b)
```

3.578 $\int \frac{1}{\sqrt{-9+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0055129, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-9+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0062782, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.002, size = 21, normalized size = 0.8

$$\ln\left(x\sqrt{b}+\sqrt{b x^{2}-9}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-9)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2-9)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40248, size = 151, normalized size = 6.04

$$\left[\frac{\log\left(2\,bx^{2}+2\,\sqrt{bx^{2}-9}\sqrt{b}x-9\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}-9}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - 9)*sqrt(b)*x - 9)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - 9))/b]
```

Sympy [A] time = 1.00639, size = 39, normalized size = 1.56

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{for}\: \frac{\left|bx^{2}\right|}{9} > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (-I*asin(sqrt(b)
*x/3)/sqrt(b), True))
```

Giac [A] time = 1.62234, size = 31, normalized size = 1.24

$$-\frac{\log \left(\left| -\sqrt{b}x + \sqrt{bx^2 - 9} \right| \right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 - 9)))/sqrt(b)
```

3.579 $\int \frac{1}{\sqrt{-9-bx^2}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0045402, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-9-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0060969, size = 26, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2-9}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-9)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-9)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29094, size = 173, normalized size = 6.65

$$\left[-\frac{\sqrt{-b}\log\left(-2\,bx^2+2\,\sqrt{-bx^2-9}\sqrt{-b}x-9\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-bx^2-9}\sqrt{b}x}{bx^2+9}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - 9)*sqrt(-b)*x - 9)/b, -arctan
(sqrt(-b*x^2 - 9)*sqrt(b)*x/(b*x^2 + 9))/sqrt(b)]
```

Sympy [C] time = 0.947742, size = 17, normalized size = 0.65

$$-\frac{i\operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-9)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/3)/sqrt(b)
```

Giac [A] time = 1.38766, size = 38, normalized size = 1.46

$$-\frac{\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2-9}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - 9)))/sqrt(-b)
```

3.580 $\int \frac{1}{\sqrt{\pi+bx^2}}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rubi [A] time = 0.005851, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi+bx^2}}\,dx = \frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0075118, size = 19, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Maple [A] time = 0.005, size = 21, normalized size = 1.1

$$\ln\left(x\sqrt{b}+\sqrt{bx^2+\pi}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+Pi)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+Pi)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35324, size = 157, normalized size = 8.26

$$\left[\frac{\log\left(-\pi - 2\,bx^2 - 2\,\sqrt{\pi + bx^2}\sqrt{b}x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{\pi+bx^2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-pi - 2*b*x^2 - 2*sqrt(pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(pi + b*x^2))/b]
```

Sympy [A] time = 0.93452, size = 17, normalized size = 0.89

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+pi)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

Giac [A] time = 3.00574, size = 30, normalized size = 1.58

$$-\frac{\log\left(-\sqrt{b}x + \sqrt{\pi + bx^2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(b)*x + sqrt(pi + b*x^2))/sqrt(b)
```

3.581 $\int \frac{1}{\sqrt{\pi-bx^2}} dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

```
[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rubi [A] time = 0.0030614, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi - b*x^2],x]

[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi - bx^2}} dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0071897, size = 19, normalized size = 1.

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi - b*x^2],x]

[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Maple [A] time = 0.006, size = 21, normalized size = 1.1

$$\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+\pi}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+Pi)^(1/2),x)
```

```
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+Pi)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28524, size = 159, normalized size = 8.37

$$\left[-\frac{\sqrt{-b}\log\left(-\pi + 2\,bx^2 - 2\,\sqrt{\pi - bx^2}\sqrt{-b}x\right)}{2\,b}, -\frac{\arctan\left(-\frac{\sqrt{b}x}{\sqrt{\pi - bx^2}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-pi + 2*b*x^2 - 2*sqrt(pi - b*x^2)*sqrt(-b)*x)/b, -arcta
n(-sqrt(b)*x/sqrt(pi - b*x^2))/sqrt(b)]
```

Sympy [A] time = 1.01061, size = 46, normalized size = 2.42

$$\begin{cases} -\frac{i\,\mathrm{acosh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{\pi} > 1 \\ \frac{\mathrm{asin}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

Giac [B] time = 2.15331, size = 38, normalized size = 2.

$$-\frac{\log\left(\left|-\sqrt{-b}x + \sqrt{\pi - bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(pi - b*x^2)))/sqrt(-b)
```

3.582 $\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0055478, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-\pi+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-\pi+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-\pi+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0062065, size = 27, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.003, size = 23, normalized size = 0.9

$$\ln\left(x\sqrt{b}+\sqrt{bx^2-\pi}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-Pi)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2-Pi)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30512, size = 178, normalized size = 6.59

$$\left[\frac{\log\left(-\pi+2\,bx^2+2\,\sqrt{-\pi+bx^2}\sqrt{b}x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(-\frac{\sqrt{-\pi+bx^2}\sqrt{-b}x}{\pi-bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-pi + 2*b*x^2 + 2*sqrt(-pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*
arctan(-sqrt(-pi + b*x^2)*sqrt(-b)*x/(pi - b*x^2))/b]
```

Sympy [A] time = 1.02321, size = 46, normalized size = 1.7

$$\begin{cases}\frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{\pi}>1\\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-pi)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (-I*asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

Giac [A] time = 1.62224, size = 34, normalized size = 1.26

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{-\pi+bx^2}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(-pi + b*x^2)))/sqrt(b)
```

3.583 $\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0052011, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-\pi-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-\pi-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0056439, size = 28, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.004, size = 23, normalized size = 0.8

$$\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2-\pi}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-Pi)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43177, size = 178, normalized size = 6.36

$$\left[-\frac{\sqrt{-b}\log\left(-\pi-2\,bx^2+2\,\sqrt{-\pi-bx^2}\sqrt{-b}x\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-\pi-bx^2}\sqrt{b}x}{\pi+bx^2}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-pi - 2*b*x^2 + 2*sqrt(-pi - b*x^2)*sqrt(-b)*x)/b, -arct
an(sqrt(-pi - b*x^2)*sqrt(b)*x/(pi + b*x^2))/sqrt(b)]
```

Sympy [C] time = 0.947158, size = 20, normalized size = 0.71

$$-\frac{i\,\text{asinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-pi)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

Giac [A] time = 1.50849, size = 41, normalized size = 1.46

$$-\frac{\log\left(\left|-\sqrt{-b}x+\sqrt{-\pi-bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-pi - b*x^2)))/sqrt(-b)
```

3.584 $\int \frac{1}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.005355, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0054723, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.001, size = 21, normalized size = 0.8

$$\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26307, size = 153, normalized size = 6.12

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 0.996455, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A] time = 1.69271, size = 31, normalized size = 1.24

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.585 $\int \frac{1}{\sqrt{a-bx^2}} \, dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0048264, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*x^2],x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a-bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1+bx^2} \, dx, x, \frac{x}{\sqrt{a-bx^2}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0055323, size = 26, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.20117, size = 171, normalized size = 6.58

$$\left[-\frac{\sqrt{-b}\log\left(2\,bx^2-2\,\sqrt{-bx^2+a}\sqrt{-b}x-a\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-bx^2+a}\sqrt{b}x}{bx^2-a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*x^2 - 2*sqrt(-b*x^2 + a)*sqrt(-b)*x - a)/b, -arctan(
sqrt(-b*x^2 + a)*sqrt(b)*x/(b*x^2 - a))/sqrt(b)]
```

Sympy [A] time = 1.05413, size = 48, normalized size = 1.85

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{|a|}>1\\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2)/Abs(a) > 1), (a
sin(sqrt(b)*x/sqrt(a))/sqrt(b), True))
```

Giac [A] time = 2.08344, size = 38, normalized size = 1.46

$$-\frac{\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2+a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/sqrt(-b)
```

3.586 $\int \frac{1}{\sqrt{-a+bx^2}} \, dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0056156, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-a+bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{-a+bx^2}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.005989, size = 27, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.002, size = 23, normalized size = 0.9

$$\ln\left(x\sqrt{b}+\sqrt{bx^2-a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2-a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31623, size = 151, normalized size = 5.59

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-a}\sqrt{b}x-a\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2-a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - a))/b]
```

Sympy [A] time = 1.06986, size = 48, normalized size = 1.78

$$\begin{cases}\frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{|a|}>1\\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2)/Abs(a) > 1), (-I*a
sin(sqrt(b)*x/sqrt(a))/sqrt(b), True))
```

Giac [A] time = 2.45779, size = 34, normalized size = 1.26

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2-a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 - a)))/sqrt(b)
```

3.587 $\int \frac{1}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.004854, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0058657, size = 28, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-b x^2-a}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36821, size = 173, normalized size = 6.18

$$\left[-\frac{\sqrt{-b}\log\left(-2\,b x^2+2\,\sqrt{-b x^2-a}\sqrt{-b}x-a\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-b x^2-a}\sqrt{b}x}{b x^2+a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - a)*sqrt(-b)*x - a)/b, -arctan
(sqrt(-b*x^2 - a)*sqrt(b)*x/(b*x^2 + a))/sqrt(b)]
```

Sympy [C] time = 1.00626, size = 20, normalized size = 0.71

$$-\frac{i\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A] time = 1.67519, size = 41, normalized size = 1.46

$$-\frac{\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2-a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - a)))/sqrt(-b)
```

3.588 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi [A] time = 0.0022067, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]

[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0033855, size = 16, normalized size = 1.

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]

[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x)
```

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Maxima [A] time = 2.80453, size = 11, normalized size = 0.69

$$\arcsin\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x/sqrt(a^2))
```

Fricas [A] time = 1.20807, size = 50, normalized size = 3.12

$$-2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [A] time = 0.986283, size = 20, normalized size = 1.25

$$\begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \frac{|x^2|}{|a^2|} > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x/a), Abs(x**2)/Abs(a**2) > 1), (asin(x/a), True))
```

Giac [A] time = 2.17221, size = 12, normalized size = 0.75

$$\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x/a)*sgn(a)
```

3.589 $\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=184

$$\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}-\frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{4ac(}{}$$

```
[Out] (-20*a^2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*c*(c*x)^(5/2)*Sqrt
[a + b*x^2])/(77*b) + (2*(c*x)^(9/2)*Sqrt[a + b*x^2])/(11*c) + (10*a^(11/4)
*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)
*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.132983, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {279, 321, 329, 220}

$$-\frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{4ac(cx)^{5/2}\sqrt{}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (-20*a^2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*c*(c*x)^(5/2)*Sqrt
[a + b*x^2])/(77*b) + (2*(c*x)^(9/2)*Sqrt[a + b*x^2])/(11*c) + (10*a^(11/4)
*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)
*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{1}{11}(2a)\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}-\frac{\left(10a^2c^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{77b}$$

$$= -\frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{\left(10a^3c^4\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{231b^2}$$

$$= -\frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{\left(20a^3c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx \cdots\right.}{231b^2}$$

$$= -\frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a\cdots}{(\sqrt{a}\cdots}}}{231b^{9/4}\sqrt{\cdots}}$$

Mathematica [C] time = 0.0619809, size = 103, normalized size = 0.56

$$\frac{2c^3\sqrt{cx}\sqrt{a+bx^2}\left(\sqrt{\frac{bx^2}{a}+1}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 +
 7*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b
^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.042, size = 152, normalized size = 0.8

$$\frac{2\,c^3}{231\,b^3x}\sqrt{cx}\left(21\,x^7b^4+5\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-ab}a^3+27\,x^5ab^3\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] 2/231/x*c^3*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(21*x^7*b^4+5*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*(-a*b)^(1/2)*a^3+27*x^5*a*b^3-4*x^3*a^2*b^2-10*x*a^3*b)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\,(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^2 + a}\sqrt{cx}c^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^3*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\,(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

3.590 $\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=301

$$-\frac{2a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+b}{\left(\sqrt{a}+\sqrt{}\right.}}}{15b^{7/4}\sqrt{a}}$$

```
[Out] (4*a*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(45*b) + (2*(c*x)^(7/2)*Sqrt[a + b*x^2]
)/(9*c) - (4*a^2*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt
[b]*x)) + (4*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]) - (2*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.229721, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 321, 329, 305, 220, 1196}

$$-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\right.}{15b^{7/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (4*a*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(45*b) + (2*(c*x)^(7/2)*Sqrt[a + b*x^2]
)/(9*c) - (4*a^2*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt
[b]*x)) + (4*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]) - (2*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{5/2}\sqrt{a+bx^2}\,dx &= \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}+\frac{1}{9}(2a)\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{\left(2a^2c^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{15b}\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{\left(4a^2c\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15b}\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{\left(4a^{5/2}c^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15b^{3/2}}+\frac{\left(4a^{5/2}c^2\right)\cdots}{\cdots}\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}+\cdots\right.}}}{15b^{7/4}\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.0459954, size = 85, normalized size = 0.28

$$
\frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyper
geometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.02, size = 221, normalized size = 0.7

$$-\frac{2c^2}{45b^2x}\sqrt{cx}\left(-5b^3x^6+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^3-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] -2/45/x*c^2*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-5*b^3*x^6+6*((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x
*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*a^3-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-7*a*b^2*x^4-2*a^2*b*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^2+a}\sqrt{cx}c^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^2*x^2, x)
```

Sympy [C] time = 52.4007, size = 46, normalized size = 0.15

$$\frac{\sqrt{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

3.591 $\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=153

$$-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}$$

```
[Out] (4*a*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b) + (2*(c*x)^(5/2)*Sqrt[a + b*x^2])/
(7*c) - (2*a^(7/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(21*b^(5/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0904466, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {279, 321, 329, 220}

$$-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (4*a*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b) + (2*(c*x)^(5/2)*Sqrt[a + b*x^2])/
(7*c) - (2*a^(7/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(21*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int (cx)^{3/2}\sqrt{a+bx^2}\,dx &= \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{1}{7}(2a)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx\\ &= \frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{\left(2a^2c^2\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{21b}\\ &= \frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{\left(4a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{21b}\\ &= \frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right|}{21b^{5/4}\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0412336, size = 85, normalized size = 0.56

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyperge
ometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.017, size = 138, normalized size = 0.9

$$-\frac{2\,c}{21\,b^2x}\sqrt{cx}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] -2/21/x*c*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(
1/2)*a^2-3*b^3*x^5-5*a*b^2*x^3-2*a^2*b*x)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2+a}\sqrt{cx}cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x, x)
```

Sympy [C] time = 4.26497, size = 46, normalized size = 0.3

$$\frac{\sqrt{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

3.592 $\int \sqrt{cx}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=269

$$\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right.\right.}{5b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] (2*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c) + (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*
Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(5/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^2]) + (2*a^(5/4)*Sqr
t[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[
a + b*x^2])
```

Rubi [A] time = 0.193773, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 329, 305, 220, 1196}

$$\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a + b*x^2],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c) + (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*
Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(5/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^2]) + (2*a^(5/4)*Sqr
t[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[
a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c} + \frac{1}{5}(2a)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c} + \frac{(4a)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5c}$$

$$= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c} + \frac{\left(4a^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5\sqrt{b}} - \frac{\left(4a^{3/2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5\sqrt{b}}$$

$$= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} +$$

Mathematica [C] time = 0.0119266, size = 56, normalized size = 0.21

$$\frac{2x\sqrt{cx}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a + b*x^2],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/
a)])/(3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.018, size = 205, normalized size = 0.8

$$\frac{2}{5\,bx}\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2-\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] 2/5*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2
)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^2+b^2*x^4+a*b*x^2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + a}\sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Sympy [C] time = 0.960508, size = 46, normalized size = 0.17

$$\frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_p
olar(I*pi)/a)/(2*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

3.593 $\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=126

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c) + (2*a^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0743437, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {279, 329, 220}

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c) + (2*a^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c} + \frac{1}{3}(2a)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx$$

$$= \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c} + \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c}$$

$$= \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c} + \frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0122697, size = 54, normalized size = 0.43

$$\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/(Sqrt
[c*x]*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.018, size = 119, normalized size = 0.9

$$\frac{2}{3b}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}a+b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+b^2*x^3+a*b
*x)/(c*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c*x), x)
```

Sympy [C] time = 0.843216, size = 46, normalized size = 0.37

$$\frac{\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi
)/a)/(2*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

3.594 $\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=263

$$\frac{2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}+\frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{I}}{c^{3/2}\sqrt{a+bx}}$$

```
[Out] (-2*Sqrt[a + b*x^2])/(c*Sqrt[c*x]) + (4*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/
(c^2*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(1/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x^2]) + (2*a^(1/4)*b^(1/4)*
(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x
^2])
```

Rubi [A] time = 0.194235, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 329, 305, 220, 1196}

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\right.}{c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(c*Sqrt[c*x]) + (4*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/
(c^2*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(1/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x^2]) + (2*a^(1/4)*b^(1/4)*
(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x
^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{(2b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{\left(4\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c^2} - \frac{\left(4\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}} + \frac{2\sqrt[4]{a}\cdots}{\cdots}$$

Mathematica [C] time = 0.0137922, size = 54, normalized size = 0.21

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/((c
*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.026, size = 194, normalized size = 0.7

$$2\frac{1}{\sqrt{bx^2+ac}\sqrt{cx}}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] 2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*
EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-b*x^2-a)/(
b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{cx}}{c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c^2*x^2), x)
```

Sympy [C] time = 1.37052, size = 49, normalized size = 0.19

$$\frac{\sqrt{a}\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(
2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

3.595 $\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

```
[Out] (-2*Sqrt[a + b*x^2])/(3*c*(c*x)^(3/2)) + (2*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(1/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0757863, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {277, 329, 220}

$$\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(3*c*(c*x)^(3/2)) + (2*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(1/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{a}c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0132762, size = 56, normalized size = 0.44

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(3*
(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.019, size = 120, normalized size = 1.

$$\frac{2}{3\,xc^2}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x-b*x^2-a)/
c^2/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] time = 4.62551, size = 49, normalized size = 0.39

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(
2*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

3.596 $\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*Sqrt[a + b*x^2])/(5*c*(c*x)^(5/2)) - (4*b*Sqrt[a + b*x^2])/(5*a*c^3*Sqr
t[c*x]) + (4*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a*c^4*(Sqrt[a] + Sqrt[b]
*x)) - (4*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*
a^(3/4)*c^(7/2)*Sqrt[a + b*x^2]) + (2*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(
a^(1/4)*Sqrt[c])], 1/2])/(5*a^(3/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.222403, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 325, 329, 305, 220, 1196}

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}+\frac{4b^{3/2}\sqrt{}}{5ac^4(}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(5*c*(c*x)^(5/2)) - (4*b*Sqrt[a + b*x^2])/(5*a*c^3*Sqr
t[c*x]) + (4*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a*c^4*(Sqrt[a] + Sqrt[b]
*x)) - (4*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*
a^(3/4)*c^(7/2)*Sqrt[a + b*x^2]) + (2*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(
a^(1/4)*Sqrt[c])], 1/2])/(5*a^(3/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} + \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{5c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{\left(2b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5ac^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5ac^5}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{\left(4b^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5\sqrt{a}c^4} - \frac{\left(4b^{3/2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{\right.}{5\sqrt{a}c^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{4b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right|}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0126724, size = 56, normalized size = 0.18

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/(5
*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.025, size = 219, normalized size = 0.7

$$\frac{2}{5\,x^2c^3a}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)x^2ab-\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
x^2*a*b-2*b^2*x^4-3*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{c^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c^4*x^4), x)
```

Sympy [C] time = 62.677, size = 53, normalized size = 0.17

$$\frac{\sqrt{a}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(7/2),x)
```

```
[Out] sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

3.597 $\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=212

$$\frac{4a^{15/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}-\frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{2(c$$

```
[Out] (-8*a^3*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (8*a^2*c*(c*x)^(5/2)*Sqr
t[a + b*x^2])/(385*b) + (4*a*(c*x)^(9/2)*Sqrt[a + b*x^2])/(55*c) + (2*(c*x)
^(9/2)*(a + b*x^2)^(3/2))/(15*c) + (4*a^(15/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.130183, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {279, 321, 329, 220}

$$-\frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{4a^{15/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{2(cx)^{9/2}(a}{15$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```


```
[Out] (-8*a^3*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (8*a^2*c*(c*x)^(5/2)*Sqr
t[a + b*x^2])/(385*b) + (4*a*(c*x)^(9/2)*Sqrt[a + b*x^2])/(55*c) + (2*(c*x)
^(9/2)*(a + b*x^2)^(3/2))/(15*c) + (4*a^(15/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (cx)^{7/2}\left(a+bx^2\right)^{3/2}dx = \frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{1}{5}(2a)\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$$

$$= \frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{1}{55}\left(4a^2\right)\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}dx$$

$$= \frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}-\frac{\left(4a^3c^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}dx}{77b}$$

$$= -\frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\cdots$$

$$= -\frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\cdots$$

$$= -\frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\cdots$$

Mathematica [C] time = 0.0772995, size = 102, normalized size = 0.48

$$\frac{2c^3\sqrt{cx}\sqrt{a+bx^2}\left(5a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(5a-11bx^2\right)\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}\right)}{165b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 +
 (b*x^2)/a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165
*b^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.019, size = 163, normalized size = 0.8

$$\frac{2\,c^3}{1155\,b^3x}\sqrt{cx}\left(77\,b^5x^9+196\,ab^4x^7+10\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-a}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] 2/1155/x*c^3*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(77*b^5*x^9+196*a*b^4*x^7+10*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
```

```
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^4+131*a^2*b^3*x^5-8*a^3*b^2*x^3-20*a^
4*b*x)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc^3x^5+ac^3x^3\right)\sqrt{bx^2+a}\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*c^3*x^5 + a*c^3*x^3)*sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

3.598 $\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=329

$$-\frac{4a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}-\frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{8a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a\cdots}{(\sqrt{a}\cdots}}}{65b^{7/4}\sqrt{\cdots}}$$

```
[Out] (8*a^2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(195*b) + (4*a*(c*x)^(7/2)*Sqrt[a + b
*x^2])/(39*c) - (8*a^3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(65*b^(3/2)*(Sqrt[a]
+ Sqrt[b]*x)) + (2*(c*x)^(7/2)*(a + b*x^2)^(3/2))/(13*c) + (8*a^(13/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(65*b^(7/4)*Sqrt[
a + b*x^2]) - (4*a^(13/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.255973, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 321, 329, 305, 220, 1196}

$$-\frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{4a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}+\frac{8a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(\cdots\right)}{65b^{7/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (8*a^2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(195*b) + (4*a*(c*x)^(7/2)*Sqrt[a + b
*x^2])/(39*c) - (8*a^3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(65*b^(3/2)*(Sqrt[a]
+ Sqrt[b]*x)) + (2*(c*x)^(7/2)*(a + b*x^2)^(3/2))/(13*c) + (8*a^(13/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(65*b^(7/4)*Sqrt[
a + b*x^2]) - (4*a^(13/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{5/2}\left(a+bx^2\right)^{3/2}\,dx &= \frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{1}{13}(6a)\int (cx)^{5/2}\sqrt{a+bx^2}\,dx\\
&= \frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{1}{39}\left(4a^2\right)\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(4a^3c^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{65b}\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(8a^3c\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{b}{c}\cdots}}\cdots\right.}{65b}\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(8a^{7/2}c^2\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{a\cdots}}\cdots\right.}{65b^{3/2}}\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}-\frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\cdots
\end{aligned}
$$

Mathematica [C] time = 0.0621499, size = 89, normalized size = 0.27

$$
\frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{13b\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*H
ypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.021, size = 232, normalized size = 0.7

$$-\frac{2c^2}{195\,b^2x}\sqrt{cx}\left(-15\,x^8b^4-40\,x^6ab^3+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)a^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] -2/195/x*c^2*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-15*x^8*b^4-40*x^6*a*b^3+12*(
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))*a^4-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4-29*x^4*a^2
*b^2-4*x^2*a^3*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bc^2x^4+ac^2x^2\right)\sqrt{bx^2+a}\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*c^2*x^4 + a*c^2*x^2)*sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Sympy [C] time = 91.753, size = 46, normalized size = 0.14

$$\frac{a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-3/2, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)
```

3.599 $\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=181

$$-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}}+\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/}}{}$$

```
[Out] (8*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(77*b) + (12*a*(c*x)^(5/2)*Sqrt[a + b*x
^2])/(77*c) + (2*(c*x)^(5/2)*(a + b*x^2)^(3/2))/(11*c) - (4*a^(11/4)*c^(3/2
)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(77*b^(5/4)*Sqrt[a
+ b*x^2])
```

Rubi [A] time = 0.104904, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {279, 321, 329, 220}

$$-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}}+\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx\right.}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (8*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(77*b) + (12*a*(c*x)^(5/2)*Sqrt[a + b*x
^2])/(77*c) + (2*(c*x)^(5/2)*(a + b*x^2)^(3/2))/(11*c) - (4*a^(11/4)*c^(3/2
)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(77*b^(5/4)*Sqrt[a
+ b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (cx)^{3/2}\left(a+bx^2\right)^{3/2}\,dx = \frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}+\frac{1}{11}(6a)\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$$

$$=\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}+\frac{1}{77}\left(12a^2\right)\int\frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

$$=\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{\left(4a^3c^2\right)\int\frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{77b}$$

$$=\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{\left(8a^3c\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\right.}{77b}$$

$$=\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}{77}$$

Mathematica [C] time = 0.0536811, size = 89, normalized size = 0.49

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hyp
ergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.011, size = 150, normalized size = 0.8

$$-\frac{2c}{77b^2x}\sqrt{cx}\left(-7x^7b^4+2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}a^3-20x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] -2/77/x*c*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-7*x^7*b^4+2*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*(-a*b)^(1/2)*a^3-20*x^5*a*b^3-17*x^3*a^2*b^2-4*x*a^3*b)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bcx^3+acx\right)\sqrt{bx^2+a}\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^3 + a*c*x)*sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Sympy [C] time = 15.5486, size = 46, normalized size = 0.25

$$\frac{a^{\frac{3}{2}}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

3.600 $\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=297

$$\frac{4a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{15b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] (4*a*(c*x)^(3/2)*Sqrt[a + b*x^2])/(15*c) + (8*a^2*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (2*(c*x)^(3/2)*(a + b*x^2)^(3/2))/(9
*c) - (8*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(15*b^(3/4)*Sqrt[a + b*x^2]) + (4*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.223943, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

```
[Out] (4*a*(c*x)^(3/2)*Sqrt[a + b*x^2])/(15*c) + (8*a^2*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (2*(c*x)^(3/2)*(a + b*x^2)^(3/2))/(9
*c) - (8*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(15*b^(3/4)*Sqrt[a + b*x^2]) + (4*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx}\left(a+bx^2\right)^{3/2}dx &= \frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{1}{3}(2a)\int \sqrt{cx}\sqrt{a+bx^2}\,dx\\ &= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{1}{15}\left(4a^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx\\ &= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{\left(8a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15c}\\ &= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{\left(8a^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15\sqrt{b}}-\frac{\left(8a^{5/2}\right)\text{Sub}}{}\\ &= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx}{(\sqrt{a}+\sqrt{}}}}{15b^{3/4}\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.0118379, size = 57, normalized size = 0.19

$$\frac{2ax\sqrt{cx}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2
)/a)])/(3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.01, size = 218, normalized size = 0.7

$$\frac{2}{45\,bx}\sqrt{cx}\left(5\,b^3x^6+12\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^3-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] 2/45*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(5*b^3*x^6+12*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*a^3-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+16*a*b^2*x^4+11*a^2*b*x^2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

Sympy [C] time = 4.2133, size = 46, normalized size = 0.15

$$\frac{a^{\frac{3}{2}} \sqrt{c} x^{\frac{3}{2}} \Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

3.601 $\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=152

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}$$

```
[Out] (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(7*c) + (2*Sqrt[c*x]*(a + b*x^2)^(3/2))/(7*
c) + (4*a^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(7*b^
(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0904508, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {279, 329, 220}

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
[Out] (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(7*c) + (2*Sqrt[c*x]*(a + b*x^2)^(3/2))/(7*
c) + (4*a^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(7*b^
(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c} + \frac{1}{7}(6a)\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx$$
$$= \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c} + \frac{1}{7}\left(4a^2\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx$$
$$= \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c} + \frac{\left(8a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{7c}$$
$$= \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c} + \frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0104361, size = 55, normalized size = 0.36

$$\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
[Out] (2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/(Sq
rt[c*x]*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.01, size = 134, normalized size = 0.9

$$\frac{2}{7b}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}a^2+b^3x^5+4ab^2x^3+3a^2bx\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(1/2),x)
```

```
[Out] 2/7/(b*x^2+a)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+b^3*x^5
+4*a*b^2*x^3+3*a^2*b*x)/b/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c*x), x)
```

Sympy [C] time = 2.83888, size = 46, normalized size = 0.3

$$\frac{a^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

3.602 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{12a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right.\right.}{5c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (12*b*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c^3) + (24*a*Sqrt[b]*Sqrt[c*x]*Sqrt[a
 + b*x^2])/(5*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(c*Sqrt[c*
x]) - (24*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1
/2])/(5*c^(3/2)*Sqrt[a + b*x^2]) + (12*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.229691, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 279, 329, 305, 220, 1196}

$$\frac{12a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
[Out] (12*b*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c^3) + (24*a*Sqrt[b]*Sqrt[c*x]*Sqrt[a
 + b*x^2])/(5*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(c*Sqrt[c*
x]) - (24*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1
/2])/(5*c^(3/2)*Sqrt[a + b*x^2]) + (12*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{3/2}}\,dx = -\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{(6b)\int \sqrt{cx}\sqrt{a+bx^2}\,dx}{c^2}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{(12ab)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5c^2}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{(24ab)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^3}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{\left(24a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^2}-\frac{\left(24a^{3/2}\sqrt{b}\right)\text{Su}}{}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}+\frac{24a\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{5c^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}}{5c^{3/2}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0127858, size = 55, normalized size = 0.19

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/(
(c*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.014, size = 208, normalized size = 0.7

$$\frac{2}{5c}\left(12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(3/2),x)
```

```
[Out] 2/5*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+b
^2*x^4-4*a*b*x^2-5*a^2)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^2*x^2), x)
```

Sympy [C] time = 2.90356, size = 49, normalized size = 0.17

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(3/2),x)
```

```
[Out] a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

3.603 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}+\frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}$$

```
[Out] (4*b*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c^3) - (2*(a + b*x^2)^(3/2))/(3*c*(c*x)^
(3/2)) + (4*a^(3/4)*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
 1/2])/(3*c^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0927667, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {277, 279, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}+\frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (4*b*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c^3) - (2*(a + b*x^2)^(3/2))/(3*c*(c*x)^
(3/2)) + (4*a^(3/4)*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
 1/2])/(3*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{5/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{(2b)\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx}{c^2}\\ &= \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{(4ab)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3c^2}\\ &= \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{(8ab)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\ &= \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0128064, size = 57, normalized size = 0.38

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])/(
3*(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.012, size = 125, normalized size = 0.8

$$\frac{2}{3\,xc^2}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}xa+b^2x^4-a^2\right)\frac{1}{\sqrt{cx}}\frac{1}{\sqrt{bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a+b^2*x
^4-a^2)/c^2/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] time = 7.81354, size = 49, normalized size = 0.32

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2}, -\frac{3}{4} \\ \frac{1}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] a**(3/2)*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

3.604 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=297

$$\frac{12\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}+\frac{24b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{24\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{}{}}}{5c^{7/2}}$$

```
[Out] (-12*b*Sqrt[a + b*x^2])/(5*c^3*Sqrt[c*x]) + (24*b^(3/2)*Sqrt[c*x]*Sqrt[a +
b*x^2])/(5*c^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(5*c*(c*x)^(5
/2)) - (24*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(5*c^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.227211, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 329, 305, 220, 1196}

$$\frac{24b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{12\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}-\frac{24\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E}{5c^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
[Out] (-12*b*Sqrt[a + b*x^2])/(5*c^3*Sqrt[c*x]) + (24*b^(3/2)*Sqrt[c*x]*Sqrt[a +
b*x^2])/(5*c^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(5*c*(c*x)^(5
/2)) - (24*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(5*c^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{7/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{(6b)\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx}{5c^2}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{\left(12b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5c^4}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{\left(24b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^5}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{\left(24\sqrt{a}b^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^4}-\frac{\left(24\sqrt{a}b^{3/2}\right)\operatorname{Subst}\left(\int\right.}{5c^4}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}+\frac{24b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}-\frac{24\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan\right.}{5c^{7/2}\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.012294, size = 57, normalized size = 0.19

$$
-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)])/
(5*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.014, size = 216, normalized size = 0.7

$$\frac{2}{5x^2c^3}\left(12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-6*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*x^2*a*b-7*b^2*x^4-8*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^4*x^4), x)
```

Sympy [C] time = 62.7262, size = 53, normalized size = 0.18

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},-\frac{5}{4}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(7/2),x)
```

```
[Out] a**(3/2)*gamma(-5/4)*hyper((-3/2, -5/4), (-1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

3.605 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}} \, dx$

Optimal. Leaf size=152

$$\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}c^{9/2}\sqrt{a+bx^2}}-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2\left(a+bx^2\right)^{3/2}}{7c(cx)^{7/2}}$$

```
[Out] (-4*b*Sqrt[a + b*x^2])/(7*c^3*(c*x)^(3/2)) - (2*(a + b*x^2)^(3/2))/(7*c*(c*
x)^(7/2)) + (4*b^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(7*a^(1/4)*c^(9/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0917747, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {277, 329, 220}

$$\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}c^{9/2}\sqrt{a+bx^2}}-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2\left(a+bx^2\right)^{3/2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
```


```
[Out] (-4*b*Sqrt[a + b*x^2])/(7*c^3*(c*x)^(3/2)) - (2*(a + b*x^2)^(3/2))/(7*c*(c*
x)^(7/2)) + (4*b^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(7*a^(1/4)*c^(9/2)*Sqrt[a + b*x^2])
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = -\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{(6b)\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx}{7c^2}$$
$$= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{(4b^2)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{7c^4}$$
$$= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{7c^5}$$
$$= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{4b^{7/4}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}c^{9/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0146829, size = 57, normalized size = 0.38

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)])/
(7*(c*x)^(9/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.023, size = 135, normalized size = 0.9

$$\frac{2}{7x^3c^4}\left(2\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^3b-3b^2x^4-4abx^2-a^2\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(9/2),x)
```

```
[Out] 2/7/(b*x^2+a)^(1/2)/x^3*(2*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(
1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b-3
*b^2*x^4-4*a*b*x^2-a^2)/c^4/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

3.606 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=331

$$\frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}+\frac{8}{1}$$

```
[Out] (-4*b*Sqrt[a + b*x^2])/(15*c^3*(c*x)^(5/2)) - (8*b^2*Sqrt[a + b*x^2])/(15*a
*c^5*Sqrt[c*x]) + (8*b^(5/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*a*c^6*(Sqrt[a]
+ Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) - (8*b^(9/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*a^(3/4)*c^(11/2)*Sqrt[a
 + b*x^2]) + (4*b^(9/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(15*a^(3/4)*c^(11/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.270284, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 325, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}+\frac{8b^{5/2}\sqrt{cx}}{15ac^6\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-4*b*Sqrt[a + b*x^2])/(15*c^3*(c*x)^(5/2)) - (8*b^2*Sqrt[a + b*x^2])/(15*a
*c^5*Sqrt[c*x]) + (8*b^(5/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*a*c^6*(Sqrt[a]
+ Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) - (8*b^(9/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*a^(3/4)*c^(11/2)*Sqrt[a
 + b*x^2]) + (4*b^(9/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(15*a^(3/4)*c^(11/2)*Sqrt[a + b*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{(2b)\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx}{3c^2}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{15c^4}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(4b^3\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{15ac^6}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(8b^3\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15ac^7}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(8b^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15\sqrt{a}c^6}-\frac{\left(8b^{5/2}\right)\cdots}{\cdots}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}+\frac{8b^{5/2}\sqrt{cx}\sqrt{a+bx^2}}{15ac^6\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+b\cdots}{\left(\sqrt{a}+\sqrt{\cdots}\right.\cdots}}\cdots}{15a^{3/4}c^{11/\cdots}}$$

Mathematica [C] time = 0.0131863, size = 57, normalized size = 0.17

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)])/
(9*(c*x)^(11/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.029, size = 234, normalized size = 0.7

$$\frac{2}{45\,ax^4c^5}\left(12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^4ab^2-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] 2/45/x^4*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b^2-6*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*x^4*a*b^2-12*b^3*x^6-23*a*b^2*x^4-16*a^2*b*x^2-5*a^3)/(b*x^2+a)^(1/2)
/a/c^5/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

3.607 $\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=128

$$-\frac{3\sqrt[4]{6}ac^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}-\frac{2}{15}c\sqrt{3a-2ax^2}(cx)^{3/2}$$

```
[Out] (-2*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/15 + (2*(c*x)^(7/2)*Sqrt[3*a - 2*a*x
^2])/(9*c) - (3*6^(1/4)*a*c^2*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sq
rt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rubi [A] time = 0.0639806, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {279, 321, 320, 319, 318, 424}

$$-\frac{3\sqrt[4]{6}ac^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}-\frac{2}{15}c\sqrt{3a-2ax^2}(cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```


```
[Out] (-2*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/15 + (2*(c*x)^(7/2)*Sqrt[3*a - 2*a*x
^2])/(9*c) - (3*6^(1/4)*a*c^2*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sq
rt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
```

```
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{1}{3}(2a)\int\frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{1}{5}\left(3ac^2\right)\int\frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{\left(3ac^2\sqrt{cx}\right)\int\frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{5\sqrt{x}}\\
&= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{\left(3ac^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int\frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}-\frac{\left(3\sqrt[4]{2}3^{3/4}ac^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int\frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\right.}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}-\frac{3\sqrt[4]{6}ac^2\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right.}{5\sqrt{x}\sqrt{3a-2ax^2}}\end{aligned}$$

Mathematica [C] time = 0.0289085, size = 74, normalized size = 0.58

$$\frac{c\sqrt{a\left(3-2x^2\right)}(cx)^{3/2}\left(3\sqrt{3}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)-\left(3-2x^2\right)^{3/2}\right)}{9\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*(c*x)^(3/2)*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeo
metric2F1[-1/2, 3/4, 7/4, (2*x^2)/3]))/(9*Sqrt[3 - 2*x^2])
```

Maple [B] time = 0.045, size = 237, normalized size = 1.9

$$\frac{c^2}{180\,x\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(80\,x^6+18\,\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/180/x*c^2*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(80*x^6+18*2^(1/2)*((2*x+2^(1/
2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))
^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2
*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-9*2^(1/2)*((2*x+2^(
1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(
(2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-168*x^4+72*x^2)/(
2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2\,ax^2 + 3\,a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-2\,ax^2 + 3\,a}\sqrt{cx}c^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x^2, x)
```

Sympy [C] time = 43.4541, size = 53, normalized size = 0.41

$$\frac{\sqrt{3}\sqrt{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), 2*
x**2*exp_polar(2*I*pi)/3)/(2*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

3.608 $\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=117

$$\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{7\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}-\frac{2}{7}c\sqrt{3a-2ax^2}\sqrt{cx}$$

```
[Out] (-2*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/7 + (2*(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]
)/(7*c) + (6^(3/4)*a*c^(3/2)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(7*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] time = 0.0774048, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {279, 321, 329, 224, 221}

$$\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{7\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}-\frac{2}{7}c\sqrt{3a-2ax^2}\sqrt{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (-2*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/7 + (2*(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]
)/(7*c) + (6^(3/4)*a*c^(3/2)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(7*Sqrt[a*(3 - 2*x^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}(6a)\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx\\ &= -\frac{2}{7}c\sqrt{cx}\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}\left(3ac^2\right)\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx\\ &= -\frac{2}{7}c\sqrt{cx}\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}(6ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)\\ &= -\frac{2}{7}c\sqrt{cx}\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{\left(2\sqrt{3}ac\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{7\sqrt{a\left(3-2x^2\right)}}\\ &= -\frac{2}{7}c\sqrt{cx}\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{7\sqrt{a\left(3-2x^2\right)}}\end{aligned}$$

Mathematica [C] time = 0.0232594, size = 74, normalized size = 0.63

$$\frac{c\sqrt{a\left(3-2x^2\right)}\sqrt{cx}\left(3\sqrt{3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{2x^2}{3}\right)-\left(3-2x^2\right)^{3/2}\right)}{7\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeome
tric2F1[-1/2, 1/4, 5/4, (2*x^2)/3]))/(7*Sqrt[3 - 2*x^2])
```

Maple [A] time = 0.028, size = 133, normalized size = 1.1

$$-\frac{c}{14x\left(2x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2x^2-3\right)}\left(-8x^5+\sqrt{\left(2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/14*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(-8*x^5+((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1
```

```
/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+20*x^3-12*x)/x/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-2\,ax^2+3\,a}\sqrt{cx}cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x, x)
```

Sympy [A] time = 4.67944, size = 53, normalized size = 0.45

$$\frac{\sqrt{3}\sqrt{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), 2*x
**2*exp_polar(2*I*pi)/3)/(2*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

3.609 $\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}$$

```
[Out] (2*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*c) - (6*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 -
 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[
3*a - 2*a*x^2])
```

Rubi [A] time = 0.0428037, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {279, 320, 319, 318, 424}

$$\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*c) - (6*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 -
 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[
3*a - 2*a*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{1}{5}(6a)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{\left(6a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{5\sqrt{x}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{\left(6a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{\left(6\sqrt[4]{2}3^{3/4}a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{6\sqrt[4]{6}a\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0134318, size = 51, normalized size = 0.52

$$\frac{2x\sqrt{a\left(3-2x^2\right)}\sqrt{cx}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)}{\sqrt{9-6x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, (2*x^2
)/3])/Sqrt[9 - 6*x^2]
```

Maple [B] time = 0.03, size = 229, normalized size = 2.3

$$\frac{1}{10\,x\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(2\,\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticE}\left(1/6\,\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/10*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*
(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*
(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
```

```
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+8*x^4-12*x^2)/x/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2\,ax^2+3\,a}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-2\,ax^2+3\,a}\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

Sympy [C] time = 0.901819, size = 53, normalized size = 0.54

$$\frac{\sqrt{3}\sqrt{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), 2*x*
*2*exp_polar(2*I*pi)/3)/(2*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2\,ax^2+3\,a}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

3.610 $\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=94

$$\frac{2\,2^{3/4}a\sqrt{3-2x^2}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}+\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}$$

```
[Out] (2*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*c) + (2*2^(3/4)*a*Sqrt[3 - 2*x^2]*Elli
pticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3^(1/4)*Sqrt[c]*Sqrt[a
*(3 - 2*x^2)])
```

Rubi [A] time = 0.0468306, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {279, 329, 224, 221}

$$\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}+\frac{2\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*c) + (2*2^(3/4)*a*Sqrt[3 - 2*x^2]*Elli
pticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3^(1/4)*Sqrt[c]*Sqrt[a
*(3 - 2*x^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\sqrt{3a-2ax^2}}{3c} + (2a)\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx$$

$$= \frac{2\sqrt{cx}\sqrt{3a-2ax^2}}{3c} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\sqrt{cx}\sqrt{3a-2ax^2}}{3c} + \frac{\left(4a\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{3}c\sqrt{a\left(3-2x^2\right)}}$$

$$= \frac{2\sqrt{cx}\sqrt{3a-2ax^2}}{3c} + \frac{2\,2^{3/4}a\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.0122879, size = 51, normalized size = 0.54

$$\frac{2x\sqrt{a\left(9-6x^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{2x^2}{3}\right)}{\sqrt{3-2x^2}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, (2*x^2)/3])/(Sqr
t[c*x]*Sqrt[3 - 2*x^2])
```

Maple [A] time = 0.029, size = 124, normalized size = 1.3

$$-\frac{1}{6x^2-9}\sqrt{-a\left(2x^2-3\right)}\left(\sqrt{\left(2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{\left(2x+\sqrt{2}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] -1/3*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((
-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*Ell
ipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/
2*2^(1/2))-4*x^3+6*x)/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(c*x), x)
```

Sympy [A] time = 0.790797, size = 53, normalized size = 0.56

$$\frac{\sqrt{3}\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(2*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-2\,ax^2+3\,a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

3.611 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{4\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(c*Sqrt[c*x]) + (4*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 - 2*
x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(c^2*Sqrt[x]*Sqrt[3
*a - 2*a*x^2])
```

Rubi [A] time = 0.0416983, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {277, 320, 319, 318, 424}

$$\frac{4\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(c*Sqrt[c*x]) + (4*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 - 2*
x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(c^2*Sqrt[x]*Sqrt[3
*a - 2*a*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}-\frac{(4a)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{c^2}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}-\frac{\left(4a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{c^2\sqrt{x}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}-\frac{\left(4a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{c^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}+\frac{\left(4\sqrt[4]{2}3^{3/4}a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}+\frac{4\sqrt[4]{6}a\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.011827, size = 51, normalized size = 0.52

$$-\frac{2x\sqrt{a\left(9-6x^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\frac{2x^2}{3}\right)}{\sqrt{3-2x^2}(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, (2*x^2)/3])/((
c*x)^(3/2)*Sqrt[3 - 2*x^2])
```

Maple [B] time = 0.034, size = 225, normalized size = 2.3

$$-\frac{1}{3c\left(2x^2-3\right)}\sqrt{-a\left(2x^2-3\right)}\left(2\sqrt{2}\sqrt{\left(2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticE}\left(1/6\sqrt{3}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] -1/3*(-a*(2*x^2-3))^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2)
)^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*
3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)
*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2)
```

```
)^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*
3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)
*3^(1/2))^(1/2),1/2*2^(1/2))+12*x^2-18)/c/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(c^2*x^2), x)
```

Sympy [C] time = 1.34588, size = 56, normalized size = 0.57

$$\frac{\sqrt{3}\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), 2*x**2*exp_polar(2*
I*pi)/3)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)
```

3.612 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{4\ 2^{3/4}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{3}c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*c*(c*x)^(3/2)) - (4*2^(3/4)*a*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rubi [A] time = 0.0482166, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {277, 329, 224, 221}

$$-\frac{4\ 2^{3/4}a\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*c*(c*x)^(3/2)) - (4*2^(3/4)*a*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{(4a)\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{3c^2} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{(8a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c^3} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{\left(8a\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{3\sqrt{3}c^3\sqrt{a\left(3-2x^2\right)}} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{4\, 2^{3/4} a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}c^{5/2}\sqrt{a\left(3-2x^2\right)}}
\end{aligned}$$

Mathematica [C] time = 0.0132876, size = 51, normalized size = 0.53

$$-\frac{2x\sqrt{a\left(3-2x^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{\sqrt{9-6x^2}(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*x*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, (2*x^2)/3])/((
c*x)^(5/2)*Sqrt[9 - 6*x^2])
```

Maple [A] time = 0.029, size = 129, normalized size = 1.3

$$\frac{2}{9\,xc^2\left(2\,x^2-3\right)}\sqrt{-a\left(2\,x^2-3\right)}\left(\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{\left(2\,x+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] 2/9*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-
2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*Elli
pticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2
*2^(1/2))*x-6*x^2+9)/x/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [A] time = 5.29622, size = 49, normalized size = 0.51

$$\frac{\sqrt{2}i\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{3}{2x^2}\right)}{2c^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] sqrt(2)*I*sqrt(a)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), 3/(2*x**2))/(2*c**
(5/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

3.613 $\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=156

$$\frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}-\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

```
[Out] (-10*a*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2])/(7*b) + (5*a^(7/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0897231, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {321, 329, 220}

$$\frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}-\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (-10*a*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2])/(7*b) + (5*a^(7/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} - \frac{\left(5ac^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{7b}$$
$$= -\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{\left(5a^2c^4\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{21b^2}$$
$$= -\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{\left(10a^2c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{21b^2}$$
$$= -\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0309965, size = 87, normalized size = 0.56

$$\frac{2c^3\sqrt{cx}\left(5a^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-5a^2-2abx^2+3b^2x^4\right)}{21b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a
]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.02, size = 141, normalized size = 0.9

$$\frac{c^3}{21\,b^3x}\sqrt{cx}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}a^2+6\,b^3x^5-4\,ab^2x^3-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/21/x*c^3*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b
)^(1/2)*a^2+6*b^3*x^5-4*a*b^2*x^3-10*a^2*b*x)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^3x^3}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^3*x^3/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 132.12, size = 44, normalized size = 0.28

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

3.614 $\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{3a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right.\right.}{5b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c^2*Sqrt[c*x]*Sqrt[a + b*x^2
])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (6*a^(5/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2]) - (3*a^(5/
4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.186499, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {321, 329, 305, 220, 1196}

$$-\frac{3a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}-\frac{6}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c^2*Sqrt[c*x]*Sqrt[a + b*x^2
])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (6*a^(5/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2]) - (3*a^(5/
4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5b}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{(6ac)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{\left(6a^{3/2}c^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b^{3/2}} + \frac{\left(6a^{3/2}c^2\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b^{3/2}}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{6ac^2\sqrt{cx}\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0246467, size = 69, normalized size = 0.25

$$\frac{2c(cx)^{3/2}\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2,
3/4, 7/4, -((b*x^2)/a)]))/(5*b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.018, size = 210, normalized size = 0.8

$$-\frac{c^2}{5\,b^2x}\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/5/x*c^2*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a
^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*b^2*x^4-2*a*b*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^2*x^2/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 33.3284, size = 44, normalized size = 0.16

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)
```

3.615 $\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=127

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.070928, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {321, 329, 220}

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b} - \frac{(ac^2)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3b}$$

$$= \frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b} - \frac{(2ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3b}$$

$$= \frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b} - \frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0234116, size = 69, normalized size = 0.54

$$\frac{2c\sqrt{cx}\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/
2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 125, normalized size = 1.

$$-\frac{c}{3\,b^2x}\sqrt{cx}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/x*c*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1
/2)*a-2*b^2*x^3-2*a*b*x)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}cx}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c*x/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 2.59322, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(a)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

3.616 $\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right|}{b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*
Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(3/4)*Sqrt
[a + b*x^2]) + (a^(1/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c
])], 1/2])/(b^(3/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.166172, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {329, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}+\frac{2}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a + b*x^2],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*
Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(3/4)*Sqrt
[a + b*x^2]) + (a^(1/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c
])], 1/2])/(b^(3/4)*Sqrt[a + b*x^2])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{b}} - \frac{\left(2\sqrt{a}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{b}}$$

$$= \frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a\cdot}{\left(\sqrt{a}\cdot\right.}}}{b^{3/4}\sqrt{}}$$

Mathematica [C] time = 0.0118667, size = 56, normalized size = 0.24

$$\frac{2x\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a + b*x^2],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^
2)/a)])/(3*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 132, normalized size = 0.6

$$\frac{a\sqrt{2}}{bx}\sqrt{cx}\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(2\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)-\text{EllipticF}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (c*x)^(1/2)/(b*x^2+a)^(1/2)*a/b*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(
1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*(2*
EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 0.768273, size = 44, normalized size = 0.19

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

3.617 $\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

```
[Out] ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(1/4)*b^(1/4)*Sqr
t[c]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0577287, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(1/4)*b^(1/4)*Sqr
t[c]*Sqrt[a + b*x^2])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{c}\\ &= \frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] time = 0.012867, size = 54, normalized size = 0.56

$$\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(S
qrt[c*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.013, size = 104, normalized size = 1.1

$$\frac{\sqrt{2}}{b}\sqrt{-ab}\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{cx}}\frac{1}{\sqrt{bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))/b/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{bcx^3+acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] time = 0.988725, size = 44, normalized size = 0.45

$$\frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*s
qrt(a)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

3.618 $\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=268

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}+$$

```
[Out] (-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(a*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b*x^2]) + (b^(1/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b
*x^2])
```

Rubi [A] time = 0.188372, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{b}\sqrt{cx}}{ac^2\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(a*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b*x^2]) + (b^(1/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b
*x^2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{b\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}c^2} - \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}} + \frac{\sqrt[4]{b}\,(}{}$$

Mathematica [C] time = 0.0120824, size = 54, normalized size = 0.2

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/
((c*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.014, size = 196, normalized size = 0.7

$$\frac{1}{ac}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a - \sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-2*b*x^2-2*a)
/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{cx}}{bc^2x^4 + ac^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] time = 1.88875, size = 48, normalized size = 0.18

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)
*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

3.619 $\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

```
[Out] (-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0730192, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {325, 329, 220}

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3ac^2}$$
$$= -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3ac^3}$$
$$= -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0126789, size = 56, normalized size = 0.43

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/
(3*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.013, size = 123, normalized size = 1.

$$-\frac{1}{3\,axc^2}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+2*b*x^2+
2*a)/a/c^2/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{bc^3x^5+ac^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] time = 8.11065, size = 48, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)
*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

3.620 $\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=306

$$-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}-\frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\right.}{5a^{7/4}c^{7/2}\sqrt{a+bx}}$$

```
[Out] (-2*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/2)) + (6*b*Sqrt[a + b*x^2])/(5*a^2*c^3
*Sqrt[c*x]) - (6*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a^2*c^4*(Sqrt[a] + S
qrt[b]*x)) + (6*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2]) - (3*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.220273, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 329, 305, 220, 1196}

$$-\frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/2)) + (6*b*Sqrt[a + b*x^2])/(5*a^2*c^3
*Sqrt[c*x]) - (6*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a^2*c^4*(Sqrt[a] + S
qrt[b]*x)) + (6*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2]) - (3*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} - \frac{(3b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{5ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{\left(3b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5a^2c^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{\left(6b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5a^2c^5}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{\left(6b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5a^{3/2}c^4} + \frac{\left(6b^{3/2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}} \cdots\right.}{5a^{3/2}c^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1} \cdots\right.}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0124074, size = 56, normalized size = 0.18

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])
/(5*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.016, size = 219, normalized size = 0.7

$$-\frac{1}{5x^2c^3a^2}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/5/x^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*x^2*a*b-6*b^2*x^4-4*a*b*x^2+2*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{bc^4x^6+ac^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c^4*x^6 + a*c^4*x^4), x)
```

Sympy [C] time = 72.705, size = 51, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\,(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

3.621 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

```
[Out] -((c*(c*x)^(5/2))/(b*Sqrt[a + b*x^2])) + (5*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/
(3*b^2) - (5*a^(3/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(6*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0866138, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {288, 321, 329, 220}

$$-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*(c*x)^(5/2))/(b*Sqrt[a + b*x^2])) + (5*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/
(3*b^2) - (5*a^(3/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(6*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{\left(5c^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2}-\frac{\left(5ac^4\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{6b^2}\\ &= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2}-\frac{\left(5ac^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3b^2}\\ &= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2}-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0313815, size = 74, normalized size = 0.48

$$\frac{c^3\sqrt{cx}\left(-5a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+5a+2bx^2\right)}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (c^3*Sqrt[c*x]*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.031, size = 128, normalized size = 0.8

$$-\frac{c^3}{6\,b^3x}\sqrt{cx}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}a-4\,b^2x^3-10\,abx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6/x*c^3*(c*x)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a-4*b^2*
x^3-10*a*b*x)/(b*x^2+a)^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^3x^3}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^3*x^3/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 159.68, size = 44, normalized size = 0.29

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

3.622 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}}+\frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}}{b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] -((c*(c*x)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/
(b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(7/4)*Sqrt[a + b*x^2]) + (3*a^(1/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(7/4)*Sqrt[a
 + b*x^2])
```

Rubi [A] time = 0.189033, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {288, 329, 305, 220, 1196}

$$\frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan\right.}{b^{7/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*(c*x)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/
(b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(7/4)*Sqrt[a + b*x^2]) + (3*a^(1/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(7/4)*Sqrt[a
 + b*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{\left(3c^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{(3c)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{\left(3\sqrt{a}c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b^{3/2}}-\frac{\left(3\sqrt{a}c^2\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b^{3/2}}$$

$$= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{a+bx^2}}+\frac{3\sqrt[4]{a}c}{}$$

Mathematica [C] time = 0.0279153, size = 60, normalized size = 0.23

$$-\frac{2c(cx)^{3/2}\left(\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (-2*c*(c*x)^(3/2)*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4
, -((b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.027, size = 197, normalized size = 0.7

$$\frac{c^2}{2\,b^2x}\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/x*c^2*(c*x)^(1/2)*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^2x^2}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^2*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 40.1507, size = 44, normalized size = 0.17

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

3.623 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

```
[Out] -((c*Sqrt[c*x])/(b*Sqrt[a + b*x^2])) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])
/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0707704, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {288, 329, 220}

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*Sqrt[c*x])/(b*Sqrt[a + b*x^2])) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])
/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0222413, size = 59, normalized size = 0.47

$$\frac{c\sqrt{cx}\left(\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c*x]*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.012, size = 115, normalized size = 0.9

$$\frac{c}{2\,b^2x}\sqrt{cx}\left(\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\sqrt{-ab}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/x*c*(c*x)^(1/2)*(EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*(-a*b)^(1/2)-2*b*x)/(b*x^2
+a)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}cx}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 2.94499, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.624 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{(\cdots}{ac\cdots}$$

```
[Out] (c*x)^(3/2)/(a*c*Sqrt[a + b*x^2]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x)) + (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.186592, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {290, 329, 305, 220, 1196}

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

```
[Out] (c*x)^(3/2)/(a*c*Sqrt[a + b*x^2]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x)) + (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac}$$

$$= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}\sqrt{b}}$$

$$= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}} - \frac{\sqrt{c}\left(\sqrt{a}\right.}{}$$

Mathematica [C] time = 0.0123387, size = 59, normalized size = 0.22

$$\frac{2x\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^
2)/a)])/(3*a*Sqrt[a + b*x^2])
```

Maple [A] time = 0.013, size = 197, normalized size = 0.7

$$-\frac{1}{2\,abx}\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a-\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(c*x)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b/x/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 1.35802, size = 44, normalized size = 0.17

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

3.625 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

```
[Out] Sqrt[c*x]/(a*c*Sqrt[a + b*x^2]) + ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(5/4)*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0700747, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {290, 329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c*x]/(a*c*Sqrt[a + b*x^2]) + ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(5/4)*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx=\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}+\frac{\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{2a}$$

$$=\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{ac}$$

$$=\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}+\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0184135, size = 59, normalized size = 0.47

$$\frac{x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+x}{a\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[c*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.017, size = 114, normalized size = 0.9

$$\frac{1}{2\,ab}\left(\text{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\sqrt{-ab}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*(EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*(-a*b)^(1/2)+2*b*x)/(b*x^2+a)^(1/2)/b/a/(c
*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2cx^5+2\,abcx^3+a^2cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x), x)
```

Sympy [C] time = 2.11185, size = 44, normalized size = 0.35

$$\frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(3/2)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

3.626 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}}+\frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\right.}{a^{7/4}c^{3/2}\sqrt{a+bx}}$$

```
[Out] 1/(a*c*Sqrt[c*x]*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(a^2*c*Sqrt[c*x]) +
 (3*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/(a^2*c^2*(Sqrt[a] + Sqrt[b]*x)) - (3
*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(7/4)*c^(
3/2)*Sqrt[a + b*x^2]) + (3*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(7/4)*c^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.219443, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$\frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\right.}{a^{7/4}c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*Sqrt[c*x]*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(a^2*c*Sqrt[c*x]) +
 (3*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/(a^2*c^2*(Sqrt[a] + Sqrt[b]*x)) - (3
*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(7/4)*c^(
3/2)*Sqrt[a + b*x^2]) + (3*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(7/4)*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}+\frac{3\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{2a}\\
&= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}+\frac{(3b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2a^2c^2}\\
&= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{a^2c^3}\\
&= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}+\frac{\left(3\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{a^{3/2}c^2}-\frac{\left(3\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}\ldots}{\sqrt{\ldots}}}{\sqrt{a+\ldots}}\ldots\right)}{a^{3/2}c^2}\\
&= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}+\frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\ldots\right.}{a^{7/4}c^{3/2}\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0125247, size = 57, normalized size = 0.19

$$
-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/
(a*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.018, size = 197, normalized size = 0.7

$$\frac{1}{2\,a^2c}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-6*b*x^
2-4*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2c^2x^6+2\,abc^2x^4+a^2c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^2*x
^2), x)
```

Sympy [C] time = 5.16216, size = 48, normalized size = 0.16

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

3.627 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

```
[Out] 1/(a*c*(c*x)^(3/2)*Sqrt[a + b*x^2]) - (5*Sqrt[a + b*x^2])/(3*a^2*c*(c*x)^(3
/2)) - (5*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(6*
a^(9/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0897644, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {290, 325, 329, 220}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```


```
[Out] 1/(a*c*(c*x)^(3/2)*Sqrt[a + b*x^2]) - (5*Sqrt[a + b*x^2])/(3*a^2*c*(c*x)^(3
/2)) - (5*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(6*
a^(9/4)*c^(5/2)*Sqrt[a + b*x^2])
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}+\frac{5\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{(5b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{6a^2c^2}$$

$$= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{(5b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3a^2c^3}$$

$$= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.012588, size = 59, normalized size = 0.38

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3a(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/
(3*a*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.017, size = 124, normalized size = 0.8

$$-\frac{1}{6\,a^2xc^2}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x+10\,bx^2+4\,a\right)\frac{1}{\sqrt{cx}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6/x*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+10*b*x^2+4*a)/(b*x^2+a
)^(1/2)/a^2/c^2/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2c^3x^7+2\,abc^3x^5+a^2c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c^3*x^7 + 2*a*b*c^3*x^5 + a^2*c^3*x
^3), x)
```

Sympy [C] time = 46.8972, size = 48, normalized size = 0.31

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

3.628 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}}-\frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{E}\ldots}{5a^{11/4}c^{7/2}\sqrt{a+\ldots}}$$

```
[Out] 1/(a*c*(c*x)^(5/2)*Sqrt[a + b*x^2]) - (7*Sqrt[a + b*x^2])/(5*a^2*c*(c*x)^(5
/2)) + (21*b*Sqrt[a + b*x^2])/(5*a^3*c^3*Sqrt[c*x]) - (21*b^(3/2)*Sqrt[c*x]
*Sqrt[a + b*x^2])/(5*a^3*c^4*(Sqrt[a] + Sqrt[b]*x)) + (21*b^(5/4)*(Sqrt[a]
+ Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(
b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(11/4)*c^(7/2)*Sqrt[a + b
*x^2]) - (21*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/
(10*a^(11/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.254267, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$-\frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan\ldots\right.}{5a^{11/4}c^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```


```
[Out] 1/(a*c*(c*x)^(5/2)*Sqrt[a + b*x^2]) - (7*Sqrt[a + b*x^2])/(5*a^2*c*(c*x)^(5
/2)) + (21*b*Sqrt[a + b*x^2])/(5*a^3*c^3*Sqrt[c*x]) - (21*b^(3/2)*Sqrt[c*x]
*Sqrt[a + b*x^2])/(5*a^3*c^4*(Sqrt[a] + Sqrt[b]*x)) + (21*b^(5/4)*(Sqrt[a]
+ Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(
b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(11/4)*c^(7/2)*Sqrt[a + b
*x^2]) - (21*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/
(10*a^(11/4)*c^(7/2)*Sqrt[a + b*x^2])
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}+\frac{7\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx}{2a}\\
&= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}-\frac{(21b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{10a^2c^2}\\
&= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}+\frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}-\frac{\left(21b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{10a^3c^4}\\
&= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}+\frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}-\frac{\left(21b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5a^3c^5}\\
&= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}+\frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}-\frac{\left(21b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5a^{5/2}c^4}\ \cdot\\
&= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}+\frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}-\frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.0128172, size = 59, normalized size = 0.18

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])
/(5*a*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.018, size = 219, normalized size = 0.7

$$-\frac{1}{10\,x^{2}c^{3}a^{3}}\left(42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^{2}ab-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/10/x^2*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-21*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*x^2*a*b-42*b^2*x^4-28*a*b*x^2+4*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/
a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^{2}+a}\sqrt{cx}}{b^{2}c^{4}x^{8}+2\,abc^{4}x^{6}+a^{2}c^{4}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c^4*x^8 + 2*a*b*c^4*x^6 + a^2*c^4*x
^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

3.629 $\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/2}} dx$

Optimal. Leaf size=155

$$\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{12\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(c*(c*x)^(5/2))/(3*b*(a + b*x^2)^(3/2)) - (5*c^3*Sqrt[c*x])/(6*b^2*Sqrt[a
+ b*x^2]) + (5*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(12*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0887926, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {288, 329, 220}

$$-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}+\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(c*(c*x)^(5/2))/(3*b*(a + b*x^2)^(3/2)) - (5*c^3*Sqrt[c*x])/(6*b^2*Sqrt[a
+ b*x^2]) + (5*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(12*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} + \frac{\left(5c^2\right)\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{6b}$$

$$= -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{\left(5c^4\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{12b^2}$$

$$= -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{\left(5c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6b^2}$$

$$= -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0510805, size = 80, normalized size = 0.52

$$\frac{c^3\sqrt{cx}\left(5\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-5a-7bx^2\right)}{6b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (c^3*Sqrt[c*x]*(-5*a - 7*b*x^2 + 5*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.031, size = 219, normalized size = 1.4

$$\frac{c^3}{12\,b^3x}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^2b+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*b+5*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a-14*b^2*x^3-10*a*b*x)/x*c^3*(c*x)^(1/2)/b^3/(b*x^2+
a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^3x^3}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^3*x^3/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3), x)
```

Sympy [C] time = 159.189, size = 44, normalized size = 0.28

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((9/4, 5/2), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

3.630 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=304

$$-\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}-$$

```
[Out] -(c*(c*x)^(3/2))/(3*b*(a + b*x^2)^(3/2)) + (c*(c*x)^(3/2))/(2*a*b*Sqrt[a +
b*x^2]) - (c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x
)) + (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3
/4)*b^(7/4)*Sqrt[a + b*x^2]) - (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x
^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4
)*Sqrt[c])], 1/2])/(4*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.215871, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {288, 290, 329, 305, 220, 1196}

$$-\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}-\frac{c^2\sqrt{cx}}{2ab^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(c*(c*x)^(3/2))/(3*b*(a + b*x^2)^(3/2)) + (c*(c*x)^(3/2))/(2*a*b*Sqrt[a +
b*x^2]) - (c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x
)) + (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3
/4)*b^(7/4)*Sqrt[a + b*x^2]) - (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x
^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4
)*Sqrt[c])], 1/2])/(4*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c^2\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx}{2b}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4ab}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2ab}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2\sqrt{a}b^{3/2}}+\frac{c^2\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right.}{2\sqrt{a}b^{3/2}}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\sqrt{cx}\sqrt{a+bx^2}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0393135, size = 74, normalized size = 0.24

$$\frac{2c(cx)^{3/2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-a\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*(c*x)^(3/2)*(-a + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.03, size = 385, normalized size = 1.3

$$-\frac{c^2}{12\,ab^2x}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
x^2*a*b+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2
-6*b^2*x^4-2*a*b*x^2)/x*c^2*(c*x)^(1/2)/b^2/a/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^2x^2}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^2*x^2/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3), x)
```

Sympy [C] time = 40.1428, size = 44, normalized size = 0.14

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

3.631 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(c*Sqrt[c*x])/(3*b*(a + b*x^2)^(3/2)) + (c*Sqrt[c*x])/(6*a*b*Sqrt[a + b*x^
2]) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^
(5/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0882295, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {288, 290, 329, 220}

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```


```
[Out] -(c*Sqrt[c*x])/(3*b*(a + b*x^2)^(3/2)) + (c*Sqrt[c*x])/(6*a*b*Sqrt[a + b*x^
2]) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^
(5/4)*b^(5/4)*Sqrt[a + b*x^2])
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c^2\int \frac{1}{\sqrt{cx}(a+bx^2)^{3/2}}\,dx}{6b}$$

$$= -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{12ab}$$

$$= -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6ab}$$

$$= -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0391637, size = 79, normalized size = 0.51

$$\frac{c\sqrt{cx}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-a+bx^2\right)}{6ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (c*Sqrt[c*x]*(-a + b*x^2 + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*a*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.012, size = 218, normalized size = 1.4

$$\frac{c}{12\,ab^2x}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}x^2b\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*b+((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*(-a*b)^(1/2)*a+2*b^2*x^3-2*a*b*x)/x*c*(c*x)^(1/2)/a/b^2/(b*x^2+a)^(3/
```

2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}cx}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2
 + a^3), x)
```

Sympy [C] time = 8.20464, size = 44, normalized size = 0.28

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.632 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{}{2a^{2}}$$

```
[Out] (c*x)^(3/2)/(3*a*c*(a + b*x^2)^(3/2)) + (c*x)^(3/2)/(2*a^2*c*Sqrt[a + b*x^2
]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a^2*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (S
qrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(7/4)*b^(
3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[
c])], 1/2])/(4*a^(7/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.223984, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {290, 329, 305, 220, 1196}

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

```
[Out] (c*x)^(3/2)/(3*a*c*(a + b*x^2)^(3/2)) + (c*x)^(3/2)/(2*a^2*c*Sqrt[a + b*x^2
]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a^2*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (S
qrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(7/4)*b^(
3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[
c])], 1/2])/(4*a^(7/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx}{2a}$$

$$= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a^2c}$$

$$= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a^{3/2}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{ac}}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right.}{2a^{3/2}\sqrt{b}}$$

$$= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\right.}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0122114, size = 59, normalized size = 0.2

$$\frac{2x\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^
2)/a)])/(3*a^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.013, size = 382, normalized size = 1.3

$$-\frac{1}{12\,a^2bx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
x^2*a*b+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2
-6*b^2*x^4-10*a*b*x^2)*(c*x)^(1/2)/b/a^2/x/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3), x)
```

Sympy [C] time = 4.78339, size = 44, normalized size = 0.15

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(5/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)
```

3.633 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

```
[Out] Sqrt[c*x]/(3*a*c*(a + b*x^2)^(3/2)) + (5*Sqrt[c*x])/(6*a^2*c*Sqrt[a + b*x^2
]) + (5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^(9/4)*b
^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0882105, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {290, 329, 220}

$$\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c*x]/(3*a*c*(a + b*x^2)^(3/2)) + (5*Sqrt[c*x])/(6*a^2*c*Sqrt[a + b*x^2
]) + (5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^(9/4)*b
^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/2}}\,dx = \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx}{6a}$$
$$= \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{12a^2}$$
$$= \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6a^2c}$$
$$= \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0352485, size = 79, normalized size = 0.5

$$\frac{5x\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+7ax+5bx^3}{6a^2\sqrt{cx}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] (7*a*x + 5*b*x^3 + 5*x*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
4, 1/2, 5/4, -((b*x^2)/a)])/(6*a^2*Sqrt[c*x]*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.019, size = 216, normalized size = 1.4

$$\frac{1}{12\,a^2b}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^2b+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*b+5*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a+10*b^2*x^3+14*a*b*x)/(c*x)^(1/2)/a^2/b/(b*x^2+a)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3cx^7+3\,ab^2cx^5+3\,a^2bcx^3+a^3cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c*x^7 + 3*a*b^2*c*x^5 + 3*a^2*b*c*x
^3 + a^3*c*x), x)
```

Sympy [C] time = 8.71567, size = 44, normalized size = 0.28

$$\frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(5/2)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

3.634 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/2}} dx$

Optimal. Leaf size=333

$$\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}}+\frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(\cdots\right.}{2a^{11/4}c^{3/2}\sqrt{a+b\cdots}}$$

```
[Out] 1/(3*a*c*Sqrt[c*x]*(a + b*x^2)^(3/2)) + 7/(6*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2
]) - (7*Sqrt[a + b*x^2])/(2*a^3*c*Sqrt[c*x]) + (7*Sqrt[b]*Sqrt[c*x]*Sqrt[a
+ b*x^2])/(2*a^3*c^2*(Sqrt[a] + Sqrt[b]*x)) - (7*b^(1/4)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(11/4)*c^(3/2)*Sqrt[a + b*x^2]) +
(7*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(11/4
)*c^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.255224, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$\frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}}-\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{11/4}c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*Sqrt[c*x]*(a + b*x^2)^(3/2)) + 7/(6*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2
]) - (7*Sqrt[a + b*x^2])/(2*a^3*c*Sqrt[c*x]) + (7*Sqrt[b]*Sqrt[c*x]*Sqrt[a
+ b*x^2])/(2*a^3*c^2*(Sqrt[a] + Sqrt[b]*x)) - (7*b^(1/4)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(11/4)*c^(3/2)*Sqrt[a + b*x^2]) +
(7*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(11/4
)*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}} + \frac{7\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx}{6a}$$

$$= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} + \frac{7\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{(7b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4a^3c^2}$$

$$= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{(7b)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a^3c^3}$$

$$= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{\left(7\sqrt{b}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right.}{2a^{5/2}c^2}$$

$$= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{\right.}{}$$

Mathematica [C] time = 0.0126359, size = 57, normalized size = 0.17

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 5/2, 3/4, -((b*x^2)/a)])/
(a^2*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.02, size = 384, normalized size = 1.2

$$\frac{1}{12\,ca^3}\left(42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-21*((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^
(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
*x^2*a*b+42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-21*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
a^2-42*b^2*x^4-70*a*b*x^2-24*a^2)/a^3/c/(c*x)^(1/2)/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3c^2x^8+3\,ab^2c^2x^6+3\,a^2bc^2x^4+a^3c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c^2*x^8 + 3*a*b^2*c^2*x^6 + 3*a^2*b
*c^2*x^4 + a^3*c^2*x^2), x)
```

Sympy [C] time = 45.9233, size = 48, normalized size = 0.14

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}} c^{\frac{3}{2}} \sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{5}{2}} (cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

3.635 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}+\frac{1}{3ac(cx)^{3/2}\left(a+\right.}$$

```
[Out] 1/(3*a*c*(c*x)^(3/2)*(a + b*x^2)^(3/2)) + 3/(2*a^2*c*(c*x)^(3/2)*Sqrt[a + b
*x^2]) - (5*Sqrt[a + b*x^2])/(2*a^3*c*(c*x)^(3/2)) - (5*b^(3/4)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(13/4)*c^(5/2)*Sqrt[a + b*x
^2])
```

Rubi [A] time = 0.110333, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {290, 325, 329, 220}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}+\frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(3/2)*(a + b*x^2)^(3/2)) + 3/(2*a^2*c*(c*x)^(3/2)*Sqrt[a + b
*x^2]) - (5*Sqrt[a + b*x^2])/(2*a^3*c*(c*x)^(3/2)) - (5*b^(3/4)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(13/4)*c^(5/2)*Sqrt[a + b*x
^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}} + \frac{3\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx}{2a}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} + \frac{15\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} - \frac{(5b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{4a^3c^2}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} - \frac{(5b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{\right.}{2a^3c^3}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} - \frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}}{4a^{13/4}c^{5/2}\sqrt{a}}$$

Mathematica [C] time = 0.013029, size = 59, normalized size = 0.32

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 5/2, 1/4, -((b*x^2)/a)])/
(3*a^2*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.018, size = 227, normalized size = 1.2

$$-\frac{1}{12\,xc^2a^3}\left(15\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^3b+15\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(15*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b+15*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*(-a*b)^(1/2)*x*a+30*b^2*x^4+42*a*b*x^2+8*a^2)/x/c^2/(c*x)^(1/2
)/a^3/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3c^3x^9+3\,ab^2c^3x^7+3\,a^2bc^3x^5+a^3c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c^3*x^9 + 3*a*b^2*c^3*x^7 + 3*a^2*b
*c^3*x^5 + a^3*c^3*x^3), x)
```

Sympy [C] time = 85.2115, size = 48, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

3.636 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}}-\frac{77b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\ldots}{10a^{15/4}c^{7/2}\ldots}$$

```
[Out] 1/(3*a*c*(c*x)^(5/2)*(a + b*x^2)^(3/2)) + 11/(6*a^2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2]) - (77*Sqrt[a + b*x^2])/(30*a^3*c*(c*x)^(5/2)) + (77*b*Sqrt[a + b*x^
2])/(10*a^4*c^3*Sqrt[c*x]) - (77*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(10*a^4
*c^4*(Sqrt[a] + Sqrt[b]*x)) + (77*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b
*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])], 1/2])/(10*a^(15/4)*c^(7/2)*Sqrt[a + b*x^2]) - (77*b^(5/4)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(20*a^(15/4)*c^(7/2)*Sq
rt[a + b*x^2])
```

Rubi [A] time = 0.295031, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$-\frac{77b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}}+\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan\ldots\right)}{10a^{15/4}c^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(5/2)*(a + b*x^2)^(3/2)) + 11/(6*a^2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2]) - (77*Sqrt[a + b*x^2])/(30*a^3*c*(c*x)^(5/2)) + (77*b*Sqrt[a + b*x^
2])/(10*a^4*c^3*Sqrt[c*x]) - (77*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(10*a^4
*c^4*(Sqrt[a] + Sqrt[b]*x)) + (77*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b
*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])], 1/2])/(10*a^(15/4)*c^(7/2)*Sqrt[a + b*x^2]) - (77*b^(5/4)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(20*a^(15/4)*c^(7/2)*Sq
rt[a + b*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx}{6a}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} + \frac{77\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx}{12a^2}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} - \frac{(77b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{20a^3c^2}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - \frac{\left(77b^2\right)\int}{20a}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - \frac{\left(77b^2\right)\text{Su}}{}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - \frac{\left(77b^{3/2}\right)\text{S}}{}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - \frac{77b^{3/2}\sqrt{c}}{10a^4c^4\left(\sqrt{}\right.}$$

Mathematica [C] time = 0.0129009, size = 59, normalized size = 0.16

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 5/2, -1/4, -((b*x^2)/a)])
/(5*a^2*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.019, size = 410, normalized size = 1.1

$$-\frac{1}{60\,x^2a^4c^3}\left(462\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^4ab^2-231\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/60*(462*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
```

```
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b^2-231*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*x^4*a*b^2+462*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*x^2*a^2*b-231*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*x^2*a^2*b-462*b^3*x^6-770*a*b^2*x^4-264*a^2*b*x^2+24*a^3)/x^
2/a^4/c^3/(c*x)^(1/2)/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3c^4x^{10}+3\,ab^2c^4x^8+3\,a^2bc^4x^6+a^3c^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c^4*x^10 + 3*a*b^2*c^4*x^8 + 3*a^2*
b*c^4*x^6 + a^3*c^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

3.637 $\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}} dx$

Optimal. Leaf size=107

$$-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

```
[Out] -(c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*a) - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[
3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*Sq
rt[x]*Sqrt[3*a - 2*a*x^2])
```

Rubi [A] time = 0.0428406, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {321, 320, 319, 318, 424}

$$-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -(c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*a) - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[
3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*Sq
rt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx &= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{1}{10}\left(9c^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{\left(9c^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{10\sqrt{x}}\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{\left(9c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{10\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{\left(9\left(\frac{3}{2}\right)^{3/4}c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{9\sqrt[4]{3}c^2\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\,2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.0319921, size = 61, normalized size = 0.57

$$\frac{c(cx)^{3/2}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)+2x^2-3\right)}{5\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*(c*x)^(3/2)*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/2, 3/4, 7/
4, (2*x^2)/3]))/(5*Sqrt[a*(3 - 2*x^2)])
```

Maple [B] time = 0.031, size = 235, normalized size = 2.2

$$\frac{c^2}{40\,ax\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(6\,\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\mathrm{EllipticE}\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/40/x*c^2*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)/a*(6*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*
3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1
/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-3*2^(1/2)*((2*x+2^(1/2)*3^
```

```
(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^
(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-16*x^4+24*x^2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}c^2x^2}{2\,ax^2-3\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x^2/(2*a*x^2 - 3*a), x)
```

Sympy [C] time = 27.6173, size = 51, normalized size = 0.48

$$\frac{\sqrt{3}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(6*sqrt(a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

3.638 $\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=88

$$\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{6}\sqrt{a\left(3-2x^2\right)}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

```
[Out] -(c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*a) + (c^(3/2)*Sqrt[3 - 2*x^2]*Ellipti
cF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[a*(3 - 2*x^2
)])
```

Rubi [A] time = 0.0466644, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {321, 329, 224, 221}

$$\frac{c^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{a\left(3-2x^2\right)}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -(c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*a) + (c^(3/2)*Sqrt[3 - 2*x^2]*Ellipti
cF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[a*(3 - 2*x^2
)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx &= -\frac{c\sqrt{cx}\sqrt{3a-2ax^2}}{3a}+\frac{1}{2}c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{c\sqrt{cx}\sqrt{3a-2ax^2}}{3a}+c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)\\
&= -\frac{c\sqrt{cx}\sqrt{3a-2ax^2}}{3a}+\frac{\left(c\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{\sqrt{3}\sqrt{a\left(3-2x^2\right)}}\\
&= -\frac{c\sqrt{cx}\sqrt{3a-2ax^2}}{3a}+\frac{c^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{a\left(3-2x^2\right)}}
\end{aligned}$$

Mathematica [C] time = 0.0214878, size = 61, normalized size = 0.69

$$\frac{c\sqrt{cx}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)+2x^2-3\right)}{3\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*Sqrt[c*x]*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (2*x^2)/3]))/(3*Sqrt[a*(3 - 2*x^2)])
```

Maple [A] time = 0.029, size = 131, normalized size = 1.5

$$-\frac{c}{12\,ax\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/12*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(
1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2),1/2*2^(1/2))+8*x^3-12*x)/x/a/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}cx}{2\,ax^2-3\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x/(2*a*x^2 - 3*a), x)
```

Sympy [A] time = 2.86659, size = 51, normalized size = 0.58

$$\frac{\sqrt{3}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(6*sqrt(a)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

3.639 $\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

```
[Out] -((6^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/S
qrt[6]], 2])/(Sqrt[x]*Sqrt[3*a - 2*a*x^2]))
```

Rubi [A] time = 0.0298237, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {320, 319, 318, 424}

$$-\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -((6^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/S
qrt[6]], 2])/(Sqrt[x]*Sqrt[3*a - 2*a*x^2]))
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] && !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{\sqrt{x}}$$
$$= \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{\sqrt{x}\sqrt{3a-2ax^2}}$$
$$= -\frac{\left(\sqrt[4]{2}3^{3/4}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$
$$= -\frac{\sqrt[4]{6}\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

Mathematica [C] time = 0.0148194, size = 53, normalized size = 0.79

$$\frac{2x\sqrt{3-2x^2}\sqrt{cx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)}{3\sqrt{a\left(9-6x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (2*x^2)/3])
/(3*Sqrt[a*(9 - 6*x^2)])
```

Maple [B] time = 0.016, size = 165, normalized size = 2.5

$$\frac{\sqrt{2}\sqrt{3}}{12\,ax\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\left(2\,\text{EllipticE}\left(1/6\,\sqrt{3}\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/12*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x
*2^(1/2)*3^(1/2))^(1/2)*(2*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(1/6*3^(1/2)*2^(1/2)*((2*
x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2)))/a/x/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{2\,ax^2-3\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*x^2 - 3*a), x)
```

Sympy [C] time = 0.735532, size = 51, normalized size = 0.76

$$\frac{\sqrt{3}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(6*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

3.640 $\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] (2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]],
 -1])/(3^(1/4)*Sqrt[c]*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] time = 0.0341655, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {329, 224, 221}

$$\frac{2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]],
 -1])/(3^(1/4)*Sqrt[c]*Sqrt[a*(3 - 2*x^2)])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{3}c\sqrt{a\left(3-2x^2\right)}}$$

$$= \frac{2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.0161984, size = 56, normalized size = 0.89

$$\frac{2x\sqrt{3-2x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)}{\sqrt{3}\sqrt{a\left(3-2x^2\right)}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3])/(Sqrt[3]*
Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

Maple [B] time = 0.025, size = 117, normalized size = 1.9

$$-\frac{1}{6a\left(2x^2-3\right)}\sqrt{-a\left(2x^2-3\right)}\sqrt{\left(2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{\left(2x+\sqrt{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/6*(-a*(2*x^2-3))^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-
2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*Elli
pticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2
*2^(1/2))/(c*x)^(1/2)/a/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{2\,acx^3-3\,acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*c*x^3 - 3*a*c*x), x)
```

Sympy [A] time = 0.963456, size = 51, normalized size = 0.81

$$\frac{\sqrt{3}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**2*exp_polar(2*I*p
i)/3)/(6*sqrt(a)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

3.641 $\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*a*c*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 -
2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sqrt
[x]*Sqrt[3*a - 2*a*x^2])
```

Rubi [A] time = 0.042602, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {325, 320, 319, 318, 424}

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*a*c*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 -
2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sqrt
[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{3c^2}\\ &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{\left(2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{3c^2\sqrt{x}}\\ &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{\left(2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{3c^2\sqrt{x}\sqrt{3a-2ax^2}}\\ &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{\left(2\sqrt[4]{\frac{2}{3}}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}\\ &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{2\sqrt[4]{2}\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}\end{aligned}$$

Mathematica [C] time = 0.0158001, size = 51, normalized size = 0.48

$$-\frac{2x\sqrt{3-2x^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{2x^2}{3}\right)}{\sqrt{a\left(9-6x^2\right)}(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-1/4, 1/2, 3/4, (2*x^2)/3])/((c*x)^
(3/2)*Sqrt[a*(9 - 6*x^2)])
```

Maple [B] time = 0.021, size = 228, normalized size = 2.1

$$-\frac{1}{18\,ac\left(2\,x^2-3\right)}\sqrt{-a\left(2\,x^2-3\right)}\left(2\,\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticE}\left(1/6\,\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/18*(-a*(2*x^2-3))^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)
*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
```

```
)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)
*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))+24*x^2-36)/c/(c*x)^(1/2)/a/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{2\,ac^2x^4-3\,ac^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*c^2*x^4 - 3*a*c^2*x^2), x)
```

Sympy [C] time = 1.94743, size = 54, normalized size = 0.5

$$\frac{\sqrt{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(6*sqrt(a)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

3.642 $\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\,2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rubi [A] time = 0.0459691, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {325, 329, 224, 221}

$$\frac{2\,2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{9c^2}$$
$$= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{9c^3}$$
$$= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{\left(4\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{9\sqrt{3}c^3\sqrt{a\left(3-2x^2\right)}}$$
$$= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{2\,2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.0146599, size = 53, normalized size = 0.54

$$-\frac{2x\sqrt{3-2x^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{3\sqrt{a\left(9-6x^2\right)}(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-3/4, 1/2, 1/4, (2*x^2)/3])/(3*(c*x
)^(5/2)*Sqrt[a*(9 - 6*x^2)])
```

Maple [A] time = 0.018, size = 132, normalized size = 1.4

$$-\frac{1}{27\,axc^2\left(2\,x^2-3\right)}\sqrt{-a\left(2\,x^2-3\right)}\left(\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/27*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(
(-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1
/2*2^(1/2))*x+12*x^2-18)/x/a/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{2\,ac^3x^5-3\,ac^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*c^3*x^5 - 3*a*c^3*x^3), x)
```

Sympy [A] time = 10.4706, size = 54, normalized size = 0.55

$$\frac{\sqrt{3}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(6*sqrt(a)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

3.643 $\int \frac{(cx)^{5/2}}{\left(3a-2ax^2\right)^{3/2}} \, dx$

Optimal. Leaf size=110

$$\frac{3\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2\ 2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

```
[Out] (c*(c*x)^(3/2))/(2*a*Sqrt[3*a - 2*a*x^2]) + (3*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[3
 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(2*2^(3/4)*a*S
qrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rubi [A] time = 0.0420811, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {288, 320, 319, 318, 424}

$$\frac{3\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2\ 2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*(c*x)^(3/2))/(2*a*Sqrt[3*a - 2*a*x^2]) + (3*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[3
 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(2*2^(3/4)*a*S
qrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{4a}\\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{4a\sqrt{x}}\\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{4a\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} + \frac{\left(3\left(\frac{3}{2}\right)^{3/4}c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{2a\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} + \frac{3\sqrt[4]{3}c^2\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.0233786, size = 59, normalized size = 0.54

$$\frac{c(cx)^{3/2}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{2x^2}{3}\right)-3\right)}{3a\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*(c*x)^(3/2)*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^
2)/3]))/(3*a*Sqrt[a*(3 - 2*x^2)])
```

Maple [B] time = 0.04, size = 230, normalized size = 2.1

$$-\frac{c^2}{16\,a^2x\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(2\,\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticE}\left(1/6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/16/x*c^2*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/
2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3
^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/
```

```
2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/
2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3
^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/
2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+8*x^2)/a^2/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}c^2x^2}{4\,a^2x^4-12\,a^2x^2+9\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x^2/(4*a^2*x^4 - 12*a^2*x^2 + 9
*a^2), x)
```

Sympy [C] time = 33.5307, size = 51, normalized size = 0.46

$$\frac{\sqrt{3}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(18*a**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

3.644 $\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{2\sqrt[4]{6}a\sqrt{a(3-2x^2)}}$$

```
[Out] (c*Sqrt[c*x])/(2*a*Sqrt[3*a - 2*a*x^2]) - (c^(3/2)*Sqrt[3 - 2*x^2]*Elliptic
F[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(2*6^(1/4)*a*Sqrt[a*(3 - 2*
x^2)])
```

Rubi [A] time = 0.0484398, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 329, 224, 221}

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}a\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
```



```
[Out] (c*Sqrt[c*x])/(2*a*Sqrt[3*a - 2*a*x^2]) - (c^(3/2)*Sqrt[3 - 2*x^2]*Elliptic
F[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(2*6^(1/4)*a*Sqrt[a*(3 - 2*
x^2)])
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx = \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{4a}$$
$$= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a}$$
$$= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{\left(c\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{2\sqrt{3}a\sqrt{a\left(3-2x^2\right)}}$$
$$= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}a\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.0170787, size = 59, normalized size = 0.63

$$-\frac{c\sqrt{cx}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)-3\right)}{6a\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] -(c*Sqrt[c*x]*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2
)/3]))/(6*a*Sqrt[a*(3 - 2*x^2)])
```

Maple [A] time = 0.033, size = 126, normalized size = 1.3

$$\frac{c}{24\,a^2x\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{(2
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] 1/24*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(
1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1
/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(
1/2))^(1/2),1/2*2^(1/2))-12*x)/x/a^2/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}cx}{4\,a^2x^4-12\,a^2x^2+9\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x/(4*a^2*x^4 - 12*a^2*x^2 + 9*a^2
), x)
```

Sympy [A] time = 3.31358, size = 51, normalized size = 0.54

$$\frac{\sqrt{3}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(18*a**(3/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

3.645 $\int \frac{\sqrt{cx}}{\left(3a-2ax^2\right)^{3/2}} \, dx$

Optimal. Leaf size=101

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}$$

```
[Out] (c*x)^(3/2)/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[c*x]*Sqrt[3 - 2*x^2]*Ellipt
icE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(6^(3/4)*a*Sqrt[x]*Sqrt[3*a -
2*a*x^2])
```

Rubi [A] time = 0.0393555, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {290, 320, 319, 318, 424}

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*x)^(3/2)/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[c*x]*Sqrt[3 - 2*x^2]*Ellipt
icE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(6^(3/4)*a*Sqrt[x]*Sqrt[3*a -
2*a*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{6a}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{6a\sqrt{x}}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{6a\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{2^{3/4}\sqrt[4]{3}a\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.0143966, size = 58, normalized size = 0.57

$$\frac{2x\left(3-2x^2\right)^{3/2}\sqrt{cx}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{2x^2}{3}\right)}{9\sqrt{3}\left(a\left(3-2x^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^2)/3
])/(9*Sqrt[3]*(a*(3 - 2*x^2))^(3/2))
```

Maple [B] time = 0.02, size = 227, normalized size = 2.3

$$-\frac{1}{72\,a^2x\left(2\,x^2-3\right)}\sqrt{cx}\sqrt{-a\left(2\,x^2-3\right)}\left(2\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/72*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^
(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)
*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(
```

```
1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^
(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)
*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(
1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+24*x^2)/a^2/x/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{4\,a^2x^4-12\,a^2x^2+9\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*x^4 - 12*a^2*x^2 + 9*a^2), x
)
```

Sympy [C] time = 1.35031, size = 51, normalized size = 0.5

$$\frac{\sqrt{3}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(18*a**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

3.646 $\int \frac{1}{\sqrt{cx}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{6}a\sqrt{c}\sqrt{a\left(3-2x^2\right)}}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

```
[Out] Sqrt[c*x]/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[3 - 2*x^2]*EllipticF[ArcSin[(
(2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*6^(1/4)*a*Sqrt[c]*Sqrt[a*(3 - 2*x^
2)])
```

Rubi [A] time = 0.0463873, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {290, 329, 224, 221}

$$\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}a\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c*x]/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[3 - 2*x^2]*EllipticF[ArcSin[(
(2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*6^(1/4)*a*Sqrt[c]*Sqrt[a*(3 - 2*x^
2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(3a-2ax^2\right)^{3/2}}\,dx=\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{6a}$$

$$=\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3ac}$$

$$=\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{3\sqrt{3}ac\sqrt{a\left(3-2x^2\right)}}$$

$$=\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}a\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.0216386, size = 59, normalized size = 0.61

$$\frac{x\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)+3\right)}{9a\sqrt{a\left(3-2x^2\right)}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (x*(3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3]))/(9*a*
Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

Maple [A] time = 0.024, size = 122, normalized size = 1.3

$$-\frac{1}{36\,a^2\left(2\,x^2-3\right)}\sqrt{-a\left(2\,x^2-3\right)}\left(\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{2}\sqrt{3}}{6}\sqrt{(2\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/36*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(
(-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1
/2*2^(1/2))+12*x)/a^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{4\,a^2cx^5-12\,a^2cx^3+9\,a^2cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*c*x^5 - 12*a^2*c*x^3 + 9*a^2
*c*x), x)
```

Sympy [A] time = 2.30421, size = 51, normalized size = 0.53

$$\frac{\sqrt{3}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**2*exp_polar(2*I*p
i)/3)/(18*a**(3/2)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

3.647 $\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

```
[Out] 1/(3*a*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]) - Sqrt[3*a - 2*a*x^2]/(3*a^2*c*Sqrt
[c*x]) + (2^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[
6]*x]/Sqrt[6]], 2])/(3^(3/4)*a*c^2*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rubi [A] time = 0.0561087, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {290, 325, 320, 319, 318, 424}

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] 1/(3*a*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]) - Sqrt[3*a - 2*a*x^2]/(3*a^2*c*Sqrt
[c*x]) + (2^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[
6]*x]/Sqrt[6]], 2])/(3^(3/4)*a*c^2*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
```

```
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}+\frac{\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx}{2a}\\
&= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{3ac^2}\\
&= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{3ac^2\sqrt{x}}\\
&= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{3ac^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\left(\sqrt[4]{\frac{2}{3}}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{ac^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\sqrt{cx}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.020038, size = 58, normalized size = 0.41

$$-\frac{2x\left(3-2x^2\right)^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};\frac{2x^2}{3}\right)}{3\sqrt{3}\left(a\left(3-2x^2\right)\right)^{3/2}(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-1/4, 3/2, 3/4, (2*x^2)/3])/(3*Sq
rt[3]*(c*x)^(3/2)*(a*(3 - 2*x^2))^(3/2))
```

Maple [B] time = 0.023, size = 228, normalized size = 1.6

$$-\frac{1}{36\,a^2c\left(2\,x^2-3\right)}\sqrt{-a\left(2\,x^2-3\right)}\left(2\,\sqrt{2}\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\mathrm{EllipticE}\left(1/6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/36*(-a*(2*x^2-3))^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)
*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)
*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))+24*x^2-24)/a^2/c/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{4\,a^2c^2x^6-12\,a^2c^2x^4+9\,a^2c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*c^2*x^6 - 12*a^2*c^2*x^4 + 9
*a^2*c^2*x^2), x)
```

Sympy [C] time = 6.01221, size = 54, normalized size = 0.39

$$\frac{\sqrt{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(18*a**(3/2)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

3.648 $\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{5\ 2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{27\sqrt[4]{3}ac^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

```
[Out] 1/(3*a*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]) - (5*Sqrt[3*a - 2*a*x^2])/(27*a^2
*c*(c*x)^(3/2)) + (5*2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(27*3^(1/4)*a*c^(5/2)*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] time = 0.0631519, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {290, 325, 329, 224, 221}

$$-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\ 2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{27\sqrt[4]{3}ac^{5/2}\sqrt{a\left(3-2x^2\right)}}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]) - (5*Sqrt[3*a - 2*a*x^2])/(27*a^2
*c*(c*x)^(3/2)) + (5*2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(27*3^(1/4)*a*c^(5/2)*Sqrt[a*(3 - 2*x^2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}+\frac{5\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx}{6a}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{27ac^2}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{10\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{27ac^3}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{\left(10\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{27\sqrt{3}ac^3\sqrt{a\left(3-2x^2\right)}}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\,2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{27\sqrt[4]{3}ac^{5/2}\sqrt{a\left(3-2x^2\right)}}
\end{aligned}
$$

Mathematica [C] time = 0.0187255, size = 58, normalized size = 0.44

$$
-\frac{2x\left(3-2x^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{9\sqrt{3}\left(a\left(3-2x^2\right)\right)^{3/2}(cx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-3/4, 3/2, 1/4, (2*x^2)/3])/(9*Sq
rt[3]*(c*x)^(5/2)*(a*(3 - 2*x^2))^(3/2))
```

Maple [A] time = 0.024, size = 133, normalized size = 1.

$$
-\frac{1}{162\,a^2xc^2\left(2\,x^2-3\right)}\sqrt{-a\left(2\,x^2-3\right)}\left(5\sqrt{\left(2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2\,x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\mathrm{EllipticF}\left(1/6\,\sqrt{3}\sqrt{2}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/162*(-a*(2*x^2-3))^(1/2)*(5*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)
*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
),1/2*2^(1/2))*x+60*x^2-36)/x/a^2/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{4\,a^2c^3x^7-12\,a^2c^3x^5+9\,a^2c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*c^3*x^7 - 12*a^2*c^3*x^5 + 9
*a^2*c^3*x^3), x)
```

Sympy [A] time = 38.3393, size = 54, normalized size = 0.41

$$\frac{\sqrt{3}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(18*a**(3/2)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right),-1\right)}{\sqrt{a}}$$

```
[Out] (2*EllipticF[ArcSin[Sqrt[a]*Sqrt[x]], -1])/Sqrt[a]
```

Rubi [A] time = 0.008636, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {329, 221}

$$\frac{2F\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*EllipticF[ArcSin[Sqrt[a]*Sqrt[x]], -1])/Sqrt[a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^4}}\,dx,x,\sqrt{x}\right)\\ &= \frac{2F\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.0065838, size = 24, normalized size = 1.14

$$2\sqrt{x}\,_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] 2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, a^2*x^2]
```

Maple [B] time = 0.045, size = 66, normalized size = 3.1

$$-\frac{1}{a\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax+1}\sqrt{-2\,ax+2}\sqrt{-ax}\mathrm{EllipticF}\left(\sqrt{ax+1},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/x^(1/2)*(-a^2*x^2+1)^(1/2)*(a*x+1)^(1/2)*(-2*a*x+2)^(1/2)*(-a*x)^(1/2)*E
llipticF((a*x+1)^(1/2),1/2*2^(1/2))/a/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\sqrt{x}}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*sqrt(x)/(a^2*x^3 - x), x)
```

Sympy [B] time = 0.67367, size = 36, normalized size = 1.71

$$\frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**2*exp_polar(2*I*pi))/(
2*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

3.650 $\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx$

Optimal. Leaf size=67

$$\frac{\left(\sqrt{a}x+1\right)\sqrt{\frac{ax^2+1}{\left(\sqrt{a}x+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{a}\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax^2+1}}$$

```
[Out] ((1 + Sqrt[a]*x)*Sqrt[(1 + a*x^2)/(1 + Sqrt[a]*x)^2]*EllipticF[2*ArcTan[a^(
1/4)*Sqrt[x]], 1/2])/(a^(1/4)*Sqrt[1 + a*x^2])
```

Rubi [A] time = 0.0377915, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {329, 220}

$$\frac{\left(\sqrt{a}x+1\right)\sqrt{\frac{ax^2+1}{\left(\sqrt{a}x+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

```
[Out] ((1 + Sqrt[a]*x)*Sqrt[(1 + a*x^2)/(1 + Sqrt[a]*x)^2]*EllipticF[2*ArcTan[a^(
1/4)*Sqrt[x]], 1/2])/(a^(1/4)*Sqrt[1 + a*x^2])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^4}}\,dx, x, \sqrt{x}\right)\\ &= \frac{\left(1+\sqrt{a}x\right)\sqrt{\frac{1+ax^2}{\left(1+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{1+ax^2}}\end{aligned}$$

Mathematica [C] time = 0.0061015, size = 23, normalized size = 0.34

$$2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

```
[Out] 2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, -(a*x^2)]
```

Maple [A] time = 0.041, size = 73, normalized size = 1.1

$$-\sqrt{2}\sqrt{-x\sqrt{-a}+1}\sqrt{x\sqrt{-a}+1}\sqrt{x\sqrt{-a}}\,\mathrm{EllipticF}\left(\sqrt{-x\sqrt{-a}+1},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{ax^2+1}}\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x^2+1)^(1/2),x)
```

```
[Out] -1/x^(1/2)/(a*x^2+1)^(1/2)*(-x*(-a)^(1/2)+1)^(1/2)*2^(1/2)*(x*(-a)^(1/2)+1)
^(1/2)*(x*(-a)^(1/2))^(1/2)*EllipticF((-x*(-a)^(1/2)+1)^(1/2),1/2*2^(1/2))/
(-a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ax^2+1}\sqrt{x}}{ax^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^2 + 1)*sqrt(x)/(a*x^3 + x), x)
```

Sympy [C] time = 0.640454, size = 32, normalized size = 0.48

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|ax^2e^{i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(a*x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a*x**2*exp_polar(I*pi))/(2*gam
ma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

3.651 $\int x^m \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\left(a+bx^2\right)^{5/2} {}_2F_1\left(1,\frac{m+6}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*(a + b*x^2)^(5/2)*Hypergeometric2F1[1, (6 + m)/2, (3 + m)/2, -((
b*x^2)/a)])/(a*(1 + m))
```

Rubi [A] time = 0.0192922, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{ax^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^m \left(a+bx^2\right)^{3/2} dx &= \frac{\left(a\sqrt{a+bx^2}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^{3/2} dx}{\sqrt{1+\frac{bx^2}{a}}}\\
&= \frac{ax^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.020168, size = 66, normalized size = 1.32

$$\frac{ax^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, 1 + (1 + m)
/2, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{m} \left(b x^{2}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^(3/2),x)
```

```
[Out] int(x^m*(b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{\frac{3}{2}} x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^{2}+a\right)^{\frac{3}{2}} x^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*x^m, x)
```

Sympy [C] time = 4.39179, size = 54, normalized size = 1.08

$$\frac{a^{\frac{3}{2}} x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-3/2, m/2 + 1/2), (m/2 + 3/2,), b*x
**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^m, x)
```

3.652 $\int x^m \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\left(a+bx^2\right)^{3/2}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*(a + b*x^2)^(3/2)*Hypergeometric2F1[1, (4 + m)/2, (3 + m)/2, -((
b*x^2)/a)])/(a*(1 + m))
```

Rubi [A] time = 0.0178072, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{a+bx^2}\,dx &= \frac{\sqrt{a+bx^2}\int x^m\sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}\\ &= \frac{x^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0130236, size = 65, normalized size = 1.3

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, 1 + (1 + m)/2
, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{m} \sqrt{b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b x^{2}+a}\, x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b x^{2}+a}\, x^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m, x)
```

Sympy [C] time = 1.06195, size = 54, normalized size = 1.08

$$\frac{\sqrt{a} x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^m, x)
```

3.653 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[1, (2 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*(1 + m))
```

Rubi [A] time = 0.0179431, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.015395, size = 65, normalized size = 1.3

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{m} \frac{1}{\sqrt{b x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\sqrt{b x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m}}{\sqrt{b x^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 0.873122, size = 53, normalized size = 1.06

$$\frac{x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 \sqrt{a} \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

3.654 $\int \frac{x^m}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, m/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m)*S
qrt[a + b*x^2])
```

Rubi [A] time = 0.0195241, antiderivative size = 66, normalized size of antiderivative = 1.38, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx^2)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0159536, size = 68, normalized size = 1.42

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x^{m} \left(b x^{2}+a \right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{{\left(b x^{2}+a\right)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{b x^{2}+a} x^{m}}{b^{2} x^{4}+2 \, a b x^{2}+a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 1.53825, size = 53, normalized size = 1.1

$$\frac{x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{2 a^{\frac{3}{2}} \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(3/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(3/2), x)
```

3.655 $\int \frac{x^m}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\left(a+bx^2\right)^{3/2}}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (-2 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
 + m)*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.0197179, antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0169601, size = 68, normalized size = 1.36

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^m \left(bx^2+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(5/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,x^m}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x
)
```

Sympy [C] time = 5.08622, size = 53, normalized size = 1.06

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(5/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((5/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(5/2), x)
```

3.656 $\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+3}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{a(m+3)}$$

```
[Out] (x^(3 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[1, (4 + m)/2, (5 + m)/2, -((b*
x^2)/a)])/(a*(3 + m))
```

Rubi [A] time = 0.0195211, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2,
 -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{2+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{3+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};-\frac{bx^2}{a}\right)}{(3+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0167157, size = 65, normalized size = 1.3

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+3}{2}+1;-\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, 1 + (3 + m
)/2, -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{2+m}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(2+m)/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m+2}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 3.89411, size = 54, normalized size = 1.08

$$\frac{x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**3*x**m*gamma(m/2 + 3/2)*hyper((1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp
_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+2}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

3.657 $\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+2}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{a(m+2)}$$

```
[Out] (x^(2 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[1, (3 + m)/2, (4 + m)/2, -((b*
x^2)/a)])/(a*(2 + m))
```

Rubi [A] time = 0.019524, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{(2+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0173393, size = 65, normalized size = 1.3

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+2}{2}+1;-\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, 1 + (2 + m
)/2, -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{1+m}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(1+m)/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m+1}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 2.14529, size = 48, normalized size = 0.96

$$\frac{x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar
(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

3.658 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[1, (2 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*(1 + m))
```

Rubi [A] time = 0.0171533, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0026274, size = 65, normalized size = 1.3

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 0.861342, size = 53, normalized size = 1.06

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

3.659 $\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=46

$$\frac{x^m\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{am}$$

```
[Out] (x^m*Sqrt[a + b*x^2]*Hypergeometric2F1[1, (1 + m)/2, (2 + m)/2, -((b*x^2)/a
)])/(a*m)
```

Rubi [A] time = 0.0186178, antiderivative size = 57, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a
)])/(m*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^m\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0118479, size = 57, normalized size = 1.24

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m}{2}+1;-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, 1 + m/2, -((b*x^2)/a)]
)/(m*Sqrt[a + b*x^2])
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int x^{-1+m}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(-1+m)/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-1}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 4.7015, size = 41, normalized size = 0.89

$$\frac{x^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(2*
sqrt(a)*gamma(m/2 + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

3.660 $\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x^{m-1}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{a(1-m)}$$

```
[Out] -((x^(-1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[1, m/2, (1 + m)/2, -((b*x^2
)/a)])/(a*(1 - m)))
```

Rubi [A] time = 0.0199039, antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] -((x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m
)/2, -((b*x^2)/a)])/((1 - m)*Sqrt[a + b*x^2]))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{-2+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= -\frac{x^{-1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0179246, size = 65, normalized size = 1.27

$$\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m-1}{2}+1;-\frac{bx^2}{a}\right)}{(m-1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, 1 + (-1
+ m)/2, -((b*x^2)/a)])/((-1 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^{-2+m}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-2}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 26.4725, size = 53, normalized size = 1.04

$$\frac{x^m\Gamma\left(\frac{m}{2}-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}-\frac{1}{2}\\ \frac{m}{2}+\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**m*gamma(m/2 - 1/2)*hyper((1/2, m/2 - 1/2), (m/2 + 1/2,), b*x**2*exp_pola
r(I*pi)/a)/(2*sqrt(a)*x*gamma(m/2 + 1/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

3.661 $\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

```
[Out] x^(2 + m)*Sqrt[a + b*x^2]
```

Rubi [A] time = 0.0113457, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {449}

$$x^{m+2}\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] x^(2 + m)*Sqrt[a + b*x^2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}}\,dx = x^{2+m}\sqrt{a+bx^2}$$

Mathematica [C] time = 0.101849, size = 104, normalized size = 6.12

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\left(b(m+3)x^2\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)+a(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2,
 (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/2,
 (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 16, normalized size = 0.9

$$x^{2+m}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x)
```

```
[Out] x^(2+m)*(b*x^2+a)^(1/2)
```

Maxima [A] time = 2.15904, size = 22, normalized size = 1.29

$$\sqrt{bx^2 + a}x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

Fricas [A] time = 1.60732, size = 39, normalized size = 2.29

$$\sqrt{bx^2 + a}xx^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(b*x^2 + a)*x*x^(m + 1)
```

Sympy [C] time = 13.1468, size = 202, normalized size = 11.88

$$\frac{\sqrt{a}mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{\sqrt{a}x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{bmx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\\frac{m}{2}+3\end{matrix}\middle|\frac{bx^2}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*(a*(2+m)+b*(3+m)*x**2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*m*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + sqrt(a)*x**2*x**m*gamma(m/2 + 1)*h
yper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/gamma(m/2 + 2) +
 b*m*x**4*x**m*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_
polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3)) + 3*b*x**4*x**m*gamma(m/2 + 2)*hy
per((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(
m/2 + 3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b(m+3)x^2 + a(m+2)\right)x^{m+1}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*(m + 3)*x^2 + a*(m + 2))*x^(m + 1)/sqrt(b*x^2 + a), x)
```

3.662 $\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

```
[Out] x^(2 + m)*Sqrt[a + b*x^2]
```

Rubi [C] time = 0.0659783, antiderivative size = 127, normalized size of antiderivative = 7.47, number of steps used = 5, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {365, 364}

$$\frac{ax^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(m+3)x^{m+4}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqrt[a +
b*x^2],x]
```

```
[Out] (a*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)])/Sqrt[a + b*x^2] + (b*(3 + m)*x^(4 + m)*Sqrt[1 + (b*x^2)/a
]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, -((b*x^2)/a)])/((4 + m)*Sqrt
[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx=(a(2+m))\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx+(b(3+m))\int \frac{x^{3+m}}{\sqrt{a+bx^2}}\,dx$$

$$=\frac{\left(a(2+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}+\frac{\left(b(3+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{3+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$

$$=\frac{ax^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{4+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\right.}{(4+m)\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0511731, size = 104, normalized size = 6.12

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\left(b(m+3)x^2\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)+a(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqr
t[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2,
 (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/2,
 (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int a\,(2+m)\,x^{1+m}\frac{1}{\sqrt{bx^2+a}}+b\,(3+m)\,x^{3+m}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x)
```

Maxima [A] time = 1.56135, size = 22, normalized size = 1.29

$$\sqrt{bx^2+a}x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

Fricas [A] time = 1.57856, size = 39, normalized size = 2.29

$$\frac{\sqrt{bx^2+a}x^{m+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 3)/x
```

Sympy [C] time = 9.51591, size = 105, normalized size = 6.18

$$\frac{\sqrt{a}x^2x^m(m+2)\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{bx^4x^m(m+3)\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\\frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x**(1+m)/(b*x**2+a)**(1/2)+b*(3+m)*x**(3+m)/(b*x**2+a)**(
1/2),x)
```

```
[Out] sqrt(a)*x**2*x**m*(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,),
b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + b*x**4*x**m*(m + 3)*gamma(m/
2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*gamma(m/2 + 3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b(m+3)x^{m+3}}{\sqrt{bx^2+a}}+\frac{a(m+2)x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b*
x^2 + a), x)
```

3.663 $\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

```
[Out] x^m/Sqrt[a + b*x^2]
```

Rubi [A] time = 0.0118946, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {449}

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] x^m/Sqrt[a + b*x^2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.106674, size = 103, normalized size = 6.87

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\left(b(m-1)x^2\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)+a(m+2)\,{}_2F_1\left(\frac{3}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
-((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$x^m \frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x)
```

```
[Out] x^m/(b*x^2+a)^(1/2)
```

Maxima [A] time = 2.42674, size = 18, normalized size = 1.2

$$\frac{x^m}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

Fricas [A] time = 1.57488, size = 39, normalized size = 2.6

$$\frac{xx^{m-1}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] x*x^(m - 1)/sqrt(b*x^2 + a)
```

Sympy [C] time = 111.034, size = 97, normalized size = 6.47

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+1\right)}+\frac{bx^2x^m(m-1)\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a*m+b*(-1+m)*x**2)/(b*x**2+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((3/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) + b*x**2*x**m*(m - 1)*gamma(m/2 + 1)*hyper((3/2,
m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b(m-1)x^2+am\right)x^{m-1}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*(m - 1)*x^2 + a*m)*x^(m - 1)/(b*x^2 + a)^(3/2), x)
```

3.664 $\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

```
[Out] x^m/Sqrt[a + b*x^2]
```

Rubi [C] time = 0.0650502, antiderivative size = 123, normalized size of antiderivative = 8.2, number of steps used = 5, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a
)])/Sqrt[a + b*x^2] - (b*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*(2 + m)*Sqrt[a + b*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) dx &= -\left(b\int \frac{x^{1+m}}{\left(a+bx^2\right)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{\left(b\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2}}+\frac{\left(m\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\
&= \frac{x^m\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{a(2+m)\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0398933, size = 103, normalized size = 6.87

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\left(b(m-1)x^2\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)+a(m+2)\,{}_2F_1\left(\frac{3}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^
2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
-((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int -bx^{1+m}\left(bx^2+a\right)^{-\frac{3}{2}}+mx^{-1+m}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x)
```

Maxima [A] time = 2.3452, size = 18, normalized size = 1.2

$$\frac{x^m}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

Fricas [A] time = 1.59138, size = 55, normalized size = 3.67

$$\frac{\sqrt{bx^2+a}x^{m+1}}{bx^3+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 1)/(b*x^3 + a*x)
```

Sympy [C] time = 11.0682, size = 94, normalized size = 6.27

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+1\right)}-\frac{bx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x**(1+m)/(b*x**2+a)**(3/2)+m*x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) - b*x**2*x**m*gamma(m/2 + 1)*hyper((3/2, m/2 + 1)
, (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{mx^{m-1}}{\sqrt{bx^2+a}}-\frac{bx^{m+1}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x)
```

3.665 $\int x^7 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=80

$$\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}$$

```
[Out] (-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (
9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)
```

Rubi [A] time = 0.0491851, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (
9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3 \sqrt[3]{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx, x, x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}\end{aligned}$$

Mathematica [A] time = 0.0274937, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{4/3}\left(108a^2bx^2-81a^3-126ab^2x^4+140b^3x^6\right)}{3640b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-81*a^3 + 108*a^2*b*x^2 - 126*a*b^2*x^4 + 140*b^3*x^6
))/(3640*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-420\,b^3x^6+378\,ab^2x^4-324\,a^2bx^2+243\,a^3}{3640\,b^4}\left(bx^2+a\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/3),x)
```

```
[Out] -3/3640*(b*x^2+a)^(4/3)*(-140*b^3*x^6+126*a*b^2*x^4-108*a^2*b*x^2+81*a^3)/b
^4
```

Maxima [A] time = 2.54927, size = 86, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{13}{3}}}{26\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{10}{3}}a}{20\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{7}{3}}a^2}{14\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{4}{3}}a^3}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/26*(b*x^2 + a)^(13/3)/b^4 - 9/20*(b*x^2 + a)^(10/3)*a/b^4 + 9/14*(b*x^2 +
 a)^(7/3)*a^2/b^4 - 3/8*(b*x^2 + a)^(4/3)*a^3/b^4
```

Fricas [A] time = 1.49548, size = 135, normalized size = 1.69

$$\frac{3\left(140\,b^4x^8+14\,ab^3x^6-18\,a^2b^2x^4+27\,a^3bx^2-81\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(140*b^4*x^8 + 14*a*b^3*x^6 - 18*a^2*b^2*x^4 + 27*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(1/3)/b^4
```

Sympy [B] time = 2.62751, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x*
*2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8
+ 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(73/3)/(3640*a*
*20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7
*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x
**12) - 1377*a**(70/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2184
0*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a
**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1458*a*
*(70/3)*b*x**2/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*
x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**
10 + 3640*a**14*b**10*x**12) - 3213*a**(67/3)*b**2*x**4*(1 + b*x**2/a)**(1/
3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800
*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a*
*14*b**10*x**12) + 3645*a**(67/3)*b**2*x**4/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) - 3927*a**(64/3)*
b**3*x**6*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 +
54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 218
40*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 4860*a**(64/3)*b**3*x**6/(3
640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**1
7*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b
**10*x**12) - 2163*a**(61/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(3640*a**20*b*
*4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6
+ 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
+ 3645*a**(61/3)*b**4*x**8/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600
*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a*
*15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1827*a**(58/3)*b**5*x**10*(1 + b
*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6
*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x*
*10 + 3640*a**14*b**10*x**12) + 1458*a**(58/3)*b**5*x**10/(3640*a**20*b**4
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5
4600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 6
573*a**(55/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**
19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*
b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(55/3
)*b**6*x**12/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x*
*4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10
 + 3640*a**14*b**10*x**12) + 8787*a**(52/3)*b**7*x**14*(1 + b*x**2/a)**(1/3
)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*
a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**
14*b**10*x**12) + 6498*a**(49/3)*b**8*x**16*(1 + b*x**2/a)**(1/3)/(3640*a**
20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*
x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x*
*12) + 2562*a**(46/3)*b**9*x**18*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2
1840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5460
0*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 420*
a**(43/3)*b**10*x**20*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
```

Giac [A] time = 1.84312, size = 77, normalized size = 0.96

$$\frac{3\left(140\left(bx^2+a\right)^{\frac{13}{3}}-546\left(bx^2+a\right)^{\frac{10}{3}}a+780\left(bx^2+a\right)^{\frac{7}{3}}a^2-455\left(bx^2+a\right)^{\frac{4}{3}}a^3\right)}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/3640*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2 + a)
^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)/b^4
```

3.666 $\int x^5 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}$$

```
[Out] (3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a
 + b*x^2)^(10/3))/(20*b^3)
```

Rubi [A] time = 0.034617, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a
 + b*x^2)^(10/3))/(20*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2 \sqrt[3]{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx, x, x^2\right)\\ &= \frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}\end{aligned}$$

Mathematica [A] time = 0.0174325, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{4/3}\left(9a^2-12abx^2+14b^2x^4\right)}{280b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(9*a^2 - 12*a*b*x^2 + 14*b^2*x^4))/(280*b^3)
```

Maple [A] time = 0.003, size = 36, normalized size = 0.6

$$\frac{42\,b^2x^4-36\,abx^2+27\,a^2}{280\,b^3}\left(bx^2+a\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/3),x)
```

```
[Out] 3/280*(b*x^2+a)^(4/3)*(14*b^2*x^4-12*a*b*x^2+9*a^2)/b^3
```

Maxima [A] time = 2.3167, size = 63, normalized size = 1.07

$$\frac{3\left(bx^2+a\right)^{\frac{10}{3}}}{20\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{7}{3}}a}{7\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{4}{3}}a^2}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^3 - 3/7*(b*x^2 + a)^(7/3)*a/b^3 + 3/8*(b*x^2 + a)
^(4/3)*a^2/b^3
```

Fricas [A] time = 1.46205, size = 105, normalized size = 1.78

$$\frac{3\left(14\,b^3x^6+2\,ab^2x^4-3\,a^2bx^2+9\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 + 2*a*b^2*x^4 - 3*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(1/3)/b^
3
```

Sympy [B] time = 1.7159, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}-\frac{27a^{\frac{34}{3}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{72a^{\frac{3}{3}}}{280a^8b^3+840a^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 84
0*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(34/3)/(280*a**8*b**3 + 840*
a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 72*a**(31/3)*b*
x**2*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b
**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(31/3)*b*x**2/(280*a**8*b**3 + 840*a
**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(28/3)*b**
2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6
*b**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(28/3)*b**2*x**4/(280*a**8*b**3 +
840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(25/3
)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840
*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(25/3)*b**3*x**6/(280*a**8*b*
*3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 135*a*
*(22/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2
 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 132*a**(19/3)*b**5*x**10*(1 +
 b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4
+ 280*a**5*b**6*x**6) + 42*a**(16/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*
a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6)
```

Giac [A] time = 2.66339, size = 58, normalized size = 0.98

$$\frac{3\left(14\left(bx^2+a\right)^{\frac{10}{3}}-40\left(bx^2+a\right)^{\frac{7}{3}}a+35\left(bx^2+a\right)^{\frac{4}{3}}a^2\right)}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/280*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/3
)*a^2)/b^3
```

3.667 $\int x^3 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}$$

```
[Out] (-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)
```

Rubi [A] time = 0.0231805, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt[3]{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx,x,x^2\right)\\ &= -\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}\end{aligned}$$

Mathematica [A] time = 0.0132055, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{4/3}\left(4bx^2-3a\right)}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-3*a + 4*b*x^2))/(56*b^2)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.7

$$-\frac{-12\,bx^2+9\,a}{56\,b^2}\left(bx^2+a\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/3),x)
```

```
[Out] -3/56*(b*x^2+a)^(4/3)*(-4*b*x^2+3*a)/b^2
```

Maxima [A] time = 1.62692, size = 41, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{4}{3}}a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^2 - 3/8*(b*x^2 + a)^(4/3)*a/b^2
```

Fricas [A] time = 1.50011, size = 78, normalized size = 2.05

$$\frac{3\left(4\,b^2x^4+abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}}{56\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/56*(4*b^2*x^4 + a*b*x^2 - 3*a^2)*(b*x^2 + a)^(1/3)/b^2
```

Sympy [B] time = 1.08524, size = 223, normalized size = 5.87

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{9a^{\frac{13}{3}}}{56a^2b^2+56ab^3x^2}-\frac{6a^{\frac{10}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{9a^{\frac{10}{3}}bx^2}{56a^2b^2+56ab^3x^2}+\frac{15a^{\frac{7}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{12a^{\frac{4}{3}}b^3x^6}{56a^2b^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(
13/3)/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*a**(10/3)*b*x**2*(1 + b*x**2/a)**
(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(10/3)*b*x**2/(56*a**2*b**2 +
56*a*b**3*x**2) + 15*a**(7/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2
```

```
 + 56*a*b**3*x**2) + 12*a**(4/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(56*a**2*b
**2 + 56*a*b**3*x**2)
```

Giac [A] time = 1.93942, size = 39, normalized size = 1.03

$$\frac{3\left(4\left(bx^2+a\right)^{\frac{7}{3}}-7\left(bx^2+a\right)^{\frac{4}{3}}a\right)}{56\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/56*(4*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a)/b^2
```

3.668 $\int x\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

Rubi [A] time = 0.003245, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Mathematica [A] time = 0.0031798, size = 18, normalized size = 1.

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{3}{8\,b}\left(bx^2+a\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/3),x)
```

```
[Out] 3/8*(b*x^2+a)^(4/3)/b
```

Maxima [A] time = 1.14705, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

Fricas [A] time = 1.41901, size = 34, normalized size = 1.89

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

Sympy [A] time = 0.183809, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\sqrt[3]{a+bx^2}}{8b} + \frac{3x^2\sqrt[3]{a+bx^2}}{8} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{a}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(1/3)/(8*b) + 3*x**2*(a + b*x**2)**(1/3)/8, Ne
(b, 0)), (a**(1/3)*x**2/2, True))
```

Giac [A] time = 2.35255, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

3.669 $\int \frac{\sqrt[3]{a+bx^2}}{x} dx$

Optimal. Leaf size=101

$$\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

```
[Out] (3*(a + b*x^2)^(1/3))/2 - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rubi [A] time = 0.0791417, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/2 - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{4}\left(3\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{1}{4}\left(3a^{2/3}\right)\operatorname{Subst}\left(\int \frac{}{a^{2/3}+\cdots}\right.\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) + \frac{1}{2}\left(3\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+\cdots}}{\sqrt[3]{a}}\right.\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.052382, size = 126, normalized size = 1.25

$$
\frac{1}{4}\left(-\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)+6\sqrt[3]{a+bx^2}+2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (6*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)] - a^(1/3)*Lo
g[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$
\int \frac{1}{x}\sqrt[3]{bx^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5347, size = 313, normalized size = 3.1

$$-\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{4}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^
(1/3)
```

Sympy [C] time = 1.06573, size = 46, normalized size = 0.46

$$-\frac{\sqrt[3]{b}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x,x)
```

```
[Out] -b**(1/3)*x**(2/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

Giac [A] time = 3.26837, size = 132, normalized size = 1.31

$$-\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{1}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/2*(b*x^2 + a)
^(1/3)
```

3.670 $\int \frac{\sqrt[3]{a+bx^2}}{x^3} \, dx$

Optimal. Leaf size=107

$$\frac{b \log \left(\sqrt[3]{a}-\sqrt[3]{a+b x^2}\right)}{4 a^{2/3}}-\frac{b \tan ^{-1}\left(\frac{2 \sqrt[3]{a+b x^2}+\sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{2 \sqrt{3} a^{2/3}}-\frac{b \log (x)}{6 a^{2/3}}-\frac{\sqrt[3]{a+b x^2}}{2 x^2}$$

```
[Out] -(a + b*x^2)^(1/3)/(2*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(2*Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(4*a^(2/3))
```

Rubi [A] time = 0.0713732, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b \log \left(\sqrt[3]{a}-\sqrt[3]{a+b x^2}\right)}{4 a^{2/3}}-\frac{b \tan ^{-1}\left(\frac{2 \sqrt[3]{a+b x^2}+\sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{2 \sqrt{3} a^{2/3}}-\frac{b \log (x)}{6 a^{2/3}}-\frac{\sqrt[3]{a+b x^2}}{2 x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(2*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(2*Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(4*a^(2/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} + \frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0074419, size = 37, normalized size = 0.35

$$\frac{3b\left(a+bx^2\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx^2}{a}+1\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x^2)/a])/(8*a^
2)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56092, size = 436, normalized size = 4.07

$$-\frac{2\sqrt{3}\left(a^{2}\right)^{\frac{1}{6}}abx^{2}\arctan\left(\frac{\left(a^{2}\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^{2}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)}{3a^{2}}\right)+\left(a^{2}\right)^{\frac{2}{3}}bx^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(bx^{2}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)\cdots}{12a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*(a^2)^(1/6)*a*b*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^
(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) -
 2*(a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^2 + a)
^(1/3)*a^2)/(a^2*x^2)
```

Sympy [C] time = 1.26436, size = 42, normalized size = 0.39

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**3,x)
```

```
[Out] -b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.671 $\int \frac{\sqrt[3]{a+bx^2}}{x^5} \, dx$

Optimal. Leaf size=135

$$-\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3}}+\frac{b^2 \log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

```
[Out] -(a + b*x^2)^(1/3)/(4*x^4) - (b*(a + b*x^2)^(1/3))/(12*a*x^2) + (b^2*ArcTan
[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) +
(b^2*Log[x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(5
/3))
```

Rubi [A] time = 0.0941841, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 51, 57, 617, 204, 31}

$$-\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3}}+\frac{b^2 \log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(4*x^4) - (b*(a + b*x^2)^(1/3))/(12*a*x^2) + (b^2*ArcTan
[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) +
(b^2*Log[x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(5
/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4} + \frac{1}{12}b\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{18a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\log(x)}{18a^{5/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{5/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\ \cdots\right)}{12a^{4/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\ \cdots\right)}{6a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}a^{5/3}} + \frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.0078034, size = 39, normalized size = 0.29

$$-\frac{3b^2\left(a+bx^2\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};\frac{bx^2}{a}+1\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + (b*x^2)/a])/(8
*a^3)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53228, size = 497, normalized size = 3.68

$$\frac{2\sqrt{3}ab^2x^4\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(\right.\right.}{36a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*a*b^2*x^4*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2)
 + (-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 +
 a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a -
(-a^2)^(2/3)) - 3*(a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^3*x^4)
```

Sympy [C] time = 1.57061, size = 42, normalized size = 0.31

$$-\frac{\sqrt[3]{b}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**5,x)
```

```
[Out] -b**(1/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(10/3)*gamma(8/3))
```

Giac [A] time = 3.35581, size = 167, normalized size = 1.24

$$\frac{1}{36}\,b^2\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(\left(bx^2+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] 1/36*b^2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(5/3) + log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(5/3) - 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x^2 + a)
^(4/3) + 2*(b*x^2 + a)^(1/3)*a)/(a*b^2*x^4))
```

3.672 $\int x^4 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=314

$$-\frac{54\, 3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{54a^2 \cdots}{\cdots}$$

```
[Out] (-54*a^2*x*(a + b*x^2)^(1/3))/(935*b^2) + (6*a*x^3*(a + b*x^2)^(1/3))/(187*
b) + (3*x^5*(a + b*x^2)^(1/3))/17 - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
+ b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.282635, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}-\frac{54\, 3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\cdots\right)}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/3),x]
```

```
[Out] (-54*a^2*x*(a + b*x^2)^(1/3))/(935*b^2) + (6*a*x^3*(a + b*x^2)^(1/3))/(187*
b) + (3*x^5*(a + b*x^2)^(1/3))/17 - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
+ b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int x^4\sqrt[3]{a+bx^2}\,dx=\frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{1}{17}(2a)\int\frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$=\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}-\frac{\left(18a^2\right)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx}{187b}$$

$$=-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{\left(54a^3\right)\int\frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{935b^2}$$

$$=-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{\left(81a^3\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{935b^3x}$$

$$=-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}}{935b^3x\sqrt{\cdots}}$$

Mathematica [C] time = 0.0512226, size = 94, normalized size = 0.3

$$\frac{3x\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-9a^2+2abx^2+11b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{187b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-9*a^2 + 2*a*b*x^2 + 11*b^2*
x^4) + 9*a^2*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)]))/(187*b^2*(1
+ (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x^4\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/3),x)
```

```
[Out] int(x^4*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*x^4, x)
```

Sympy [A] time = 0.811777, size = 29, normalized size = 0.09

$$\frac{\sqrt[3]{a}x^5{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*x**5*hyper((-1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*x^4, x)
```

3.673 $\int x^2 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=290

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3}{11}x^3\sqrt[3]{a+$$

```
[Out] (6*a*x*(a + b*x^2)^(1/3))/(55*b) + (3*x^3*(a + b*x^2)^(1/3))/11 + (6*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/
3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^2*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.161355, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/3),x]
```

```
[Out] (6*a*x*(a + b*x^2)^(1/3))/(55*b) + (3*x^3*(a + b*x^2)^(1/3))/11 + (6*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/
3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^2*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt[3]{a+bx^2}\,dx &= \frac{3}{11}x^3\sqrt[3]{a+bx^2}+\frac{1}{11}(2a)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}-\frac{\left(6a^2\right)\int\frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{55b}\\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}-\frac{\left(9a^2\sqrt{bx^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{55b^2x}\\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}+\frac{6\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\cdots\right)}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^{\cdots}}}}
\end{aligned}$$

Mathematica [C] time = 0.0452349, size = 62, normalized size = 0.21

$$\frac{3x\sqrt[3]{a+bx^2}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}+a+bx^2\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*(a + b*x^2 - (a*Hypergeometric2F1[-1/3, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(11*b)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^2\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/3),x)
```

```
[Out] int(x^2*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{3}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*x^2, x)
```

Sympy [A] time = 0.709148, size = 29, normalized size = 0.1

$$\frac{\sqrt[3]{a}x^3 {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*x^2, x)
```

3.674 $\int \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=266

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] (3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a +
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.124485, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 236, 219}

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a +
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{a+bx^2}\,dx &= \frac{3}{5}x\sqrt[3]{a+bx^2}+\frac{1}{5}(2a)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{3}{5}x\sqrt[3]{a+bx^2}+\frac{\left(3a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{5bx}\\
&= \frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right|}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.0051491, size = 46, normalized size = 0.17

$$\frac{x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3),x]
```

```
[Out] (x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/3)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 0.664355, size = 26, normalized size = 0.1

$$\sqrt[3]{a}x{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3), x)
```

3.675 $\int \frac{\sqrt[3]{a+bx^2}}{x^2} dx$

Optimal. Leaf size=260

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{x}$$

```
[Out] -((a + b*x^2)^(1/3)/x) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.125427, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 236, 219}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/3)/x) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt[3]{a+bx^2}}{x}+\frac{1}{3}(2b)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx^2}}{x}+\frac{\sqrt{bx^2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{x}\\
&= -\frac{\sqrt[3]{a+bx^2}}{x}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-\right.}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.0084361, size = 49, normalized size = 0.19

$$-\frac{\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/3)))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/x^2, x)
```

Sympy [A] time = 0.711676, size = 29, normalized size = 0.11

$$-\frac{\sqrt[3]{a}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**2,x)
```

```
[Out] -a**(1/3)*hyper((-1/2, -1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^2, x)
```

3.676 $\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=290

$$\frac{2\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\cdots$$

```
[Out] -(a + b*x^2)^(1/3)/(3*x^3) - (2*b*(a + b*x^2)^(1/3))/(9*a*x) + (2*Sqrt[2 -
Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2
)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2)])
```

Rubi [A] time = 0.157859, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 325, 236, 219}

$$\frac{2\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^4,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(3*x^3) - (2*b*(a + b*x^2)^(1/3))/(9*a*x) + (2*Sqrt[2 -
Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2
)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx &= -\frac{\sqrt[3]{a+bx^2}}{3x^3}+\frac{1}{9}(2b)\int \frac{1}{x^2\left(a+bx^2\right)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{\left(2b^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{27a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{\left(b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9ax}\\
&= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}+\frac{2\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{9\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0087967, size = 51, normalized size = 0.18

$$
-\frac{\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^4,x]
```

```
[Out] -((a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, -((b*x^2)/a)])/(3*x
^3*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$
\int \frac{1}{x^4}\sqrt[3]{bx^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^4,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/x^4, x)
```

Sympy [A] time = 0.82812, size = 34, normalized size = 0.12

$$-\frac{\sqrt[3]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**4,x)
```

```
[Out] -a**(1/3)*hyper((-3/2, -1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^4, x)
```

3.677 $\int x^7 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=80

$$\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}$$

```
[Out] (-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) -
(9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)
```

Rubi [A] time = 0.0473386, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) -
(9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{2/3}dx&=\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{2/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx,x,x^2\right)\\&=-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}\end{aligned}$$

Mathematica [A] time = 0.0246458, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{5/3}\left(135a^2bx^2-81a^3-180ab^2x^4+220b^3x^6\right)}{6160b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(-81*a^3 + 135*a^2*b*x^2 - 180*a*b^2*x^4 + 220*b^3*x^6
))/(6160*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-660\,b^3x^6+540\,ab^2x^4-405\,a^2bx^2+243\,a^3}{6160\,b^4}\left(bx^2+a\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(2/3),x)
```

```
[Out] -3/6160*(b*x^2+a)^(5/3)*(-220*b^3*x^6+180*a*b^2*x^4-135*a^2*b*x^2+81*a^3)/b
^4
```

Maxima [A] time = 1.59909, size = 86, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{14}{3}}}{28\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{11}{3}}a}{22\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{8}{3}}a^2}{16\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a^3}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/28*(b*x^2 + a)^(14/3)/b^4 - 9/22*(b*x^2 + a)^(11/3)*a/b^4 + 9/16*(b*x^2 +
 a)^(8/3)*a^2/b^4 - 3/10*(b*x^2 + a)^(5/3)*a^3/b^4
```

Fricas [A] time = 1.71919, size = 135, normalized size = 1.69

$$\frac{3\left(220\,b^4x^8+40\,ab^3x^6-45\,a^2b^2x^4+54\,a^3bx^2-81\,a^4\right)\left(bx^2+a\right)^{\frac{2}{3}}}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/6160*(220*b^4*x^8 + 40*a*b^3*x^6 - 45*a^2*b^2*x^4 + 54*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(2/3)/b^4
```

Sympy [B] time = 2.93274, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x*
*2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 243*a**(74/3)/(6160*a
**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b*
*7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10
*x**12) - 1296*a**(71/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36
960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 9240
0*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458
*a**(71/3)*b*x**2/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) - 2808*a**(68/3)*b**2*x**4*(1 + b*x**2/a)*
*(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 1
23200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 61
60*a**14*b**10*x**12) + 3645*a**(68/3)*b**2*x**4/(6160*a**20*b**4 + 36960*a
**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**
16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 3120*a**(
65/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x
**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**
8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4860*a**(65/3)*b**3*
x**6/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123
200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160
*a**14*b**10*x**12) - 1050*a**(62/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(6160*
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 3645*a**(62/3)*b**4*x**8/(6160*a**20*b**4 + 36960*a**19*b**5*x**
2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
+ 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4032*a**(59/3)*b**5*x*
*10*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*
a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a*
*15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458*a**(59/3)*b**5*x**10/(6160*
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 11004*a**(56/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**
4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6
+ 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12)
+ 243*a**(56/3)*b**6*x**12/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400
*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a
**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 14352*a**(53/3)*b**7*x**14*(1 +
 b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) + 10485*a**(50/3)*b**8*x**16*(1 + b*x**2/a
)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 +
 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 +
6160*a**14*b**10*x**12) + 4080*a**(47/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(
6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a*
*17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14
*b**10*x**12) + 660*a**(44/3)*b**10*x**20*(1 + b*x**2/a)**(2/3)/(6160*a**20
*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x
**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**
12)
```

Giac [A] time = 2.97849, size = 77, normalized size = 0.96

$$\frac{3\left(220\left(bx^2+a\right)^{\frac{14}{3}}-840\left(bx^2+a\right)^{\frac{11}{3}}a+1155\left(bx^2+a\right)^{\frac{8}{3}}a^2-616\left(bx^2+a\right)^{\frac{5}{3}}a^3\right)}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/6160*(220*(b*x^2 + a)^(14/3) - 840*(b*x^2 + a)^(11/3)*a + 1155*(b*x^2 + a
)^(8/3)*a^2 - 616*(b*x^2 + a)^(5/3)*a^3)/b^4
```

3.678 $\int x^5 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}$$

```
[Out] (3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(
a + b*x^2)^(11/3))/(22*b^3)
```

Rubi [A] time = 0.0359452, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(
a + b*x^2)^(11/3))/(22*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{2/3} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{2/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}\end{aligned}$$

Mathematica [A] time = 0.0165565, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{5/3}\left(9a^2-15abx^2+20b^2x^4\right)}{440b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(9*a^2 - 15*a*b*x^2 + 20*b^2*x^4))/(440*b^3)
```

Maple [A] time = 0.004, size = 36, normalized size = 0.6

$$\frac{60\,b^2x^4-45\,abx^2+27\,a^2}{440\,b^3}\left(bx^2+a\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(2/3),x)
```

```
[Out] 3/440*(b*x^2+a)^(5/3)*(20*b^2*x^4-15*a*b*x^2+9*a^2)/b^3
```

Maxima [A] time = 1.96401, size = 63, normalized size = 1.07

$$\frac{3\left(bx^2+a\right)^{\frac{11}{3}}}{22\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{8}{3}}a}{8\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a^2}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^2 + a)^(11/3)/b^3 - 3/8*(b*x^2 + a)^(8/3)*a/b^3 + 3/10*(b*x^2 + a
)^(5/3)*a^2/b^3
```

Fricas [A] time = 1.72563, size = 105, normalized size = 1.78

$$\frac{3\left(20\,b^3x^6+5\,ab^2x^4-6\,a^2bx^2+9\,a^3\right)\left(bx^2+a\right)^{\frac{2}{3}}}{440\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/440*(20*b^3*x^6 + 5*a*b^2*x^4 - 6*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(2/3)/b^
3
```

Sympy [B] time = 1.8706, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}-\frac{27a^{\frac{35}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{}{440a^8b^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1
320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(35/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 63*a**(32/3
)*b*x**2*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*
a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(32/3)*b*x**2/(440*a**8*b**3 +
 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 42*a**(2
9/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(29/3)*b**2*x**4/(440*a*
*8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) +
 78*a**(26/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b*
*4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(26/3)*b**3*x**
6/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**
6*x**6) + 207*a**(23/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 195*a**(20/
3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 60*a**(17/3)*b**6*x**12*(1 + b*
x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 +
 440*a**5*b**6*x**6)
```

Giac [A] time = 2.1507, size = 58, normalized size = 0.98

$$\frac{3\left(20\left(bx^2+a\right)^{\frac{11}{3}}-55\left(bx^2+a\right)^{\frac{8}{3}}a+44\left(bx^2+a\right)^{\frac{5}{3}}a^2\right)}{440\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/440*(20*(b*x^2 + a)^(11/3) - 55*(b*x^2 + a)^(8/3)*a + 44*(b*x^2 + a)^(5/3
)*a^2)/b^3
```

3.679 $\int x^3 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}$$

```
[Out] (-3*a*(a + b*x^2)^(5/3))/(10*b^2) + (3*(a + b*x^2)^(8/3))/(16*b^2)
```

Rubi [A] time = 0.0234761, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(5/3))/(10*b^2) + (3*(a + b*x^2)^(8/3))/(16*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{2/3}dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^{2/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx,x,x^2\right)\\&=-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}\end{aligned}$$

Mathematica [A] time = 0.0127471, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{5/3}\left(5bx^2-3a\right)}{80b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(-3*a + 5*b*x^2))/(80*b^2)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$-\frac{-15\,bx^2+9\,a}{80\,b^2}\left(bx^2+a\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(2/3),x)
```

```
[Out] -3/80*(b*x^2+a)^(5/3)*(-5*b*x^2+3*a)/b^2
```

Maxima [A] time = 2.46676, size = 41, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{8}{3}}}{16\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^2 - 3/10*(b*x^2 + a)^(5/3)*a/b^2
```

Fricas [A] time = 1.76861, size = 81, normalized size = 2.13

$$\frac{3\left(5\,b^2x^4+2\,abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3)/b^2
```

Sympy [A] time = 0.737089, size = 66, normalized size = 1.74

$$\begin{cases} -\frac{9a^2\left(a+bx^2\right)^{\frac{2}{3}}}{80b^2}+\frac{3ax^2\left(a+bx^2\right)^{\frac{2}{3}}}{40b}+\frac{3x^4\left(a+bx^2\right)^{\frac{2}{3}}}{16} & \text{for } b\neq 0 \\ \frac{a^{\frac{2}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)**(
2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/4, T
```

```
rue))
```

Giac [A] time = 2.88715, size = 39, normalized size = 1.03

$$\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}-8\left(bx^2+a\right)^{\frac{5}{3}}a\right)}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/80*(5*(b*x^2 + a)^(8/3) - 8*(b*x^2 + a)^(5/3)*a)/b^2
```

3.680 $\int x\left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

Rubi [A] time = 0.0036006, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Mathematica [A] time = 0.0033351, size = 18, normalized size = 1.

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{3}{10\,b}\left(bx^2+a\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(2/3),x)
```

```
[Out] 3/10*(b*x^2+a)^(5/3)/b
```

Maxima [A] time = 2.06763, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

Fricas [A] time = 1.73614, size = 35, normalized size = 1.94

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

Sympy [A] time = 0.358824, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\left(a+bx^2\right)^{\frac{2}{3}}}{10b} + \frac{3x^2\left(a+bx^2\right)^{\frac{2}{3}}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(2/3)/(10*b) + 3*x**2*(a + b*x**2)**(2/3)/10,
Ne(b, 0)), (a**(2/3)*x**2/2, True))
```

Giac [A] time = 2.45134, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

3.681 $\int \frac{(a+bx^2)^{2/3}}{x} \, dx$

Optimal. Leaf size=101

$$\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rubi [A] time = 0.0664327, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 50, 55, 617, 204, 31}

$$\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3} - \frac{1}{2}a^{2/3}\log(x) - \frac{1}{4}\left(3a^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right) + \frac{1}{4}(3a)\operatorname{Subst}\left(\int \right.\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3} - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) - \frac{1}{2}\left(3a^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3} + \frac{1}{2}\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0351165, size = 93, normalized size = 0.92

$$\frac{1}{4}\left(3\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\left(a+bx^2\right)^{2/3}\right)+2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2a^{2/3}\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*
a^(2/3)*Log[x] + 3*((a + b*x^2)^(2/3) + a^(2/3)*Log[a^(1/3) - (a + b*x^2)^(
1/3)]))/4
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8373, size = 358, normalized size = 3.54

$$\frac{1}{2}\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{4}\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)+\frac{1}{2}\left(a^2\right)^{\frac{1}{3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)
*(a^2)^(1/3))/a) - 1/4*(a^2)^(1/3)*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a
+ (b*x^2 + a)^(1/3)*(a^2)^(2/3)) + 1/2*(a^2)^(1/3)*log((b*x^2 + a)^(1/3)*a
- (a^2)^(2/3)) + 3/4*(b*x^2 + a)^(2/3)
```

Sympy [C] time = 1.11012, size = 46, normalized size = 0.46

$$-\frac{b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x,x)
```

```
[Out] -b**(2/3)*x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(1/3))
```

Giac [A] time = 4.26245, size = 132, normalized size = 1.31

$$\frac{1}{2}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{2}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/2*a^(2/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/4*(b*x^2 + a)^
(2/3)
```

3.682 $\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

```
[Out] -(a + b*x^2)^(2/3)/(2*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3)
 - (a + b*x^2)^(1/3)])/(2*a^(1/3))
```

Rubi [A] time = 0.0663066, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 47, 55, 617, 204, 31}

$$-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(2*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3)
 - (a + b*x^2)^(1/3)])/(2*a^(1/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{2/3}}{2x^2} + \frac{1}{3} b\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{2/3}}{2x^2} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{1}{2} b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,d\ldots\right.}{2\sqrt[3]{a}}\\
&= -\frac{(a+bx^2)^{2/3}}{2x^2} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}} - \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}\\
&= -\frac{(a+bx^2)^{2/3}}{2x^2} + \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}
\end{aligned}$$

Mathematica [C] time = 0.0078281, size = 37, normalized size = 0.36

$$\frac{3b\left(a+bx^2\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx^2}{a}+1\right)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x^2)/a])/(10*a
^2)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8534, size = 819, normalized size = 7.88

$$\left[\frac{3\sqrt{\frac{1}{3}}abx^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-a^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}\right.}{6\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x^2*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*b*x^2*log((b*x^2 + a)^(2
/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^2*log((b*x^2 + a
)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^2), 1/6*(6*sqrt(1/3)*a^(2/
3)*b*x^2*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3
)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^
(2/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^
2)]
```

Sympy [C] time = 1.31117, size = 42, normalized size = 0.4

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**3,x)
```

```
[Out] -b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(2/3)*gamma(4/3))
```

Giac [A] time = 5.31429, size = 144, normalized size = 1.38

$$\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{bx^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(1/3) - log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1
/3) + 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^2 + a)^(2/3)
/(b*x^2))*b
```

3.683 $\int \frac{(a+bx^2)^{2/3}}{x^5} dx$

Optimal. Leaf size=135

$$-\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{4/3}}+\frac{b^2 \log(x)}{18a^{4/3}}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{\left(a+bx^2\right)^{2/3}}{4x^4}$$

```
[Out] -(a + b*x^2)^(2/3)/(4*x^4) - (b*(a + b*x^2)^(2/3))/(6*a*x^2) - (b^2*ArcTan[
(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) + (
b^2*Log[x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(4/
3))
```

Rubi [A] time = 0.0877556, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 51, 55, 617, 204, 31}

$$-\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{4/3}}+\frac{b^2 \log(x)}{18a^{4/3}}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{\left(a+bx^2\right)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(4*x^4) - (b*(a + b*x^2)^(2/3))/(6*a*x^2) - (b^2*ArcTan[
(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) + (
b^2*Log[x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(4/
3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}+\frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{18a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}+\frac{b^2\log(x)}{18a^{4/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2\,\text{Subst}\left(\int \cdots\right.}{\cdots}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\cdots\right.}{6a^{4/3}}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.008453, size = 39, normalized size = 0.29

$$
-\frac{3b^2\left(a+bx^2\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};\frac{bx^2}{a}+1\right)}{10a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^5,x]
```

[Out] (-3*b^2*(a + b*x^2)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + (b*x^2)/a])/(1
0*a^3)

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(2/3)/x^5,x)

[Out] int((b*x^2+a)^(2/3)/x^5,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.85463, size = 994, normalized size = 7.36

$$\left[\frac{3\sqrt{\frac{1}{3}}ab^2x^4\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^2}\right)+(-a)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}-\right.}{36a^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="fricas")

[Out] [1/36*(3*sqrt(1/3)*a*b^2*x^4*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*
(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt(
(-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b^2
*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2
*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x^2 + 3*
a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4), -1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-(-a)^
(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/
3)/a)) - (-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^
(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/
3)) + 3*(2*a*b*x^2 + 3*a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4)]

Sympy [C] time = 1.64532, size = 42, normalized size = 0.31

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**5,x)
```

```
[Out] -b**(2/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(8/3)*gamma(7/3))
```

Giac [A] time = 4.25879, size = 170, normalized size = 1.26

$$-\frac{1}{36}b^2\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*b^2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(4/3) + 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x^2 +
 a)^(5/3) + (b*x^2 + a)^(2/3)*a)/(a*b^2*x^4))
```

3.684 $\int x^4 \left(a+bx^2\right)^{2/3} \, dx$

Optimal. Leaf size=601

$$-\frac{108\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{1729b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{108a^2x\left(a+\right.}{1729b}$$

[Out] (-108*a^2*x*(a + b*x^2)^(2/3))/(1729*b^2) + (12*a*x^3*(a + b*x^2)^(2/3))/(247*b) + (3*x^5*(a + b*x^2)^(2/3))/19 - (324*a^3*x)/(1729*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rubi [A] time = 0.462292, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {279, 321, 235, 304, 219, 1879}

$$-\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}-\frac{324a^3x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{108\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F}{1729b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}\right.}}}$$

Antiderivative was successfully verified.

[In] Int[x^4*(a + b*x^2)^(2/3),x]

[Out] (-108*a^2*x*(a + b*x^2)^(2/3))/(1729*b^2) + (12*a*x^3*(a + b*x^2)^(2/3))/(247*b) + (3*x^5*(a + b*x^2)^(2/3))/19 - (324*a^3*x)/(1729*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+bx^2\right)^{2/3}dx &= \frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{1}{19}(4a)\int\frac{x^4}{\sqrt[3]{a+bx^2}}dx\\
&= \frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{\left(36a^2\right)\int\frac{x^2}{\sqrt[3]{a+bx^2}}dx}{247b}\\
&= -\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{\left(108a^3\right)\int\frac{1}{\sqrt[3]{a+bx^2}}dx}{1729b^2}\\
&= -\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{\left(162a^3\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\right.}{1729b^3x}\\
&= -\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{\left(162a^3\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\sqrt{-a+x}}\right.}{1729b^3x}\\
&= -\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{324a^3x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}
\end{aligned}
$$

Mathematica [C] time = 0.0491592, size = 94, normalized size = 0.16

$$\frac{3x\left(a+bx^2\right)^{2/3}\left(\left(\frac{bx^2}{a}+1\right)^{2/3}\left(-9a^2+4abx^2+13b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{247b^2\left(\frac{bx^2}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3)*((1 + (b*x^2)/a)^(2/3)*(-9*a^2 + 4*a*b*x^2 + 13*b^2*
x^4) + 9*a^2*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)]))/(247*b^2*(1
+ (b*x^2)/a)^(2/3))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x^4\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^4*(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{2}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{2}{3}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^4, x)
```

Sympy [A] time = 0.984561, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}}x^5{}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{2}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^4, x)
```

3.685 $\int x^2\left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=577

$$\frac{12\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}\ldots}{\ldots}$$

```
[Out] (12*a*x*(a + b*x^2)^(2/3))/(91*b) + (3*x^3*(a + b*x^2)^(2/3))/13 + (36*a^2*
x)/(91*b*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (18*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^2*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2)]) + (12*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(91*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.368626, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {279, 321, 235, 304, 219, 1879}

$$\frac{12\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(2/3),x]
```

```
[Out] (12*a*x*(a + b*x^2)^(2/3))/(91*b) + (3*x^3*(a + b*x^2)^(2/3))/13 + (36*a^2*
x)/(91*b*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (18*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^2*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2)]) + (12*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(91*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{2/3}dx=\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{1}{13}(4a)\int\frac{x^2}{\sqrt[3]{a+bx^2}}dx$$
$$=\frac{12ax\left(a+bx^2\right)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}-\frac{\left(12a^2\right)\int\frac{1}{\sqrt[3]{a+bx^2}}dx}{91b}$$
$$=\frac{12ax\left(a+bx^2\right)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}-\frac{\left(18a^2\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a+bx^2}\right)}{91b^2x}$$
$$=\frac{12ax\left(a+bx^2\right)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{\left(18a^2\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a+bx^2}\right)}{91b^2x}-\frac{(18}{}$$
$$=\frac{12ax\left(a+bx^2\right)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{36a^2x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.051442, size = 62, normalized size = 0.11

$$\frac{3x\left(a+bx^2\right)^{2/3}\left(-\frac{a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{2/3}}+a+bx^2\right)}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3)*(a + b*x^2 - (a*Hypergeometric2F1[-2/3, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(2/3)))/(13*b)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^2*(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{2}{3}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^2, x)
```

Sympy [A] time = 0.809248, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}}x^3{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

3.686 $\int \left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=550

$$-\frac{4\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\cdots}{\cdots}$$

```
[Out] (3*x*(a + b*x^2)^(2/3))/7 - (12*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))) + (6*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.307188, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {195, 235, 304, 219, 1879}

$$-\frac{4\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3))/7 - (12*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))) + (6*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{2/3} dx &= \frac{3}{7}x\left(a+bx^2\right)^{2/3}+\frac{1}{7}(4a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx \\
&= \frac{3}{7}x\left(a+bx^2\right)^{2/3}+\frac{\left(6a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{7bx} \\
&= \frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{\left(6a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{7bx}+\frac{\left(6\sqrt{2\left(2+\sqrt{3}\right)}a^{4/3}\sqrt{bx^2}\right)\text{S}\cdots}{\cdots} \\
&= \frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a\cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{7bx\sqrt{-\frac{\cdots}{\left(\left(\cdots\right.\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.0056107, size = 46, normalized size = 0.08

$$\frac{x\left(a+bx^2\right)^{2/3}{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3),x]
```

```
[Out] (x*(a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(2/3)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 0.697659, size = 26, normalized size = 0.05

$$a^{\frac{2}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3), x)
```

3.687 $\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=538

$$-\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\cdots\right.}{\cdots}$$

```
[Out] -((a + b*x^2)^(2/3)/x) - (4*b*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1
/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.303027, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 235, 304, 219, 1879}

$$-\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x^2)^(2/3)/x) - (4*b*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1
/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^2}\,dx = -\frac{\left(a+bx^2\right)^{2/3}}{x}+\frac{1}{3}(4b)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{x}+\frac{\left(2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{x}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{x}-\frac{\left(2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{x}+\frac{\left(2\sqrt{2\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{bx^2}\right)\mathrm{Sub}}{x}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{x}-\frac{4bx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right)}}}{x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{}\right)}}}$$

Mathematica [C] time = 0.0100983, size = 49, normalized size = 0.09

$$-\frac{\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, -1/2, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(2/3)))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/x^2, x)
```

Sympy [A] time = 0.754195, size = 29, normalized size = 0.05

$$-\frac{a^{\frac{2}{3}}{}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**2,x)
```

```
[Out] -a**(2/3)*hyper((-2/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{\frac{2}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

3.688 $\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx$

Optimal. Leaf size=575

$$-\frac{4\sqrt{2}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt{2+\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{\right.}\cdots}{\cdots}$$

```
[Out] -(a + b*x^2)^(2/3)/(3*x^3) - (4*b*(a + b*x^2)^(2/3))/(9*a*x) - (4*b^2*x)/(9
*a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (2*Sqrt[2 + Sqrt[3]]*b*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2
)]) - (4*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^(2/3
)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.373007, antiderivative size = 575, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {277, 325, 235, 304, 219, 1879}

$$-\frac{4\sqrt{2}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt{2+\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^4,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(3*x^3) - (4*b*(a + b*x^2)^(2/3))/(9*a*x) - (4*b^2*x)/(9
*a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (2*Sqrt[2 + Sqrt[3]]*b*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2
)]) - (4*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^(2/3
)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^4}\,dx = -\frac{\left(a+bx^2\right)^{2/3}}{3x^3} + \frac{1}{9}(4b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3} - \frac{4b\left(a+bx^2\right)^{2/3}}{9ax} + \frac{\left(4b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{27a}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3} - \frac{4b\left(a+bx^2\right)^{2/3}}{9ax} + \frac{\left(2b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{9ax}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3} - \frac{4b\left(a+bx^2\right)^{2/3}}{9ax} - \frac{\left(2b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{9ax} + \frac{\left(2\sqrt{2\left(2+\sqrt{3}\right.}\right.}{}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3} - \frac{4b\left(a+bx^2\right)^{2/3}}{9ax} - \frac{4b^2x}{9a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{2\sqrt{2+\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{}}{3}$$

Mathematica [C] time = 0.0086728, size = 51, normalized size = 0.09

$$-\frac{\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^4,x]
```

```
[Out] -((a + b*x^2)^(2/3)*Hypergeometric2F1[-3/2, -2/3, -1/2, -((b*x^2)/a)])/(3*x
^3*(1 + (b*x^2)/a)^(2/3))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^4,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/x^4, x)
```

Sympy [A] time = 0.904695, size = 34, normalized size = 0.06

$$-\frac{a^{\frac{2}{3}}{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{2}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**4,x)
```

```
[Out] -a**(2/3)*hyper((-3/2, -2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

3.689 $\int x^7 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=80

$$\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}$$

```
[Out] (-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) -
 (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)
```

Rubi [A] time = 0.048907, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) -
 (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{4/3} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{4/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}\end{aligned}$$

Mathematica [A] time = 0.026091, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{7/3}\left(189a^2bx^2-81a^3-315ab^2x^4+455b^3x^6\right)}{14560b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-81*a^3 + 189*a^2*b*x^2 - 315*a*b^2*x^4 + 455*b^3*x^6
))/(14560*b^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.6

$$-\frac{-1365\,b^3x^6+945\,ab^2x^4-567\,a^2bx^2+243\,a^3}{14560\,b^4}\left(bx^2+a\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(4/3),x)
```

```
[Out] -3/14560*(b*x^2+a)^(7/3)*(-455*b^3*x^6+315*a*b^2*x^4-189*a^2*b*x^2+81*a^3)/
b^4
```

Maxima [A] time = 1.85292, size = 86, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{16}{3}}}{32\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{13}{3}}a}{26\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{10}{3}}a^2}{20\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{7}{3}}a^3}{14\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/32*(b*x^2 + a)^(16/3)/b^4 - 9/26*(b*x^2 + a)^(13/3)*a/b^4 + 9/20*(b*x^2 +
 a)^(10/3)*a^2/b^4 - 3/14*(b*x^2 + a)^(7/3)*a^3/b^4
```

Fricas [A] time = 1.74078, size = 162, normalized size = 2.02

$$\frac{3\left(455\,b^5x^{10}+595\,ab^4x^8+14\,a^2b^3x^6-18\,a^3b^2x^4+27\,a^4bx^2-81\,a^5\right)\left(bx^2+a\right)^{\frac{1}{3}}}{14560\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/14560*(455*b^5*x^10 + 595*a*b^4*x^8 + 14*a^2*b^3*x^6 - 18*a^3*b^2*x^4 + 2
7*a^4*b*x^2 - 81*a^5)*(b*x^2 + a)^(1/3)/b^4
```

Sympy [A] time = 5.78559, size = 136, normalized size = 1.7

$$\begin{cases} -\frac{243a^5\sqrt[3]{a+bx^2}}{14560b^4}+\frac{81a^4x^2\sqrt[3]{a+bx^2}}{14560b^3}-\frac{27a^3x^4\sqrt[3]{a+bx^2}}{7280b^2}+\frac{3a^2x^6\sqrt[3]{a+bx^2}}{1040b}+\frac{51ax^8\sqrt[3]{a+bx^2}}{416}+\frac{3bx^{10}\sqrt[3]{a+bx^2}}{32} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-243*a**5*(a + b*x**2)**(1/3)/(14560*b**4) + 81*a**4*x**2*(a + b
*x**2)**(1/3)/(14560*b**3) - 27*a**3*x**4*(a + b*x**2)**(1/3)/(7280*b**2) +
 3*a**2*x**6*(a + b*x**2)**(1/3)/(1040*b) + 51*a*x**8*(a + b*x**2)**(1/3)/4
16 + 3*b*x**10*(a + b*x**2)**(1/3)/32, Ne(b, 0)), (a**(4/3)*x**8/8, True))
```

Giac [B] time = 2.53501, size = 181, normalized size = 2.26

$$\frac{3\left(\frac{4\left(140\left(bx^2+a\right)^{\frac{13}{3}}-546\left(bx^2+a\right)^{\frac{10}{3}}a+780\left(bx^2+a\right)^{\frac{7}{3}}a^2-455\left(bx^2+a\right)^{\frac{4}{3}}a^3\right)a}{b^3}+\frac{455\left(bx^2+a\right)^{\frac{16}{3}}-2240\left(bx^2+a\right)^{\frac{13}{3}}a+4368\left(bx^2+a\right)^{\frac{10}{3}}a^2-4160\left(bx^2+a\right)^{\frac{7}{3}}a^3+1820}{b^3}\right.}{14560\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14560*(4*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2
+ a)^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)*a/b^3 + (455*(b*x^2 + a)^(16/3)
 - 2240*(b*x^2 + a)^(13/3)*a + 4368*(b*x^2 + a)^(10/3)*a^2 - 4160*(b*x^2 +
a)^(7/3)*a^3 + 1820*(b*x^2 + a)^(4/3)*a^4)/b^3)/b
```

3.690 $\int x^5 \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}$$

```
[Out] (3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3
*(a + b*x^2)^(13/3))/(26*b^3)
```

Rubi [A] time = 0.0377544, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3
*(a + b*x^2)^(13/3))/(26*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{4/3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{4/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}\end{aligned}$$

Mathematica [A] time = 0.0184864, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{7/3}\left(9a^2-21abx^2+35b^2x^4\right)}{910b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)
```

Maple [A] time = 0.004, size = 36, normalized size = 0.6

$$\frac{105\,b^2x^4 - 63\,abx^2 + 27\,a^2}{910\,b^3}\left(bx^2 + a\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(4/3),x)
```

```
[Out] 3/910*(b*x^2+a)^(7/3)*(35*b^2*x^4-21*a*b*x^2+9*a^2)/b^3
```

Maxima [A] time = 1.96662, size = 63, normalized size = 1.07

$$\frac{3\left(bx^2 + a\right)^{\frac{13}{3}}}{26\,b^3} - \frac{3\left(bx^2 + a\right)^{\frac{10}{3}}a}{10\,b^3} + \frac{3\left(bx^2 + a\right)^{\frac{7}{3}}a^2}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/26*(b*x^2 + a)^(13/3)/b^3 - 3/10*(b*x^2 + a)^(10/3)*a/b^3 + 3/14*(b*x^2 +
 a)^(7/3)*a^2/b^3
```

Fricas [A] time = 1.67103, size = 128, normalized size = 2.17

$$\frac{3\left(35\,b^4x^8 + 49\,ab^3x^6 + 2\,a^2b^2x^4 - 3\,a^3bx^2 + 9\,a^4\right)\left(bx^2 + a\right)^{\frac{1}{3}}}{910\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/910*(35*b^4*x^8 + 49*a*b^3*x^6 + 2*a^2*b^2*x^4 - 3*a^3*b*x^2 + 9*a^4)*(b*
x^2 + a)^(1/3)/b^3
```

Sympy [A] time = 3.7216, size = 112, normalized size = 1.9

$$\begin{cases} \frac{27a^4\sqrt[3]{a+bx^2}}{910b^3} - \frac{9a^3x^2\sqrt[3]{a+bx^2}}{910b^2} + \frac{3a^2x^4\sqrt[3]{a+bx^2}}{455b} + \frac{21ax^6\sqrt[3]{a+bx^2}}{130} + \frac{3bx^8\sqrt[3]{a+bx^2}}{26} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((27*a**4*(a + b*x**2)**(1/3)/(910*b**3) - 9*a**3*x**2*(a + b*x**2
)**(1/3)/(910*b**2) + 3*a**2*x**4*(a + b*x**2)**(1/3)/(455*b) + 21*a*x**6*(
a + b*x**2)**(1/3)/130 + 3*b*x**8*(a + b*x**2)**(1/3)/26, Ne(b, 0)), (a**(4
/3)*x**6/6, True))
```

Giac [B] time = 2.73441, size = 143, normalized size = 2.42

$$\frac{3\left(\frac{13\left(14\left(bx^2+a\right)^{\frac{10}{3}}-40\left(bx^2+a\right)^{\frac{7}{3}}a+35\left(bx^2+a\right)^{\frac{4}{3}}a^2\right)a}{b^2}+\frac{140\left(bx^2+a\right)^{\frac{13}{3}}-546\left(bx^2+a\right)^{\frac{10}{3}}a+780\left(bx^2+a\right)^{\frac{7}{3}}a^2-455\left(bx^2+a\right)^{\frac{4}{3}}a^3}{b^2}\right)}{3640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/3640*(13*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)
^(4/3)*a^2)*a/b^2 + (140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 78
0*(b*x^2 + a)^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)/b^2)/b
```

3.691 $\int x^3 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}$$

```
[Out] (-3*a*(a + b*x^2)^(7/3))/(14*b^2) + (3*(a + b*x^2)^(10/3))/(20*b^2)
```

Rubi [A] time = 0.0242434, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(7/3))/(14*b^2) + (3*(a + b*x^2)^(10/3))/(20*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{4/3} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{4/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx,x,x^2\right)\\ &= -\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}\end{aligned}$$

Mathematica [A] time = 0.0140855, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{7/3}\left(7bx^2-3a\right)}{140b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-3*a + 7*b*x^2))/(140*b^2)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$-\frac{-21\,bx^{2}+9\,a}{140\,b^{2}}\left(bx^{2}+a\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(4/3),x)
```

```
[Out] -3/140*(b*x^2+a)^(7/3)*(-7*b*x^2+3*a)/b^2
```

Maxima [A] time = 2.11117, size = 41, normalized size = 1.08

$$\frac{3\left(bx^{2}+a\right)^{\frac{10}{3}}}{20\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}a}{14\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^2 - 3/14*(b*x^2 + a)^(7/3)*a/b^2
```

Fricas [A] time = 1.59479, size = 103, normalized size = 2.71

$$\frac{3\left(7\,b^{3}x^{6}+11\,ab^{2}x^{4}+a^{2}bx^{2}-3\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{140\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/140*(7*b^3*x^6 + 11*a*b^2*x^4 + a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3)/b^2
```

Sympy [A] time = 2.32836, size = 88, normalized size = 2.32

$$\begin{cases}-\frac{9a^{3}\sqrt[3]{a+bx^{2}}}{140b^{2}}+\frac{3a^{2}x^{2}\sqrt[3]{a+bx^{2}}}{140b}+\frac{33ax^{4}\sqrt[3]{a+bx^{2}}}{140}+\frac{3bx^{6}\sqrt[3]{a+bx^{2}}}{20} & \text{for}\: b\neq 0\\ \frac{a^{\frac{4}{3}}x^{4}}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x**2)**(1/3)/(140*b**2) + 3*a**2*x**2*(a + b*x**2
)**(1/3)/(140*b) + 33*a*x**4*(a + b*x**2)**(1/3)/140 + 3*b*x**6*(a + b*x**2
```

```
)**(1/3)/20, Ne(b, 0)), (a**(4/3)*x**4/4, True))
```

Giac [B] time = 1.78949, size = 105, normalized size = 2.76

$$\frac{3\left(\frac{5\left(4\left(bx^2+a\right)^{\frac{7}{3}}-7\left(bx^2+a\right)^{\frac{4}{3}}a\right)a}{b}+\frac{14\left(bx^2+a\right)^{\frac{10}{3}}-40\left(bx^2+a\right)^{\frac{7}{3}}a+35\left(bx^2+a\right)^{\frac{4}{3}}a^2}{b}\right)}{280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/280*(5*(4*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a)*a/b + (14*(b*x^2 + a
)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/3)*a^2)/b)/b
```

3.692 $\int x\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Rubi [A] time = 0.0038096, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Mathematica [A] time = 0.0044949, size = 18, normalized size = 1.

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Maple [A] time = 0.001, size = 15, normalized size = 0.8

$$\frac{3}{14\,b}\left(bx^2+a\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(4/3),x)
```

```
[Out] 3/14*(b*x^2+a)^(7/3)/b
```

Maxima [A] time = 1.85676, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

Fricas [B] time = 1.82185, size = 73, normalized size = 4.06

$$\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)/b
```

Sympy [A] time = 1.34041, size = 65, normalized size = 3.61

$$\begin{cases} \frac{3a^2\sqrt[3]{a+bx^2}}{14b}+\frac{3ax^2\sqrt[3]{a+bx^2}}{7}+\frac{3bx^4\sqrt[3]{a+bx^2}}{14} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((3*a**2*(a + b*x**2)**(1/3)/(14*b) + 3*a*x**2*(a + b*x**2)**(1/3)
/7 + 3*b*x**4*(a + b*x**2)**(1/3)/14, Ne(b, 0)), (a**(4/3)*x**2/2, True))
```

Giac [A] time = 2.65785, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

3.693 $\int \frac{(a+bx^2)^{4/3}}{x}\,dx$

Optimal. Leaf size=117

$$\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 - (Sqrt[3]*a^(4/3)*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(4/3)*Log[x])
/2 + (3*a^(4/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)])/4
```

Rubi [A] time = 0.0819629, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 - (Sqrt[3]*a^(4/3)*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(4/3)*Log[x])
/2 + (3*a^(4/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{4/3}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx,x,x^2\right)\\ &= \frac{3}{8}\left(a+bx^2\right)^{4/3}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^2\right)\\ &= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\ &= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{4}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)-\frac{1}{4}\left(3a^{5}\right.\\ &= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{}{-3}\right.\\ &= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+}\right.\end{aligned}$$

Mathematica [A] time = 0.0460494, size = 144, normalized size = 1.23

$$\frac{1}{8}\left(4a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-4\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+3bx^2\sqrt[3]{a+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (15*a*(a + b*x^2)^(1/3) + 3*b*x^2*(a + b*x^2)^(1/3) - 4*Sqrt[3]*a^(4/3)*Arc
Tan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) -
(a + b*x^2)^(1/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
 + b*x^2)^(2/3)])/8
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08342, size = 332, normalized size = 2.84

$$-\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{4}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + 5*a
)*(b*x^2 + a)^(1/3)
```

Sympy [C] time = 1.39364, size = 49, normalized size = 0.42

$$-\frac{b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x,x)
```

```
[Out] -b**(4/3)*x**(8/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(-1/3))
```

Giac [A] time = 4.92671, size = 149, normalized size = 1.27

$$-\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{4}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/8*(b*x^2 + a)
^(4/3) + 3/2*(b*x^2 + a)^(1/3)*a
```

3.694 $\int \frac{(a+bx^2)^{4/3}}{x^3} dx$

Optimal. Leaf size=116

$$-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2\sqrt[3]{a}b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}b\log(x)$$

```
[Out] 2*b*(a + b*x^2)^(1/3) - (a + b*x^2)^(4/3)/(2*x^2) - (2*a^(1/3)*b*ArcTan[(a^
(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log
[x])/3 + a^(1/3)*b*Log[a^(1/3) - (a + b*x^2)^(1/3)]
```

Rubi [A] time = 0.0840763, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 50, 57, 617, 204, 31}

$$-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2\sqrt[3]{a}b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] 2*b*(a + b*x^2)^(1/3) - (a + b*x^2)^(4/3)/(2*x^2) - (2*a^(1/3)*b*ArcTan[(a^
(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log
[x])/3 + a^(1/3)*b*Log[a^(1/3) - (a + b*x^2)^(1/3)]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{4/3}}{2x^2}+\frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^2\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}+\frac{1}{3}(2ab)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}-\frac{2}{3}\sqrt[3]{a}b\log(x)-\left(\sqrt[3]{a}b\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)-\left(a^{2/3}b\right)\text{Su}\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}-\frac{2}{3}\sqrt[3]{a}b\log(x)+\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\left(2\sqrt[3]{a}b\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}-\frac{2\sqrt[3]{a}b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}b\log(x)+\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0088158, size = 37, normalized size = 0.32

$$
\frac{3b\left(a+bx^2\right)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{bx^2}{a}+1\right)}{14a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 + (b*x^2)/a])/(14*
a^2)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(bx^2+a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05872, size = 359, normalized size = 3.09

$$-\frac{4\sqrt{3}a^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2a^{\frac{1}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-\right.}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/
3) + sqrt(3)*a)/a) + 2*a^(1/3)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) - 4*a^(1/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) -
 3*(3*b*x^2 - a)*(b*x^2 + a)^(1/3))/x^2
```

Sympy [C] time = 1.60822, size = 46, normalized size = 0.4

$$-\frac{b^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**3,x)
```

```
[Out] -b**(4/3)*x**(2/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

Giac [A] time = 4.62452, size = 161, normalized size = 1.39

$$-\frac{1}{6}\left(4\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)+2a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/
a^(1/3)) + 2*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) - 4*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) - 9*(b*x^2 + a)^(1
/3) + 3*(b*x^2 + a)^(1/3)*a/(b*x^2))*b
```

3.695 $\int \frac{(a+bx^2)^{4/3}}{x^5} \, dx$

Optimal. Leaf size=132

$$\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}-\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{\left(a+bx^2\right)^{4/3}}{4x^4}$$

```
[Out] -(b*(a + b*x^2)^(1/3))/(3*x^2) - (a + b*x^2)^(4/3)/(4*x^4) - (b^2*ArcTan[(a
^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^
2*Log[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(2/3))
```

Rubi [A] time = 0.087383, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}-\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{\left(a+bx^2\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^5,x]
```


```
[Out] -(b*(a + b*x^2)^(1/3))/(3*x^2) - (a + b*x^2)^(4/3)/(4*x^4) - (b^2*ArcTan[(a
^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^
2*Log[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(2/3))
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{4/3}}{4x^4} + \frac{1}{3}b\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4} + \frac{1}{9}b^2\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4} - \frac{b^2\log(x)}{9a^{2/3}} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{2/3}} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x \cdots}\right.}{6\cdots}\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{\cdots}}{\cdots}\right.}{3a^{2/3}}\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4} - \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{2/3}} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.009028, size = 39, normalized size = 0.3

$$-\frac{3b^2\left(a+bx^2\right)^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};\frac{bx^2}{a}+1\right)}{14a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, 1 + (b*x^2)/a])/(
14*a^3)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74868, size = 471, normalized size = 3.57

$$-\frac{4\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+2\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)\right.}{36a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/36*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 2*(a^2)^(2/3)*b
^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2
/3)) - 4*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*
a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^2*x^4)
```

Sympy [C] time = 1.84654, size = 42, normalized size = 0.32

$$-\frac{b^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**5,x)
```

```
[Out] -b**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

Giac [A] time = 4.57951, size = 167, normalized size = 1.27

$$-\frac{1}{36}b^2\left(\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*b^2*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + 2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(2/3) - 4*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*x^2
 + a)^(4/3) - 4*(b*x^2 + a)^(1/3)*a)/(b^2*x^4))
```

3.696 $\int x^4 \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=335

$$-\frac{432\, 3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{21505b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-432$$

```
[Out] (-432*a^3*x*(a + b*x^2)^(1/3))/(21505*b^2) + (48*a^2*x^3*(a + b*x^2)^(1/3))
/(4301*b) + (24*a*x^5*(a + b*x^2)^(1/3))/391 + (3*x^5*(a + b*x^2)^(4/3))/23
 - (432*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(21505*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.228282, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$-\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}-\frac{432\, 3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+\right.}{21505b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(4/3),x]
```

```
[Out] (-432*a^3*x*(a + b*x^2)^(1/3))/(21505*b^2) + (48*a^2*x^3*(a + b*x^2)^(1/3))
/(4301*b) + (24*a*x^5*(a + b*x^2)^(1/3))/391 + (3*x^5*(a + b*x^2)^(4/3))/23
 - (432*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(21505*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int x^4 \left(a+bx^2\right)^{4/3} dx = \frac{3}{23}x^5\left(a+bx^2\right)^{4/3} + \frac{1}{23}(8a)\int x^4\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{24}{391}ax^5\sqrt[3]{a+bx^2} + \frac{3}{23}x^5\left(a+bx^2\right)^{4/3} + \frac{1}{391}\left(16a^2\right)\int \frac{x^4}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b} + \frac{24}{391}ax^5\sqrt[3]{a+bx^2} + \frac{3}{23}x^5\left(a+bx^2\right)^{4/3} - \frac{\left(144a^3\right)\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}dx}{4301b}$$

$$= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2} + \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b} + \frac{24}{391}ax^5\sqrt[3]{a+bx^2} + \frac{3}{23}x^5\left(a+bx^2\right)^{4/3} + \frac{\left(432a^4\right)\int \frac{1}{\left(a+bx^2\right.}}{21505b^2}$$

$$= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2} + \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b} + \frac{24}{391}ax^5\sqrt[3]{a+bx^2} + \frac{3}{23}x^5\left(a+bx^2\right)^{4/3} + \frac{\left(648a^4\sqrt{bx^2}\right)\text{Su}}{}$$

$$= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2} + \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b} + \frac{24}{391}ax^5\sqrt[3]{a+bx^2} + \frac{3}{23}x^5\left(a+bx^2\right)^{4/3} - \frac{432\ 3^{3/4}\sqrt{2-\sqrt{}}}{}$$

Mathematica [C] time = 0.0746132, size = 79, normalized size = 0.24

$$\frac{3x\sqrt[3]{a+bx^2}\left(\frac{9a^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}} - \left(9a-17bx^2\right)\left(a+bx^2\right)^2\right)}{391b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*(-((9*a - 17*b*x^2)*(a + b*x^2)^2) + (9*a^3*Hypergeo
metric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(391*b^2)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x^4 \left(bx^2+a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^4*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^6+ax^4\right)\left(bx^2+a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^6 + a*x^4)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 1.51659, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}} x^5 {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*x**5*hyper((-4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*x^4, x)
```

3.697 $\int x^2 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=311

$$\frac{48\, 3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{935b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{48a^2x}{9}$$

```
[Out] (48*a^2*x*(a + b*x^2)^(1/3))/(935*b) + (24*a*x^3*(a + b*x^2)^(1/3))/187 + (
3*x^3*(a + b*x^2)^(4/3))/17 + (48*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1/3) -
(a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^
(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.185407, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$\frac{48\, 3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{935b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{48a^2x\sqrt[3]{a+b}}{935b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(4/3),x]
```

```
[Out] (48*a^2*x*(a + b*x^2)^(1/3))/(935*b) + (24*a*x^3*(a + b*x^2)^(1/3))/187 + (
3*x^3*(a + b*x^2)^(4/3))/17 + (48*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1/3) -
(a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^
(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{4/3}dx &= \frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{1}{17}(8a)\int x^2\sqrt[3]{a+bx^2}\,dx\\
&= \frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{1}{187}\left(16a^2\right)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}dx\\
&= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}-\frac{\left(48a^3\right)\int\frac{1}{\left(a+bx^2\right)^{2/3}}dx}{935b}\\
&= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}-\frac{\left(72a^3\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a}\right.}{935b^2x}\\
&= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{48\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{}}{}
\end{aligned}$$

93

Mathematica [C] time = 0.0626881, size = 67, normalized size = 0.22

$$\frac{3x\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2-\frac{a^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*((a + b*x^2)^2 - (a^2*Hypergeometric2F1[-4/3, 1/2, 3
/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(17*b)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^2*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^4 + ax^2)(bx^2 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a*x^2)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 1.16534, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}} x^3 {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*x**3*hyper((-4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*x^2, x)
```

3.698 $\int (a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=285

$$-\frac{16\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{24}{55}ax\sqrt[3]{a$$

```
[Out] (24*a*x*(a + b*x^2)^(1/3))/55 + (3*x*(a + b*x^2)^(4/3))/11 - (16*3^(3/4)*Sq
rt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(
a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))^2)])
```

Rubi [A] time = 0.166135, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 236, 219}

$$-\frac{16\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{24}{55}ax\sqrt[3]{a+bx^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3),x]
```

```
[Out] (24*a*x*(a + b*x^2)^(1/3))/55 + (3*x*(a + b*x^2)^(4/3))/11 - (16*3^(3/4)*Sq
rt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(
a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{4/3}dx &= \frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{1}{11}(8a)\int \sqrt[3]{a+bx^2}\,dx\\ &= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{1}{55}\left(16a^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}dx\\ &= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{\left(24a^2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a+bx^2}\right)}{55bx}\\ &= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}-\frac{16\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}\right.}}}\end{aligned}$$

Mathematica [C] time = 0.0061998, size = 47, normalized size = 0.16

$$\frac{ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3),x]
```

```
[Out] (a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(1/3)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3), x)
```

Sympy [A] time = 0.921562, size = 26, normalized size = 0.09

$$a^{\frac{4}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*x*hyper((-4/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3), x)
```

3.699 $\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx$

Optimal. Leaf size=280

$$-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{(a+bx^2)^{4/}}{x}$$

```
[Out] (8*b*x*(a + b*x^2)^(1/3))/5 - (a + b*x^2)^(4/3)/x - (16*Sqrt[2 - Sqrt[3]]*a
*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.15408, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 236, 219}

$$-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{(a+bx^2)^{4/3}}{x}+\frac{8}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^2,x]
```

```
[Out] (8*b*x*(a + b*x^2)^(1/3))/5 - (a + b*x^2)^(4/3)/x - (16*Sqrt[2 - Sqrt[3]]*a
*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x^2}\,dx &= -\frac{\left(a+bx^2\right)^{4/3}}{x}+\frac{1}{3}(8b)\int \sqrt[3]{a+bx^2}\,dx\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{x}+\frac{1}{15}(16ab)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{x}+\frac{\left(8a\sqrt{bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{5x}\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{x}-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+}{(1-}\right.\right.}{5\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.0085901, size = 50, normalized size = 0.18

$$-\frac{a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^2,x]
```

```
[Out] -((a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/2, 1/2, -((b*x^2)/a)])/(x
*(1 + (b*x^2)/a)^(1/3)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)/x^2, x)
```

Sympy [A] time = 1.0987, size = 29, normalized size = 0.1

$$-\frac{a^{\frac{4}{3}}\,{}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**2,x)
```

```
[Out] -a**(4/3)*hyper((-4/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^2, x)
```

3.700 $\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx$

Optimal. Leaf size=284

$$-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{(a+bx^2)^{4/3}}{3x^3}-$$

```
[Out] (-8*b*(a + b*x^2)^(1/3))/(9*x) - (a + b*x^2)^(4/3)/(3*x^3) - (16*Sqrt[2 - S
qrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((a^(1/
3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2)])
```

Rubi [A] time = 0.150323, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 236, 219}

$$-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{(a+bx^2)^{4/3}}{3x^3}-\frac{8b\sqrt[3]{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^4,x]
```

```
[Out] (-8*b*(a + b*x^2)^(1/3))/(9*x) - (a + b*x^2)^(4/3)/(3*x^3) - (16*Sqrt[2 - S
qrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((a^(1/
3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = -\frac{(a+bx^2)^{4/3}}{3x^3} + \frac{1}{9}(8b)\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx$$

$$= -\frac{8b\sqrt[3]{a+bx^2}}{9x} - \frac{(a+bx^2)^{4/3}}{3x^3} + \frac{1}{27}\left(16b^2\right)\int \frac{1}{(a+bx^2)^{2/3}}\,dx$$

$$= -\frac{8b\sqrt[3]{a+bx^2}}{9x} - \frac{(a+bx^2)^{4/3}}{3x^3} + \frac{\left(8b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{9x}$$

$$= -\frac{8b\sqrt[3]{a+bx^2}}{9x} - \frac{(a+bx^2)^{4/3}}{3x^3} - \frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F(\sin^{-1}}{9\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.0090037, size = 52, normalized size = 0.18

$$-\frac{a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^4,x]
```

```
[Out] -(a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -4/3, -1/2, -((b*x^2)/a)])/(3
*x^3*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^4,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{4}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{4}{3}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)/x^4, x)
```

Sympy [A] time = 1.11716, size = 34, normalized size = 0.12

$$-\frac{a^{\frac{4}{3}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2}, -\frac{4}{3}\\ -\frac{1}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**4,x)
```

```
[Out] -a**(4/3)*hyper((-3/2, -4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{4}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^4, x)
```

3.701 $\int x\left(-1+x^2\right)^{7/3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

```
[Out] (3*(-1 + x^2)^(10/3))/20
```

Rubi [A] time = 0.0021335, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(7/3),x]
```

```
[Out] (3*(-1 + x^2)^(10/3))/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^{7/3}\,dx=\frac{3}{20}\left(-1+x^2\right)^{10/3}$$

Mathematica [A] time = 0.0044398, size = 13, normalized size = 1.

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(7/3),x]
```

```
[Out] (3*(-1 + x^2)^(10/3))/20
```

Maple [A] time = 0.001, size = 16, normalized size = 1.2

$$\frac{(3+3x)(-1+x)}{20}\left(x^2-1\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(7/3),x)
```

```
[Out] 3/20*(1+x)*(-1+x)*(x^2-1)^(7/3)
```

Maxima [A] time = 1.84928, size = 12, normalized size = 0.92

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="maxima")
```

```
[Out] 3/20*(x^2 - 1)^(10/3)
```

Fricas [B] time = 1.65653, size = 65, normalized size = 5.

$$\frac{3}{20}\left(x^6-3\,x^4+3\,x^2-1\right)\left(x^2-1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="fricas")
```

```
[Out] 3/20*(x^6 - 3*x^4 + 3*x^2 - 1)*(x^2 - 1)^(1/3)
```

Sympy [B] time = 2.75557, size = 56, normalized size = 4.31

$$\frac{3x^6\sqrt[3]{x^2-1}}{20}-\frac{9x^4\sqrt[3]{x^2-1}}{20}+\frac{9x^2\sqrt[3]{x^2-1}}{20}-\frac{3\sqrt[3]{x^2-1}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(7/3),x)
```

```
[Out] 3*x**6*(x**2 - 1)**(1/3)/20 - 9*x**4*(x**2 - 1)**(1/3)/20 + 9*x**2*(x**2 -
1)**(1/3)/20 - 3*(x**2 - 1)**(1/3)/20
```

Giac [A] time = 2.7084, size = 12, normalized size = 0.92

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="giac")
```

```
[Out] 3/20*(x^2 - 1)^(10/3)
```

3.702 $\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=80

$$\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}$$

```
[Out] (-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (
9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)
```

Rubi [A] time = 0.0463976, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (
9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}
\end{aligned}$$

Mathematica [A] time = 0.0280746, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{2/3}\left(54a^2bx^2-81a^3-45ab^2x^4+40b^3x^6\right)}{880b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(-81*a^3 + 54*a^2*b*x^2 - 45*a*b^2*x^4 + 40*b^3*x^6))/
(880*b^4)
```

Maple [A] time = 0.004, size = 47, normalized size = 0.6

$$-\frac{-120\,b^3x^6+135\,ab^2x^4-162\,a^2bx^2+243\,a^3}{880\,b^4}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(1/3),x)
```

```
[Out] -3/880*(b*x^2+a)^(2/3)*(-40*b^3*x^6+45*a*b^2*x^4-54*a^2*b*x^2+81*a^3)/b^4
```

Maxima [A] time = 1.72394, size = 86, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{11}{3}}}{22\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{8}{3}}a}{16\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{5}{3}}a^2}{10\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^2 + a)^(11/3)/b^4 - 9/16*(b*x^2 + a)^(8/3)*a/b^4 + 9/10*(b*x^2 +
a)^(5/3)*a^2/b^4 - 3/4*(b*x^2 + a)^(2/3)*a^3/b^4
```

Fricas [A] time = 1.65236, size = 109, normalized size = 1.36

$$\frac{3\left(40\,b^3x^6-45\,ab^2x^4+54\,a^2bx^2-81\,a^3\right)\left(bx^2+a\right)^{\frac{2}{3}}}{880\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/880*(40*b^3*x^6 - 45*a*b^2*x^4 + 54*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(2/3)
/b^4
```

Sympy [B] time = 2.49541, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2
 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 +
5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 243*a**(71/3)/(880*a**20*b
**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6
+ 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) -
1296*a**(68/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b*
*5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*
x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(68/3)*b*x*
*2/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a
**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*
b**10*x**12) - 2808*a**(65/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(880*a**20*b*
*4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 +
 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 3
645*a**(65/3)*b**2*x**4/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**1
8*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b*
*9*x**10 + 880*a**14*b**10*x**12) - 3120*a**(62/3)*b**3*x**6*(1 + b*x**2/a)
**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17
600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a
**14*b**10*x**12) + 4860*a**(62/3)*b**3*x**6/(880*a**20*b**4 + 5280*a**19*b
**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8
*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 1710*a**(59/3)*b**
4*x**8*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200
*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**
15*b**9*x**10 + 880*a**14*b**10*x**12) + 3645*a**(59/3)*b**4*x**8/(880*a**2
0*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x*
*6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)
 + 72*a**(56/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**
19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*
b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(56/3)
*b**5*x**10/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4
+ 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 8
80*a**14*b**10*x**12) + 1104*a**(53/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(88
0*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b
**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*
x**12) + 243*a**(53/3)*b**6*x**12/(880*a**20*b**4 + 5280*a**19*b**5*x**2 +
13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 528
0*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1152*a**(50/3)*b**7*x**14*(1
+ b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**
6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x*
*10 + 880*a**14*b**10*x**12) + 585*a**(47/3)*b**8*x**16*(1 + b*x**2/a)**(2/
3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a
**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*
b**10*x**12) + 120*a**(44/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(880*a**20*b*
*4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 +
 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)
```

Giac [A] time = 2.48188, size = 77, normalized size = 0.96

$$\frac{3\left(40\left(bx^2+a\right)^{\frac{11}{3}}-165\left(bx^2+a\right)^{\frac{8}{3}}a+264\left(bx^2+a\right)^{\frac{5}{3}}a^2-220\left(bx^2+a\right)^{\frac{2}{3}}a^3\right)}{880\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/880*(40*(b*x^2 + a)^(11/3) - 165*(b*x^2 + a)^(8/3)*a + 264*(b*x^2 + a)^(5
/3)*a^2 - 220*(b*x^2 + a)^(2/3)*a^3)/b^4
```

3.703 $\int \frac{x^5}{\sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^3}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^3}$$

```
[Out] (3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a
 + b*x^2)^(8/3))/(16*b^3)
```

Rubi [A] time = 0.0343872, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^3}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a
 + b*x^2)^(8/3))/(16*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{a+bx^2}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}} \, dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right) dx, x, x^2\right) \\
&= \frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^3}
\end{aligned}$$

Mathematica [A] time = 0.0222005, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{2/3}\left(9a^2-6abx^2+5b^2x^4\right)}{80b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.6

$$\frac{15\,b^2x^4 - 18\,abx^2 + 27\,a^2}{80\,b^3}\left(bx^2 + a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/3),x)
```

```
[Out] 3/80*(b*x^2+a)^(2/3)*(5*b^2*x^4-6*a*b*x^2+9*a^2)/b^3
```

Maxima [A] time = 1.66832, size = 63, normalized size = 1.07

$$\frac{3\left(bx^2 + a\right)^{\frac{8}{3}}}{16\,b^3} - \frac{3\left(bx^2 + a\right)^{\frac{5}{3}}a}{5\,b^3} + \frac{3\left(bx^2 + a\right)^{\frac{2}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^3 - 3/5*(b*x^2 + a)^(5/3)*a/b^3 + 3/4*(b*x^2 + a)^
(2/3)*a^2/b^3
```

Fricas [A] time = 1.68285, size = 81, normalized size = 1.37

$$\frac{3\left(5\,b^2x^4 - 6\,abx^2 + 9\,a^2\right)\left(bx^2 + a\right)^{\frac{2}{3}}}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^2*x^4 - 6*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(2/3)/b^3
```

Sympy [B] time = 1.57727, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{32}{3}}\left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} - \frac{27a^{\frac{32}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} + \frac{63a^{\frac{29}{3}}bx^2\left(1\right.}{80a^8b^3 + 240a^7b^4x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240
*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(32/3)/(80*a**8*b**3 + 240*a**
7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 63*a**(29/3)*b*x**2
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) - 81*a**(29/3)*b*x**2/(80*a**8*b**3 + 240*a**7*b**
4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 42*a**(26/3)*b**2*x**4*(
1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**
4 + 80*a**5*b**6*x**6) - 81*a**(26/3)*b**2*x**4/(80*a**8*b**3 + 240*a**7*b*
*4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 18*a**(23/3)*b**3*x**6*
(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x*
*4 + 80*a**5*b**6*x**6) - 27*a**(23/3)*b**3*x**6/(80*a**8*b**3 + 240*a**7*b
**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 27*a**(20/3)*b**4*x**8
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(8
0*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6)
```

Giac [A] time = 1.47091, size = 58, normalized size = 0.98

$$\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}-16\left(bx^2+a\right)^{\frac{5}{3}}a+20\left(bx^2+a\right)^{\frac{2}{3}}a^2\right)}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/80*(5*(b*x^2 + a)^(8/3) - 16*(b*x^2 + a)^(5/3)*a + 20*(b*x^2 + a)^(2/3)*a
^2)/b^3
```

3.704 $\int \frac{x^3}{\sqrt[3]{a+bx^2}} dx$

Optimal. Leaf size=38

$$\frac{3(a+bx^2)^{5/3}}{10b^2} - \frac{3a(a+bx^2)^{2/3}}{4b^2}$$

```
[Out] (-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)
```

Rubi [A] time = 0.0225681, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3(a+bx^2)^{5/3}}{10b^2} - \frac{3a(a+bx^2)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{a+bx^2}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{b}\right) dx, x, x^2\right) \\
&= -\frac{3a(a+bx^2)^{2/3}}{4b^2} + \frac{3(a+bx^2)^{5/3}}{10b^2}
\end{aligned}$$

Mathematica [A] time = 0.0117262, size = 28, normalized size = 0.74

$$\frac{3(a+bx^2)^{2/3}(2bx^2-3a)}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(-3*a + 2*b*x^2))/(20*b^2)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$-\frac{-6\,bx^2+9\,a}{20\,b^2}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/3),x)
```

```
[Out] -3/20*(b*x^2+a)^(2/3)*(-2*b*x^2+3*a)/b^2
```

Maxima [A] time = 1.94062, size = 41, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

Fricas [A] time = 1.69788, size = 59, normalized size = 1.55

$$\frac{3\left(2\,bx^2-3\,a\right)\left(bx^2+a\right)^{\frac{2}{3}}}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/20*(2*b*x^2 - 3*a)*(b*x^2 + a)^(2/3)/b^2
```

Sympy [B] time = 1.00957, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}+\frac{9a^{\frac{11}{3}}}{20a^2b^2+20ab^3x^2}-\frac{3a^{\frac{8}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}+\frac{9a^{\frac{8}{3}}bx^2}{20a^2b^2+20ab^3x^2}+\frac{6a^{\frac{5}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(
11/3)/(20*a**2*b**2 + 20*a*b**3*x**2) - 3*a**(8/3)*b*x**2*(1 + b*x**2/a)**(
2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(8/3)*b*x**2/(20*a**2*b**2 + 20
*a*b**3*x**2) + 6*a**(5/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 +
20*a*b**3*x**2)
```

Giac [A] time = 2.58322, size = 39, normalized size = 1.03

$$\frac{3\left(2\left(bx^2+a\right)^{\frac{5}{3}}-5\left(bx^2+a\right)^{\frac{2}{3}}a\right)}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/20*(2*(b*x^2 + a)^(5/3) - 5*(b*x^2 + a)^(2/3)*a)/b^2
```

3.705 $\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

```
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Rubi [A] time = 0.0032547, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(1/3),x]

[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx=\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Mathematica [A] time = 0.0025281, size = 18, normalized size = 1.

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(1/3),x]

[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{3}{4\,b}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/3),x)
```

```
[Out] 3/4*(b*x^2+a)^(2/3)/b
```

Maxima [A] time = 1.72255, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

Fricas [A] time = 1.67, size = 34, normalized size = 1.89

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

Sympy [A] time = 0.387948, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\left(a+bx^2\right)^{\frac{2}{3}}}{4b} & \text{for}\ b \neq 0 \\ \frac{x^2}{2\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(2/3)/(4*b), Ne(b, 0)), (x**2/(2*a**(1/3)), True
))
```

Giac [A] time = 2.07744, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

3.706 $\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(1
/3)) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3)
)
```

Rubi [A] time = 0.0508861, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(1
/3)) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3)
)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$
$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$

Mathematica [A] time = 0.0294161, size = 70, normalized size = 0.81

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\log(x)}{4\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(1/3)),x]

[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[x] +
 3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/3),x)

[Out] int(1/x/(b*x^2+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77851, size = 679, normalized size = 7.9

$$\left[\frac{\sqrt{3}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b*x^2 + a)^(2/3
)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)
^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a, 1/4
*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
 + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] time = 1.01144, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(2/3)*gamma(4/3))
```

Giac [A] time = 4.02423, size = 117, normalized size = 1.36

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{1}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2\,a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

3.707 $\int \frac{1}{x^3 \sqrt[3]{a+bx^2}}\, dx$

Optimal. Leaf size=110

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

```
[Out] -(a + b*x^2)^(2/3)/(2*a*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(
1/3) - (a + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rubi [A] time = 0.065413, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(2*a*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(
1/3) - (a + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{6a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+b}\right.}{4a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{4/3}}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0071402, size = 37, normalized size = 0.34

$$\frac{3b\left(a+bx^2\right)^{2/3}{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx^2}{a}+1\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] (3*b*(a + b*x^2)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x^2)/a])/(4*a^
2)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89782, size = 929, normalized size = 8.45

$$\left[\frac{3\sqrt{\frac{1}{3}}abx^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^2}\right)+(-a)^{\frac{2}{3}}\,bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}\right.}{12\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(1/3)*a*b*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-
a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a
)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^2 + a)^(2/3)*a)/
(a^2*x^2), -1/12*(6*sqrt(1/3)*a*b*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*
(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x^2*
log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)
^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(
a^2*x^2)]
```

Sympy [C] time = 1.26251, size = 41, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(8/3)*gamma(7/3))
```

Giac [A] time = 4.55557, size = 149, normalized size = 1.35

$$-\frac{1}{12}b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6\left(bx^2+\right.}{abx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/12*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) - log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x^2 + a)^(
2/3)/(a*b*x^2))
```

3.708 $\int \frac{1}{x^5 \sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=138

$$\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}-\frac{b^2 \log(x)}{9a^{7/3}}+\frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2}-\frac{\left(a+bx^2\right)^{2/3}}{4ax^4}$$

```
[Out] -(a + b*x^2)^(2/3)/(4*a*x^4) + (b*(a + b*x^2)^(2/3))/(3*a^2*x^2) + (b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3))
 - (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(
7/3))
```

Rubi [A] time = 0.0933155, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}-\frac{b^2 \log(x)}{9a^{7/3}}+\frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2}-\frac{\left(a+bx^2\right)^{2/3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(4*a*x^4) + (b*(a + b*x^2)^(2/3))/(3*a^2*x^2) + (b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3))
 - (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(
7/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{9a^2}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{7/3}} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{}{a^{2/3}+\cdots}\right.}{}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\cdots\right.}{3a^{7/3}}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}$$

Mathematica [C] time = 0.0071379, size = 39, normalized size = 0.28

$$-\frac{3b^2\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{bx^2}{a}+1\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + (b*x^2)/a])/(4
*a^3)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88683, size = 899, normalized size = 6.51

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^4\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-2a^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+\right.\right.}{36a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)
) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 2*a^(2/3)*b^2*x^4*log((b*x^2
+ a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((
b*x^2 + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3
*x^4), 1/36*(12*sqrt(1/3)*a^(2/3)*b^2*x^4*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(
1/3) + a^(1/3))/a^(1/3)) - 2*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) + (b*x^2
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) -
a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3*x^4)]
```

Sympy [C] time = 1.59983, size = 41, normalized size = 0.3

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}x^{\frac{14}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1
/3)*x**(14/3)*gamma(10/3))
```

Giac [A] time = 4.83445, size = 171, normalized size = 1.24

$$\frac{1}{36}\,b^2\left(\frac{4\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2\,\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4\,\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4\left(b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/36*b^2*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(7/3) - 2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(7/3) + 4*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x^2
+ a)^(5/3) - 7*(b*x^2 + a)^(2/3)*a)/(a^2*b^2*x^4))
```

3.709 $\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=580

$$-\frac{27\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\quad}{91b^2\left(\left(1-\right.\right.}$$

```
[Out] (-27*a*x*(a + b*x^2)^(2/3))/(91*b^2) + (3*x^3*(a + b*x^2)^(2/3))/(13*b) - (
81*a^2*x)/(91*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (81*3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(182*b^3
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)]) - (27*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(91*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.366373, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 235, 304, 219, 1879}

$$-\frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{27\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/3),x]
```


```
[Out] (-27*a*x*(a + b*x^2)^(2/3))/(91*b^2) + (3*x^3*(a + b*x^2)^(2/3))/(13*b) - (
81*a^2*x)/(91*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (81*3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(182*b^3
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)]) - (27*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(91*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```


<u>Rule 321</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \frac{3x^3\left(a+bx^2\right)^{2/3}}{13b} - \frac{(9a)\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx}{13b}$$
$$= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2} + \frac{3x^3\left(a+bx^2\right)^{2/3}}{13b} + \frac{\left(27a^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{91b^2}$$
$$= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2} + \frac{3x^3\left(a+bx^2\right)^{2/3}}{13b} + \frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{182b^3x}$$
$$= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2} + \frac{3x^3\left(a+bx^2\right)^{2/3}}{13b} - \frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{182b^3x} + \frac{\left(81\sqrt{\right.}}{}$$
$$= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2} + \frac{3x^3\left(a+bx^2\right)^{2/3}}{13b} - \frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.0241447, size = 79, normalized size = 0.14

$$\frac{3\left(9a^2x\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-2abx^3+7b^2x^5\right)}{91b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(-9*a^2*x - 2*a*b*x^3 + 7*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)]))/(91*b^2*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/3),x)
```

```
[Out] int(x^4/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 0.709002, size = 27, normalized size = 0.05

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/3),x)
```

```
[Out] x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/3), x)
```

3.710 $\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=556

$$\frac{3\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{}$$

```
[Out] (3*x*(a + b*x^2)^(2/3))/(7*b) + (9*a*x)/(7*b*((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(14*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3*Sqrt[2]*3^(3/4)*a^(4/
3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
+ (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.309479, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 235, 304, 219, 1879}

$$\frac{3\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3))/(7*b) + (9*a*x)/(7*b*((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(14*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3*Sqrt[2]*3^(3/4)*a^(4/
3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
+ (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \frac{3x\left(a+bx^2\right)^{2/3}}{7b} - \frac{(3a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{7b}$$

$$= \frac{3x\left(a+bx^2\right)^{2/3}}{7b} - \frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{14b^2x}$$

$$= \frac{3x\left(a+bx^2\right)^{2/3}}{7b} + \frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{14b^2x} - \frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}a^{4/3}\sqrt{bx^2}\right)\text{Subst}}{7b^2x}$$

$$= \frac{3x\left(a+bx^2\right)^{2/3}}{7b} + \frac{9ax}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}\ldots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\ldots\right.}}}{14b^2x\sqrt{-\frac{\sqrt[3]{a}\ldots}{\left(\left(1-\sqrt{\ldots}\right.\right.}}}$$

Mathematica [C] time = 0.0179898, size = 62, normalized size = 0.11

$$\frac{3x\left(-a\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{7b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2,
-((b*x^2)/a)]))/(7*b*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/3),x)
```

```
[Out] int(x^2/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 0.652025, size = 27, normalized size = 0.05

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/3),x)
```

```
[Out] x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)
```

3.711 $\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=529

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\cdots}{\cdots}$$

```
[Out] (-3*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3*3^(1/4)*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2)]) - (Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.249889, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {235, 304, 219, 1879}

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\,(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/3),x]
```

```
[Out] (-3*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3*3^(1/4)*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2)]) - (Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx &= \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx}\\ &= -\frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx}+\frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{\ldots}\right.}{bx}\\ &= -\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\ldots}{(1-\ldots}\right.\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\end{aligned}$$

Mathematica [C] time = 0.0053152, size = 46, normalized size = 0.09

$$\frac{x\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/3),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/3)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b x^{2}+a}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(b x^{2}+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-1/3), x)
```

Sympy [A] time = 0.637249, size = 24, normalized size = 0.05

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3),x)
```

```
[Out] x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

3.712 $\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=546

$$-\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\right.}{}$$

```
[Out] -((a + b*x^2)^(2/3)/(a*x)) - (b*x)/(a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(2*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (Sqrt[2]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.299705, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 235, 304, 219, 1879}

$$-\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/3)),x]
```


```
[Out] -((a + b*x^2)^(2/3)/(a*x)) - (b*x)/(a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(2*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (Sqrt[2]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{2/3}}{ax}+\frac{b\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{3a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{ax}+\frac{\sqrt{bx^2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{2ax}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{ax}-\frac{\sqrt{bx^2}\,\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{2ax}+\frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}} \cdots\right.}{a^{2/3}x}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{ax}-\frac{bx}{a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\cdots}{2a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.008582, size = 49, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/3)))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b*x^4 + a*x^2), x)
```

Sympy [A] time = 0.711596, size = 27, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/3),x)
```

```
[Out] -hyper((-1/2, 1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/3)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

3.713 $\int \frac{1}{x^4 \sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=578

$$\frac{5\sqrt{2}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{5b^2x}{9a^2\left((1-\sqrt{3})\sqrt[3]{a}-\cdots\right.}$$

```
[Out] -(a + b*x^2)^(2/3)/(3*a*x^3) + (5*b*(a + b*x^2)^(2/3))/(9*a^2*x) + (5*b^2*x
)/(9*a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (5*Sqrt[2 + Sqrt[3]
]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
 + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*3^(3/4)*a^(5/3)*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))^2)]) + (5*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1
/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*
a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.357196, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 235, 304, 219, 1879}

$$\frac{5b^2x}{9a^2\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}+\frac{5\sqrt{2}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\cdots}{(1-\sqrt{3})\cdots}\right)\cdots\right.}{9\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(3*a*x^3) + (5*b*(a + b*x^2)^(2/3))/(9*a^2*x) + (5*b^2*x
)/(9*a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (5*Sqrt[2 + Sqrt[3]
]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
 + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*3^(3/4)*a^(5/3)*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))^2)]) + (5*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1
/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*
a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} - \frac{(5b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{9a}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} - \frac{\left(5b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{27a^2}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} - \frac{\left(5b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{18a^2x}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} + \frac{\left(5b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{18a^2x} - \frac{\left(5\sqrt{\frac{1}{2}\left(2+\right.}\right.}{}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} + \frac{5b^2x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{5\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{}$$

Mathematica [C] time = 0.0086942, size = 51, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/3)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, -((b*x^2)/a)])/(
3*x^3*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{2}{3}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b*x^6 + a*x^4), x)
```

Sympy [A] time = 0.827645, size = 32, normalized size = 0.06

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/3),x)
```

```
[Out] -hyper((-3/2, 1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

3.714 $\int \frac{x^7}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=80

$$\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}$$

```
[Out] (-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9
*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)
```

Rubi [A] time = 0.045896, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9
*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}\end{aligned}$$

Mathematica [A] time = 0.0239664, size = 50, normalized size = 0.62

$$\frac{3\sqrt[3]{a+bx^2}\left(27a^2bx^2-81a^3-18ab^2x^4+14b^3x^6\right)}{280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-81*a^3 + 27*a^2*b*x^2 - 18*a*b^2*x^4 + 14*b^3*x^6))/
(280*b^4)
```

Maple [A] time = 0.006, size = 47, normalized size = 0.6

$$-\frac{-42\,b^3x^6+54\,ab^2x^4-81\,a^2bx^2+243\,a^3}{280\,b^4}\sqrt[3]{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/280*(b*x^2+a)^(1/3)*(-14*b^3*x^6+18*a*b^2*x^4-27*a^2*b*x^2+81*a^3)/b^4
```

Maxima [A] time = 2.11981, size = 86, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{10}{3}}}{20\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{7}{3}}a}{14\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{4}{3}}a^2}{8\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^4 - 9/14*(b*x^2 + a)^(7/3)*a/b^4 + 9/8*(b*x^2 + a
)^(4/3)*a^2/b^4 - 3/2*(b*x^2 + a)^(1/3)*a^3/b^4
```

Fricas [A] time = 1.69348, size = 109, normalized size = 1.36

$$\frac{3\left(14\,b^3x^6-18\,ab^2x^4+27\,a^2bx^2-81\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 - 18*a*b^2*x^4 + 27*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(1/3)
/b^4
```

Sympy [B] time = 2.52817, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 168
0*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(70/3)/(280*a**20*b**4
 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 420
0*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 1377*a
**(67/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**
2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 16
80*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(67/3)*b*x**2/(280*a
**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x
**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12)
 - 3213*a**(64/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a*
*19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b*
*8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 3645*a**(64/3)*b
**2*x**4/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 56
00*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**
14*b**10*x**12) - 3927*a**(61/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**20
*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6
+ 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 4
860*a**(61/3)*b**3*x**6/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18
*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*
x**10 + 280*a**14*b**10*x**12) - 2583*a**(58/3)*b**4*x**8*(1 + b*x**2/a)**(
1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a
**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b
**10*x**12) + 3645*a**(58/3)*b**4*x**8/(280*a**20*b**4 + 1680*a**19*b**5*x*
*2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1
680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 693*a**(55/3)*b**5*x**10*(1
 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**
6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**1
0 + 280*a**14*b**10*x**12) + 1458*a**(55/3)*b**5*x**10/(280*a**20*b**4 + 16
80*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**
16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 273*a**(52/
3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
+ 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680
*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(52/3)*b**6*x**12/(280*
a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*
x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12
) + 387*a**(49/3)*b**7*x**14*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a
**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b
**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 198*a**(46/3)*b
**8*x**16*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 42
00*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**
15*b**9*x**10 + 280*a**14*b**10*x**12) + 42*a**(43/3)*b**9*x**18*(1 + b*x**
2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 +
 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*
a**14*b**10*x**12)
```

Giac [A] time = 1.79455, size = 77, normalized size = 0.96

$$\frac{3\left(14\left(bx^2+a\right)^{\frac{10}{3}}-60\left(bx^2+a\right)^{\frac{7}{3}}a+105\left(bx^2+a\right)^{\frac{4}{3}}a^2-140\left(bx^2+a\right)^{\frac{1}{3}}a^3\right)}{280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/280*(14*(b*x^2 + a)^(10/3) - 60*(b*x^2 + a)^(7/3)*a + 105*(b*x^2 + a)^(4/
3)*a^2 - 140*(b*x^2 + a)^(1/3)*a^3)/b^4
```

3.715 $\int \frac{x^5}{\left(a+bx^2\right)^{2/3}} \, dx$

Optimal. Leaf size=59

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}$$

```
[Out] (3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a
 + b*x^2)^(7/3))/(14*b^3)
```

Rubi [A] time = 0.0335988, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a
 + b*x^2)^(7/3))/(14*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}\end{aligned}$$

Mathematica [A] time = 0.0174606, size = 39, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx^2}\left(9a^2-3abx^2+2b^2x^4\right)}{28b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)
```

Maple [A] time = 0.006, size = 36, normalized size = 0.6

$$\frac{6\,b^2x^4 - 9\,abx^2 + 27\,a^2}{28\,b^3}\sqrt[3]{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(2/3),x)
```

```
[Out] 3/28*(b*x^2+a)^(1/3)*(2*b^2*x^4-3*a*b*x^2+9*a^2)/b^3
```

Maxima [A] time = 1.19484, size = 63, normalized size = 1.07

$$\frac{3\left(bx^2 + a\right)^{\frac{7}{3}}}{14\,b^3} - \frac{3\left(bx^2 + a\right)^{\frac{4}{3}}a}{4\,b^3} + \frac{3\left(bx^2 + a\right)^{\frac{1}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^3 - 3/4*(b*x^2 + a)^(4/3)*a/b^3 + 3/2*(b*x^2 + a)^
(1/3)*a^2/b^3
```

Fricas [A] time = 1.64501, size = 81, normalized size = 1.37

$$\frac{3\left(2\,b^2x^4 - 3\,abx^2 + 9\,a^2\right)\left(bx^2 + a\right)^{\frac{1}{3}}}{28\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/28*(2*b^2*x^4 - 3*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(1/3)/b^3
```

Sympy [B] time = 1.61195, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1 + \frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} - \frac{27a^{\frac{31}{3}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} + \frac{72a^{\frac{28}{3}}bx^2\sqrt[3]{1 + \frac{bx}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) - 27*a**(31/3)/(28*a**8*b**3 + 84*a**7*b
**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 72*a**(28/3)*b*x**2*(1
+ b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
28*a**5*b**6*x**6) - 81*a**(28/3)*b*x**2/(28*a**8*b**3 + 84*a**7*b**4*x**2
+ 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 60*a**(25/3)*b**2*x**4*(1 + b*x*
*2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**
5*b**6*x**6) - 81*a**(25/3)*b**2*x**4/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 8
4*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 18*a**(22/3)*b**3*x**6*(1 + b*x**2/
a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b
**6*x**6) - 27*a**(22/3)*b**3*x**6/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) + 9*a**(19/3)*b**4*x**8*(1 + b*x**2/a)**
(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*
x**6) + 6*a**(16/3)*b**5*x**10*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**
7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6)
```

Giac [A] time = 2.74925, size = 58, normalized size = 0.98

$$\frac{3\left(2\left(bx^2+a\right)^{\frac{7}{3}}-7\left(bx^2+a\right)^{\frac{4}{3}}a+14\left(bx^2+a\right)^{\frac{1}{3}}a^2\right)}{28\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/28*(2*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a + 14*(b*x^2 + a)^(1/3)*a^
2)/b^3
```

3.716 $\int \frac{x^3}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

```
[Out] (-3*a*(a + b*x^2)^(1/3))/(2*b^2) + (3*(a + b*x^2)^(4/3))/(8*b^2)
```

Rubi [A] time = 0.0235036, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(2/3),x]

[Out] (-3*a*(a + b*x^2)^(1/3))/(2*b^2) + (3*(a + b*x^2)^(4/3))/(8*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx,x,x^2\right)\\ &= -\frac{3a\sqrt[3]{a+bx^2}}{2b^2}+\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.0129794, size = 27, normalized size = 0.71

$$\frac{3\left(bx^2-3a\right)\sqrt[3]{a+bx^2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(-3*a + b*x^2)*(a + b*x^2)^(1/3))/(8*b^2)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.7

$$-\frac{-3\,bx^2+9\,a}{8\,b^2}\sqrt[3]{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/8*(b*x^2+a)^(1/3)*(-b*x^2+3*a)/b^2
```

Maxima [A] time = 1.73467, size = 41, normalized size = 1.08

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

Fricas [A] time = 1.74131, size = 55, normalized size = 1.45

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}\left(bx^2-3\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/8*(b*x^2 + a)^(1/3)*(b*x^2 - 3*a)/b^2
```

Sympy [B] time = 1.01786, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{10}{3}}}{8a^2b^2+8ab^3x^2}-\frac{6a^{\frac{7}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{7}{3}}bx^2}{8a^2b^2+8ab^3x^2}+\frac{3a^{\frac{4}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(10
/3)/(8*a**2*b**2 + 8*a*b**3*x**2) - 6*a**(7/3)*b*x**2*(1 + b*x**2/a)**(1/3)
```

```
/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(7/3)*b*x**2/(8*a**2*b**2 + 8*a*b**3*
x**2) + 3*a**(4/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*
x**2)
```

Giac [A] time = 2.1225, size = 36, normalized size = 0.95

$$\frac{3\left(\left(bx^2+a\right)^{\frac{4}{3}}-4\left(bx^2+a\right)^{\frac{1}{3}}a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/8*((b*x^2 + a)^(4/3) - 4*(b*x^2 + a)^(1/3)*a)/b^2
```

3.717 $\int \frac{x}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=18

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

Rubi [A] time = 0.0035487, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}}{2b}$$

Mathematica [A] time = 0.0023881, size = 18, normalized size = 1.

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{3}{2\,b}\sqrt[3]{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(2/3),x)
```

```
[Out] 3/2*(b*x^2+a)^(1/3)/b
```

Maxima [A] time = 2.15942, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

Fricas [A] time = 1.86528, size = 34, normalized size = 1.89

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

Sympy [A] time = 0.412376, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(1/3)/(2*b), Ne(b, 0)), (x**2/(2*a**(2/3)), True
))
```

Giac [A] time = 1.5118, size = 19, normalized size = 1.06

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

3.718 $\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=86

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

```
[Out] -(Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(
2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(2/3
))
```

Rubi [A] time = 0.0518777, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 57, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(
2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(2/3
))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}\\
&= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}\\
&= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0310799, size = 101, normalized size = 1.17

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{4a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^2)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(4*a^(2/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78634, size = 356, normalized size = 4.14

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^2 + a)^(2/3
)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log((b
*x^2 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] time = 1.06003, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(4/3)*gamma(5/3))
```

Giac [A] time = 4.04531, size = 117, normalized size = 1.36

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{2}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{2}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

3.719 $\int \frac{1}{x^3(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=107

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

```
[Out] -(a + b*x^2)^(1/3)/(2*a*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(2*a^(5/3))
```

Rubi [A] time = 0.0658551, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 57, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(2*a*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(2*a^(5/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{3a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\log(x)}{3a^{5/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2a^{5/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a}\right.}{2a^{4/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.0072726, size = 37, normalized size = 0.35

$$\frac{3b\sqrt[3]{a+bx^2}\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx^2}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*b*(a + b*x^2)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x^2)/a])/(2*a^
2)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8237, size = 466, normalized size = 4.36

$$\frac{2\sqrt{3}abx^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^2+\right.\right.}{6a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*b*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3
)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) +
(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 + a)^(
1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (-a^2)^
(2/3)) - 3*(b*x^2 + a)^(1/3)*a^2)/(a^3*x^2)
```

Sympy [C] time = 1.33437, size = 41, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(10/3)*gamma(8/3))
```

Giac [A] time = 4.16456, size = 147, normalized size = 1.37

$$\frac{1}{6}b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(bx^2+\right.}{abx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(5/3) - 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^2 + a)^(1/
3)/(a*b*x^2))
```

3.720 $\int \frac{1}{x^5\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=138

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

```
[Out] -(a + b*x^2)^(1/3)/(4*a*x^4) + (5*b*(a + b*x^2)^(1/3))/(12*a^2*x^2) - (5*b^
2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(
8/3)) - (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3
)])/(12*a^(8/3))
```

Rubi [A] time = 0.0883603, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 57, 617, 204, 31}

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(4*a*x^4) + (5*b*(a + b*x^2)^(1/3))/(12*a^2*x^2) - (5*b^
2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(
8/3)) - (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3
)])/(12*a^(8/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{18a^2}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2\log(x)}{18a^{8/3}} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{8/3}} - \left(5b^2\right)\text{Subs}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{6a^{8/3}}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}a^{8/3}} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}$$

Mathematica [C] time = 0.0070714, size = 39, normalized size = 0.28

$$-\frac{3b^2\sqrt[3]{a+bx^2}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx^2}{a}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x^2)/a])/(2
*a^3)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77968, size = 474, normalized size = 3.43

$$-\frac{10\,\sqrt{3}\left(a^2\right)^{\frac{1}{6}} a b^2 x^4 \arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}} a+2\,\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3\,a^2}\right)+5\left(a^2\right)^{\frac{2}{3}} b^2 x^4 \log\left(\left(bx^2+a\right)^{\frac{2}{3}} a+\left(a^2\right)^{\frac{1}{3}} a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right.}{36\,a^4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/36*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 5*(a^2)^(2/3)*
b^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(
2/3)) - 10*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) - 3*(
5*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3))/(a^4*x^4)
```

Sympy [C] time = 1.73589, size = 41, normalized size = 0.3

$$-\frac{\Gamma\left(\frac{8}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3}, \frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{a e^{i\pi}}{bx^2}\right)}{2 b^{\frac{2}{3}} x^{\frac{16}{3}} \Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*x**(16/3)*gamma(11/3))
```

Giac [A] time = 4.09054, size = 171, normalized size = 1.24

$$-\frac{1}{36}b^2\left(\frac{10\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/36*b^2*(10*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(8/3) + 5*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(8/3) - 10*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(b*x
^2 + a)^(4/3) - 8*(b*x^2 + a)^(1/3)*a)/(a^2*b^2*x^4))
```

3.721 $\int \frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=293

$$-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{27ax\sqrt[3]{a}}{55b}$$

```
[Out] (-27*a*x*(a + b*x^2)^(1/3))/(55*b^2) + (3*x^3*(a + b*x^2)^(1/3))/(11*b) - (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55
*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.157527, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 236, 219}

$$-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(2/3),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(1/3))/(55*b^2) + (3*x^3*(a + b*x^2)^(1/3))/(11*b) - (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55
*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{3x^3\sqrt[3]{a+bx^2}}{11b} - \frac{(9a)\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx}{11b}$$

$$= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2} + \frac{3x^3\sqrt[3]{a+bx^2}}{11b} + \frac{\left(27a^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{55b^2}$$

$$= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2} + \frac{3x^3\sqrt[3]{a+bx^2}}{11b} + \frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{110b^3x}$$

$$= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2} + \frac{3x^3\sqrt[3]{a+bx^2}}{11b} - \frac{27\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}\right.}}}$$

Mathematica [C] time = 0.0250044, size = 79, normalized size = 0.27

$$\frac{3\left(9a^2x\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-4abx^3+5b^2x^5\right)}{55b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(-9*a^2*x - 4*a*b*x^3 + 5*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(2/3)*Hyperg
eometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)]))/(55*b^2*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^4\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^4/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 0.739556, size = 27, normalized size = 0.09

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(2/3),x)
```

```
[Out] x**5*hyper((2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(2/3), x)
```

3.722 $\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=269

$$\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3x\sqrt[3]{a+bx^2}}{5b}$$

```
[Out] (3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (
a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.123467, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 236, 219}

$$\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3x\sqrt[3]{a+bx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(2/3),x]
```


```
[Out] (3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (
a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{3x\sqrt[3]{a+bx^2}}{5b} - \frac{(3a)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{5b}$$

$$= \frac{3x\sqrt[3]{a+bx^2}}{5b} - \frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{10b^2x}$$

$$= \frac{3x\sqrt[3]{a+bx^2}}{5b} + \frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.0189708, size = 62, normalized size = 0.23

$$\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
-((b*x^2)/a)]))/(5*b*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^2/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 0.665896, size = 27, normalized size = 0.1

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(2/3),x)
```

```
[Out] x**3*hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(2/3), x)
```

3.723 $\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=246

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)]))
```

Rubi [A] time = 0.0955809, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {236, 219}

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-2/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)]))
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx}$$

$$= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\middle|-7+4\sqrt{3}\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.0070344, size = 46, normalized size = 0.19

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-2/3),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(2/3)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-2/3), x)
```

Sympy [A] time = 0.6436, size = 24, normalized size = 0.1

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(2/3),x)
```

```
[Out] x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-2/3), x)
```

3.724 $\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=265

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

```
[Out] -((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.120547, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 236, 219}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(2/3)),x]
```

```
[Out] -((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^2}}{ax} - \frac{b\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{3a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{ax} - \frac{\sqrt{bx^2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2ax}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{ax} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)| -\right.}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.0086949, size = 49, normalized size = 0.18

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(2/3)))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/(b*x^4 + a*x^2), x)
```

Sympy [A] time = 0.772984, size = 27, normalized size = 0.1

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((-1/2, 2/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(2/3)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

3.725 $\int \frac{1}{x^4(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=293

$$-\frac{7\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-$$

```
[Out] -(a + b*x^2)^(1/3)/(3*a*x^3) + (7*b*(a + b*x^2)^(1/3))/(9*a^2*x) - (7*Sqrt[
2 - Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*Sqrt
[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])
```

Rubi [A] time = 0.149903, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 236, 219}

$$\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+b}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(3*a*x^3) + (7*b*(a + b*x^2)^(1/3))/(9*a^2*x) - (7*Sqrt[
2 - Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*Sqrt
[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^2}}{3ax^3}-\frac{(7b)\int \frac{1}{x^2\left(a+bx^2\right)^{2/3}}\,dx}{9a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}+\frac{\left(7b^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{27a^2}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}+\frac{\left(7b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{18a^2x}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{9\sqrt[4]{3}a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.0093624, size = 51, normalized size = 0.17

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(2/3)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, -((b*x^2)/a)])/(
3*x^3*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/(b*x^6 + a*x^4), x)
```

Sympy [A] time = 0.952189, size = 32, normalized size = 0.11

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{2}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((-3/2, 2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

3.726 $\int \frac{x^7}{\left(a+bx^2\right)^{4/3}} \, dx$

Optimal. Leaf size=80

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

```
[Out] (3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*
a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)
```

Rubi [A] time = 0.0438257, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*
a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^{4/3}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right) dx,x,x^2\right)\\
&= \frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}
\end{aligned}$$

Mathematica [A] time = 0.0226396, size = 50, normalized size = 0.62

$$\frac{3\left(27a^2bx^2+81a^3-9ab^2x^4+5b^3x^6\right)}{80b^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^(
1/3))
```

Maple [A] time = 0.006, size = 47, normalized size = 0.6

$$\frac{15\,b^3x^6 - 27\,ab^2x^4 + 81\,a^2bx^2 + 243\,a^3}{80\,b^4}\frac{1}{\sqrt[3]{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(4/3),x)
```

```
[Out] 3/80/(b*x^2+a)^(1/3)*(5*b^3*x^6-9*a*b^2*x^4+27*a^2*b*x^2+81*a^3)/b^4
```

Maxima [A] time = 1.22713, size = 86, normalized size = 1.08

$$\frac{3\left(bx^2 + a\right)^{\frac{8}{3}}}{16\,b^4} - \frac{9\left(bx^2 + a\right)^{\frac{5}{3}}a}{10\,b^4} + \frac{9\left(bx^2 + a\right)^{\frac{2}{3}}a^2}{4\,b^4} + \frac{3\,a^3}{2\left(bx^2 + a\right)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^4 - 9/10*(b*x^2 + a)^(5/3)*a/b^4 + 9/4*(b*x^2 + a)
^(2/3)*a^2/b^4 + 3/2*a^3/((b*x^2 + a)^(1/3)*b^4)
```

Fricas [A] time = 1.68212, size = 124, normalized size = 1.55

$$\frac{3\left(5\,b^3x^6 - 9\,ab^2x^4 + 27\,a^2bx^2 + 81\,a^3\right)\left(bx^2 + a\right)^{\frac{2}{3}}}{80\left(b^5x^2 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^3*x^6 - 9*a*b^2*x^4 + 27*a^2*b*x^2 + 81*a^3)*(b*x^2 + a)^(2/3)/(b
^5*x^2 + a*b^4)
```

Sympy [B] time = 2.64196, size = 1584, normalized size = 19.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 +
1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a*
*15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(68/3)/(80*a**20*b**4 + 480
*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16
*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 1296*a**(65/3)*
b*x**2*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a*
*18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**
9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(65/3)*b*x**2/(80*a**20*b**4 + 48
0*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**1
6*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 2808*a**(62/3)
*b**2*x**4*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 120
0*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15
*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(62/3)*b**2*x**4/(80*a**20*b*
*4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 12
00*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 3120*a*
*(59/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**
2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 48
0*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 4860*a**(59/3)*b**3*x**6/(80*a
**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x*
*6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) +
1830*a**(56/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b
**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x*
*8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(56/3)*b**4*x**
8/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*
b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x*
*12) + 528*a**(53/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*
a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*
b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(53/3)*b
**5*x**10/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 160
0*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*
b**10*x**12) + 96*a**(50/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4
 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200
*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(5
0/3)*b**6*x**12/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4
 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*
a**14*b**10*x**12) + 48*a**(47/3)*b**7*x**14*(1 + b*x**2/a)**(2/3)/(80*a**2
0*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6
+ 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 15*
a**(44/3)*b**8*x**16*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*
x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 +
 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12)
```

Giac [A] time = 2.00577, size = 77, normalized size = 0.96

$$\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}-24\left(bx^2+a\right)^{\frac{5}{3}}a+60\left(bx^2+a\right)^{\frac{2}{3}}a^2+\frac{40\,a^3}{\left(bx^2+a\right)^{\frac{1}{3}}}\right)}{80\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/80*(5*(b*x^2 + a)^(8/3) - 24*(b*x^2 + a)^(5/3)*a + 60*(b*x^2 + a)^(2/3)*a
^2 + 40*a^3/(b*x^2 + a)^(1/3))/b^4
```

3.727 $\int \frac{x^5}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=59

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

```
[Out] (-3*a^2)/(2*b^3*(a + b*x^2)^(1/3)) - (3*a*(a + b*x^2)^(2/3))/(2*b^3) + (3*(
a + b*x^2)^(5/3))/(10*b^3)
```

Rubi [A] time = 0.0332049, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^2)/(2*b^3*(a + b*x^2)^(1/3)) - (3*a*(a + b*x^2)^(2/3))/(2*b^3) + (3*(
a + b*x^2)^(5/3))/(10*b^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx,x,x^2\right)\\
&= -\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}
\end{aligned}$$

Mathematica [A] time = 0.015648, size = 38, normalized size = 0.64

$$\frac{3\left(-9a^2-3abx^2+b^2x^4\right)}{10b^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(-9*a^2 - 3*a*b*x^2 + b^2*x^4))/(10*b^3*(a + b*x^2)^(1/3))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.6

$$-\frac{-3\,b^{2}x^{4}+9\,ab x^{2}+27\,a^{2}}{10\,b^{3}}\frac{1}{\sqrt[3]{b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(4/3),x)
```

```
[Out] -3/10/(b*x^2+a)^(1/3)*(-b^2*x^4+3*a*b*x^2+9*a^2)/b^3
```

Maxima [A] time = 1.65963, size = 63, normalized size = 1.07

$$\frac{3\left(b x^{2}+a\right)^{\frac{5}{3}}}{10\,b^{3}}-\frac{3\left(b x^{2}+a\right)^{\frac{2}{3}}a}{2\,b^{3}}-\frac{3\,a^{2}}{2\left(b x^{2}+a\right)^{\frac{1}{3}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b^3 - 3/2*(b*x^2 + a)^(2/3)*a/b^3 - 3/2*a^2/((b*x^2
+ a)^(1/3)*b^3)
```

Fricas [A] time = 1.74528, size = 97, normalized size = 1.64

$$\frac{3\left(b^{2}x^{4}-3\,ab x^{2}-9\,a^{2}\right)\left(b x^{2}+a\right)^{\frac{2}{3}}}{10\left(b^{4}x^{2}+ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/10*(b^2*x^4 - 3*a*b*x^2 - 9*a^2)*(b*x^2 + a)^(2/3)/(b^4*x^2 + a*b^3)
```

Sympy [B] time = 1.63656, size = 561, normalized size = 9.51

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx^{2}}{a}\right)^{\frac{2}{3}}}{10a^{8}b^{3}+30a^{7}b^{4}x^{2}+30a^{6}b^{5}x^{4}+10a^{5}b^{6}x^{6}}+\frac{27a^{\frac{29}{3}}}{10a^{8}b^{3}+30a^{7}b^{4}x^{2}+30a^{6}b^{5}x^{4}+10a^{5}b^{6}x^{6}}-\frac{63a^{\frac{26}{3}}bx^{2}\left(1+\right.}{10a^{8}b^{3}+30a^{7}b^{4}x^{2}+30a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*
a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 27*a**(29/3)/(10*a**8*b**3 + 30*a**7*
b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 63*a**(26/3)*b*x**2*(1
 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 +
 10*a**5*b**6*x**6) + 81*a**(26/3)*b*x**2/(10*a**8*b**3 + 30*a**7*b**4*x**2
 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 42*a**(23/3)*b**2*x**4*(1 + b*x
**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a*
*5*b**6*x**6) + 81*a**(23/3)*b**2*x**4/(10*a**8*b**3 + 30*a**7*b**4*x**2 +
30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 3*a**(20/3)*b**3*x**6*(1 + b*x**2/
a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b
**6*x**6) + 27*a**(20/3)*b**3*x**6/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a
**6*b**5*x**4 + 10*a**5*b**6*x**6) + 3*a**(17/3)*b**4*x**8*(1 + b*x**2/a)**
(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*
x**6)
```

Giac [A] time = 2.76319, size = 55, normalized size = 0.93

$$\frac{3\left(\left(bx^2+a\right)^{\frac{5}{3}}-5\left(bx^2+a\right)^{\frac{2}{3}}a-\frac{5a^2}{\left(bx^2+a\right)^{\frac{1}{3}}}\right)}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/10*((b*x^2 + a)^(5/3) - 5*(b*x^2 + a)^(2/3)*a - 5*a^2/(b*x^2 + a)^(1/3))/
b^3
```

3.728 $\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=38

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{4b^2}$$

```
[Out] (3*a)/(2*b^2*(a + b*x^2)^(1/3)) + (3*(a + b*x^2)^(2/3))/(4*b^2)
```

Rubi [A] time = 0.0234592, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a)/(2*b^2*(a + b*x^2)^(1/3)) + (3*(a + b*x^2)^(2/3))/(4*b^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{4/3}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx,x,x^2\right)\\ &= \frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.0113592, size = 27, normalized size = 0.71

$$\frac{3(3a+bx^2)}{4b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(3*a + b*x^2))/(4*b^2*(a + b*x^2)^(1/3))
```

Maple [A] time = 0.004, size = 24, normalized size = 0.6

$$\frac{3\,b x^{2}+9\,a}{4\,b^{2}}\frac{1}{\sqrt[3]{b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(4/3),x)
```

```
[Out] 3/4/(b*x^2+a)^(1/3)*(b*x^2+3*a)/b^2
```

Maxima [A] time = 2.24349, size = 41, normalized size = 1.08

$$\frac{3\left(b x^{2}+a\right)^{\frac{2}{3}}}{4\,b^{2}}+\frac{3\,a}{2\left(b x^{2}+a\right)^{\frac{1}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b^2 + 3/2*a/((b*x^2 + a)^(1/3)*b^2)
```

Fricas [A] time = 1.72117, size = 74, normalized size = 1.95

$$\frac{3\left(b x^{2}+3\,a\right)\left(b x^{2}+a\right)^{\frac{2}{3}}}{4\left(b^{3} x^{2}+a b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x^2 + 3*a)*(b*x^2 + a)^(2/3)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.648634, size = 46, normalized size = 1.21

$$\begin{cases}\frac{9a}{4b^{2}\sqrt[3]{a+bx^{2}}}+\frac{3x^{2}}{4b\sqrt[3]{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4a^{\frac{4}{3}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)**(1/
3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))
```

Giac [A] time = 1.99315, size = 36, normalized size = 0.95

$$\frac{3\left((bx^2+a)^{\frac{2}{3}}+\frac{2a}{(bx^2+a)^{\frac{1}{3}}}\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/4*((b*x^2 + a)^(2/3) + 2*a/(b*x^2 + a)^(1/3))/b^2
```

3.729 $\int \frac{x}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=18

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Rubi [A] time = 0.0035819, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(4/3),x]
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{4/3}}\,dx=-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Mathematica [A] time = 0.0026457, size = 18, normalized size = 1.

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(4/3),x]
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$-\frac{3}{2\,b}\frac{1}{\sqrt[3]{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(4/3),x)
```

```
[Out] -3/2/b/(b*x^2+a)^(1/3)
```

Maxima [A] time = 1.53318, size = 19, normalized size = 1.06

$$-\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

Fricas [A] time = 1.72524, size = 54, normalized size = 3.

$$-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x^2 + a)^(2/3)/(b^2*x^2 + a*b)
```

Sympy [A] time = 0.603189, size = 26, normalized size = 1.44

$$\begin{cases} -\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), Tru
e))
```

Giac [A] time = 2.32453, size = 19, normalized size = 1.06

$$-\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

3.730 $\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=104

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

```
[Out] 3/(2*a*(a + b*x^2)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))
/(Sqrt[3]*a^(1/3))])/(2*a^(4/3)) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rubi [A] time = 0.0648134, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*(a + b*x^2)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))
/(Sqrt[3]*a^(1/3))])/(2*a^(4/3)) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}}-\frac{\log(x)}{2a^{4/3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+\dots}\right.}{4a}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{4/3}}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0063843, size = 36, normalized size = 0.35

$$\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^2)/a])/(2*a*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(4/3),x)
```

[Out] int(1/x/(b*x^2+a)^(4/3),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.81512, size = 894, normalized size = 8.6

$$\left[\frac{\sqrt{3}\left(abx^2+a^2\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-\left(bx^2+a\right)a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2\right.\right.}{4\left(a^2bx^2+a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="fricas")

[Out] [1/4*(sqrt(3)*(a*b*x^2 + a^2)*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - (b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3), -1/4*((b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x^2 + a^2)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3)]

Sympy [C] time = 1.18214, size = 41, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**2+a)**(4/3),x)

[Out] -gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4/3)*x**(8/3)*gamma(7/3))

Giac [A] time = 4.0003, size = 136, normalized size = 1.31

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{4}{3}}}+\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/2/((b*x^2 + a)
^(1/3)*a)
```

3.731 $\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b}{a^2\sqrt[3]{a+bx^2}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{1}{2ax^2\sqrt[3]{a+bx^2}}$$

```
[Out] (-2*b)/(a^2*(a + b*x^2)^(1/3)) - 1/(2*a*x^2*(a + b*x^2)^(1/3)) - (2*b*ArcTa
n[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (
2*b*Log[x])/(3*a^(7/3)) - (b*Log[a^(1/3) - (a + b*x^2)^(1/3)])/a^(7/3)
```

Rubi [A] time = 0.0801575, antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}+\frac{3}{2ax^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^2*(a + b*x^2)^(1/3)) - (2*(a + b*x^2)^(2/3))/(a^2*x^2) - (2*b*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) +
(2*b*Log[x])/(3*a^(7/3)) - (b*Log[a^(1/3) - (a + b*x^2)^(1/3)])/a^(7/3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3 \left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}+\frac{2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{a}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{3a^2}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}+\frac{2b\log(x)}{3a^{7/3}}+\frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{b\operatorname{Subst}\left(\int\right.}{}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}+\frac{2b\log(x)}{3a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}+\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{a^{7/3}}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0070998, size = 37, normalized size = 0.3

$$-\frac{3b\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] (-3*b*Hypergeometric2F1[-1/3, 2, 2/3, 1 + (b*x^2)/a])/(2*a^2*(a + b*x^2)^(1
/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89612, size = 1127, normalized size = 9.16

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^2x^4+a^2bx^2\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^2}\right)+2\left(b^2x^4+abx^2\right)(-}{6\left(a^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt((-a)^(1/3)/a)*log((2*b*x^2 -
 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(
1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + 2
*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(
-a)^(1/3) + (-a)^(2/3)) - 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^
(1/3) + (-a)^(1/3)) - 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a
^4*x^2), -1/6*(12*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt(-(-a)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(
b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(1
/3) + (-a)^(1/3)) + 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a^4
*x^2)]
```

Sympy [C] time = 1.58406, size = 41, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{14}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(7/3)*hyper((4/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4
/3)*x**(14/3)*gamma(10/3))
```

Giac [A] time = 4.92595, size = 171, normalized size = 1.39

$$-\frac{1}{6}b\left(\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{}{\left(\left(bx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -1/6*b*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*b*x^2 + a
)/(((b*x^2 + a)^(4/3) - (b*x^2 + a)^(1/3)*a)*a^2))
```

3.732 $\int \frac{1}{x^5(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=159

$$\frac{7b^2}{3a^3\sqrt[3]{a+bx^2}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}+\frac{7b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b}{12a^2x^2\sqrt[3]{a+bx^2}}-\frac{1}{4ax^4\sqrt[3]{a+bx^2}}$$

```
[Out] (7*b^2)/(3*a^3*(a + b*x^2)^(1/3)) - 1/(4*a*x^4*(a + b*x^2)^(1/3)) + (7*b)/(
12*a^2*x^2*(a + b*x^2)^(1/3)) + (7*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3
))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) +
 (7*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(10/3))
```

Rubi [A] time = 0.105579, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}+\frac{7b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2}-\frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4}+\frac{3}{2ax^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^4*(a + b*x^2)^(1/3)) - (7*(a + b*x^2)^(2/3))/(4*a^2*x^4) + (7*b*(a
 + b*x^2)^(2/3))/(3*a^3*x^2) + (7*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3)
)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) +
(7*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(10/3))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{4/3}}\,dx, x, x^2\right)$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a^2}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{9a^3}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{\ldots}\right.}{6a^{10/3}}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}} - \ldots$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} + \frac{7b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{10/3}} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2 \ldots}{\ldots}$$

Mathematica [C] time = 0.0074407, size = 39, normalized size = 0.25

$$\frac{3b^2\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 3, 2/3, 1 + (b*x^2)/a])/(2*a^3*(a + b*x^2)^(
1/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8441, size = 1107, normalized size = 6.96

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^3x^6+a^2b^2x^4\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-14\left(b^3x^6+ab^2x^4\right)a^{\frac{2}{3}}\log\left(\cdots\right.}{36\left(a^4bx\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/36*(42*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*sqrt(-1/a^(2/3))*log((2*b*x^2
 + 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3)
)*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 14*(b^3*x^6
+ a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3))
+ 3*(28*a*b^2*x^4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^
5*x^4), -1/36*(14*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*
x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b
*x^2 + a)^(1/3) - a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*arctan(
sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^
4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^5*x^4)]
```

Sympy [C] time = 1.99542, size = 41, normalized size = 0.26

$$-\frac{\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{20}{3}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(10/3)*hyper((4/3, 10/3), (13/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**
(4/3)*x**(20/3)*gamma(13/3))
```

Giac [A] time = 3.47891, size = 190, normalized size = 1.19

$$\frac{1}{36}b^2\left(\frac{28\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{10}{3}}}-\frac{14\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{10}{3}}}+\frac{28\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{10}{3}}}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 1/36*b^2*(28*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(10/3) - 14*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(10/3) + 28*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(10/3) + 54/((b
*x^2 + a)^(1/3)*a^3) + 3*(10*(b*x^2 + a)^(5/3) - 13*(b*x^2 + a)^(2/3)*a)/(a
^3*b^2*x^4))
```

3.733 $\int \frac{x^4}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=577

$$\frac{27\ 3^{3/4} a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{7\sqrt{2}b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/}}{}$$

[Out] (-3*x^3)/(2*b*(a + b*x^2)^(1/3)) + (27*x*(a + b*x^2)^(2/3))/(14*b^2) + (81*a*x)/(14*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (81*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(28*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (27*3^(3/4)*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*Sqrt[2]*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rubi [A] time = 0.365301, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {288, 321, 235, 304, 219, 1879}

$$\frac{27\ 3^{3/4} a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7\sqrt{2}b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b*x^2)^(4/3),x]

[Out] (-3*x^3)/(2*b*(a + b*x^2)^(1/3)) + (27*x*(a + b*x^2)^(2/3))/(14*b^2) + (81*a*x)/(14*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (81*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(28*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (27*3^(3/4)*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*Sqrt[2]*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{4/3}}\,dx = -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt[3]{a+bx^2}}dx}{2b}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} - \frac{(27a)\int \frac{1}{\sqrt[3]{a+bx^2}}dx}{14b^2}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} - \frac{\left(81a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}dx, x, \sqrt[3]{a+bx^2}\right)}{28b^3x}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} + \frac{\left(81a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}dx, x, \sqrt[3]{a+bx^2}\right)}{28b^3x} - \frac{\left(81\sqrt{\frac{1}{2}\left(2+\right.}\right.}{}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} + \frac{81ax}{14b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.024062, size = 65, normalized size = 0.11

$$\frac{3x\left(-9a\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+9a+2bx^2\right)}{14b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(9*a + 2*b*x^2 - 9*a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2,
 3/2, -((b*x^2)/a)]))/(14*b^2*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int x^4\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^4/(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}x^4}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 0.747874, size = 27, normalized size = 0.05

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(4/3),x)
```

```
[Out] x**5*hyper((4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

3.734 $\int \frac{x^2}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=553

$$-\frac{3\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt{2}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}}{}$$

```
[Out] (-3*x)/(2*b*(a + b*x^2)^(1/3)) - (9*x)/(2*b*((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(4*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (3*3^(3/4)*a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.303232, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 235, 304, 219, 1879}

$$-\frac{3\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*x)/(2*b*(a + b*x^2)^(1/3)) - (9*x)/(2*b*((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(4*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (3*3^(3/4)*a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{4/3}}\,dx = -\frac{3x}{2b\sqrt[3]{a+bx^2}} + \frac{3\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{2b}$$

$$= -\frac{3x}{2b\sqrt[3]{a+bx^2}} + \frac{\left(9\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4b^2x}$$

$$= -\frac{3x}{2b\sqrt[3]{a+bx^2}} - \frac{\left(9\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4b^2x} + \frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{bx^2}\right)\text{Sub}\cdots}{2b\cdots}$$

$$= -\frac{3x}{2b\sqrt[3]{a+bx^2}} - \frac{9x}{2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a\cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{4b^2x\sqrt{-\frac{\cdots}{\left(\left(\cdots\right.\right.}}}$$

Mathematica [C] time = 0.0158215, size = 55, normalized size = 0.1

$$\frac{3x\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)
/a)]))/(2*b*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^2/(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}x^2}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 0.747548, size = 27, normalized size = 0.05

$$\frac{x^3{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(4/3),x)
```

```
[Out] x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

3.735 $\int \frac{1}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=552

$$\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{}$$

```
[Out] (3*x)/(2*a*(a + b*x^2)^(1/3)) + (3*x)/(2*a*((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(4*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3^(3/4)*(a^(1/3) - (a + b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(Sqrt[2]*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x
^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.297732, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {199, 235, 304, 219, 1879}

$$\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-4/3),x]
```

```
[Out] (3*x)/(2*a*(a + b*x^2)^(1/3)) + (3*x)/(2*a*((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(4*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3^(3/4)*(a^(1/3) - (a + b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(Sqrt[2]*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x
^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx = \frac{3x}{2a\sqrt[3]{a+bx^2}} - \frac{\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} - \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4abx}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4abx} - \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{bx^2}\right)\text{Subst}\left(\int \cdots\right.}{2a^{2/3}bx\cdots}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}}}{4a^{2/3}bx\sqrt{-\frac{\sqrt[3]{\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

Mathematica [C] time = 0.0107076, size = 58, normalized size = 0.11

$$\frac{3x - x\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-4/3),x]
```

```
[Out] (3*x - x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a
)])/(2*a*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 0.721874, size = 24, normalized size = 0.04

$$\frac{x{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(4/3),x)
```

```
[Out] x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

3.736 $\int \frac{1}{x^2\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=571

$$-\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt{2}\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\right.}$$

```
[Out] 3/(2*a*x*(a + b*x^2)^(1/3)) - (5*(a + b*x^2)^(2/3))/(2*a^2*x) - (5*b*x)/(2*
a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt
[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3
) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellip
ticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a^(5/3)*x*Sqrt[-((a^(1/3)*(a
^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)
]) - (5*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(
1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*3^(1/4)*a^(5/3)*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.352225, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {290, 325, 235, 304, 219, 1879}

$$-\frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{5\left(a+bx^2\right)^{2/3}}{2a^2x}-\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{\sqrt{2}\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x*(a + b*x^2)^(1/3)) - (5*(a + b*x^2)^(2/3))/(2*a^2*x) - (5*b*x)/(2*
a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt
[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3
) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellip
ticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a^(5/3)*x*Sqrt[-((a^(1/3)*(a
^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)
]) - (5*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(
1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*3^(1/4)*a^(5/3)*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{4/3}}\,dx = \frac{3}{2ax\sqrt[3]{a+bx^2}} + \frac{5\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} + \frac{(5b)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{6a^2}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} + \frac{\left(5\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^2x}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} - \frac{\left(5\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^2x} + \frac{\left(5\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right.}\right.}{}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} - \frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{}$$

Mathematica [C] time = 0.0095934, size = 52, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(4/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/3)))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b^2x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [A] time = 0.944672, size = 27, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(4/3),x)
```

```
[Out] -hyper((-1/2, 4/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(4/3)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

3.737 $\int \frac{1}{x^4\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=599

$$\frac{55b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt{2}\sqrt[4]{3}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\right.}}$$

```
[Out] 3/(2*a*x^3*(a + b*x^2)^(1/3)) - (11*(a + b*x^2)^(2/3))/(6*a^2*x^3) + (55*b*
(a + b*x^2)^(2/3))/(18*a^3*x) + (55*b^2*x)/(18*a^3*((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))) - (55*Sqrt[2 + Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(12*3^(3/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (55*b*(a^(1/3) - (a
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.418246, antiderivative size = 599, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {290, 325, 235, 304, 219, 1879}

$$\frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x}-\frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3}+\frac{55b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\cdots\right)}{9\sqrt{2}\sqrt[4]{3}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\cdots\right)}{\left(\left(1-\sqrt{3}\right)\cdots\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^3*(a + b*x^2)^(1/3)) - (11*(a + b*x^2)^(2/3))/(6*a^2*x^3) + (55*b*
(a + b*x^2)^(2/3))/(18*a^3*x) + (55*b^2*x)/(18*a^3*((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))) - (55*Sqrt[2 + Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(12*3^(3/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (55*b*(a^(1/3) - (a
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{4/3}}\,dx = \frac{3}{2ax^3\sqrt[3]{a+bx^2}} + \frac{11\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} - \frac{(55b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{18a^2}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} - \frac{\left(55b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{54a^3}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} - \frac{\left(55b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx}\right.}{36a^3x}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} + \frac{\left(55b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a}\right.}{36a^3x}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} + \frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{55\sqrt{2+}}{}$$

Mathematica [C] time = 0.0092524, size = 54, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(4/3)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 4/3, -1/2, -((b*x^2)/a)])/(
3*a*x^3*(a + b*x^2)^(1/3))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [A] time = 1.17494, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{4}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(4/3),x)
```

```
[Out] -hyper((-3/2, 4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

3.738 $\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=195

$$-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}-\frac{5a^3c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{108b^{8/3}}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{54\sqrt{3}b^{8/3}}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{ac}{}$$

```
[Out] (-5*a^2*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(108*b^2) + (a*c*(c*x)^(10/3)*(a
 + b*x^2)^(1/3))/(36*b) + ((c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*c) - (5*a^3*c
^(13/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sq
rt[3]])/(54*Sqrt[3]*b^(8/3)) - (5*a^3*c^(13/3)*Log[b^(1/3)*(c*x)^(2/3) - c^
(2/3)*(a + b*x^2)^(1/3)])/(108*b^(8/3))
```

Rubi [A] time = 0.390868, antiderivative size = 275, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}-\frac{5a^3c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}}+\frac{5a^3c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{324b^{8/3}}-\frac{5a^3c^{13/3}\tan^{-1}\left(\ldots\right)}{54\sqrt{3}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (-5*a^2*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(108*b^2) + (a*c*(c*x)^(10/3)*(a
 + b*x^2)^(1/3))/(36*b) + ((c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*c) - (5*a^3*c
^(13/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[
3]*c^(2/3))])/(54*Sqrt[3]*b^(8/3)) - (5*a^3*c^(13/3)*Log[c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(162*b^(8/3)) + (5*a^3*c^(13/3)*Log[c^(4/
3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3)])/(324*b^(8/3))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{1}{9}a\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$=\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}-\frac{\left(5a^2c^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{54b}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{\left(5a^3c^4\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{81b^2}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{\left(5a^3c^3\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\right.}{27b^2}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{\left(5a^3c^3\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\right.}{54b^2}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{\left(5a^3c^3\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx,\right.}{54b^2}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{\left(5a^3c^{11/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\right.}{162b^{7/3}}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}-\frac{5a^3c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}-\frac{5a^3c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}}$$

$$=-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{54\sqrt{3}b^{8/3}}$$

Mathematica [C] time = 0.0699317, size = 102, normalized size = 0.52

$$\frac{c^3(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-5a^2+abx^2+6b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{36b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-5*a^2 + a*b*x^2
 + 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^2)/a)]))/(36
*b^2*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (cx)^{\frac{13}{3}} \sqrt[3]{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{13}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{13}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

3.739 $\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=164

$$\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}$$

```
[Out] (a*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b) + ((c*x)^(10/3)*(a + b*x^2)^(1/3
))/(4*c) + (a^2*c^(7/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b
*x^2)^(1/3)))/Sqrt[3]])/(6*Sqrt[3]*b^(5/3)) + (a^2*c^(7/3)*Log[b^(1/3)*(c*x
)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(12*b^(5/3))
```

Rubi [A] time = 0.301706, antiderivative size = 244, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}-\frac{a^2c^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{36b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{6\sqrt{3}b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (a*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b) + ((c*x)^(10/3)*(a + b*x^2)^(1/3
))/(4*c) + (a^2*c^(7/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^
2)^(1/3))/(Sqrt[3]*c^(2/3))])/(6*Sqrt[3]*b^(5/3)) + (a^2*c^(7/3)*Log[c^(2/3
) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(18*b^(5/3)) - (a^2*c^(7/3)*L
og[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)])/(36*b^(5/3))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{1}{6}a\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{3b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx,x,(cx)^{2/3}\right)}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{4/3}}+\frac{\left(a^2c^{5/3}\right)\text{Subst}}{}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}+\frac{\left(a^2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b}{}}\right.}{12b^{4/3}}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}-\frac{a^2c^{7/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\right.}{36b^{5/3}}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}b^{5/3}}+\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}$$

Mathematica [C] time = 0.0484638, size = 85, normalized size = 0.52

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{4b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hyp
ergeometric2F1[-1/3, 2/3, 5/3, -((b*x^2)/a)]))/(4*b*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{3}}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

3.740 $\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

```
[Out] ((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) - (a*c^(1/3)*ArcTan[(1 + (2*b^(1/3)*(
c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(2*Sqrt[3]*b^(2/3)) - (a
*c^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(4*b^(2/3))
```

Rubi [A] time = 0.273921, antiderivative size = 211, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}}+\frac{a\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{12b^{2/3}}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2\sqrt{3}b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) - (a*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(
1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*Sqrt[3]*b^(2/3)
) - (a*c^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(6*b
^(2/3)) + (a*c^(1/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3)
+ (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(12*b^(2/3))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{1}{3}a\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6\sqrt[3]{b}\sqrt[3]{c}} - \frac{a\,\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6\sqrt[3]{b}\sqrt[3]{c}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\sqrt[3]{c}} + \frac{\left(a\sqrt[3]{c}\right)\text{Sub}}{}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{12b^{2/3}} + \frac{\left(a\sqrt[3]{c}\right)\text{Su}}{}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{2/3}} - \frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\right.}{12b^{2/3}}$$

Mathematica [C] time = 0.0134915, size = 56, normalized size = 0.42

$$\frac{3x\sqrt[3]{cx}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)}{4\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(c*x)^(1/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x
^2)/a)])/(4*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 1.74784, size = 46, normalized size = 0.35

$$\frac{\sqrt[3]{a}\sqrt[3]{c}x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

3.741 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{5/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(1 + (2*
b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(2*c^(5/3)) - (
3*b^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(4*c^(5/3))
```

Rubi [A] time = 0.274435, antiderivative size = 208, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {277, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4c^{5/3}}-\frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(c^(2/3)
 + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*c^(5/3
)) - (b^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(2*c^
(5/3)) + (b^(1/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(4*c^(5/3))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b^{2/3}\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{7/3}} - \frac{b^{2/3}\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{7/3}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} - \frac{\left(3b^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{7/3}} + \frac{\sqrt[3]{b}\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{5/3}} + \frac{\left(3\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{\cdots}{-3 \cdots}\right.}{\cdots}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{5/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}\cdots}{\sqrt[3]{a\cdots}}\right.}{4c^{5/3}}$$

Mathematica [C] time = 0.013499, size = 56, normalized size = 0.43

$$-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^2}{a}\right)}{2(cx)^{5/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -((b*x^2)/a)])/(
2*(c*x)^(5/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2+a}\,(cx)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 5.20801, size = 49, normalized size = 0.37

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(5/3),x)
```

```
[Out] a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

3.742 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

Rubi [A] time = 0.0056703, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]

[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx=-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Mathematica [A] time = 0.0086013, size = 26, normalized size = 0.93

$$-\frac{3x\left(a+bx^2\right)^{4/3}}{8a(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]

[Out] (-3*x*(a + b*x^2)^(4/3))/(8*a*(c*x)^(11/3))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$-\frac{3\,x}{8\,a}\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{11}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(11/3),x)
```

```
[Out] -3/8*x*(b*x^2+a)^(4/3)/a/(c*x)^(11/3)
```

Maxima [A] time = 1.92746, size = 47, normalized size = 1.68

$$-\frac{3\left(bc^{\frac{1}{3}}x^3 + ac^{\frac{1}{3}}x\right)\left(bx^2 + a\right)^{\frac{1}{3}}}{8\,ac^4x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="maxima")
```

```
[Out] -3/8*(b*c^(1/3)*x^3 + a*c^(1/3)*x)*(b*x^2 + a)^(1/3)/(a*c^4*x^(11/3))
```

Fricas [A] time = 2.50888, size = 65, normalized size = 2.32

$$-\frac{3\left(bx^2 + a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{8\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] -3/8*(b*x^2 + a)^(4/3)*(c*x)^(1/3)/(a*c^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(11/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x)
```

3.743 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}} \, dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2
*c*(c*x)^(14/3))
```

Rubi [A] time = 0.0147292, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2
*c*(c*x)^(14/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}} \, dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}-\frac{3\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}} \, dx}{4a}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}
\end{aligned}$$

Mathematica [A] time = 0.0169664, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\left(3bx^2-4a\right)}{56a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(4/3)*(-4*a + 3*b*x^2))/(56*a^2*c^6*x^5)
```

Maple [A] time = 0.004, size = 31, normalized size = 0.5

$$-\frac{3x\left(-3bx^2+4a\right)}{56a^2}\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{17}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(17/3),x)
```

```
[Out] -3/56*x*(b*x^2+a)^(4/3)*(-3*b*x^2+4*a)/a^2/(c*x)^(17/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

Fricas [A] time = 2.49901, size = 108, normalized size = 1.89

$$\frac{3\left(3b^2x^4-abx^2-4a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{56a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="fricas")
```

```
[Out] 3/56*(3*b^2*x^4 - a*b*x^2 - 4*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^6*x
^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(17/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

3.744 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2
*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))
```

Rubi [A] time = 0.0247993, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2
*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}-\frac{3\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx}{2a}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}+\frac{9\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{14a^2}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}
\end{aligned}$$

Mathematica [A] time = 0.0180083, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\left(14a^2-12abx^2+9b^2x^4\right)}{280a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(c*x)^(1/3)*(a + b*x^2)^(4/3)*(14*a^2 - 12*a*b*x^2 + 9*b^2*x^4))/(280*a
^3*c^8*x^7)
```

Maple [A] time = 0.004, size = 42, normalized size = 0.5

$$-\frac{3x\left(9b^2x^4-12abx^2+14a^2\right)}{280a^3}\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{23}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(23/3),x)
```

```
[Out] -3/280*x*(b*x^2+a)^(4/3)*(9*b^2*x^4-12*a*b*x^2+14*a^2)/a^3/(c*x)^(23/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

Fricas [A] time = 1.76081, size = 136, normalized size = 1.6

$$-\frac{3\left(9b^3x^6-3ab^2x^4+2a^2bx^2+14a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] -3/280*(9*b^3*x^6 - 3*a*b^2*x^4 + 2*a^2*b*x^2 + 14*a^3)*(b*x^2 + a)^(1/3)*(
c*x)^(1/3)/(a^3*c^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(23/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

3.745 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=113

$$\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a + b*x^2)^(7/3))/(56*a^
2*c*(c*x)^(26/3)) - (81*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243
*(a + b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))
```

Rubi [A] time = 0.0394777, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a + b*x^2)^(7/3))/(56*a^
2*c*(c*x)^(26/3)) - (81*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243
*(a + b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}-\frac{9\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx}{4a}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}+\frac{27\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{29/3}}\,dx}{14a^2}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}-\frac{81\int \frac{\left(a+bx^2\right)^{10/3}}{(cx)^{29/3}}\,dx}{140a^3}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}
\end{aligned}$$

Mathematica [A] time = 0.0170239, size = 63, normalized size = 0.56

$$\frac{3\left(a+bx^2\right)^{4/3}\left(126a^2bx^2-140a^3-108ab^2x^4+81b^3x^6\right)}{3640a^4c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-140*a^3 + 126*a^2*b*x^2 - 108*a*b^2*x^4 + 81*b^3*x^6
))/(3640*a^4*c^9*x^8*(c*x)^(2/3))
```

Maple [A] time = 0.006, size = 53, normalized size = 0.5

$$-\frac{3\,x\left(-81\,b^3x^6+108\,ab^2x^4-126\,a^2bx^2+140\,a^3\right)}{3640\,a^4}\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{29}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(29/3),x)
```

```
[Out] -3/3640*x*(b*x^2+a)^(4/3)*(-81*b^3*x^6+108*a*b^2*x^4-126*a^2*b*x^2+140*a^3)
/a^4/(c*x)^(29/3)
```

Maxima [A] time = 1.38082, size = 86, normalized size = 0.76

$$\frac{3\left(81\,b^4x^9-27\,ab^3x^7+18\,a^2b^2x^5-14\,a^3bx^3-140\,a^4x\right)\left(bx^2+a\right)^{\frac{1}{3}}}{3640\,a^4c^{\frac{29}{3}}x^{\frac{29}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="maxima")
```

```
[Out] 3/3640*(81*b^4*x^9 - 27*a*b^3*x^7 + 18*a^2*b^2*x^5 - 14*a^3*b*x^3 - 140*a^4
*x)*(b*x^2 + a)^(1/3)/(a^4*c^(29/3)*x^(29/3))
```

Fricas [A] time = 1.69862, size = 166, normalized size = 1.47

$$\frac{3\left(81\,b^4x^8-27\,ab^3x^6+18\,a^2b^2x^4-14\,a^3bx^2-140\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{3640\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(81*b^4*x^8 - 27*a*b^3*x^6 + 18*a^2*b^2*x^4 - 14*a^3*b*x^2 - 140*a^4
)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^10*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x)
```

3.746 $\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=451

$$\frac{7a^2c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{14}{}$$

```
[Out] (-14*a^2*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b^2) + (2*a*c*(c*x)^(7/3)*
(a + b*x^2)^(1/3))/(45*b) + ((c*x)^(13/3)*(a + b*x^2)^(1/3))/(5*c) + (7*a^2
*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b
^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/
(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.970588, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 321, 329, 241, 225}

$$-\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2}+\frac{7a^2c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\right.}{135\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (-14*a^2*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b^2) + (2*a*c*(c*x)^(7/3)*
(a + b*x^2)^(1/3))/(45*b) + ((c*x)^(13/3)*(a + b*x^2)^(1/3))/(5*c) + (7*a^2
*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b
^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/
(a + b*x^2)^(1/3))^2))])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{1}{15}(2a)\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} - \frac{\left(14a^2c^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{135b}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{\left(14a^3c^4\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}}{405b^2}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{\left(14a^3c^3\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right.}\right.}{135b^2}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{\left(14a^3c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx}{c^2}}}\right.}{135b^2\sqrt{\frac{a}{a+bx^2}}\sqrt{a}}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{7a^2c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}\right.}{}$$

Mathematica [C] time = 0.0577519, size = 103, normalized size = 0.23

$$\frac{c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-7a^2+2abx^2+9b^2x^4\right)+7a^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{45b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-7*a^2 + 2*a*b*x
^2 + 9*b^2*x^4) + 7*a^2*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(
45*b^2*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (cx)^{\frac{10}{3}}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}c^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c^3*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

3.747 $\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=418

$$-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{(cx}{\ }$$

```
[Out] (2*a*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b) + ((c*x)^(7/3)*(a + b*x^2)^(1/3
))/(3*c) - (a*c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x
^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1
 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^
(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - (
(1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(
9*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x
)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.713411, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 321, 329, 241, 225}

$$-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{(cx)^{7/3}\sqrt[3]{a+}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (2*a*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b) + ((c*x)^(7/3)*(a + b*x^2)^(1/3
))/(3*c) - (a*c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x
^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1
 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^
(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - (
(1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(
9*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x
)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c} + \frac{1}{9}(2a)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b} + \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c} - \frac{\left(2a^2c^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{27b}$$

$$= \frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b} + \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c} - \frac{\left(2a^2c\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{9b}$$

$$= \frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b} + \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c} - \frac{\left(2a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{9b\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= \frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b} + \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c} - \frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2}\cdots}{\sqrt[3]{\cdots}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{\cdots}}{\sqrt[3]{a+bx^2}}\right.\cdots}}}{9\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c\cdots\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\cdots\right.}}}$$

Mathematica [C] time = 0.0396052, size = 85, normalized size = 0.2

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hyp
ergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (cx)^{\frac{4}{3}}\sqrt[3]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}cx,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c*x, x)
```

Sympy [C] time = 19.8912, size = 46, normalized size = 0.11

$$\frac{\sqrt[3]{a}c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-1/3, 7/6), (13/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

3.748 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$

Optimal. Leaf size=381

$$\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\sqrt[3]{a+}}{c}$$

```
[Out] ((c*x)^(1/3)*(a + b*x^2)^(1/3))/c + ((c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3)
 - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^
(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Ellip
ticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3
))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 +
 Sqrt[3])/4])/(3^(1/4)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1
/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sq
rt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.6586, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {279, 329, 241, 225}

$$\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

```
[Out] ((c*x)^(1/3)*(a + b*x^2)^(1/3))/c + ((c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3)
 - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^
(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Ellip
ticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3
))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 +
 Sqrt[3])/4])/(3^(1/4)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1
/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sq
rt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c} + \frac{1}{3}(2a)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{c\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c} + \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right)\middle|\frac{1}{4}\ldots}{\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.0105573, size = 54, normalized size = 0.14

$$\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)])/((c
*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^{2}+a}\,(cx)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^{2}+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c*x), x)
```

Sympy [C] time = 1.38295, size = 46, normalized size = 0.12

$$\frac{\sqrt[3]{a}\sqrt[3]{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\,\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(2/3),x)
```

```
[Out] a**(1/3)*x**(1/3)*gamma(1/6)*hyper((-1/3, 1/6), (7/6,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(2/3)*gamma(7/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

3.749 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx$

Optimal. Leaf size=391

$$\frac{3^{3/4} b \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a\cdots}}{5c(\cdots}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*c*(c*x)^(5/3)) + (3^(3/4)*b*(c*x)^(1/3)*(a + b*x^
2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3)
+ (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(
a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))], (2 + Sqrt[3])/4])/(5*a*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c
^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.664521, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {277, 329, 241, 225}

$$\frac{3^{3/4} b \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*c*(c*x)^(5/3)) + (3^(3/4)*b*(c*x)^(1/3)*(a + b*x^
2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3)
+ (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(
a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))], (2 + Sqrt[3])/4])/(5*a*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c
^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{(2b)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{5c^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{(6b)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{5c^3}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{(6b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5c^3\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right.\right.}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0137101, size = 56, normalized size = 0.14

$$
-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{3};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-5/6, -1/3, 1/6, -((b*x^2)/a)])/(
5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2+a}\,(cx)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)*(c*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

Sympy [C] time = 49.8976, size = 32, normalized size = 0.08

$$-\frac{\sqrt[3]{b}\,{}_2F_1\left(\begin{matrix}-\frac{1}{3}, \frac{1}{2} \\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{8}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(8/3),x)
```

```
[Out] -b**(1/3)*hyper((-1/3, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(8/3)*
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

3.750 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx$

Optimal. Leaf size=422

$$-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*c*(c*x)^(11/3)) - (6*b*(a + b*x^2)^(1/3))/(55*a*
c^3*(c*x)^(5/3)) - (3*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/
3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c
^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Elliptic
F[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/
(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sq
rt[3])/4])/(55*a^2*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.71974, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 325, 329, 241, 225}

$$-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{6b\sqrt[3]{a+}}{55ac^3(cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```


```
[Out] (-3*(a + b*x^2)^(1/3))/(11*c*(c*x)^(11/3)) - (6*b*(a + b*x^2)^(1/3))/(55*a*
c^3*(c*x)^(5/3)) - (3*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/
3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c
^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Elliptic
F[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/
(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sq
rt[3])/4])/(55*a^2*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} + \frac{(2b)\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx}{11c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{\left(6b^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{55ac^4}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{\left(18b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{55ac^5}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{\left(18b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55ac^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{3\, 3^{3/4} b^2 \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(c\cdots\right.}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/\cdots}}{\sqrt[3]{a+bx^2}}\right.\cdots}}}$$

Mathematica [C] time = 0.0124458, size = 56, normalized size = 0.13

$$-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -1/3, -5/6, -((b*x^2)/a)])
/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2+a}\,(cx)^{-\frac{14}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{c^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(14/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

3.751 $\int (cx)^{2/3} \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] (3*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*x^
2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] time = 0.0173999, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*x^
2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx &= \frac{\sqrt[3]{a+bx^2}\int (cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= \frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0105107, size = 56, normalized size = 0.97

$$\frac{3x(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (cx)^{\frac{2}{3}} \sqrt[3]{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Sympy [C] time = 2.79991, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{a} c^{\frac{2}{3}} x^{\frac{5}{3}} \Gamma\left(\frac{5}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-1/3, 5/6), (11/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

3.752 $\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}} \, dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] (3*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2
)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] time = 0.0174453, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

```
[Out] (3*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2
)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}} \, dx &= \frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{1+\frac{bx^2}{a}}}{\sqrt[3]{cx}} \, dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\
&= \frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.0119162, size = 56, normalized size = 0.97

$$\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2)/a)])/(2*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2+a}\frac{1}{\sqrt[3]{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c*x), x)
```

Sympy [C] time = 1.05199, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{a}x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(1/3),x)
```

```
[Out] a**(1/3)*x**(2/3)*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(1/3)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

3.753 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] (-3*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(c*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] time = 0.0182894, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{3\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(c*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx &= \frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{1+\frac{bx^2}{a}}}{(cx)^{4/3}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= -\frac{3\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0113679, size = 54, normalized size = 0.96

$$-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(
(c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2+a}\,(cx)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

Sympy [C] time = 3.09779, size = 49, normalized size = 0.88

$$\frac{\sqrt[3]{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(4/3),x)
```

```
[Out] a**(1/3)*gamma(-1/6)*hyper((-1/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

3.754 $\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=223

$$-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}-\frac{5a^4c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{324b^{8/3}}-\frac{5a^4c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{162\sqrt{3}b^{8/3}}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+$$

```
[Out] (-5*a^3*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(324*b^2) + (a^2*c*(c*x)^(10/3)*
(a + b*x^2)^(1/3))/(108*b) + (a*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(18*c) + ((
c*x)^(16/3)*(a + b*x^2)^(4/3))/(8*c) - (5*a^4*c^(13/3)*ArcTan[(1 + (2*b^(1/
3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(162*Sqrt[3]*b^(8/3)
) - (5*a^4*c^(13/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(
324*b^(8/3))
```

Rubi [A] time = 0.366338, antiderivative size = 303, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}-\frac{5a^4c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{486b^{8/3}}+\frac{5a^4c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{972b^{8/3}}-\frac{5a^4c^{13/3}\tan^{-1}\left(\frac{2\sqrt[3]{}}{}\right)}{162\sqrt{3}b^{8}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (-5*a^3*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(324*b^2) + (a^2*c*(c*x)^(10/3)*
(a + b*x^2)^(1/3))/(108*b) + (a*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(18*c) + ((
c*x)^(16/3)*(a + b*x^2)^(4/3))/(8*c) - (5*a^4*c^(13/3)*ArcTan[(c^(2/3) + (2
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(162*Sqrt[3]*b
^(8/3)) - (5*a^4*c^(13/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3)])/(486*b^(8/3)) + (5*a^4*c^(13/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/
(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(972*
b^(8/3))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (cx)^{13/3}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+\frac{1}{3}a\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$$
$$= \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+\frac{1}{27}a^2\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}dx$$
$$= \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}-\frac{\left(5a^3c^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}dx}{162b}$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}-$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}-$$
$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}-$$

Mathematica [C] time = 0.0775268, size = 102, normalized size = 0.46

$$\frac{c^3(cx)^{4/3}\sqrt[3]{a+bx^2}\left(5a^3\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)-\left(5a-9bx^2\right)\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}\right)}{72b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3)*(-((5*a - 9*b*x^2)*(a + b*x^2)^2*(1 + (b
*x^2)/a)^(1/3)) + 5*a^3*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b*x^2)/a)]))/(
72*b^2*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (cx)^{\frac{13}{3}} \left(b x^2+a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2+a\right)^{\frac{4}{3}} (cx)^{\frac{13}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2+a\right)^{\frac{4}{3}} (cx)^{\frac{13}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

3.755 $\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=192

$$\frac{a^3 c^{7/3} \log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27 b^{5/3}}+\frac{2 a^3 c^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3} b^{5/3}}+\frac{a^2 c (cx)^{4/3}\sqrt[3]{a+bx^2}}{27 b}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+a(\ldots$$

```
[Out] (a^2*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b) + (a*(c*x)^(10/3)*(a + b*x^2)^
(1/3))/(9*c) + ((c*x)^(10/3)*(a + b*x^2)^(4/3))/(6*c) + (2*a^3*c^(7/3)*ArcT
an[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(27*
Sqrt[3]*b^(5/3)) + (a^3*c^(7/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^
2)^(1/3)])/(27*b^(5/3))
```

Rubi [A] time = 0.322993, antiderivative size = 272, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{2 a^3 c^{7/3} \log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{81 b^{5/3}}-\frac{a^3 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{81 b^{5/3}}+\frac{2 a^3 c^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{27\sqrt{3} b^{5/3}}+\frac{a^2 c (cx)^{4/3}\sqrt[3]{a\ldots}}{27 b}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a^2*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b) + (a*(c*x)^(10/3)*(a + b*x^2)^
(1/3))/(9*c) + ((c*x)^(10/3)*(a + b*x^2)^(4/3))/(6*c) + (2*a^3*c^(7/3)*ArcT
an[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))]
)/(27*Sqrt[3]*b^(5/3)) + (2*a^3*c^(7/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)])/(81*b^(5/3)) - (a^3*c^(7/3)*Log[c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)])/(81*b^(5/3))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (cx)^{7/3}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} + \frac{1}{9}(4a)\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} + \frac{1}{27}\left(2a^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(4a^3c^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}dx}{81b}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(4a^3c\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)}\right.}{27b}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(2a^3c\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)}\right.}{27b}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(2a^3c\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}d\right.}{27b}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(2a^3c^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}}{c^{2/}}}\right.}{81b^{4/3}}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} + \frac{2a^3c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)}{\sqrt[3]{a+b}}\right.}{81b^{5/3}}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} + \frac{2a^3c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)}{\sqrt[3]{a+b}}\right.}{81b^{5/3}}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c} + \frac{2a^3c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^2}{c^{2/3}\sqrt[3]{a+b}}}{\sqrt{3}}\right.}{27\sqrt{3}b^{5/3}}$$

Mathematica [C] time = 0.0628249, size = 89, normalized size = 0.46

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{6b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^2
*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b*x^2)/a)]))/(6*b*(1 + (b*x^2)/a)^(1/
3))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{3}} \left(b x^2 + a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

3.756 $\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=163

$$-\frac{a^2\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{6b^{2/3}}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

```
[Out] (a*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(3*c) + ((c*x)^(4/3)*(a + b*x^2)^(4/3))/(
4*c) - (a^2*c^(1/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2
)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) - (a^2*c^(1/3)*Log[b^(1/3)*(c*x)^(2
/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(6*b^(2/3))
```

Rubi [A] time = 0.289962, antiderivative size = 243, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}+\frac{a^2\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{18b^{2/3}}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{a(cx)^{4/3}\sqrt[3]{a+b}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(3*c) + ((c*x)^(4/3)*(a + b*x^2)^(4/3))/(
4*c) - (a^2*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(Sqrt[3]*c^(2/3))])/(3*Sqrt[3]*b^(2/3)) - (a^2*c^(1/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(9*b^(2/3)) + (a^2*c^(1/3)*Log[c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3)])/(18*b^(2/3))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{3}(2a)\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{9}\left(2a^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}dx,x,\sqrt[3]{cx}\right)}{3c}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}dx,x,(cx)^{2/3}\right)}{3c}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9\sqrt[3]{b}\sqrt[3]{c}}-\frac{a^2\,\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\cdots}\cdots\right)}{6\sqrt[3]{b}\cdots}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}+\frac{a^2\sqrt[3]{c}\log\left(c^{4/3}+\frac{b^{2/3}(\cdots}{(a+b\cdots}\right.}{18b^{2}\cdots}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx\cdots}{\sqrt[3]{a+b\cdots}}\right.}{9b^{2/3}}$$

Mathematica [C] time = 0.0134284, size = 57, normalized size = 0.35

$$\frac{3ax\sqrt[3]{cx}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)}{4\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*x*(c*x)^(1/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b
*x^2)/a)])/(4*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx}\left(bx^2+a\right)^{\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 20.347, size = 46, normalized size = 0.28

$$\frac{a^{\frac{4}{3}}\sqrt[3]{c}x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

3.757 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=153

$$\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{a\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

```
[Out] (2*b*(c*x)^(4/3)*(a + b*x^2)^(1/3))/c^3 - (3*(a + b*x^2)^(4/3))/(2*c*(c*x)^
(2/3)) - (2*a*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x
^2)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)) - (a*b^(1/3)*Log[b^(1/3)*(c*x)^(2/3
) - c^(2/3)*(a + b*x^2)^(1/3)])/c^(5/3)
```

Rubi [A] time = 0.287007, antiderivative size = 233, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {277, 279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a\sqrt[3]{b}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{3c^{5/3}}+\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{\sqrt{3}c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (2*b*(c*x)^(4/3)*(a + b*x^2)^(1/3))/c^3 - (3*(a + b*x^2)^(4/3))/(2*c*(c*x)^
(2/3)) - (2*a*b^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))/(Sqrt[3]*c^(2/3))])/(Sqrt[3]*c^(5/3)) - (2*a*b^(1/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(3*c^(5/3)) + (a*b^(1/3)*Log[c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3)])/(3*c^(5/3))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{5/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4b)\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx}{c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{3c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{\left(2ab^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{7/3}} - \frac{\left(2ab^{2/3}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} - \frac{\left(ab^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\right.}{c^{7/3}}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} + \frac{a\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\right.}{3c^{5/3}}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}} - \frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} +$$

Mathematica [C] time = 0.0129541, size = 57, normalized size = 0.37

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^2}{a}\right)}{2(cx)^{5/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/3, 2/3, -((b*x^2)/a)])
/(2*(c*x)^(5/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 22.1967, size = 49, normalized size = 0.32

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(5/3),x)
```

```
[Out] a**(4/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

3.758 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=157

$$-\frac{3b^{4/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{11/3}}-\frac{\sqrt{3}b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

```
[Out] (-3*b*(a + b*x^2)^(1/3))/(2*c^3*(c*x)^(2/3)) - (3*(a + b*x^2)^(4/3))/(8*c*(
c*x)^(8/3)) - (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)
*(a + b*x^2)^(1/3)))/Sqrt[3]])/(2*c^(11/3)) - (3*b^(4/3)*Log[b^(1/3)*(c*x)^
(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(4*c^(11/3))
```

Rubi [A] time = 0.290834, antiderivative size = 234, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {277, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}}+\frac{b^{4/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4c^{11/3}}-\frac{\sqrt{3}b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3\left(a \cdots\right.}{8c \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*b*(a + b*x^2)^(1/3))/(2*c^3*(c*x)^(2/3)) - (3*(a + b*x^2)^(4/3))/(8*c*(
c*x)^(8/3)) - (Sqrt[3]*b^(4/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*c^(11/3)) - (b^(4/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(2*c^(11/3)) + (b^(4/3)*Log[c^(4/
3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3)])/(4*c^(11/3))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{b\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx}{c^2}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{b^2\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{c^4}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{b^{5/3}\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{13/3}} - \frac{b^{5/3}\,\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx\right.}{2c^{13/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} - \frac{(3b^{5/3})\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)}{\sqrt[3]{a+}}\right.}{4c^{13/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{11/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{\sqrt{3}b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{11/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3}\log\left(\right.}{}$$

Mathematica [C] time = 0.0147145, size = 57, normalized size = 0.36

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{bx^2}{a}\right)}{8(cx)^{11/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -4/3, -1/3, -((b*x^2)/a)]
)/(8*(c*x)^(11/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{11}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(11/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

3.759 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{14/3}}$$

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))
```

Rubi [A] time = 0.0060914, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx=-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{14/3}}$$

Mathematica [A] time = 0.0101296, size = 26, normalized size = 0.93

$$-\frac{3x\left(a+bx^2\right)^{7/3}}{14a(cx)^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(7/3))/(14*a*(c*x)^(17/3))
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$-\frac{3\,x}{14\,a}\left(bx^2+a\right)^{\frac{7}{3}}(cx)^{-\frac{17}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(17/3),x)
```

```
[Out] -3/14*x*(b*x^2+a)^(7/3)/a/(c*x)^(17/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

Fricas [A] time = 2.1771, size = 104, normalized size = 3.71

$$-\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{14\,ac^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="fricas")
```

```
[Out] -3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(17/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

3.760 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{20/3}}$$

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*
a^2*c*(c*x)^(20/3))
```

Rubi [A] time = 0.0148183, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*
a^2*c*(c*x)^(20/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx&=-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{20/3}}-\frac{3\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{7a}\\&=-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{140a^2c(cx)^{20/3}}\end{aligned}$$

Mathematica [A] time = 0.0179027, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{7/3}\left(3bx^2-7a\right)}{140a^2c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(7/3)*(-7*a + 3*b*x^2))/(140*a^2*c^8*x^7)
```

Maple [A] time = 0.004, size = 31, normalized size = 0.5

$$-\frac{3x\left(-3bx^2+7a\right)}{140a^2}\left(bx^2+a\right)^{\frac{7}{3}}(cx)^{-\frac{23}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(23/3),x)
```

```
[Out] -3/140*x*(b*x^2+a)^(7/3)*(-3*b*x^2+7*a)/a^2/(c*x)^(23/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

Fricas [A] time = 2.11848, size = 132, normalized size = 2.32

$$\frac{3\left(3b^3x^6-ab^2x^4-11a^2bx^2-7a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{140a^2c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] 3/140*(3*b^3*x^6 - a*b^2*x^4 - 11*a^2*b*x^2 - 7*a^3)*(b*x^2 + a)^(1/3)*(c*x
)^(1/3)/(a^2*c^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(23/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

3.761 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}} \, dx$

Optimal. Leaf size=85

$$-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}$$

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(26/3)) + (9*(a + b*x^2)^(10/3))/(70*a
^2*c*(c*x)^(26/3)) - (27*(a + b*x^2)^(13/3))/(910*a^3*c*(c*x)^(26/3))
```

Rubi [A] time = 0.027316, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(26/3)) + (9*(a + b*x^2)^(10/3))/(70*a
^2*c*(c*x)^(26/3)) - (27*(a + b*x^2)^(13/3))/(910*a^3*c*(c*x)^(26/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}} \, dx &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}-\frac{6\int \frac{(a+bx^2)^{7/3}}{(cx)^{29/3}} \, dx}{7a}\\
&= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}+\frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}}+\frac{9\int \frac{(a+bx^2)^{10/3}}{(cx)^{29/3}} \, dx}{35a^2}\\
&= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}+\frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}
\end{aligned}$$

Mathematica [A] time = 0.0159202, size = 52, normalized size = 0.61

$$-\frac{3(a+bx^2)^{7/3}(35a^2-21abx^2+9b^2x^4)}{910a^3c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3)*(35*a^2 - 21*a*b*x^2 + 9*b^2*x^4))/(910*a^3*c^9*x^8*(
c*x)^(2/3))
```

Maple [A] time = 0.005, size = 42, normalized size = 0.5

$$-\frac{3x\left(9b^2x^4-21abx^2+35a^2\right)}{910a^3}\left(bx^2+a\right)^{\frac{7}{3}}(cx)^{-\frac{29}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(29/3),x)
```

```
[Out] -3/910*x*(b*x^2+a)^(7/3)*(9*b^2*x^4-21*a*b*x^2+35*a^2)/a^3/(c*x)^(29/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

Fricas [A] time = 2.08289, size = 161, normalized size = 1.89

$$-\frac{3\left(9b^4x^8-3ab^3x^6+2a^2b^2x^4+49a^3bx^2+35a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{910a^3c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] -3/910*(9*b^4*x^8 - 3*a*b^3*x^6 + 2*a^2*b^2*x^4 + 49*a^3*b*x^2 + 35*a^4)*(b
*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^10*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{29}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

3.762 $\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=479

$$\frac{8a^3c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{405\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{16a^3c}{}$$

```
[Out] (-16*a^3*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(405*b^2) + (16*a^2*c*(c*x)^(7/
3)*(a + b*x^2)^(1/3))/(945*b) + (8*a*(c*x)^(13/3)*(a + b*x^2)^(1/3))/(105*c
) + ((c*x)^(13/3)*(a + b*x^2)^(4/3))/(7*c) + (8*a^3*c^(7/3)*(c*x)^(1/3)*(a
+ b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(405*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)
^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1
/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.829501, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 321, 329, 241, 225}

$$-\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{8a^3c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2\right.\right.}{405\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```


```
[Out] (-16*a^3*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(405*b^2) + (16*a^2*c*(c*x)^(7/
3)*(a + b*x^2)^(1/3))/(945*b) + (8*a*(c*x)^(13/3)*(a + b*x^2)^(1/3))/(105*c
) + ((c*x)^(13/3)*(a + b*x^2)^(4/3))/(7*c) + (8*a^3*c^(7/3)*(c*x)^(1/3)*(a
+ b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(405*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)
^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1
/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (cx)^{10/3}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{1}{21}(8a)\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{1}{315}\left(16a^2\right)\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}-\frac{\left(16a^3c^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}dx}{405b}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

Mathematica [C] time = 0.070091, size = 102, normalized size = 0.21

$$\frac{c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(7a^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)-\left(7a-15bx^2\right)\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}\right)}{105b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-((7*a - 15*b*x^2)*(a + b*x^2)^2*(1 + (
b*x^2)/a)^(1/3)) + 7*a^3*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/
(105*b^2*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (cx)^{\frac{10}{3}}\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{10}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc^3x^5+ac^3x^3\right)\left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*c^3*x^5 + a*c^3*x^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{10}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

3.763 $\int (cx)^{4/3}\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=448

$$-\frac{8a^2\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+16a^2$$

```
[Out] (16*a^2*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b) + (8*a*(c*x)^(7/3)*(a + b*
x^2)^(1/3))/(45*c) + ((c*x)^(7/3)*(a + b*x^2)^(4/3))/(5*c) - (8*a^2*c^(1/3)
*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*
c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3]
)*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b*Sqrt[-(
(b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/(
(a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2))])
```

Rubi [A] time = 0.745642, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 321, 329, 241, 225}

$$-\frac{8a^2\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a}}{135b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (16*a^2*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b) + (8*a*(c*x)^(7/3)*(a + b*
x^2)^(1/3))/(45*c) + ((c*x)^(7/3)*(a + b*x^2)^(4/3))/(5*c) - (8*a^2*c^(1/3)
*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*
c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3]
)*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b*Sqrt[-(
(b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/(
(a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2))])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (cx)^{4/3}\left(a+bx^2\right)^{4/3}dx=\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}+\frac{1}{15}(8a)\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$$
$$=\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}+\frac{1}{135}\left(16a^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}dx$$
$$=\frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{\left(16a^3c^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}dx}{405b}$$
$$=\frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{\left(16a^3c\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\right.}{135b}$$
$$=\frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{\left(16a^3c\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}dx\right.}{135b\sqrt{\frac{a}{a+bx^2}}\sqrt{a+b}}$$
$$=\frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{8a^2\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\right.}{}$$

Mathematica [C] time = 0.0477848, size = 89, normalized size = 0.2

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{5b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^2
*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/(5*b*(1 + (b*x^2)/a)^(1/
3))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (cx)^{\frac{4}{3}}\left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bcx^3+acx\right)\left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^3 + a*c*x)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Sympy [C] time = 64.5754, size = 46, normalized size = 0.1

$$\frac{a^{\frac{4}{3}} c^{\frac{4}{3}} x^{\frac{7}{3}} \Gamma\left(\frac{7}{6}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{4}{3}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-4/3, 7/6), (13/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

3.764 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx$

Optimal. Leaf size=414

$$\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\left(a+\right.}{3c}$$

```
[Out] (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*c) + ((c*x)^(1/3)*(a + b*x^2)^(4/3))
/(3*c) + (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4
)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.694529, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {279, 329, 241, 225}

$$\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

```
[Out] (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*c) + ((c*x)^(1/3)*(a + b*x^2)^(4/3))
/(3*c) + (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4
)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c} + \frac{1}{9}(8a)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$$

$$= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c} + \frac{1}{27}\left(16a^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c} + \frac{\left(16a^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{9c}$$

$$= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c} + \frac{\left(16a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{9c\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c} + \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}{9\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+}{}\right.}}}$$

Mathematica [C] time = 0.0107466, size = 55, normalized size = 0.13

$$\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)])/(
(c*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c*x), x)
```

Sympy [C] time = 14.3585, size = 46, normalized size = 0.11

$$\frac{a^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(2/3),x)
```

```
[Out] a**(4/3)*x**(1/3)*gamma(1/6)*hyper((-4/3, 1/6), (7/6,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(2/3)*gamma(7/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

3.765 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx$

Optimal. Leaf size=414

$$\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{8b\sqrt[3]{cx}\sqrt[3]{a}}{5c^3}$$

```
[Out] (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(5*c^3) - (3*(a + b*x^2)^(4/3))/(5*c*(c
*x)^(5/3)) + (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^
(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + S
qrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3
) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 +
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(5*3^
(1/4)*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.697927, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 279, 329, 241, 225}

$$\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

```
[Out] (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(5*c^3) - (3*(a + b*x^2)^(4/3))/(5*c*(c
*x)^(5/3)) + (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^
(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + S
qrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3
) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 +
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(5*3^
(1/4)*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{8/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}dx}{5c^2}\\
&= \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(16ab)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}dx}{15c^2}\\
&= \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(16ab)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}dx,x,\sqrt[3]{cx}\right)}{5c^3}\\
&= \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(16ab)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5c^3\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\mathrm{co}\ldots\right.}{5\sqrt[4]{3}c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/\ldots}}{\sqrt[3]{a+bx^{\ldots}}}\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx\ldots}{\sqrt[3]{a+bx^2}}\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.0125823, size = 57, normalized size = 0.14

$$
-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{5}{6};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -5/6, 1/6, -((b*x^2)/a)])
/(5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$
\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{8}{3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

Sympy [C] time = 71.727, size = 32, normalized size = 0.08

$$\frac{b^{\frac{4}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(8/3),x)
```

```
[Out] b**(4/3)*x*hyper((-4/3, -1/2), (1/2,), a*exp_polar(I*pi)/(b*x**2))/c**(8/3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

3.766 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx$

Optimal. Leaf size=419

$$\frac{8\ 3^{3/4} b^2 \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{24b\sqrt[3]{\cdots}}{55c\cdots}$$

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(55*c^3*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(11*
c*(c*x)^(11/3)) + (8*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3
))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF
[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(
c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqr
t[3])/4]]/(55*a*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c
*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.70216, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {277, 329, 241, 225}

$$\frac{8\ 3^{3/4} b^2 \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(55*c^3*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(11*
c*(c*x)^(11/3)) + (8*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3
))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF
[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(
c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqr
t[3])/4]]/(55*a*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c
*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{14/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}} + \frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx}{11c^2}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}} + \frac{\left(16b^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{55c^4}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}} + \frac{\left(48b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{55c^5}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}} + \frac{\left(48b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55c^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}} + \frac{8\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\cdots}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\cdots}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\cdots\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\cdots}{\cdots}\right)}}}$$

Mathematica [C] time = 0.0135436, size = 57, normalized size = 0.14

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{11}{6},-\frac{4}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -4/3, -5/6, -((b*x^2)/a)
])/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{-\frac{14}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{1}{3}}}{c^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(14/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

3.767 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx$

Optimal. Leaf size=450

$$-\frac{24\, 3^{3/4} b^3 \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\mathrm{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{935a^2c^{23/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{48}{93}$$

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(187*c^3*(c*x)^(11/3)) - (48*b^2*(a + b*x^2)^(1/3
))/(935*a*c^5*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) - (2
4*3^(3/4)*b^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(935*a^2*
c^(23/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.766648, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 325, 329, 241, 225}

$$-\frac{24\, 3^{3/4} b^3 \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{935a^2c^{23/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{48b^2\sqrt[3]{a+}}{935ac^5(cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(187*c^3*(c*x)^(11/3)) - (48*b^2*(a + b*x^2)^(1/3
))/(935*a*c^5*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) - (2
4*3^(3/4)*b^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(935*a^2*
c^(23/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{20/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} + \frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx}{17c^2}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} + \frac{\left(16b^2\right)\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx}{187c^4}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{\left(48b^3\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{935ac^6}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{\left(144b^3\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{935ac^7}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{\left(144b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{935ac^7\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{24\,3^{3/4}b^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^2}{(a}\cdots}{\left(c^{2/3}\cdots\right.}}}{935a^2c^{23/3}\sqrt{-\cdots}}$$

Mathematica [C] time = 0.0132303, size = 57, normalized size = 0.13

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{17}{6},-\frac{4}{3};-\frac{11}{6};-\frac{bx^2}{a}\right)}{17(cx)^{20/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-17/6, -4/3, -11/6, -((b*x^2)/a
)])/(17*(c*x)^(20/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{-\frac{20}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{c^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^7*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(20/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

3.768 $\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=59

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] (3*a*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] time = 0.0192229, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{2/3}\left(a+bx^2\right)^{4/3} dx &= \frac{\left(a\sqrt[3]{a+bx^2}\right)\int (cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{4/3} dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= \frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0106084, size = 57, normalized size = 0.97

$$\frac{3ax(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((
b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int (cx)^{\frac{2}{3}} \left(b x^{2}+a\right)^{\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{\frac{4}{3}} (cx)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^{2}+a\right)^{\frac{4}{3}} (cx)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Sympy [C] time = 30.3366, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}} c^{\frac{2}{3}} x^{\frac{5}{3}} \Gamma\left(\frac{5}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-4/3, 5/6), (11/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{4}{3}} (cx)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

3.769 $\int \frac{\left(a+bx^2\right)^{4/3}}{\sqrt[3]{cx}}\,dx$

Optimal. Leaf size=59

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] (3*a*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x
^2)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] time = 0.0185268, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

```
[Out] (3*a*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x
^2)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{4/3}}{\sqrt[3]{cx}}\,dx &= \frac{\left(a\sqrt[3]{a+bx^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}}{\sqrt[3]{cx}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= \frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0119165, size = 57, normalized size = 0.97

$$\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^2)/a)])/(
2*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}\frac{1}{\sqrt[3]{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c*x), x)
```

Sympy [C] time = 11.6606, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(1/3),x)
```

```
[Out] a**(4/3)*x**(2/3)*gamma(1/3)*hyper((-4/3, 1/3), (4/3,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(1/3)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

3.770 $\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{4/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{3a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] (-3*a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])/(
c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] time = 0.0189494, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{3a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])/(
c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{4/3}}\,dx &= \frac{\left(a\sqrt[3]{a+bx^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}}{(cx)^{4/3}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= -\frac{3a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0126199, size = 55, normalized size = 0.96

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])
/((c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

Sympy [C] time = 14.4061, size = 49, normalized size = 0.86

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(4/3),x)
```

```
[Out] a**(4/3)*gamma(-1/6)*hyper((-4/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

3.771 $\int \frac{(cx)^{19/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=198

$$\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}+\frac{10a^3c^{19/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{11/3}}+\frac{20a^3c^{19/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}b^{11/3}}-\frac{2ac^3(cx)^{10/3}\sqrt[3]{a}}{9b^2}$$

```
[Out] (10*a^2*c^5*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b^3) - (2*a*c^3*(c*x)^(10/3)
*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*b) + (2
0*a^3*c^(19/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/
3)))/Sqrt[3]])/(27*Sqrt[3]*b^(11/3)) + (10*a^3*c^(19/3)*Log[b^(1/3)*(c*x)^(
2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(27*b^(11/3))
```

Rubi [A] time = 0.337918, antiderivative size = 278, normalized size of antiderivative = 1.4, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}+\frac{20a^3c^{19/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{81b^{11/3}}-\frac{10a^3c^{19/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{81b^{11/3}}+\frac{20a^3c^{19/3}\tan}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (10*a^2*c^5*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b^3) - (2*a*c^3*(c*x)^(10/3)
*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*b) + (2
0*a^3*c^(19/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(Sqrt[3]*c^(2/3))])/(27*Sqrt[3]*b^(11/3)) + (20*a^3*c^(19/3)*Log[c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(81*b^(11/3)) - (10*a^3*c^(19/3)
*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3)])/(81*b^(11/3))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(cx)^{19/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(8ac^2\right)\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= -\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{\left(20a^2c^4\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{27b^2}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(40a^3c^6\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{81b^3}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(40a^3c^5\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)}\right.}{27b^3}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^5\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)}\right.}{27b^3}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^5\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\right.}{27b^3}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^{17/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{}}{c^2}}\right.}{81b^{10/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx}{\sqrt[3]{a+b}}\right.}{81b^{11/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx}{\sqrt[3]{a+b}}\right.}{81b^{11/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^2}{c^{2/3}\sqrt[3]{a+}}}{\sqrt{3}}\right.}{27\sqrt{3}b^{11/3}}$$

Mathematica [C] time = 0.0276539, size = 87, normalized size = 0.44

$$\frac{c^5(cx)^{4/3}\left(-20a^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)+8a^2bx^2+20a^3-3ab^2x^4+9b^3x^6\right)}{54b^3\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^5*(c*x)^(4/3)*(20*a^3 + 8*a^2*b*x^2 - 3*a*b^2*x^4 + 9*b^3*x^6 - 20*a^3*H
ypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b*x^2)]))/(54*b^3*(a + b*x^2)^(2
/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (cx)^{\frac{19}{3}} \left(bx^2+a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{19}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(19/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{19}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

3.772 $\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}b^{8/3}}-\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}$$

```
[Out] (-5*a*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b^2) + (c*(c*x)^(10/3)*(a + b*
x^2)^(1/3))/(4*b) - (5*a^2*c^(13/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^
(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(6*Sqrt[3]*b^(8/3)) - (5*a^2*c^(13/3)*L
og[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(12*b^(8/3))
```

Rubi [A] time = 0.289967, antiderivative size = 247, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}}+\frac{5a^2c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{36b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{6\sqrt{3}b^{8/3}}-\frac{5ac^3(cx)^{4/3}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (-5*a*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b^2) + (c*(c*x)^(10/3)*(a + b*
x^2)^(1/3))/(4*b) - (5*a^2*c^(13/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(6*Sqrt[3]*b^(8/3)) - (5*a^2*c^(1
3/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(18*b^(8/3)) +
 (5*a^2*c^(13/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b
^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(36*b^(8/3))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{\left(5ac^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{6b}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^4\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{3b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^{11/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{7/3}} - \frac{\left(5a^2c^{11/3}\right)\cdots}{\cdots}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} - \frac{\left(5a^2c^{11/3}\right)\text{Subst}\left(\int \cdots\right)}{12\cdots}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} + \frac{5a^2c^{13/3}\log\left(c^{4/3}+\frac{b^{2/3}\cdots}{(a+\cdots}\cdots\right)}{36b^{8/3}\cdots}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}b^{8/3}} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(\cdots}{\sqrt[3]{a\cdots}}\right)}{18b^{8/3}}$$

Mathematica [C] time = 0.0206255, size = 76, normalized size = 0.46

$$\frac{c^3(cx)^{4/3}\left(5a^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)-5a^2-2abx^2+3b^2x^4\right)}{12b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Hypergeometric2F1[
2/3, 1, 5/3, (b*x^2)/(a + b*x^2)]))/(12*b^2*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (cx)^{\frac{13}{3}}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

3.773 $\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=131

$$\frac{ac^{7/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) + (a*c^(7/3)*ArcTan[(1 + (2*b^(1/3)
*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(5/3)) + (a
*c^(7/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(2*b^(5/3))
```

Rubi [A] time = 0.26161, antiderivative size = 209, normalized size of antiderivative = 1.6, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}}-\frac{ac^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{6b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{\sqrt{3}b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) + (a*c^(7/3)*ArcTan[(c^(2/3) + (2*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(Sqrt[3]*b^(5/3)
) + (a*c^(7/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(3*b
^(5/3)) - (a*c^(7/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3)
+ (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(6*b^(5/3))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(2ac^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{3b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(2ac)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{4/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)}{\sqrt[3]{a+}}\right.}{3b^{4/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{4/3}} - \frac{\left(ac^{7/}\right.}{}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{5/3}} - \frac{\left(ac^{7/3}\right)\text{Su}}{}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4}}{\left(a+bx^2\right)^{2}}\right.}{6b^{5/3}}$$

Mathematica [C] time = 0.0175675, size = 58, normalized size = 0.44

$$\frac{c(cx)^{4/3}\left(-a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)+a+bx^2\right)}{2b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2 - a*Hypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b
*x^2)]))/(2*b*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{3}}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 147.3, size = 44, normalized size = 0.34

$$\frac{c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(7/3)*x**(10/3)*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

3.774 $\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=106

$$-\frac{3\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2b^{2/3}}$$

```
[Out] -(Sqrt[3]*c^(1/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^
(1/3)))/Sqrt[3]])/(2*b^(2/3)) - (3*c^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3
)*(a + b*x^2)^(1/3)])/(4*b^(2/3))
```

Rubi [A] time = 0.244622, antiderivative size = 183, normalized size of antiderivative = 1.73, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}}+\frac{\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] -(Sqrt[3]*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(Sqrt[3]*c^(2/3))])/(2*b^(2/3)) - (c^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)
^(2/3))/(a + b*x^2)^(1/3)])/(2*b^(2/3)) + (c^(1/3)*Log[c^(4/3) + (b^(2/3)*(
c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3)])/(4*b^(2/3))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{3\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c} \\
&= \frac{3\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c} \\
&= \frac{3\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\sqrt[3]{c}} - \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\sqrt[3]{c}} \\
&= -\frac{\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\sqrt[3]{c}} + \frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt[3]{b}}{c^{2/3}}+\frac{2b^{2/3}x}{c^{4/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,d\right.}{4b^{2/3}} \\
&= -\frac{\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4b^{2/3}} + \frac{\left(3\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{2b^{2/3}} \\
&= -\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2b^{2/3}} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4b^{2/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0093289, size = 45, normalized size = 0.42

$$\frac{3x\sqrt[3]{cx}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)}{4\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(c*x)^(1/3)*Hypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b*x^2)])/(4*(a
 + b*x^2)^(2/3))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 1.68814, size = 44, normalized size = 0.42

$$\frac{\sqrt[3]{c}x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(1/3)*x**(4/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(2/3)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

3.775 $\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

Rubi [A] time = 0.0062157, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Mathematica [A] time = 0.0055331, size = 26, normalized size = 0.93

$$-\frac{3x\sqrt[3]{a+bx^2}}{2a(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3))/(2*a*(c*x)^(5/3))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$-\frac{3\,x}{2\,a}\sqrt[3]{bx^2+a}\,(cx)^{-\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/2*x*(b*x^2+a)^(1/3)/a/(c*x)^(5/3)
```

Maxima [A] time = 2.29757, size = 47, normalized size = 1.68

$$-\frac{3\left(bc^{\frac{1}{3}}x^3 + ac^{\frac{1}{3}}x\right)}{2\left(bx^2 + a\right)^{\frac{2}{3}}ac^2x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -3/2*(b*c^(1/3)*x^3 + a*c^(1/3)*x)/((b*x^2 + a)^(2/3)*a*c^2*x^(5/3))
```

Fricas [A] time = 2.01498, size = 62, normalized size = 2.21

$$-\frac{3\left(bx^2 + a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{2\,ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^2*x)
```

Sympy [A] time = 9.03116, size = 36, normalized size = 1.29

$$\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2} + 1}\Gamma\left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-1/3)/(2*a*c**(5/3)*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{2}{3}}(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)), x)
```

3.776 $\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(8/3)) + (9*(a + b*x^2)^(4/3))/(8*a^2*c
*(c*x)^(8/3))
```

Rubi [A] time = 0.0153678, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(8/3)) + (9*(a + b*x^2)^(4/3))/(8*a^2*c
*(c*x)^(8/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}-\frac{3\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx}{a}\\ &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}\end{aligned}$$

Mathematica [A] time = 0.0180092, size = 34, normalized size = 0.6

$$-\frac{3x\left(a-3bx^2\right)\sqrt[3]{a+bx^2}}{8a^2(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/(8*a^2*(c*x)^(11/3))
```

Maple [A] time = 0.003, size = 29, normalized size = 0.5

$$-\frac{3\,x\left(-3\,bx^2+a\right)}{8\,a^2}\sqrt[3]{bx^2+a}\,(cx)^{-\frac{11}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/8*x*(b*x^2+a)^(1/3)*(-3*b*x^2+a)/a^2/(c*x)^(11/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

Fricas [A] time = 2.04231, size = 85, normalized size = 1.49

$$\frac{3\left(3\,bx^2-a\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{8\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/8*(3*b*x^2 - a)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

3.777 $\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(4/3))/(4*a^2*
c*(c*x)^(14/3)) - (27*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(14/3))
```

Rubi [A] time = 0.0254695, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(4/3))/(4*a^2*
c*(c*x)^(14/3)) - (27*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(14/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}-\frac{6\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx}{a}\\ &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}+\frac{9\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx}{2a^2}\\ &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}\end{aligned}$$

Mathematica [A] time = 0.0238583, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(2a^2-3abx^2+9b^2x^4\right)}{28a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*
c^6*x^5)
```

Maple [A] time = 0.004, size = 42, normalized size = 0.5

$$-\frac{3x\left(9b^2x^4-3abx^2+2a^2\right)}{28a^3}\sqrt[3]{bx^2+a}\,(cx)^{-\frac{17}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/28*x*(b*x^2+a)^(1/3)*(9*b^2*x^4-3*a*b*x^2+2*a^2)/a^3/(c*x)^(17/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

Fricas [A] time = 2.03088, size = 112, normalized size = 1.32

$$-\frac{3\left(9b^2x^4-3abx^2+2a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{28a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/28*(9*b^2*x^4 - 3*a*b*x^2 + 2*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^
6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(17/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

3.778 $\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=113

$$\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(20/3)) + (27*(a + b*x^2)^(4/3))/(8*a^2
*c*(c*x)^(20/3)) - (81*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(20/3)) + (243*(a
 + b*x^2)^(10/3))/(280*a^4*c*(c*x)^(20/3))
```

Rubi [A] time = 0.0393894, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(20/3)) + (27*(a + b*x^2)^(4/3))/(8*a^2
*c*(c*x)^(20/3)) - (81*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(20/3)) + (243*(a
 + b*x^2)^(10/3))/(280*a^4*c*(c*x)^(20/3))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}-\frac{9\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx}{a}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}+\frac{27\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx}{2a^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}-\frac{81\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{14a^3}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}
\end{aligned}$$

Mathematica [A] time = 0.0291058, size = 63, normalized size = 0.56

$$\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(18a^2bx^2-14a^3-27ab^2x^4+81b^3x^6\right)}{280a^4c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-14*a^3 + 18*a^2*b*x^2 - 27*a*b^2*x^4 + 8
1*b^3*x^6))/(280*a^4*c^8*x^7)
```

Maple [A] time = 0.005, size = 53, normalized size = 0.5

$$-\frac{3\,x\left(-81\,b^3x^6+27\,ab^2x^4-18\,a^2bx^2+14\,a^3\right)}{280\,a^4}\sqrt[3]{bx^2+a}\,(cx)^{-\frac{23}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/280*x*(b*x^2+a)^(1/3)*(-81*b^3*x^6+27*a*b^2*x^4-18*a^2*b*x^2+14*a^3)/a^4
/(c*x)^(23/3)
```

Maxima [A] time = 1.38595, size = 86, normalized size = 0.76

$$\frac{3\left(81\,b^4x^9+54\,ab^3x^7-9\,a^2b^2x^5+4\,a^3bx^3-14\,a^4x\right)}{280\left(bx^2+a\right)^{\frac{2}{3}}a^4c^{\frac{23}{3}}x^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/280*(81*b^4*x^9 + 54*a*b^3*x^7 - 9*a^2*b^2*x^5 + 4*a^3*b*x^3 - 14*a^4*x)/
((b*x^2 + a)^(2/3)*a^4*c^(23/3)*x^(23/3))
```

Fricas [A] time = 1.54331, size = 139, normalized size = 1.23

$$\frac{3\left(81\,b^3x^6-27\,ab^2x^4+18\,a^2bx^2-14\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^4c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(81*b^3*x^6 - 27*a*b^2*x^4 + 18*a^2*b*x^2 - 14*a^3)*(b*x^2 + a)^(1/3)
*(c*x)^(1/3)/(a^4*c^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(23/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{2}{3}} (cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)), x)
```

3.779 $\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=421

$$\frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{18\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{7ac}{}$$

```
[Out] (-7*a*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(7/3)*(a + b*x^
2)^(1/3))/(3*b) + (7*a*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^
(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))
/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[A
rcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[
3])/4])/(18*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.688803, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {321, 329, 241, 225}

$$-\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2}+\frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2\right.\right.}{18\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```


```
[Out] (-7*a*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(7/3)*(a + b*x^
2)^(1/3))/(3*b) + (7*a*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^
(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))
/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[A
rcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[
3])/4])/(18*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{\left(7ac^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}\\
&= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}+\frac{\left(7a^2c^4\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{27b^2}\\
&= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}+\frac{\left(7a^2c^3\right)\operatorname{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{9b^2}\\
&= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}+\frac{\left(7a^2c^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{9b^2\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}+\frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}\cdots}{\sqrt[3]{a\cdots}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/\cdots}}{\sqrt[3]{a+bx^2}}\cdots\right.}}}{18\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2}\cdots\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\cdots\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.0300166, size = 87, normalized size = 0.21

$$\frac{c^3\sqrt[3]{cx}\left(7a^2\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)-7a^2-4abx^2+3b^2x^4\right)}{9b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^3*(c*x)^(1/3)*(-7*a^2 - 4*a*b*x^2 + 3*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(2
/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)]))/(9*b^2*(a + b*x^2)^(2/
3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (cx)^{\frac{10}{3}}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{10}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx)^{\frac{1}{3}}c^3x^3}{\left(bx^2+a\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/3)*c^3*x^3/(b*x^2 + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{10}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

3.780 $\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=388

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}(2\right.}{2\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3
)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2
/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)
)^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((
1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.639557, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {321, 329, 241, 225}

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```


```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3
)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2
/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)
)^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((
1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b} - \frac{\left(ac^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{3b}$$

$$= \frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{b\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= \frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b} - \frac{\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right.\right.}{2\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.0229326, size = 66, normalized size = 0.17

$$\frac{c\sqrt[3]{cx}\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)+a+bx^2\right)}{b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6,
2/3, 7/6, -((b*x^2)/a)]))/(b*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int (cx)^{\frac{4}{3}} \left(bx^2+a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{4}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{3}}cx}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/3)*c*x/(b*x^2 + a)^(2/3), x)
```

Sympy [C] time = 12.0156, size = 44, normalized size = 0.11

$$\frac{c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(4/3)*x**(7/3)*gamma(7/6)*hyper((2/3, 7/6), (13/6,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(13/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{4}{3}}}{\left(bx^2 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

3.781 $\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=364

$$\frac{3^{3/4}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] (3^(3/4)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(2*a*c^(5/3)*S
qrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.591276, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {329, 241, 225}

$$\frac{3^{3/4}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3^(3/4)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(2*a*c^(5/3)*S
qrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2))])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(cx)^{2/3} \left(a+bx^2\right)^{2/3}} \, dx = \frac{3 \operatorname{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}} \, dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{3 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}} \, dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{c\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= \frac{3^{3/4} \sqrt[3]{cx}\sqrt[3]{a+bx^2} \left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right) \sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}} F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right)|\frac{1}{4}\left(2\right.\right.}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.0140878, size = 54, normalized size = 0.15

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)])/
((c*x)^(2/3)*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{2}{3}} \left(b x^2 + a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^2 + a\right)^{\frac{2}{3}} (cx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}}{b c x^3 + a c x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] time = 2.27123, size = 31, normalized size = 0.09

$$-\frac{{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{a e^{i\pi}}{b x^2}\right)}{b^{\frac{2}{3}} c^{\frac{2}{3}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(2/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((1/2, 2/3), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(2/3)*c**(2/3)*x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

3.782 $\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=394

$$-\frac{3\,3^{3/4} b \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3}{5}$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(a +
b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(10*a^2*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(
2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3
)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.641643, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {325, 329, 241, 225}

$$-\frac{3\,3^{3/4} b \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a+bx}}{5ac(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(a +
b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(10*a^2*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(
2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3
)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{(3b)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{5ac^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{(9b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{5ac^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{(9b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5ac^3\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{3\,3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\cdots}{c^{2/3}-\cdots}\right.\right.\cdots}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.012114, size = 56, normalized size = 0.14

$$-\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},\frac{2}{3};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-5/6, 2/3, 1/6, -((b*x^2)/a)]
)/(5*(c*x)^(8/3)*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{8}{3}} \left(bx^2+a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}} (cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}}{bc^3x^5+ac^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] time = 116.922, size = 48, normalized size = 0.12

$$\frac{\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, \frac{2}{3} \\ \frac{1}{6} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}c^{\frac{8}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(8/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-5/6)*hyper((-5/6, 2/3), (1/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3
)*c**(8/3)*x**(5/3)*gamma(1/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

3.783 $\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=425

$$\frac{27\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+$$

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*a*c*(c*x)^(11/3)) + (27*b*(a + b*x^2)^(1/3))/(55
*a^2*c^3*(c*x)^(5/3)) + (27*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2
/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*El
lipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (
2 + Sqrt[3])/4])/(110*a^3*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1
+ Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rubi [A] time = 0.691575, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {325, 329, 241, 225}

$$\frac{27\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{27b\sqrt[3]{a+}}{55a^2c^3(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*a*c*(c*x)^(11/3)) + (27*b*(a + b*x^2)^(1/3))/(55
*a^2*c^3*(c*x)^(5/3)) + (27*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2
/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*El
lipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (
2 + Sqrt[3])/4])/(110*a^3*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1
+ Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}-\frac{(9b)\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx}{11ac^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{\left(27b^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{55a^2c^4}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{\left(81b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{55a^2c^5}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{\left(81b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55a^2c^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{27\;3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2}\cdots}{\sqrt[3]{\cdots}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)\cdots}{\sqrt[3]{a+bx^2}}\cdots\right)}}}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c\cdots\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\cdots\right)}}}
\end{aligned}
$$

Mathematica [C] time = 0.0129282, size = 56, normalized size = 0.13

$$-\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(-\frac{11}{6},\frac{2}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-11/6, 2/3, -5/6, -((b*x^2)/a
)])/(11*(c*x)^(14/3)*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{14}{3}}\left(bx^2+a\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{bc^5x^7+ac^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^5*x^7 + a*c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(14/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

3.784 $\int \frac{(cx)^{2/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{5/3}\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}$$

```
[Out] (3*(c*x)^(5/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(2/3))
```

Rubi [A] time = 0.0183455, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{5/3}\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(c*x)^(5/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(2/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{2/3}}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{(cx)^{2/3}}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\left(a+bx^2\right)^{2/3}}\\
&= \frac{3(cx)^{5/3}\left(1+\frac{bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0118511, size = 56, normalized size = 0.97

$$\frac{3x(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -(
(b*x^2)/a)])/(5*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (cx)^{\frac{2}{3}}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{2}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx)^{\frac{2}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Sympy [C] time = 2.03337, size = 44, normalized size = 0.76

$$\frac{c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(2/3)*x**(5/3)*gamma(5/6)*hyper((2/3, 5/6), (11/6,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(11/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{2}{3}}}{\left(bx^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

3.785 $\int \frac{1}{\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\left(a+bx^2\right)^{2/3}}$$

```
[Out] (3*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*
x^2)/a)])/(2*c*(a + b*x^2)^(2/3))
```

Rubi [A] time = 0.0189458, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*
x^2)/a)])/(2*c*(a + b*x^2)^(2/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{\sqrt[3]{cx}\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\left(a+bx^2\right)^{2/3}}\\
&= \frac{3(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\left(a+bx^2\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0104509, size = 56, normalized size = 0.97

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^2)/a)])/
(2*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{cx}}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}\left(cx\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{2}{3}}}{bcx^3+acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] time = 1.51933, size = 46, normalized size = 0.79

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}\sqrt[3]{c}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-1/3)*hyper((1/3, 2/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*c**(1/3)*x**(2/3)*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

3.786 $\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}$$

```
[Out] (-3*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)])/
(c*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Rubi [A] time = 0.0193969, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{3\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)])/
(c*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{(cx)^{4/3}\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\left(a+bx^2\right)^{2/3}}\\ &= -\frac{3\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.0109105, size = 54, normalized size = 0.96

$$-\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)]
)/((c*x)^(4/3)*(a + b*x^2)^(2/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{4}{3}}\left(bx^2+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{bc^2x^4+ac^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] time = 5.38381, size = 48, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{2}{3}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}c^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-1/6)*hyper((-1/6, 2/3), (5/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3
)*c**(4/3)*x**(1/3)*gamma(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

3.787 $\int x^4 \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=121

$$\frac{8a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}$$

```
[Out] (-4*a^2*x*(a + b*x^2)^(1/4))/(77*b^2) + (2*a*x^3*(a + b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a + b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0487949, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 233, 231}

$$-\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{8a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/4),x]
```


```
[Out] (-4*a^2*x*(a + b*x^2)^(1/4))/(77*b^2) + (2*a*x^3*(a + b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a + b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a + b*x^2)^(3/4))
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int x^4 \sqrt[4]{a+bx^2}\,dx = \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{1}{11}a\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} - \frac{\left(6a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx}{77b}$$

$$= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2} + \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{\left(4a^3\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2} + \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{\left(4a^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^2\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2} + \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{8a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0509471, size = 93, normalized size = 0.77

$$\frac{2x\sqrt[4]{a+bx^2}\left(\sqrt[4]{\frac{bx^2}{a}+1}\left(-6a^2+abx^2+7b^2x^4\right)+6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-6*a^2 + a*b*x^2 + 7*b^2*x^4
) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)]))/(77*b^2*(1 + (b
*x^2)/a)^(1/4))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x^4\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4*(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*x^4, x)
```

Sympy [C] time = 0.888968, size = 29, normalized size = 0.24

$$\frac{\sqrt[4]{a}x^5{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

3.788 $\int x^2 \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=97

$$-\frac{4a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}+\frac{2ax\sqrt[4]{a+bx^2}}{21b}$$

```
[Out] (2*a*x*(a + b*x^2)^(1/4))/(21*b) + (2*x^3*(a + b*x^2)^(1/4))/7 - (4*a^(5/2)
*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^(
3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0329361, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 233, 231}

$$-\frac{4a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}+\frac{2ax\sqrt[4]{a+bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*a*x*(a + b*x^2)^(1/4))/(21*b) + (2*x^3*(a + b*x^2)^(1/4))/7 - (4*a^(5/2)
*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^(
3/2)*(a + b*x^2)^(3/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sqrt[4]{a+bx^2}\,dx &= \frac{2}{7}x^3\sqrt[4]{a+bx^2}+\frac{1}{7}a\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{\left(2a^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{21b}\\
&= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{\left(2a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21b\left(a+bx^2\right)^{3/4}}\\
&= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{4a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0453466, size = 62, normalized size = 0.64

$$
\frac{2x\sqrt[4]{a+bx^2}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{\frac{bx^2}{a}+1}}+a+bx^2\right)}{7b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*(a + b*x^2 - (a*Hypergeometric2F1[-1/4, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)))/(7*b)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$
\int x^2\sqrt[4]{bx^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2*(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(bx^2+a\right)^{\frac{1}{4}}x^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*x^2, x)
```

Sympy [C] time = 0.765711, size = 29, normalized size = 0.3

$$\frac{\sqrt[4]{a}x^3{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

3.789 $\int \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

```
[Out] (2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0178442, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 233, 231}

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt[4]{a+bx^2}\,dx = \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{1}{3}a\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{\left(a\left(1+\frac{bx^2}{a}\right)\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0046881, size = 46, normalized size = 0.61

$$\frac{x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4),x]
```

```
[Out] (x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/4)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4),x)
```

```
[Out] int((b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 0.713273, size = 26, normalized size = 0.35

$$\sqrt[4]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4), x)
```

3.790 $\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

```
[Out] -((a + b*x^2)^(1/4)/x) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(3/4)
```

Rubi [A] time = 0.0205531, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 233, 231}

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/4)/x) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(3/4)
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a+bx^2}}{x} + \frac{1}{2}b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{a+bx^2}}{x} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{2\left(a+bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{x} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0090033, size = 49, normalized size = 0.68

$$-\frac{\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/4)))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^2,x)
```

```
[Out] int((b*x^2+a)^(1/4)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/x^2, x)
```

Sympy [C] time = 0.750253, size = 29, normalized size = 0.4

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**2,x)
```

```
[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^2, x)
```

3.791 $\int \frac{\sqrt[4]{a+bx^2}}{x^4} dx$

Optimal. Leaf size=99

$$-\frac{b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\sqrt[4]{a+bx^2}}{3x^3}$$

```
[Out] -(a + b*x^2)^(1/4)/(3*x^3) - (b*(a + b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 + b*x^2)^(3/4))
```

Rubi [A] time = 0.0310681, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 325, 233, 231}

$$-\frac{b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\sqrt[4]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^4,x]
```

```
[Out] -(a + b*x^2)^(1/4)/(3*x^3) - (b*(a + b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 + b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{b^2\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{12a}\\
&= -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\left(b^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a\left(a+bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0087689, size = 51, normalized size = 0.52

$$
-\frac{\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^4,x]
```

```
[Out] -((a + b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x^2)/a)])/(3*x
^3*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$
\int \frac{1}{x^4}\sqrt[4]{bx^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^4,x)
```

```
[Out] int((b*x^2+a)^(1/4)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
```

[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/4)/x^4, x)

Sympy [C] time = 0.913614, size = 34, normalized size = 0.34

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/4)/x**4,x)

[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)

3.792 $\int \frac{\sqrt[4]{a+bx^2}}{x^6} \, dx$

Optimal. Leaf size=123

$$\frac{b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

```
[Out] -(a + b*x^2)^(1/4)/(5*x^5) - (b*(a + b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a + b
*x^2)^(1/4))/(12*a^2*x) + (b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0463112, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 325, 233, 231}

$$\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^6,x]
```

```
[Out] -(a + b*x^2)^(1/4)/(5*x^5) - (b*(a + b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a + b
*x^2)^(1/4))/(12*a^2*x) + (b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a + b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{5x^5}+\frac{1}{10}b\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}-\frac{b^2\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx}{12a}\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^3\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{24a^2}\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{\left(b^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{24a^2\left(a+bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0091232, size = 51, normalized size = 0.41

$$-\frac{\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^6,x]
```

```
[Out] -((a + b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x^2)/a)])/(5*x
^5*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^6,x)
```

```
[Out] int((b*x^2+a)^(1/4)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/x^6, x)
```

Sympy [C] time = 1.18949, size = 34, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2}, -\frac{1}{4}\\ -\frac{3}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**6,x)
```

```
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```

3.793 $\int x^4 \sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=126

$$\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}$$

```
[Out] (-4*a^2*x*(a - b*x^2)^(1/4))/(77*b^2) - (2*a*x^3*(a - b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a - b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0458994, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {279, 321, 233, 232}

$$-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}+\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^2)^(1/4),x]
```


```
[Out] (-4*a^2*x*(a - b*x^2)^(1/4))/(77*b^2) - (2*a*x^3*(a - b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a - b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a - b*x^2)^(3/4))
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int x^4\sqrt[4]{a-bx^2}\,dx=\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{1}{11}a\int\frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx$$
$$=-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{\left(6a^2\right)\int\frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx}{77b}$$
$$=-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{\left(4a^3\right)\int\frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{77b^2}$$
$$=-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{\left(4a^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^2\left(a-bx^2\right)^{3/4}}$$
$$=-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0555366, size = 95, normalized size = 0.75

$$-\frac{2x\sqrt[4]{a-bx^2}\left(\sqrt[4]{1-\frac{bx^2}{a}}\left(6a^2+abx^2-7b^2x^4\right)-6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*((1 - (b*x^2)/a)^(1/4)*(6*a^2 + a*b*x^2 - 7*b^2*x^4
) - 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a]))/(77*b^2*(1 - (b*x^
2)/a)^(1/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^4\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4*(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*x^4, x)
```

Sympy [C] time = 0.916091, size = 31, normalized size = 0.25

$$\frac{\sqrt[4]{a}x^5 {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

3.794 $\int x^2 \sqrt[4]{a-bx^2}\, dx$

Optimal. Leaf size=101

$$\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}-\frac{2ax\sqrt[4]{a-bx^2}}{21b}$$

```
[Out] (-2*a*x*(a - b*x^2)^(1/4))/(21*b) + (2*x^3*(a - b*x^2)^(1/4))/7 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^
(3/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0329024, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {279, 321, 233, 232}

$$\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}-\frac{2ax\sqrt[4]{a-bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*a*x*(a - b*x^2)^(1/4))/(21*b) + (2*x^3*(a - b*x^2)^(1/4))/7 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^
(3/2)*(a - b*x^2)^(3/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int x^2\sqrt[4]{a-bx^2}\,dx=\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{1}{7}a\int\frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx$$
$$=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{\left(2a^2\right)\int\frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{21b}$$
$$=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{\left(2a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21b\left(a-bx^2\right)^{3/4}}$$
$$=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0509617, size = 64, normalized size = 0.63

$$\frac{2x\sqrt[4]{a-bx^2}\left(\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}-a+bx^2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-1/4, 1/2, 3/2, (
b*x^2)/a])/(1 - (b*x^2)/a)^(1/4)))/(7*b)
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x^2\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2*(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-bx^2+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*x^2, x)
```

Sympy [C] time = 0.787977, size = 31, normalized size = 0.31

$$\frac{\sqrt[4]{a}x^3{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-bx^2+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

3.795 $\int \sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

```
[Out] (2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0178637, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {195, 233, 232}

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \sqrt[4]{a-bx^2}\,dx = \frac{2}{3}x\sqrt[4]{a-bx^2} + \frac{1}{3}a\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx$$
$$= \frac{2}{3}x\sqrt[4]{a-bx^2} + \frac{\left(a\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3\left(a-bx^2\right)^{3/4}}$$
$$= \frac{2}{3}x\sqrt[4]{a-bx^2} + \frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0066705, size = 47, normalized size = 0.6

$$\frac{x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4),x]
```

```
[Out] (x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*
x^2)/a)^(1/4)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4),x)
```

```
[Out] int((-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 0.727895, size = 27, normalized size = 0.35

$$\sqrt[4]{a}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2 + a\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4), x)
```

3.796 $\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}$$

```
[Out] -((a - b*x^2)^(1/4)/x) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(3/4)
```

Rubi [A] time = 0.0198882, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {277, 233, 232}

$$-\frac{\sqrt[4]{a-bx^2}}{x}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^2,x]
```

```
[Out] -((a - b*x^2)^(1/4)/x) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(3/4)
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{1}{2}b\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{\left(b\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{2\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0090417, size = 50, normalized size = 0.66

$$-\frac{\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^2,x]
```

```
[Out] -(((a - b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (b*x^2)/a])/(x*(1 -
 (b*x^2)/a)^(1/4)))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^2,x)
```

```
[Out] int((-b*x^2+a)^(1/4)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^2, x)
```

Sympy [C] time = 0.775789, size = 31, normalized size = 0.41

$$-\frac{\sqrt[4]{a}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**2,x)
```

```
[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

3.797 $\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx$

Optimal. Leaf size=103

$$-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

```
[Out] -(a - b*x^2)^(1/4)/(3*x^3) + (b*(a - b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 -
(b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 - b*x^2)^(3/4))
```

Rubi [A] time = 0.0319621, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {277, 325, 233, 232}

$$-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^4,x]
```

```
[Out] -(a - b*x^2)^(1/4)/(3*x^3) + (b*(a - b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 -
(b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 - b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx &= -\frac{\sqrt[4]{a-bx^2}}{3x^3}-\frac{1}{6}b\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{b^2\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{12a}\\
&= -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\left(b^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a\left(a-bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.010061, size = 52, normalized size = 0.5

$$
-\frac{\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^4,x]
```

```
[Out] -((a - b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (b*x^2)/a])/(3*x^3*
(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$
\int \frac{1}{x^4}\sqrt[4]{-bx^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^4,x)
```

```
[Out] int((-b*x^2+a)^(1/4)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^4, x)
```

Sympy [C] time = 0.923354, size = 36, normalized size = 0.35

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**4,x)
```

```
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

3.798 $\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx$

Optimal. Leaf size=128

$$-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

```
[Out] -(a - b*x^2)^(1/4)/(5*x^5) + (b*(a - b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a - b
*x^2)^(1/4))/(12*a^2*x) - (b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0455706, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {277, 325, 233, 232}

$$\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^6,x]
```


```
[Out] -(a - b*x^2)^(1/4)/(5*x^5) + (b*(a - b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a - b
*x^2)^(1/4))/(12*a^2*x) - (b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a - b*x^2)^(3/4))
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx &= -\frac{\sqrt[4]{a-bx^2}}{5x^5}-\frac{1}{10}b\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}-\frac{b^2\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx}{12a}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^3\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{24a^2}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{\left(b^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{24a^2\left(a-bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0103868, size = 52, normalized size = 0.41

$$-\frac{\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^6,x]
```

```
[Out] -((a - b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (b*x^2)/a])/(5*x^5*
(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^6,x)
```

```
[Out] int((-b*x^2+a)^(1/4)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^6, x)
```

Sympy [C] time = 1.19517, size = 36, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2}, -\frac{1}{4}\\ -\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**6,x)
```

```
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

3.799 $\int x^4 \left(a+bx^2\right)^{3/4} dx$

Optimal. Leaf size=143

$$-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{8a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^2}}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}$$

```
[Out] (8*a^3*x)/(65*b^2*(a + b*x^2)^(1/4)) - (4*a^2*x*(a + b*x^2)^(3/4))/(65*b^2)
 + (2*a*x^3*(a + b*x^2)^(3/4))/(39*b) + (2*x^5*(a + b*x^2)^(3/4))/13 - (8*a
^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
65*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0514642, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 229, 227, 196}

$$-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{8a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^2}}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/4),x]
```

```
[Out] (8*a^3*x)/(65*b^2*(a + b*x^2)^(1/4)) - (4*a^2*x*(a + b*x^2)^(3/4))/(65*b^2)
 + (2*a*x^3*(a + b*x^2)^(3/4))/(39*b) + (2*x^5*(a + b*x^2)^(3/4))/13 - (8*a
^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
65*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \left(a+bx^2\right)^{3/4} dx &= \frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{1}{13}(3a)\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx \\
&= \frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\right)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{13b} \\
&= -\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{\left(4a^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{65b^2} \\
&= -\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{\left(4a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{65b^2\sqrt[4]{a+bx^2}} \\
&= \frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{\left(4a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int}{65b^2\sqrt[4]{a}} \\
&= \frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{8a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}}E}{65b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0522239, size = 93, normalized size = 0.65

$$
\frac{2x\left(a+bx^2\right)^{3/4}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-2a^2+abx^2+3b^2x^4\right)+2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{39b^2\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(3/4)*((1 + (b*x^2)/a)^(3/4)*(-2*a^2 + a*b*x^2 + 3*b^2*x^4
) + 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)]))/(39*b^2*(1 + (b
*x^2)/a)^(3/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$
\int x^4\left(bx^2+a\right)^{\frac{3}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4*(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{3}{4}} x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^4, x)
```

Sympy [C] time = 1.35365, size = 29, normalized size = 0.2

$$\frac{a^{\frac{3}{4}} x^5 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.800 $\int x^2 \left(a+bx^2\right)^{3/4} dx$

Optimal. Leaf size=119

$$\frac{4a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}$$

```
[Out] (-4*a^2*x)/(15*b*(a + b*x^2)^(1/4)) + (2*a*x*(a + b*x^2)^(3/4))/(15*b) + (2
*x^3*(a + b*x^2)^(3/4))/9 + (4*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0377391, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 229, 227, 196}

$$\frac{4a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/4),x]
```

```
[Out] (-4*a^2*x)/(15*b*(a + b*x^2)^(1/4)) + (2*a*x*(a + b*x^2)^(3/4))/(15*b) + (2
*x^3*(a + b*x^2)^(3/4))/9 + (4*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{3/4}dx &= \frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{1}{3}a\int\frac{x^2}{\sqrt[4]{a+bx^2}}dx\\
&= \frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\right)\int\frac{1}{\sqrt[4]{a+bx^2}}dx}{15b}\\
&= \frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{15b\sqrt[4]{a+bx^2}}\\
&= -\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{\left(2a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{15b\sqrt[4]{a+bx^2}}\\
&= -\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0505306, size = 62, normalized size = 0.52

$$\frac{2x\left(a+bx^2\right)^{3/4}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}+a+bx^2\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(3/4)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2*(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{3}{4}}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^2, x)
```

Sympy [C] time = 1.09418, size = 29, normalized size = 0.24

$$\frac{a^{\frac{3}{4}}x^3{}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.801 $\int \left(a+bx^2\right)^{3/4} \, dx$

Optimal. Leaf size=92

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

```
[Out] (6*a*x)/(5*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/5 - (6*a^(3/2)*(1 +
 (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(
a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0209407, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {195, 229, 227, 196}

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4),x]
```

```
[Out] (6*a*x)/(5*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/5 - (6*a^(3/2)*(1 +
 (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(
a + b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/4} dx = \frac{2}{5}x\left(a+bx^2\right)^{3/4} + \frac{1}{5}(3a)\int \frac{1}{\sqrt[4]{a+bx^2}}dx$$
$$= \frac{2}{5}x\left(a+bx^2\right)^{3/4} + \frac{\left(3a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{5\sqrt[4]{a+bx^2}}$$
$$= \frac{6ax}{5\sqrt[4]{a+bx^2}} + \frac{2}{5}x\left(a+bx^2\right)^{3/4} - \frac{\left(3a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{5\sqrt[4]{a+bx^2}}$$
$$= \frac{6ax}{5\sqrt[4]{a+bx^2}} + \frac{2}{5}x\left(a+bx^2\right)^{3/4} - \frac{6a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0047591, size = 46, normalized size = 0.5

$$\frac{x\left(a+bx^2\right)^{3/4} {}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4),x]
```

```
[Out] (x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(3/4)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4),x)
```

```
[Out] int((b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{3}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 0.896532, size = 26, normalized size = 0.28

$$a^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4), x)
```

3.802 $\int \frac{(a+bx^2)^{3/4}}{x^2} \, dx$

Optimal. Leaf size=88

$$\frac{3bx}{\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{x}-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

```
[Out] (3*b*x)/(a + b*x^2)^(1/4) - (a + b*x^2)^(3/4)/x - (3*Sqrt[a]*Sqrt[b]*(1 + (
b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(1
/4)
```

Rubi [A] time = 0.0238425, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 229, 227, 196}

$$\frac{3bx}{\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{x}-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^2,x]
```

```
[Out] (3*b*x)/(a + b*x^2)^(1/4) - (a + b*x^2)^(3/4)/x - (3*Sqrt[a]*Sqrt[b]*(1 + (
b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(1
/4)
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = -\frac{(a+bx^2)^{3/4}}{x} + \frac{1}{2}(3b)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$$
$$= -\frac{(a+bx^2)^{3/4}}{x} + \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0082458, size = 49, normalized size = 0.56

$$-\frac{(a+bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(3/4)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^2,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^2, x)
```

Sympy [C] time = 1.02385, size = 29, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**2,x)
```

```
[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.803 $\int \frac{(a+bx^2)^{3/4}}{x^4} dx$

Optimal. Leaf size=121

$$\frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^2}} - \frac{b\left(a+bx^2\right)^{3/4}}{2ax} - \frac{\left(a+bx^2\right)^{3/4}}{3x^3}$$

```
[Out] (b^2*x)/(2*a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*x^3) - (b*(a + b*x^2
)^(3/4))/(2*a*x) - (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0378355, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 229, 227, 196}

$$\frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^2}} - \frac{b\left(a+bx^2\right)^{3/4}}{2ax} - \frac{\left(a+bx^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^4,x]
```

```
[Out] (b^2*x)/(2*a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*x^3) - (b*(a + b*x^2
)^(3/4))/(2*a*x) - (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a + b*x^2)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/4}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{3/4}}{3x^3}+\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx\\
&= -\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}+\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a}\\
&= -\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}+\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a\sqrt[4]{a+bx^2}}\\
&= \frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}-\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a\sqrt[4]{a+bx^2}}\\
&= \frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}-\frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0103978, size = 51, normalized size = 0.42

$$
-\frac{\left(a+bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^4,x]
```

```
[Out] -((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^2)/a)])/(3*x
^3*(1 + (b*x^2)/a)^(3/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$
\int \frac{1}{x^4}\left(bx^2+a\right)^{\frac{3}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^4,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^4, x)
```

Sympy [C] time = 1.14974, size = 34, normalized size = 0.28

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**4,x)
```

```
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.804 $\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx$

Optimal. Leaf size=145

$$-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}$$

```
[Out] (-3*b^3*x)/(20*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*x^5) - (b*(a +
 b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a + b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a
^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0505805, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 229, 227, 196}

$$-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^6,x]
```

```
[Out] (-3*b^3*x)/(20*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*x^5) - (b*(a +
 b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a + b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a
^(3/2)*(a + b*x^2)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/4}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}+\frac{1}{10}(3b)\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}-\frac{\left(3b^2\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{20a} \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}-\frac{\left(3b^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{40a^2} \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}-\frac{\left(3b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{40a^2\sqrt[4]{a+bx^2}} \\
&= -\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{\left(3b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{40a^2\sqrt[4]{a+bx^2}} \\
&= -\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{20a^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0089544, size = 51, normalized size = 0.35

$$
-\frac{\left(a+bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^6,x]
```

```
[Out] -((a + b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x^2)/a)])/(5*x
^5*(1 + (b*x^2)/a)^(3/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$
\int \frac{1}{x^6}\left(bx^2+a\right)^{\frac{3}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^6,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^6, x)
```

Sympy [C] time = 1.40899, size = 34, normalized size = 0.23

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**6,x)
```

```
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.805 $\int x^4 \left(a-bx^2\right)^{3/4} dx$

Optimal. Leaf size=126

$$-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}$$

```
[Out] (-4*a^2*x*(a - b*x^2)^(3/4))/(65*b^2) - (2*a*x^3*(a - b*x^2)^(3/4))/(39*b)
+ (2*x^5*(a - b*x^2)^(3/4))/13 + (8*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^(5/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0445159, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {279, 321, 229, 228}

$$-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^2)^(3/4),x]
```

```
[Out] (-4*a^2*x*(a - b*x^2)^(3/4))/(65*b^2) - (2*a*x^3*(a - b*x^2)^(3/4))/(39*b)
+ (2*x^5*(a - b*x^2)^(3/4))/13 + (8*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^(5/2)*(a - b*x^2)^(1/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a-bx^2\right)^{3/4}dx &= \frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{1}{13}(3a)\int\frac{x^4}{\sqrt[4]{a-bx^2}}dx\\
&=-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{\left(2a^2\right)\int\frac{x^2}{\sqrt[4]{a-bx^2}}dx}{13b}\\
&=-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{\left(4a^3\right)\int\frac{1}{\sqrt[4]{a-bx^2}}dx}{65b^2}\\
&=-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{\left(4a^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{65b^2\sqrt[4]{a-bx^2}}\\
&=-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right|}{65b^{5/2}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0581087, size = 95, normalized size = 0.75

$$-\frac{2x\left(a-bx^2\right)^{3/4}\left(\left(1-\frac{bx^2}{a}\right)^{3/4}\left(2a^2+abx^2-3b^2x^4\right)-2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{39b^2\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4)*((1 - (b*x^2)/a)^(3/4)*(2*a^2 + a*b*x^2 - 3*b^2*x^4
) - 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a]))/(39*b^2*(1 - (b*x^
2)/a)^(3/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int x^4\left(-bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4*(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-bx^2+a\right)^{\frac{3}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{3}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^4, x)
```

Sympy [C] time = 1.39667, size = 31, normalized size = 0.25

$$\frac{a^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.806 $\int x^2 \left(a-bx^2\right)^{3/4} dx$

Optimal. Leaf size=101

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}$$

```
[Out] (-2*a*x*(a - b*x^2)^(3/4))/(15*b) + (2*x^3*(a - b*x^2)^(3/4))/9 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^
(3/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0329896, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {279, 321, 229, 228}

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*a*x*(a - b*x^2)^(3/4))/(15*b) + (2*x^3*(a - b*x^2)^(3/4))/9 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^
(3/2)*(a - b*x^2)^(1/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a-bx^2\right)^{3/4}dx&=\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{1}{3}a\int\frac{x^2}{\sqrt[4]{a-bx^2}}dx\\&=-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{\left(2a^2\right)\int\frac{1}{\sqrt[4]{a-bx^2}}dx}{15b}\\&=-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{\left(2a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{15b\sqrt[4]{a-bx^2}}\\&=-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0519357, size = 64, normalized size = 0.63

$$\frac{2x\left(a-bx^2\right)^{3/4}\left(\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}-a+bx^2\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(3/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-3/4, 1/2, 3/2, (
b*x^2)/a])/(1 - (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^2\left(-bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2*(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-bx^2+a\right)^{\frac{3}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{3}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^2, x)
```

Sympy [C] time = 1.11439, size = 31, normalized size = 0.31

$$\frac{a^{\frac{3}{4}}x^3{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.807 $\int \left(a-bx^2\right)^{3/4} dx$

Optimal. Leaf size=78

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

```
[Out] (2*x*(a - b*x^2)^(3/4))/5 + (6*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0187742, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {195, 229, 228}

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(3/4))/5 + (6*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \left(a-bx^2\right)^{3/4} dx = \frac{2}{5}x\left(a-bx^2\right)^{3/4} + \frac{1}{5}(3a)\int \frac{1}{\sqrt[4]{a-bx^2}}dx$$
$$= \frac{2}{5}x\left(a-bx^2\right)^{3/4} + \frac{\left(3a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5\sqrt[4]{a-bx^2}}$$
$$= \frac{2}{5}x\left(a-bx^2\right)^{3/4} + \frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0062014, size = 47, normalized size = 0.6

$$\frac{x\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4),x]
```

```
[Out] (x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*
x^2)/a)^(3/4)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4),x)
```

```
[Out] int((-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 0.886928, size = 27, normalized size = 0.35

$$a^{\frac{3}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{b x^{2} e^{2 i \pi}}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-b x^{2} + a\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4), x)
```

3.808 $\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx$

Optimal. Leaf size=76

$$-\frac{(a-bx^2)^{3/4}}{x}-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

```
[Out] -((a - b*x^2)^(3/4)/x) - (3*Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(1/4)
```

Rubi [A] time = 0.0204751, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {277, 229, 228}

$$-\frac{(a-bx^2)^{3/4}}{x}-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^2,x]
```

```
[Out] -((a - b*x^2)^(3/4)/x) - (3*Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(1/4)
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{3/4}}{x^2}\,dx = -\frac{\left(a-bx^2\right)^{3/4}}{x} - \frac{1}{2}(3b)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{x} - \frac{\left(3b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0087876, size = 50, normalized size = 0.66

$$-\frac{\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^2,x]
```

```
[Out] -(((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (b*x^2)/a])/(x*(1 -
 (b*x^2)/a)^(3/4)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(-bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^2,x)
```

```
[Out] int((-b*x^2+a)^(3/4)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/x^2, x)
```

Sympy [C] time = 1.03792, size = 31, normalized size = 0.41

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**2,x)
```

```
[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.809 $\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=103

$$\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a-bx^2}}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}-\frac{\left(a-bx^2\right)^{3/4}}{3x^3}$$

```
[Out] -(a - b*x^2)^(3/4)/(3*x^3) + (b*(a - b*x^2)^(3/4))/(2*a*x) + (b^(3/2)*(1 -
(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a
 - b*x^2)^(1/4))
```

Rubi [A] time = 0.0317913, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {277, 325, 229, 228}

$$\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a-bx^2}}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}-\frac{\left(a-bx^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^4,x]
```


```
[Out] -(a - b*x^2)^(3/4)/(3*x^3) + (b*(a - b*x^2)^(3/4))/(2*a*x) + (b^(3/2)*(1 -
(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a
 - b*x^2)^(1/4))
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^2\right)^{3/4}}{x^4}\,dx &= -\frac{\left(a-bx^2\right)^{3/4}}{3x^3}-\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{3x^3}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}+\frac{b^2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{3x^3}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}+\frac{\left(b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a\sqrt[4]{a-bx^2}}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{3x^3}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}+\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0094421, size = 52, normalized size = 0.5

$$-\frac{\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^4,x]
```

```
[Out] -((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (b*x^2)/a])/(3*x^3*
(1 - (b*x^2)/a)^(3/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(-bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^4,x)
```

```
[Out] int((-b*x^2+a)^(3/4)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/x^4, x)
```

Sympy [C] time = 1.18824, size = 36, normalized size = 0.35

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2}, -\frac{3}{4}\\ -\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**4,x)
```

```
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.810 $\int \frac{(a-bx^2)^{3/4}}{x^6} dx$

Optimal. Leaf size=128

$$\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}+\frac{b(a-bx^2)^{3/4}}{10ax^3}-\frac{(a-bx^2)^{3/4}}{5x^5}$$

```
[Out] -(a - b*x^2)^(3/4)/(5*x^5) + (b*(a - b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a -
 b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(3/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0450154, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {277, 325, 229, 228}

$$\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}+\frac{b(a-bx^2)^{3/4}}{10ax^3}-\frac{(a-bx^2)^{3/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^6,x]
```

```
[Out] -(a - b*x^2)^(3/4)/(5*x^5) + (b*(a - b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a -
 b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(3/2)*(a - b*x^2)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx &= -\frac{(a-bx^2)^{3/4}}{5x^5} - \frac{1}{10}(3b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5} + \frac{b(a-bx^2)^{3/4}}{10ax^3} - \frac{(3b^2)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a} \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5} + \frac{b(a-bx^2)^{3/4}}{10ax^3} + \frac{3b^2(a-bx^2)^{3/4}}{20a^2x} + \frac{(3b^3)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^2} \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5} + \frac{b(a-bx^2)^{3/4}}{10ax^3} + \frac{3b^2(a-bx^2)^{3/4}}{20a^2x} + \frac{\left(3b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{40a^2\sqrt[4]{a-bx^2}} \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5} + \frac{b(a-bx^2)^{3/4}}{10ax^3} + \frac{3b^2(a-bx^2)^{3/4}}{20a^2x} + \frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0094402, size = 52, normalized size = 0.41

$$-\frac{(a-bx^2)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^6,x]
```

```
[Out] -((a - b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (b*x^2)/a])/(5*x^5*
(1 - (b*x^2)/a)^(3/4))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(-bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^6,x)
```

```
[Out] int((-b*x^2+a)^(3/4)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/x^6, x)
```

Sympy [C] time = 1.45972, size = 36, normalized size = 0.28

$$-\frac{a^{\frac{3}{4}}{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},-\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**6,x)
```

```
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.811 $\int \left(a+bx^2\right)^{5/4} dx$

Optimal. Leaf size=92

$$\frac{10a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

```
[Out] (10*a*x*(a + b*x^2)^(1/4))/21 + (2*x*(a + b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0242793, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 233, 231}

$$\frac{10a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4),x]
```

```
[Out] (10*a*x*(a + b*x^2)^(1/4))/21 + (2*x*(a + b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a + b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/4} dx = \frac{2}{7}x\left(a+bx^2\right)^{5/4} + \frac{1}{7}(5a)\int \sqrt[4]{a+bx^2}\, dx$$
$$= \frac{10}{21}ax\sqrt[4]{a+bx^2} + \frac{2}{7}x\left(a+bx^2\right)^{5/4} + \frac{1}{21}\left(5a^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}} dx$$
$$= \frac{10}{21}ax\sqrt[4]{a+bx^2} + \frac{2}{7}x\left(a+bx^2\right)^{5/4} + \frac{\left(5a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}} dx}{21\left(a+bx^2\right)^{3/4}}$$
$$= \frac{10}{21}ax\sqrt[4]{a+bx^2} + \frac{2}{7}x\left(a+bx^2\right)^{5/4} + \frac{10a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0085423, size = 47, normalized size = 0.51

$$\frac{ax\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/4),x]
```

```
[Out] (a*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(1/4)
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4),x)
```

```
[Out] int((b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{5}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/4), x)
```

Sympy [C] time = 1.33677, size = 26, normalized size = 0.28

$$a^{\frac{5}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/4),x)
```

```
[Out] a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/4), x)
```

3.812 $\int \left(a-bx^2\right)^{5/4} dx$

Optimal. Leaf size=96

$$\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

```
[Out] (10*a*x*(a - b*x^2)^(1/4))/21 + (2*x*(a - b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0251112, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {195, 233, 232}

$$\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/4),x]
```

```
[Out] (10*a*x*(a - b*x^2)^(1/4))/21 + (2*x*(a - b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a - b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \left(a-bx^2\right)^{5/4} dx &= \frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{1}{7}(5a)\int \sqrt[4]{a-bx^2}\,dx \\
&= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{1}{21}\left(5a^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx \\
&= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{\left(5a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{21\left(a-bx^2\right)^{3/4}} \\
&= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0095724, size = 48, normalized size = 0.5

$$
\frac{ax\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/4),x]
```

```
[Out] (a*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/4)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$
\int \left(-bx^2+a\right)^{\frac{5}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/4),x)
```

```
[Out] int((-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(-bx^2+a\right)^{\frac{5}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{5}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(5/4), x)
```

Sympy [C] time = 1.3201, size = 27, normalized size = 0.28

$$a^{\frac{5}{4}}x\,{}_2F_1\left(\begin{matrix} -\frac{5}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/4),x)
```

```
[Out] a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/4), x)
```

3.813 $\int \left(a+bx^2\right)^{7/4}\,dx$

Optimal. Leaf size=111

$$\frac{14a^2x}{15\sqrt[4]{a+bx^2}}-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}$$

```
[Out] (14*a^2*x)/(15*(a + b*x^2)^(1/4)) + (14*a*x*(a + b*x^2)^(3/4))/45 + (2*x*(a
 + b*x^2)^(7/4))/9 - (14*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0291233, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {195, 229, 227, 196}

$$\frac{14a^2x}{15\sqrt[4]{a+bx^2}}-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4),x]
```

```
[Out] (14*a^2*x)/(15*(a + b*x^2)^(1/4)) + (14*a*x*(a + b*x^2)^(3/4))/45 + (2*x*(a
 + b*x^2)^(7/4))/9 - (14*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{7/4} dx &= \frac{2}{9}x\left(a+bx^2\right)^{7/4}+\frac{1}{9}(7a)\int \left(a+bx^2\right)^{3/4} dx \\
&= \frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}+\frac{1}{15}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a+bx^2}}dx \\
&= \frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}+\frac{\left(7a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{15\sqrt[4]{a+bx^2}} \\
&= \frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}-\frac{\left(7a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{15\sqrt[4]{a+bx^2}} \\
&= \frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}-\frac{14a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0071599, size = 47, normalized size = 0.42

$$
\frac{ax\left(a+bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/4),x]
```

```
[Out] (a*x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-7/4, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(3/4)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$
\int \left(bx^2+a\right)^{\frac{7}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4),x)
```

```
[Out] int((b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(bx^2+a\right)^{\frac{7}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{7}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(7/4), x)
```

Sympy [C] time = 2.20992, size = 26, normalized size = 0.23

$$a^{\frac{7}{4}}x\,{}_2F_1\left(\begin{matrix} -\frac{7}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4),x)
```

```
[Out] a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.814 $\int \left(a-bx^2\right)^{7/4} dx$

Optimal. Leaf size=96

$$\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}$$

```
[Out] (14*a*x*(a - b*x^2)^(3/4))/45 + (2*x*(a - b*x^2)^(7/4))/9 + (14*a^(5/2)*(1
- (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]
*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0256642, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {195, 229, 228}

$$\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(7/4),x]
```

```
[Out] (14*a*x*(a - b*x^2)^(3/4))/45 + (2*x*(a - b*x^2)^(7/4))/9 + (14*a^(5/2)*(1
- (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]
*(a - b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \left(a-bx^2\right)^{7/4} dx = \frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{1}{9}(7a)\int \left(a-bx^2\right)^{3/4} dx$$
$$= \frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{1}{15}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$$
$$= \frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{\left(7a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{15\sqrt[4]{a-bx^2}}$$
$$= \frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0076569, size = 48, normalized size = 0.5

$$\frac{ax\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(7/4),x]
```

```
[Out] (a*x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-7/4, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/4)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(7/4),x)
```

```
[Out] int((-b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{7}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(7/4), x)
```

Sympy [C] time = 2.26042, size = 27, normalized size = 0.28

$$a^{\frac{7}{4}}x_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(7/4),x)
```

```
[Out] a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.815 $\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3}-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{16a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^2}}-\frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2}+\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}$$

```
[Out] (-16*a^3*x)/(39*b^3*(a + b*x^2)^(1/4)) + (8*a^2*x*(a + b*x^2)^(3/4))/(39*b^
3) - (20*a*x^3*(a + b*x^2)^(3/4))/(117*b^2) + (2*x^5*(a + b*x^2)^(3/4))/(13
*b) + (16*a^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(39*b^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0518924, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 229, 227, 196}

$$\frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3}-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{16a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^2}}-\frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2}+\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(1/4),x]
```


```
[Out] (-16*a^3*x)/(39*b^3*(a + b*x^2)^(1/4)) + (8*a^2*x*(a + b*x^2)^(3/4))/(39*b^
3) - (20*a*x^3*(a + b*x^2)^(3/4))/(117*b^2) + (2*x^5*(a + b*x^2)^(3/4))/(13
*b) + (16*a^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(39*b^(7/2)*(a + b*x^2)^(1/4))
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} - \frac{(10a)\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx}{13b}$$

$$= -\frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} + \frac{\left(20a^2\right)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{39b^2}$$

$$= \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} - \frac{\left(8a^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{39b^3}$$

$$= \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} - \frac{\left(8a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{39b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}} + \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} + \frac{\left(8a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int -}{39b^3\sqrt[4]{a+b}}$$

$$= -\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}} + \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} + \frac{16a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\right.}{39b^{7/2}\sqrt[4]{a}}$$

Mathematica [C] time = 0.0328298, size = 90, normalized size = 0.62

$$\frac{2\left(-12a^3x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+2a^2bx^3+12a^3x-ab^2x^5+9b^3x^7\right)}{117b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*(12*a^3*x + 2*a^2*b*x^3 - a*b^2*x^5 + 9*b^3*x^7 - 12*a^3*x*(1 + (b*x^2)/
a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(117*b^3*(a + b*x
^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^6\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^6/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 0.917057, size = 27, normalized size = 0.18

$$\frac{x^7 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/4),x)
```

```
[Out] x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/4), x)
```

3.816 $\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}-\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2}+\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}$$

```
[Out] (8*a^2*x)/(15*b^2*(a + b*x^2)^(1/4)) - (4*a*x*(a + b*x^2)^(3/4))/(15*b^2) +
 (2*x^3*(a + b*x^2)^(3/4))/(9*b) - (8*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0362143, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 229, 227, 196}

$$\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}-\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2}+\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/4),x]
```

```
[Out] (8*a^2*x)/(15*b^2*(a + b*x^2)^(1/4)) - (4*a*x*(a + b*x^2)^(3/4))/(15*b^2) +
 (2*x^3*(a + b*x^2)^(3/4))/(9*b) - (8*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} - \frac{(2a)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{3b}$$
$$= -\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{15b^2}$$
$$= -\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{15b^2\sqrt[4]{a+bx^2}}$$
$$= \frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}} - \frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} - \frac{\left(4a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{15b^2\sqrt[4]{a+bx^2}}$$
$$= \frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}} - \frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} - \frac{8a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0209872, size = 79, normalized size = 0.65

$$\frac{2\left(6a^2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-6a^2x-abx^3+5b^2x^5\right)}{45b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*(-6*a^2*x - a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(45*b^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^4\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 0.765393, size = 27, normalized size = 0.22

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/4),x)
```

```
[Out] x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

3.817 $\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=98

$$\frac{4a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x\left(a+bx^2\right)^{3/4}}{5b}$$

```
[Out] (-4*a*x)/(5*b*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/(5*b) + (4*a^(3/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^
(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0239423, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 229, 227, 196}

$$\frac{4a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x\left(a+bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/4),x]
```

```
[Out] (-4*a*x)/(5*b*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/(5*b) + (4*a^(3/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^
(3/2)*(a + b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x\left(a+bx^2\right)^{3/4}}{5b} - \frac{(2a)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5b}$$
$$= \frac{2x\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(2a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5b\sqrt[4]{a+bx^2}}$$
$$= -\frac{4ax}{5b\sqrt[4]{a+bx^2}} + \frac{2x\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(2a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5b\sqrt[4]{a+bx^2}}$$
$$= -\frac{4ax}{5b\sqrt[4]{a+bx^2}} + \frac{2x\left(a+bx^2\right)^{3/4}}{5b} + \frac{4a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0155101, size = 62, normalized size = 0.63

$$\frac{2x\left(-a\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
-((b*x^2)/a)]))/(5*b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 0.691345, size = 27, normalized size = 0.28

$$\frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/4),x)
```

```
[Out] x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

3.818 $\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

```
[Out] (2*x)/(a + b*x^2)^(1/4) - (2*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0138555, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {229, 227, 196}

$$\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/4),x]
```

```
[Out] (2*x)/(a + b*x^2)^(1/4) - (2*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0060906, size = 46, normalized size = 0.65

$$\frac{x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/4),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/4)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-1/4), x)
```

Sympy [C] time = 0.678049, size = 24, normalized size = 0.34

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4),x)
```

```
[Out] x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/4), x)
```

3.819 $\int \frac{1}{x^2 \sqrt[4]{a+bx^2}} \, dx$

Optimal. Leaf size=93

$$\frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

```
[Out] (b*x)/(a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(a*x) - (Sqrt[b]*(1 + (b*x^
2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^
2)^(1/4))
```

Rubi [A] time = 0.0252727, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 229, 227, 196}

$$\frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/4)),x]
```

```
[Out] (b*x)/(a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(a*x) - (Sqrt[b]*(1 + (b*x^
2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^
2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{3/4}}{ax} + \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{2a}$$
$$= -\frac{\left(a+bx^2\right)^{3/4}}{ax} + \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2a\sqrt[4]{a+bx^2}}$$
$$= \frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{ax} - \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a\sqrt[4]{a+bx^2}}$$
$$= \frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0084933, size = 49, normalized size = 0.53

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/4)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/4)))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^4 + a*x^2), x)
```

Sympy [C] time = 0.761748, size = 27, normalized size = 0.29

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/4)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

3.820 $\int \frac{1}{x^4 \sqrt[4]{a+bx^2}} dx$

Optimal. Leaf size=124

$$-\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}+\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}+\frac{b\left(a+bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

```
[Out] -(b^2*x)/(2*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*a*x^3) + (b*(a +
b*x^2)^(3/4))/(2*a^2*x) + (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0361695, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 229, 227, 196}

$$-\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}+\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}+\frac{b\left(a+bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/4)),x]
```

```
[Out] -(b^2*x)/(2*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*a*x^3) + (b*(a +
b*x^2)^(3/4))/(2*a^2*x) + (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(a + b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{3/4}}{3ax^3} - \frac{b\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{2a}$$
$$= -\frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} - \frac{b^2\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a^2}$$
$$= -\frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} - \frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} + \frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} + \frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0085505, size = 51, normalized size = 0.41

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/4)),x]

[Out] -((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x^2)/a)])/(
3*x^3*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/4),x)

[Out] int(1/x^4/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^6 + a*x^4), x)
```

Sympy [C] time = 0.921953, size = 32, normalized size = 0.26

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

3.821 $\int \frac{1}{x^6 \sqrt[4]{a+bx^2}} dx$

Optimal. Leaf size=148

$$\frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} - \frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

```
[Out] (7*b^3*x)/(20*a^3*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*a*x^5) + (7*b*(
a + b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^2*(a + b*x^2)^(3/4))/(20*a^3*x) - (7*
b^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(20*a^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0508354, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 229, 227, 196}

$$\frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} - \frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(1/4)),x]
```


```
[Out] (7*b^3*x)/(20*a^3*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*a*x^5) + (7*b*(
a + b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^2*(a + b*x^2)^(3/4))/(20*a^3*x) - (7*
b^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(20*a^(5/2)*(a + b*x^2)^(1/4))
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5} - \frac{(7b)\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx}{10a}$$

$$= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} + \frac{\left(7b^2\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{20a^2}$$

$$= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x} + \frac{\left(7b^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{40a^3}$$

$$= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x} + \frac{\left(7b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{40a^3\sqrt[4]{a+bx^2}}$$

$$= \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{5ax^5} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x} - \frac{\left(7b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,d}{40a^3\sqrt[4]{a+bx^2}}$$

$$= \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{5ax^5} + \frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\right.\right.}{20a^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0087274, size = 51, normalized size = 0.34

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(1/4)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x^2)/a)])/(
5*x^5*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{bx^8+ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^8 + a*x^6), x)
```

Sympy [C] time = 1.18404, size = 32, normalized size = 0.22

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2},\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[4]{a}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

3.822 $\int \frac{x^6}{\sqrt[4]{a-bx^2}} dx$

Optimal. Leaf size=129

$$-\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}+\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

```
[Out] (-8*a^2*x*(a - b*x^2)^(3/4))/(39*b^3) - (20*a*x^3*(a - b*x^2)^(3/4))/(117*b
^2) - (2*x^5*(a - b*x^2)^(3/4))/(13*b) + (16*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*
EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0447899, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 229, 228}

$$-\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}+\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(1/4),x]
```

```
[Out] (-8*a^2*x*(a - b*x^2)^(3/4))/(39*b^3) - (20*a*x^3*(a - b*x^2)^(3/4))/(117*b
^2) - (2*x^5*(a - b*x^2)^(3/4))/(13*b) + (16*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*
EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a - b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = -\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b} + \frac{(10a)\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx}{13b}$$
$$= -\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2} - \frac{2x^5\left(a-bx^2\right)^{3/4}}{13b} + \frac{\left(20a^2\right)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{39b^2}$$
$$= -\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2} - \frac{2x^5\left(a-bx^2\right)^{3/4}}{13b} + \frac{\left(8a^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{39b^3}$$
$$= -\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2} - \frac{2x^5\left(a-bx^2\right)^{3/4}}{13b} + \frac{\left(8a^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{39b^3\sqrt[4]{a-bx^2}}$$
$$= -\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2} - \frac{2x^5\left(a-bx^2\right)^{3/4}}{13b} + \frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0304395, size = 89, normalized size = 0.69

$$\frac{2x\left(12a^3\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+2a^2bx^2-12a^3+ab^2x^4+9b^3x^6\right)}{117b^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(-12*a^3 + 2*a^2*b*x^2 + a*b^2*x^4 + 9*b^3*x^6 + 12*a^3*(1 - (b*x^2)/a
)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(117*b^3*(a - b*x^2)^
(1/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^6\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^6}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*x^6/(b*x^2 - a), x)
```

Sympy [C] time = 0.950227, size = 29, normalized size = 0.22

$$\frac{x^7{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

3.823 $\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

```
[Out] (-4*a*x*(a - b*x^2)^(3/4))/(15*b^2) - (2*x^3*(a - b*x^2)^(3/4))/(9*b) + (8*
a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(15*b^(5/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0336114, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 229, 228}

$$\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(1/4),x]
```

```
[Out] (-4*a*x*(a - b*x^2)^(3/4))/(15*b^2) - (2*x^3*(a - b*x^2)^(3/4))/(9*b) + (8*
a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(15*b^(5/2)*(a - b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = -\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{(2a)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{3b}$$
$$= -\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2} - \frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{15b^2}$$
$$= -\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2} - \frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{15b^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2} - \frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0222595, size = 79, normalized size = 0.76

$$\frac{2\left(6a^2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-6a^2x+abx^3+5b^2x^5\right)}{45b^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(1/4),x]
```

```
[Out] (2*(-6*a^2*x + a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(45*b^2*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^4\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*x^4/(b*x^2 - a), x)
```

Sympy [C] time = 0.787897, size = 29, normalized size = 0.28

$$\frac{x^5 {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

3.824 $\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.021667, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 229, 228}

$$\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{2x\left(a-bx^2\right)^{3/4}}{5b} + \frac{(2a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5b} \\
&= -\frac{2x\left(a-bx^2\right)^{3/4}}{5b} + \frac{\left(2a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5b\sqrt[4]{a-bx^2}} \\
&= -\frac{2x\left(a-bx^2\right)^{3/4}}{5b} + \frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0183005, size = 64, normalized size = 0.79

$$
\frac{2x\left(a\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-a+bx^2\right)}{5b\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 (b*x^2)/a]))/(5*b*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$
\int x^2 \frac{1}{\sqrt[4]{-bx^2+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^2}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*x^2/(b*x^2 - a), x)
```

Sympy [C] time = 0.72358, size = 29, normalized size = 0.36

$$\frac{x^3{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

3.825 $\int \frac{1}{\sqrt[4]{a-bx^2}} \, dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0112114, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {229, 228}

$$\frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1/4),x]
```

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-bx^2}} \, dx &= \frac{\sqrt[4]{1-\frac{bx^2}{a}} \int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}} \, dx}{\sqrt[4]{a-bx^2}}\\ &= \frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0069895, size = 47, normalized size = 0.81

$$\frac{x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1/4),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(a -
b*x^2)^(1/4)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^2 - a), x)
```

Sympy [C] time = 0.689039, size = 26, normalized size = 0.45

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/4),x)
```

```
[Out] x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-1/4), x)
```

3.826 $\int \frac{1}{x^2 \sqrt[4]{a-bx^2}} dx$

Optimal. Leaf size=79

$$-\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}$$

```
[Out] -((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0202877, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 229, 228}

$$-\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(1/4)),x]
```

```
[Out] -((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = -\frac{(a-bx^2)^{3/4}}{ax} - \frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{2a}$$
$$= -\frac{(a-bx^2)^{3/4}}{ax} - \frac{\left(b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2a\sqrt[4]{a-bx^2}}$$
$$= -\frac{(a-bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0088997, size = 50, normalized size = 0.63

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (b*x^2)/a])/(x*(
a - b*x^2)^(1/4)))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^4-ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^4 - a*x^2), x)
```

Sympy [C] time = 0.790655, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(1/4)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

3.827 $\int \frac{1}{x^4 \sqrt[4]{a-bx^2}} dx$

Optimal. Leaf size=106

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

```
[Out] -(a - b*x^2)^(3/4)/(3*a*x^3) - (b*(a - b*x^2)^(3/4))/(2*a^2*x) - (b^(3/2)*(
1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2
)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0313681, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 229, 228}

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(a - b*x^2)^(3/4)/(3*a*x^3) - (b*(a - b*x^2)^(3/4))/(2*a^2*x) - (b^(3/2)*(
1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2
)*(a - b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}dx = -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} + \frac{b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}dx}{2a}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} - \frac{b\left(a-bx^2\right)^{3/4}}{2a^2x} - \frac{b^2\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{4a^2}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} - \frac{b\left(a-bx^2\right)^{3/4}}{2a^2x} - \frac{\left(b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{4a^2\sqrt[4]{a-bx^2}}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} - \frac{b\left(a-bx^2\right)^{3/4}}{2a^2x} - \frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0098855, size = 52, normalized size = 0.49

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(1/4)),x]
```

```
[Out] -((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (b*x^2)/a])/(3*x
^3*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^4/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^6-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^6 - a*x^4), x)
```

Sympy [C] time = 0.954187, size = 34, normalized size = 0.32

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

3.828 $\int \frac{1}{x^6 \sqrt[4]{a-bx^2}} dx$

Optimal. Leaf size=131

$$-\frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

```
[Out] -(a - b*x^2)^(3/4)/(5*a*x^5) - (7*b*(a - b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^
2*(a - b*x^2)^(3/4))/(20*a^3*x) - (7*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(5/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0443256, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 229, 228}

$$-\frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(a - b*x^2)^(3/4)/(5*a*x^5) - (7*b*(a - b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^
2*(a - b*x^2)^(3/4))/(20*a^3*x) - (7*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(5/2)*(a - b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} + \frac{(7b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{10a}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} + \frac{\left(7b^2\right)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a^2}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x} - \frac{\left(7b^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^3}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x} - \frac{\left(7b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{40a^3\sqrt[4]{a-bx^2}}$$

$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0098481, size = 52, normalized size = 0.4

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(1/4)),x]
```

```
[Out] -((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (b*x^2)/a])/(5*x
^5*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^6/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^8-ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^8 - a*x^6), x)
```

Sympy [C] time = 1.19079, size = 34, normalized size = 0.26

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2},\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

3.829 $\int \frac{x^6}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{80a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}+\frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2}+\frac{2x^5\sqrt[4]{a+bx^2}}{11b}$$

```
[Out] (40*a^2*x*(a + b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a + b*x^2)^(1/4))/(77*b^
2) + (2*x^5*(a + b*x^2)^(1/4))/(11*b) - (80*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*E
llipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0429373, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 233, 231}

$$\frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3}-\frac{80a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}-\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2}+\frac{2x^5\sqrt[4]{a+bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(3/4),x]
```

```
[Out] (40*a^2*x*(a + b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a + b*x^2)^(1/4))/(77*b^
2) + (2*x^5*(a + b*x^2)^(1/4))/(11*b) - (80*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*E
llipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a + b*x^2)^(3/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{(10a)\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx}{11b}$$

$$= -\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} + \frac{\left(60a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{\left(40a^3\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{77b^3}$$

$$= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{\left(40a^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^3\left(a+bx^2\right)^{3/4}}$$

$$= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{80a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.035175, size = 90, normalized size = 0.73

$$\frac{2\left(-20a^3x\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+10a^2bx^3+20a^3x-3ab^2x^5+7b^3x^7\right)}{77b^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*(20*a^3*x + 10*a^2*b*x^3 - 3*a*b^2*x^5 + 7*b^3*x^7 - 20*a^3*x*(1 + (b*x^
2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(77*b^3*(a + b
*x^2)^(3/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^6\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^6/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 0.902338, size = 27, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{7}{2} \\ \frac{9}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(3/4),x)
```

```
[Out] x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

3.830 $\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=100

$$\frac{8a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b}$$

```
[Out] (-4*a*x*(a + b*x^2)^(1/4))/(7*b^2) + (2*x^3*(a + b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0314355, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 233, 231}

$$\frac{8a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/4),x]
```

```
[Out] (-4*a*x*(a + b*x^2)^(1/4))/(7*b^2) + (2*x^3*(a + b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a + b*x^2)^(3/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{2x^3\sqrt[4]{a+bx^2}}{7b} - \frac{(6a)\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx}{7b}$$

$$= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{\left(4a^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{7b^2}$$

$$= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{\left(4a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{7b^2\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{8a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0265728, size = 78, normalized size = 0.78

$$\frac{2\left(2a^2x\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)-2a^2x-abx^3+b^2x^5\right)}{7b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*(-2*a^2*x - a*b*x^3 + b^2*x^5 + 2*a^2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeome
tric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(7*b^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^4\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 0.77111, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/4),x)
```

```
[Out] x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

3.831 $\int \frac{x^2}{\left(a+bx^2\right)^{3/4}} \, dx$

Optimal. Leaf size=78

$$\frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0195595, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 233, 231}

$$\frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{(2a)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{3b}$$
$$= \frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{\left(2a\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3b\,(a+bx^2)^{3/4}}$$
$$= \frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{4a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\,(a+bx^2)^{3/4}}$$

Mathematica [C] time = 0.0168684, size = 62, normalized size = 0.79

$$\frac{2x\left(-a\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
-((b*x^2)/a)]))/(3*b*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 0.71829, size = 27, normalized size = 0.35

$$\frac{x^3{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/4),x)
```

```
[Out] x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

3.832 $\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0096891, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {233, 231}

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{\left(a+bx^2\right)^{3/4}}\\ &= \frac{2\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.006091, size = 46, normalized size = 0.82

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/4}\,_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/4),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(3/4)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx^2+a)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2+a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-3/4), x)
```

Sympy [C] time = 0.68075, size = 24, normalized size = 0.43

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4),x)
```

```
[Out] x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^{2}+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-3/4), x)
```

3.833 $\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}$$

```
[Out] -((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0186475, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 233, 231}

$$-\frac{\sqrt[4]{a+bx^2}}{ax}-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/4)),x]
```

```
[Out] -((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{2a}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{2a\left(a+bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0092633, size = 49, normalized size = 0.64

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/4)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(3/4)))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^4 + a*x^2), x)
```

Sympy [C] time = 0.878916, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(3/4)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

3.834 $\int \frac{1}{x^4 \left(a+bx^2\right)^{3/4}} dx$

Optimal. Leaf size=102

$$\frac{5b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

```
[Out] -(a + b*x^2)^(1/4)/(3*a*x^3) + (5*b*(a + b*x^2)^(1/4))/(6*a^2*x) + (5*b^(3/
2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^
(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0314378, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 233, 231}

$$\frac{5b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/4)),x]
```


```
[Out] -(a + b*x^2)^(1/4)/(3*a*x^3) + (5*b*(a + b*x^2)^(1/4))/(6*a^2*x) + (5*b^(3/
2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^
(3/2)*(a + b*x^2)^(3/4))
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{3/4}} \, dx = -\frac{\sqrt[4]{a+bx^2}}{3ax^3} - \frac{(5b) \int \frac{1}{x^2 \left(a+bx^2\right)^{3/4}} \, dx}{6a}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{3ax^3} + \frac{5b\sqrt[4]{a+bx^2}}{6a^2x} + \frac{\left(5b^2\right) \int \frac{1}{\left(a+bx^2\right)^{3/4}} \, dx}{12a^2}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{3ax^3} + \frac{5b\sqrt[4]{a+bx^2}}{6a^2x} + \frac{\left(5b^2 \left(1+\frac{bx^2}{a}\right)^{3/4}\right) \int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}} \, dx}{12a^2 \left(a+bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{3ax^3} + \frac{5b\sqrt[4]{a+bx^2}}{6a^2x} + \frac{5b^{3/2} \left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \middle| 2\right)}{6a^{3/2} \left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0091158, size = 51, normalized size = 0.5

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3 \left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/4)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x^2)/a)])/(
3*x^3*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \left(bx^2+a\right)^{-\frac{3}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{bx^6+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^6 + a*x^4), x)
```

Sympy [C] time = 1.09522, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

3.835 $\int \frac{1}{x^6\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{3b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

```
[Out] -(a + b*x^2)^(1/4)/(5*a*x^5) + (3*b*(a + b*x^2)^(1/4))/(10*a^2*x^3) - (3*b^
2*(a + b*x^2)^(1/4))/(4*a^3*x) - (3*b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0427865, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 233, 231}

$$-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x}-\frac{3b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(3/4)),x]
```

```
[Out] -(a + b*x^2)^(1/4)/(5*a*x^5) + (3*b*(a + b*x^2)^(1/4))/(10*a^2*x^3) - (3*b^
2*(a + b*x^2)^(1/4))/(4*a^3*x) - (3*b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a + b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{3/4}} \, dx = -\frac{\sqrt[4]{a+bx^2}}{5ax^5} - \frac{(9b) \int \frac{1}{x^4 \left(a+bx^2\right)^{3/4}} \, dx}{10a}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} + \frac{\left(3b^2\right) \int \frac{1}{x^2 \left(a+bx^2\right)^{3/4}} \, dx}{4a^2}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{\left(3b^3\right) \int \frac{1}{\left(a+bx^2\right)^{3/4}} \, dx}{8a^3}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{\left(3b^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right) \int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}} \, dx}{8a^3 \left(a+bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{3b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.009011, size = 51, normalized size = 0.4

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(3/4)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x^2)/a)])/(
5*x^5*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \left(bx^2+a\right)^{-\frac{3}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}} x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{bx^8+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^8 + a*x^6), x)
```

Sympy [C] time = 1.35798, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

3.836 $\int \frac{x^6}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=129

$$\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}-\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

```
[Out] (-40*a^2*x*(a - b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a - b*x^2)^(1/4))/(77*b
^2) - (2*x^5*(a - b*x^2)^(1/4))/(11*b) + (80*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*
EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0464821, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 233, 232}

$$-\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}+\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(3/4),x]
```

```
[Out] (-40*a^2*x*(a - b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a - b*x^2)^(1/4))/(77*b
^2) - (2*x^5*(a - b*x^2)^(1/4))/(11*b) + (80*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*
EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a - b*x^2)^(3/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{(10a)\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx}{11b}$$

$$= -\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{\left(60a^2\right)\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{\left(40a^3\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{77b^3}$$

$$= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{\left(40a^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0351946, size = 91, normalized size = 0.71

$$\frac{2\left(20a^3x\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)+10a^2bx^3-20a^3x+3ab^2x^5+7b^3x^7\right)}{77b^3\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*(-20*a^3*x + 10*a^2*b*x^3 + 3*a*b^2*x^5 + 7*b^3*x^7 + 20*a^3*x*(1 - (b*x
^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(77*b^3*(a - b*x
^2)^(3/4))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int x^6\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}x^6}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(1/4)*x^6/(b*x^2 - a), x)
```

Sympy [C] time = 0.910052, size = 29, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(3/4),x)

[Out] x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

3.837 $\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

```
[Out] (-4*a*x*(a - b*x^2)^(1/4))/(7*b^2) - (2*x^3*(a - b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0318736, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 233, 232}

$$\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(3/4),x]
```

```
[Out] (-4*a*x*(a - b*x^2)^(1/4))/(7*b^2) - (2*x^3*(a - b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a - b*x^2)^(3/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2x^3\sqrt[4]{a-bx^2}}{7b} + \frac{(6a)\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx}{7b}$$

$$= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2} - \frac{2x^3\sqrt[4]{a-bx^2}}{7b} + \frac{\left(4a^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{7b^2}$$

$$= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2} - \frac{2x^3\sqrt[4]{a-bx^2}}{7b} + \frac{\left(4a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{7b^2\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2} - \frac{2x^3\sqrt[4]{a-bx^2}}{7b} + \frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0265884, size = 77, normalized size = 0.74

$$\frac{2x\left(2a^2\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right) - 2a^2 + abx^2 + b^2x^4\right)}{7b^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2*a^2 + a*b*x^2 + b^2*x^4 + 2*a^2*(1 - (b*x^2)/a)^(3/4)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(7*b^2*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int x^4\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^{2}+a\right)^{\frac{1}{4}}x^{4}}{bx^{2}-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*x^4/(b*x^2 - a), x)
```

Sympy [C] time = 0.775516, size = 29, normalized size = 0.28

$$\frac{x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(3/4),x)
```

```
[Out] x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{4}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

3.838 $\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=81

$$\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

```
[Out] (-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0216295, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 233, 232}

$$\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{(2a)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{3b}$$

$$=-\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{\left(2a\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3b\left(a-bx^2\right)^{3/4}}$$

$$=-\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0195983, size = 64, normalized size = 0.79

$$\frac{2x\left(a\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)-a+bx^2\right)}{3b\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 (b*x^2)/a]))/(3*b*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x^2\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}x^2}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*x^2/(b*x^2 - a), x)
```

Sympy [C] time = 0.725392, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(3/4),x)
```

```
[Out] x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

3.839 $\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.010857, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {233, 232}

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx &= \frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{\left(a-bx^2\right)^{3/4}}\\ &= \frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.0067723, size = 47, normalized size = 0.81

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3/4),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(a -
b*x^2)^(3/4)
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^2 - a), x)
```

Sympy [C] time = 0.692853, size = 26, normalized size = 0.45

$$\frac{x_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(3/4),x)
```

```
[Out] x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(3/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2 + a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-3/4), x)
```

3.840 $\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}$$

```
[Out] -((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0209561, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 233, 232}

$$\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(3/4)),x]
```

```
[Out] -((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2 (a-bx^2)^{3/4}} \, dx = -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{b \int \frac{1}{(a-bx^2)^{3/4}} \, dx}{2a}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\left(b \left(1-\frac{bx^2}{a}\right)^{3/4}\right) \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}} \, dx}{2a (a-bx^2)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\sqrt{b} \left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a} (a-bx^2)^{3/4}}$$

Mathematica [C] time = 0.0094599, size = 50, normalized size = 0.64

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{1}{2}, \frac{3}{4}; \frac{1}{2}; \frac{bx^2}{a}\right)}{x (a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(3/4)),x]
```

```
[Out] -(((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (b*x^2)/a])/(x*(
a - b*x^2)^(3/4)))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(-bx^2+a\right)^{-\frac{3}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^4-ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^4 - a*x^2), x)
```

Sympy [C] time = 0.88379, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(3/4)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

3.841 $\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=106

$$\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

```
[Out] -(a - b*x^2)^(1/4)/(3*a*x^3) - (5*b*(a - b*x^2)^(1/4))/(6*a^2*x) + (5*b^(3/
2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^
(3/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0323221, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 233, 232}

$$\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(3/4)),x]
```

```
[Out] -(a - b*x^2)^(1/4)/(3*a*x^3) - (5*b*(a - b*x^2)^(1/4))/(6*a^2*x) + (5*b^(3/
2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^
(3/2)*(a - b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{3ax^3} + \frac{(5b)\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx}{6a}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{\left(5b^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{12a^2}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{\left(5b^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a^2\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0099567, size = 52, normalized size = 0.49

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(3/4)),x]
```

```
[Out] -((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (b*x^2)/a])/(3*x
^3*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^4/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^6-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^6 - a*x^4), x)
```

Sympy [C] time = 1.10797, size = 34, normalized size = 0.32

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ -\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

3.842 $\int \frac{1}{x^6\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=131

$$\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

```
[Out] -(a - b*x^2)^(1/4)/(5*a*x^5) - (3*b*(a - b*x^2)^(1/4))/(10*a^2*x^3) - (3*b^
2*(a - b*x^2)^(1/4))/(4*a^3*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.046621, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 233, 232}

$$-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(3/4)),x]
```

```
[Out] -(a - b*x^2)^(1/4)/(5*a*x^5) - (3*b*(a - b*x^2)^(1/4))/(10*a^2*x^3) - (3*b^
2*(a - b*x^2)^(1/4))/(4*a^3*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a - b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a-bx^2\right)^{3/4}} \, dx = -\frac{\sqrt[4]{a-bx^2}}{5ax^5}+\frac{(9b)\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}} \, dx}{10a}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}+\frac{\left(3b^2\right)\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}} \, dx}{4a^2}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{\left(3b^3\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}} \, dx}{8a^3}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{\left(3b^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}} \, dx}{8a^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0096728, size = 52, normalized size = 0.4

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(3/4)),x]
```

```
[Out] -((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (b*x^2)/a])/(5*x
^5*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(-bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^6/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^8-ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^8 - a*x^6), x)
```

Sympy [C] time = 1.36884, size = 34, normalized size = 0.26

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

3.843 $\int \frac{x^6}{(a+bx^2)^{5/4}} dx$

Optimal. Leaf size=124

$$\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

```
[Out] (8*a^2*x)/(3*b^3*(a + b*x^2)^(1/4)) - (4*a*x^3)/(9*b^2*(a + b*x^2)^(1/4)) +
 (2*x^5)/(9*b*(a + b*x^2)^(1/4)) - (16*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipt
icE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0434474, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {285, 197, 196}

$$\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/4),x]
```

```
[Out] (8*a^2*x)/(3*b^3*(a + b*x^2)^(1/4)) - (4*a*x^3)/(9*b^2*(a + b*x^2)^(1/4)) +
 (2*x^5)/(9*b*(a + b*x^2)^(1/4)) - (16*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipt
icE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{(10a)\int \frac{x^4}{\left(a+bx^2\right)^{5/4}}\,dx}{9b}$$
$$= -\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} + \frac{\left(4a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx}{3b^2}$$
$$= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{\left(8a^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{3b^3}$$
$$= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{\left(8a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{3b^3\sqrt[4]{a+bx^2}}$$
$$= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{16a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0284226, size = 78, normalized size = 0.63

$$\frac{2\left(12a^2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-12a^2x-2abx^3+b^2x^5\right)}{9b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*(-12*a^2*x - 2*a*b*x^3 + b^2*x^5 + 12*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hyperg
eometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(9*b^3*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x^6\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/4),x)
```

```
[Out] int(x^6/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{3}{4}}x^6}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.902676, size = 27, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/4),x)
```

```
[Out] x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

3.844 $\int \frac{x^4}{\left(a+bx^2\right)^{5/4}} \, dx$

Optimal. Leaf size=100

$$\frac{24a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}-\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}$$

```
[Out] (-12*a*x)/(5*b^2*(a + b*x^2)^(1/4)) + (2*x^3)/(5*b*(a + b*x^2)^(1/4)) + (24
*a^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(5*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0314686, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {285, 197, 196}

$$\frac{24a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}-\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/4),x]
```

```
[Out] (-12*a*x)/(5*b^2*(a + b*x^2)^(1/4)) + (2*x^3)/(5*b*(a + b*x^2)^(1/4)) + (24
*a^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(5*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{2x^3}{5b\sqrt[4]{a+bx^2}} - \frac{(6a)\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx}{5b}$$

$$= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{\left(12a^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{5b^2}$$

$$= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{\left(12a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5b^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{24a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0203067, size = 65, normalized size = 0.65

$$\frac{2\left(-6ax\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+6ax+bx^3\right)}{5b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*(6*a*x + b*x^3 - 6*a*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2,
 3/2, -((b*x^2)/a)]))/(5*b^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x^4\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/4),x)
```

```
[Out] int(x^4/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}x^4}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.819233, size = 27, normalized size = 0.27

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/4),x)
```

```
[Out] x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)
```

3.845 $\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=74

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (2*x)/(b*(a + b*x^2)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0190679, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {285, 197, 196}

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x)/(b*(a + b*x^2)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{(2a)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{b}$$

$$= \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{\left(2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{b\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0168254, size = 53, normalized size = 0.72

$$\frac{2x\left(\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*(-1 + (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)
/a)]))/(b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}x^2}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.803203, size = 27, normalized size = 0.36

$$\frac{x^3{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/4),x)
```

```
[Out] x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

3.846 $\int \frac{1}{(a+bx^2)^{5/4}} \, dx$

Optimal. Leaf size=56

$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

```
[Out] (2*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt
[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0101319, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {197, 196}

$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/4),x]
```

```
[Out] (2*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt
[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/4}} \, dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}} \int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}} \, dx}{a\sqrt[4]{a+bx^2}}$$
$$= \frac{2\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01149, size = 55, normalized size = 0.98

$$\frac{2x - x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/4),x]
```

```
[Out] (2*x - x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a
)])/(a*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.783372, size = 24, normalized size = 0.43

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4),x)
```

```
[Out] x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-5/4), x)
```

3.847 $\int \frac{1}{x^2(a+bx^2)^{5/4}} dx$

Optimal. Leaf size=76

$$-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

```
[Out] -(1/(a*x*(a + b*x^2)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0201927, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {286, 197, 196}

$$-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/4)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^2)^(1/4))
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx = -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{(3b)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{2a}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{3\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0086232, size = 52, normalized size = 0.68

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/4)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/4)))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{3}{4}}}{b^2x^6+2\,abx^4+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] time = 1.0892, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/4)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

3.848 $\int \frac{1}{x^4(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=102

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(1/4)) + (7*b)/(6*a^2*x*(a + b*x^2)^(1/4)) + (7*b^(
3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*
a^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0303397, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {286, 197, 196}

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(1/4)) + (7*b)/(6*a^2*x*(a + b*x^2)^(1/4)) + (7*b^(
3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*
a^(5/2)*(a + b*x^2)^(1/4))
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx = -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} - \frac{(7b)\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx}{6a}$$

$$= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} + \frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{\left(7b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{4a^2}$$

$$= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} + \frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{\left(7b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} + \frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{7b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0090307, size = 54, normalized size = 0.53

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/4)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x^2)/a)])/(
3*a*x^3*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] time = 1.36842, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

3.849 $\int \frac{1}{x^6(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

```
[Out] -1/(5*a*x^5*(a + b*x^2)^(1/4)) + (11*b)/(30*a^2*x^3*(a + b*x^2)^(1/4)) - (7
7*b^2)/(60*a^3*x*(a + b*x^2)^(1/4)) - (77*b^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ell
ipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0424427, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {286, 197, 196}

$$-\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/(5*a*x^5*(a + b*x^2)^(1/4)) + (11*b)/(30*a^2*x^3*(a + b*x^2)^(1/4)) - (7
7*b^2)/(60*a^3*x*(a + b*x^2)^(1/4)) - (77*b^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ell
ipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a + b*x^2)^(1/4))
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/4}} dx = -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} - \frac{(11b) \int \frac{1}{x^4\left(a+bx^2\right)^{5/4}} dx}{10a}$$

$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} + \frac{\left(77b^2\right) \int \frac{1}{x^2\left(a+bx^2\right)^{5/4}} dx}{60a^2}$$

$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x \sqrt[4]{a+bx^2}} - \frac{\left(77b^3\right) \int \frac{1}{\left(a+bx^2\right)^{5/4}} dx}{40a^3}$$

$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x \sqrt[4]{a+bx^2}} - \frac{\left(77b^3 \sqrt[4]{1+\frac{bx^2}{a}}\right) \int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}} dx}{40a^4 \sqrt[4]{a+bx^2}}$$

$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x \sqrt[4]{a+bx^2}} - \frac{77b^{5/2} \sqrt[4]{1+\frac{bx^2}{a}} E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2} \sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0089902, size = 54, normalized size = 0.43

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\, {}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5ax^5 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(5/4)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x^2)/a)])/(
5*a*x^5*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \left(bx^2+a\right)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^{10}+2\,abx^8+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] time = 1.76872, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{5}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

3.850 $\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

```
[Out] (2*x^5)/(b*(a - b*x^2)^(1/4)) + (8*a*x*(a - b*x^2)^(3/4))/(3*b^3) + (20*x^3
*(a - b*x^2)^(3/4))/(9*b^2) - (16*a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0474167, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {288, 321, 229, 228}

$$-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(5/4),x]
```


```
[Out] (2*x^5)/(b*(a - b*x^2)^(1/4)) + (8*a*x*(a - b*x^2)^(3/4))/(3*b^3) + (20*x^3
*(a - b*x^2)^(3/4))/(9*b^2) - (16*a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a - b*x^2)^(1/4))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx}{b}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{(20a)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{3b^2}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{\left(8a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{3b^3}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{\left(8a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{3b^3\sqrt[4]{a-bx^2}}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0287999, size = 78, normalized size = 0.63

$$-\frac{2x\left(12a^2\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-12a^2+2abx^2+b^2x^4\right)}{9b^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(5/4),x]
```

```
[Out] (-2*x*(-12*a^2 + 2*a*b*x^2 + b^2*x^4 + 12*a^2*(1 - (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(9*b^3*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int x^6\left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^6}{b^2x^4-2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.93804, size = 29, normalized size = 0.23

$$\frac{x^7\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(5/4),x)
```

```
[Out] x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

3.851 $\int \frac{x^4}{\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}+\frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

```
[Out] (2*x^3)/(b*(a - b*x^2)^(1/4)) + (12*x*(a - b*x^2)^(3/4))/(5*b^2) - (24*a^(3
/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b
^(5/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0313051, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {288, 321, 229, 228}

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}+\frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x^3)/(b*(a - b*x^2)^(1/4)) + (12*x*(a - b*x^2)^(3/4))/(5*b^2) - (24*a^(3
/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b
^(5/2)*(a - b*x^2)^(1/4))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2x^3}{b\sqrt[4]{a-bx^2}}-\frac{6\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{b}\\
&= \frac{2x^3}{b\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}-\frac{(12a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5b^2}\\
&= \frac{2x^3}{b\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}-\frac{\left(12a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5b^2\sqrt[4]{a-bx^2}}\\
&= \frac{2x^3}{b\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0199998, size = 66, normalized size = 0.65

$$-\frac{2\left(6ax\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-6ax+bx^3\right)}{5b^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(5/4),x]
```

```
[Out] (-2*(-6*a*x + b*x^3 + 6*a*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/
2, 3/2, (b*x^2)/a]))/(5*b^2*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int x^4\left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(x^4/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.847994, size = 29, normalized size = 0.29

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(5/4),x)
```

```
[Out] x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

3.852 $\int \frac{x^2}{(a-bx^2)^{5/4}} \, dx$

Optimal. Leaf size=77

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] (2*x)/(b*(a - b*x^2)^(1/4)) - (4*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.021773, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {288, 229, 228}

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(5/4),x]
```



```
[Out] (2*x)/(b*(a - b*x^2)^(1/4)) - (4*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a - b*x^2)^(1/4))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{b}$$

$$= \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{\left(2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{b\sqrt[4]{a-bx^2}}$$

$$= \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0154405, size = 56, normalized size = 0.73

$$\frac{2x-2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x - 2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a
])/(b*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^2\left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^2}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.827378, size = 29, normalized size = 0.38

$$\frac{x^3{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(5/4),x)
```

```
[Out] x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

3.853 $\int \frac{1}{\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

```
[Out] (2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0193488, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {199, 229, 228}

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5/4),x]
```

```
[Out] (2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{a}$$
$$= \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{a\sqrt[4]{a-bx^2}}$$
$$= \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0130012, size = 56, normalized size = 0.73

$$\frac{2x - x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{a\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5/4),x]
```

```
[Out] (2*x - x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])
/(a*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 0.801041, size = 26, normalized size = 0.34

$$\frac{x\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/4),x)
```

```
[Out] x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(5/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```

3.854 $\int \frac{1}{x^2(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=99

$$-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}+\frac{2}{ax\sqrt[4]{a-bx^2}}$$

```
[Out] 2/(a*x*(a - b*x^2)^(1/4)) - (3*(a - b*x^2)^(3/4))/(a^2*x) - (3*Sqrt[b]*(1 -
 (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a
- b*x^2)^(1/4))
```

Rubi [A] time = 0.031739, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {290, 325, 229, 228}

$$-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}+\frac{2}{ax\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(5/4)),x]
```


```
[Out] 2/(a*x*(a - b*x^2)^(1/4)) - (3*(a - b*x^2)^(3/4))/(a^2*x) - (3*Sqrt[b]*(1 -
 (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a
- b*x^2)^(1/4))
```


Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx = \frac{2}{ax\sqrt[4]{a-bx^2}}+\frac{3\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{a}$$

$$=\frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{(3b)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{2a^2}$$

$$=\frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{\left(3b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2a^2\sqrt[4]{a-bx^2}}$$

$$=\frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0102177, size = 53, normalized size = 0.54

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{ax\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(5/4)),x]
```

```
[Out] -(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (b*x^2)/a])/(a*x
*(a - b*x^2)^(1/4)))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^6-2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] time = 1.13905, size = 29, normalized size = 0.29

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(5/4)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

3.855 $\int \frac{1}{x^4\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

```
[Out] 2/(a*x^3*(a - b*x^2)^(1/4)) - (7*(a - b*x^2)^(3/4))/(3*a^2*x^3) - (7*b*(a -
 b*x^2)^(3/4))/(2*a^3*x) - (7*b^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(5/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0451057, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {290, 325, 229, 228}

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x^3*(a - b*x^2)^(1/4)) - (7*(a - b*x^2)^(3/4))/(3*a^2*x^3) - (7*b*(a -
 b*x^2)^(3/4))/(2*a^3*x) - (7*b^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(5/2)*(a - b*x^2)^(1/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2}{ax^3\sqrt[4]{a-bx^2}}+\frac{7\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{a}\\
&= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}+\frac{(7b)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{2a^2}\\
&= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{\left(7b^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a^3}\\
&= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{\left(7b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a^3\sqrt[4]{a-bx^2}}\\
&= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0107757, size = 55, normalized size = 0.44

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(5/4)),x]
```

```
[Out] -((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (b*x^2)/a])/(3*a
*x^3*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^4/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^8-2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] time = 1.38744, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

3.856 $\int \frac{1}{x^6 (a-bx^2)^{5/4}} \, dx$

Optimal. Leaf size=151

$$-\frac{77b^2 (a-bx^2)^{3/4}}{20a^4x} - \frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}} - \frac{77b (a-bx^2)^{3/4}}{30a^3x^3} - \frac{11 (a-bx^2)^{3/4}}{5a^2x^5} + \frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

```
[Out] 2/(a*x^5*(a - b*x^2)^(1/4)) - (11*(a - b*x^2)^(3/4))/(5*a^2*x^5) - (77*b*(a
 - b*x^2)^(3/4))/(30*a^3*x^3) - (77*b^2*(a - b*x^2)^(3/4))/(20*a^4*x) - (77
*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(20*a^(7/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0610959, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {290, 325, 229, 228}

$$-\frac{77b^2 (a-bx^2)^{3/4}}{20a^4x} - \frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}} - \frac{77b (a-bx^2)^{3/4}}{30a^3x^3} - \frac{11 (a-bx^2)^{3/4}}{5a^2x^5} + \frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x^5*(a - b*x^2)^(1/4)) - (11*(a - b*x^2)^(3/4))/(5*a^2*x^5) - (77*b*(a
 - b*x^2)^(3/4))/(30*a^3*x^3) - (77*b^2*(a - b*x^2)^(3/4))/(20*a^4*x) - (77
*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(20*a^(7/2)*(a - b*x^2)^(1/4))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6\left(a-bx^2\right)^{5/4}}\,dx = \frac{2}{ax^5\sqrt[4]{a-bx^2}}+\frac{11\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx}{a}$$
$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}+\frac{(77b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{10a^2}$$
$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}-\frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3}+\frac{\left(77b^2\right)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a^3}$$
$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}-\frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3}-\frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x}-\frac{\left(77b^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^4}$$
$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}-\frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3}-\frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x}-\frac{\left(77b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}}{40a^4\sqrt[4]{a-bx^2}}$$
$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}-\frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3}-\frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x}-\frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-}\right.}{20a^{7/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0102303, size = 55, normalized size = 0.36

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5ax^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(5/4)),x]

[Out] -((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, (b*x^2)/a])/(5*a
*x^5*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(-bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(5/4),x)

[Out] int(1/x^6/(-b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^{10}-2\,abx^8+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] time = 1.78238, size = 34, normalized size = 0.23

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{5}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)
```

3.857 $\int \frac{1}{(a+bx^2)^{7/4}} \, dx$

Optimal. Leaf size=78

$$\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2x}{3a\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0166227, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {199, 233, 231}

$$\frac{2x}{3a\left(a+bx^2\right)^{3/4}}+\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-7/4),x]
```

```
[Out] (2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx = \frac{2x}{3a\left(a+bx^2\right)^{3/4}} + \frac{\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{3a}$$

$$= \frac{2x}{3a\left(a+bx^2\right)^{3/4}} + \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2x}{3a\left(a+bx^2\right)^{3/4}} + \frac{2\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0216339, size = 55, normalized size = 0.71

$$\frac{x\left(\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+2\right)}{3a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-7/4),x]
```

```
[Out] (x*(2 + (1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)
]))/(3*a*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 1.1028, size = 24, normalized size = 0.31

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4),x)
```

```
[Out] x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

3.858 $\int \frac{1}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=78

$$\frac{6\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{2x}{5a\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*x)/(5*a*(a + b*x^2)^(5/4)) + (6*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0173645, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {199, 197, 196}

$$\frac{6\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{2x}{5a\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/4),x]
```

```
[Out] (2*x)/(5*a*(a + b*x^2)^(5/4)) + (6*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2x}{5a\left(a+bx^2\right)^{5/4}} + \frac{3\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{5a}$$

$$= \frac{2x}{5a\left(a+bx^2\right)^{5/4}} + \frac{\left(3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5a^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{5a\left(a+bx^2\right)^{5/4}} + \frac{6\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0313522, size = 72, normalized size = 0.92

$$\frac{-3x\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+8ax+6bx^3}{5a^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/4),x]
```

```
[Out] (8*a*x + 6*b*x^3 - 3*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, -((b*x^2)/a)])/(5*a^2*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4),x)
```

```
[Out] int(1/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] time = 1.77887, size = 24, normalized size = 0.31

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4),x)
```

```
[Out] x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

3.859 $\int \frac{1}{(a+bx^2)^{11/4}}\,dx$

Optimal. Leaf size=97

$$\frac{10\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{10x}{21a^2\left(a+bx^2\right)^{3/4}}+\frac{2x}{7a\left(a+bx^2\right)^{7/4}}$$

```
[Out] (2*x)/(7*a*(a + b*x^2)^(7/4)) + (10*x)/(21*a^2*(a + b*x^2)^(3/4)) + (10*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0235896, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {199, 233, 231}

$$\frac{10x}{21a^2\left(a+bx^2\right)^{3/4}}+\frac{10\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2x}{7a\left(a+bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-11/4),x]
```

```
[Out] (2*x)/(7*a*(a + b*x^2)^(7/4)) + (10*x)/(21*a^2*(a + b*x^2)^(3/4)) + (10*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}}\,dx = \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{5\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx}{7a}$$

$$= \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a+bx^2\right)^{3/4}} + \frac{5\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{21a^2}$$

$$= \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a+bx^2\right)^{3/4}} + \frac{\left(5\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a+bx^2\right)^{3/4}} + \frac{10\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0309899, size = 75, normalized size = 0.77

$$\frac{5x\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+2x\left(8a+5bx^2\right)}{21a^2\left(a+bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-11/4),x]
```

```
[Out] (2*x*(8*a + 5*b*x^2) + 5*x*(a + b*x^2)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(21*a^2*(a + b*x^2)^(7/4))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4),x)
```

```
[Out] int(1/(b*x^2+a)^(11/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="maxima")
```

[Out] integrate((b*x^2 + a)^(-11/4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)

Sympy [C] time = 2.94785, size = 24, normalized size = 0.25

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)**(11/4),x)

[Out] x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/4)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(-11/4), x)

3.860 $\int \frac{1}{(a-bx^2)^{7/4}} dx$

Optimal. Leaf size=81

$$\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2x}{3a\left(a-bx^2\right)^{3/4}}$$

```
[Out] (2*x)/(3*a*(a - b*x^2)^(3/4)) + (2*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0178061, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {199, 233, 232}

$$\frac{2x}{3a\left(a-bx^2\right)^{3/4}}+\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-7/4),x]
```

```
[Out] (2*x)/(3*a*(a - b*x^2)^(3/4)) + (2*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{7/4}}\,dx = \frac{2x}{3a\left(a-bx^2\right)^{3/4}} + \frac{\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{3a}$$

$$= \frac{2x}{3a\left(a-bx^2\right)^{3/4}} + \frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a\left(a-bx^2\right)^{3/4}}$$

$$= \frac{2x}{3a\left(a-bx^2\right)^{3/4}} + \frac{2\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0201655, size = 56, normalized size = 0.69

$$\frac{x\left(\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)+2\right)}{3a\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-7/4),x]
```

```
[Out] (x*(2 + (1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))
/(3*a*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 1.10458, size = 26, normalized size = 0.32

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/4),x)
```

```
[Out] x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(7/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

3.861 $\int \frac{1}{(a-bx^2)^{9/4}} \, dx$

Optimal. Leaf size=101

$$\frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}} + \frac{2x}{5a\left(a-bx^2\right)^{5/4}}$$

```
[Out] (2*x)/(5*a*(a - b*x^2)^(5/4)) + (6*x)/(5*a^2*(a - b*x^2)^(1/4)) - (6*(1 - (
b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqr
t[b]*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.02682, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {199, 229, 228}

$$\frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}} + \frac{2x}{5a\left(a-bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-9/4),x]
```

```
[Out] (2*x)/(5*a*(a - b*x^2)^(5/4)) + (6*x)/(5*a^2*(a - b*x^2)^(1/4)) - (6*(1 - (
b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqr
t[b]*(a - b*x^2)^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{9/4}}dx = \frac{2x}{5a\left(a-bx^2\right)^{5/4}} + \frac{3\int \frac{1}{\left(a-bx^2\right)^{5/4}}dx}{5a}$$

$$= \frac{2x}{5a\left(a-bx^2\right)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{3\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5a^2}$$

$$= \frac{2x}{5a\left(a-bx^2\right)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{\left(3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5a^2\sqrt[4]{a-bx^2}}$$

$$= \frac{2x}{5a\left(a-bx^2\right)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0332388, size = 74, normalized size = 0.73

$$\frac{-3x\left(a-bx^2\right)\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+8ax-6bx^3}{5a^2\left(a-bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-9/4),x]
```

```
[Out] (8*a*x - 6*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (b*x^2)/a])/(5*a^2*(a - b*x^2)^(5/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(9/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x
)
```

Sympy [C] time = 1.78914, size = 26, normalized size = 0.26

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(9/4),x)
```

```
[Out] x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(9/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```

3.862 $\int \frac{1}{(a-bx^2)^{11/4}}\,dx$

Optimal. Leaf size=101

$$\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10x}{21a^2\left(a-bx^2\right)^{3/4}}+\frac{2x}{7a\left(a-bx^2\right)^{7/4}}$$

```
[Out] (2*x)/(7*a*(a - b*x^2)^(7/4)) + (10*x)/(21*a^2*(a - b*x^2)^(3/4)) + (10*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0258074, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {199, 233, 232}

$$\frac{10x}{21a^2\left(a-bx^2\right)^{3/4}}+\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2x}{7a\left(a-bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-11/4),x]
```

```
[Out] (2*x)/(7*a*(a - b*x^2)^(7/4)) + (10*x)/(21*a^2*(a - b*x^2)^(3/4)) + (10*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{11/4}}\,dx = \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{5\int \frac{1}{\left(a-bx^2\right)^{7/4}}\,dx}{7a}$$

$$= \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a-bx^2\right)^{3/4}} + \frac{5\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{21a^2}$$

$$= \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a-bx^2\right)^{3/4}} + \frac{\left(5\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2\left(a-bx^2\right)^{3/4}}$$

$$= \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a-bx^2\right)^{3/4}} + \frac{10\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0342762, size = 77, normalized size = 0.76

$$\frac{5x\left(a-bx^2\right)\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right) + 2x\left(8a-5bx^2\right)}{21a^2\left(a-bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-11/4),x]
```

```
[Out] (2*x*(8*a - 5*b*x^2) + 5*x*(a - b*x^2)*(1 - (b*x^2)/a)^(3/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(21*a^2*(a - b*x^2)^(7/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(11/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(11/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-11/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x
)
```

Sympy [C] time = 2.93662, size = 26, normalized size = 0.26

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(11/4),x)
```

```
[Out] x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(11/4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-11/4), x)
```

3.863 $\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=99

$$\frac{2}{39}\left(3x^2+2\right)^{3/4}x^5-\frac{40\left(3x^2+2\right)^{3/4}x^3}{1053}+\frac{32\left(3x^2+2\right)^{3/4}x}{1053}-\frac{128x}{1053\sqrt[4]{3x^2+2}}+\frac{128\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

```
[Out] (-128*x)/(1053*(2 + 3*x^2)^(1/4)) + (32*x*(2 + 3*x^2)^(3/4))/1053 - (40*x^3
*(2 + 3*x^2)^(3/4))/1053 + (2*x^5*(2 + 3*x^2)^(3/4))/39 + (128*2^(1/4)*Elli
pticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])
```

Rubi [A] time = 0.0282477, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 227, 196}

$$\frac{2}{39}\left(3x^2+2\right)^{3/4}x^5-\frac{40\left(3x^2+2\right)^{3/4}x^3}{1053}+\frac{32\left(3x^2+2\right)^{3/4}x}{1053}-\frac{128x}{1053\sqrt[4]{3x^2+2}}+\frac{128\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (-128*x)/(1053*(2 + 3*x^2)^(1/4)) + (32*x*(2 + 3*x^2)^(3/4))/1053 - (40*x^3
*(2 + 3*x^2)^(3/4))/1053 + (2*x^5*(2 + 3*x^2)^(3/4))/39 + (128*2^(1/4)*Elli
pticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} - \frac{20}{39}\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx$$
$$= \frac{32x\left(2+3x^2\right)^{3/4}}{1053} - \frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} - \frac{64\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx}{1053}$$
$$= -\frac{128x}{1053\sqrt[4]{2+3x^2}} + \frac{32x\left(2+3x^2\right)^{3/4}}{1053} - \frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} + \frac{128\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx}{1053}$$
$$= -\frac{128x}{1053\sqrt[4]{2+3x^2}} + \frac{32x\left(2+3x^2\right)^{3/4}}{1053} - \frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} + \frac{128\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\right.\right.}{1053\sqrt{3}}$$

Mathematica [C] time = 0.026938, size = 54, normalized size = 0.55

$$\frac{2x\left(\left(3x^2+2\right)^{3/4}\left(27x^4-20x^2+16\right)-16\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(3/4)*(16 - 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric2
F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/1053
```

Maple [C] time = 0.028, size = 43, normalized size = 0.4

$$\frac{2x\left(27x^4-20x^2+16\right)}{1053}\left(3x^2+2\right)^{\frac{3}{4}} - \frac{32\,2^{3/4}x}{1053}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4-20*x^2+16)*(3*x^2+2)^(3/4)-32/1053*2^(3/4)*x*hypergeom([1/
4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(3\,x^2+2)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] time = 0.786577, size = 27, normalized size = 0.27

$$\frac{2^{\frac{3}{4}}x^7{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

3.864 $\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{2}{27}\left(3x^2+2\right)^{3/4}x^3-\frac{8}{135}\left(3x^2+2\right)^{3/4}x+\frac{32x}{135\sqrt[4]{3x^2+2}}-\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

```
[Out] (32*x)/(135*(2 + 3*x^2)^(1/4)) - (8*x*(2 + 3*x^2)^(3/4))/135 + (2*x^3*(2 +
3*x^2)^(3/4))/27 - (32*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(135*Sq
rt[3])
```

Rubi [A] time = 0.0201116, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 227, 196}

$$\frac{2}{27}\left(3x^2+2\right)^{3/4}x^3-\frac{8}{135}\left(3x^2+2\right)^{3/4}x+\frac{32x}{135\sqrt[4]{3x^2+2}}-\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (32*x)/(135*(2 + 3*x^2)^(1/4)) - (8*x*(2 + 3*x^2)^(3/4))/135 + (2*x^3*(2 +
3*x^2)^(3/4))/27 - (32*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(135*Sq
rt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx &= \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} - \frac{4}{9}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx \\ &= -\frac{8}{135}x\left(2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} + \frac{16}{135}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx \\ &= \frac{32x}{135\sqrt[4]{2+3x^2}} - \frac{8}{135}x\left(2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} - \frac{32}{135}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx \\ &= \frac{32x}{135\sqrt[4]{2+3x^2}} - \frac{8}{135}x\left(2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} - \frac{32\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0170571, size = 49, normalized size = 0.6

$$\frac{2}{135}x\left(4\ 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+\left(3x^2+2\right)^{3/4}\left(5x^2-4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(3/4)*(-4 + 5*x^2) + 4*2^(3/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, (-3*x^2)/2]))/135
```

Maple [C] time = 0.02, size = 38, normalized size = 0.5

$$\frac{2\,x\left(5\,x^2-4\right)}{135}\left(3\,x^2+2\right)^{\frac{3}{4}}+\frac{8\,2^{3/4}x}{135}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2+2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2-4)*(3*x^2+2)^(3/4)+8/135*2^(3/4)*x*hypergeom([1/4,1/2],[3/2]
,-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(3\,x^2+2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] time = 0.663713, size = 27, normalized size = 0.33

$$\frac{2^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

3.865 $\int \frac{x^2}{\sqrt[4]{2+3x^2}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{15}\left(3x^2+2\right)^{3/4}x-\frac{8x}{15\sqrt[4]{3x^2+2}}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

```
[Out] (-8*x)/(15*(2 + 3*x^2)^(1/4)) + (2*x*(2 + 3*x^2)^(3/4))/15 + (8*2^(1/4)*Ell
ipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rubi [A] time = 0.012527, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 227, 196}

$$\frac{2}{15}\left(3x^2+2\right)^{3/4}x-\frac{8x}{15\sqrt[4]{3x^2+2}}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (-8*x)/(15*(2 + 3*x^2)^(1/4)) + (2*x*(2 + 3*x^2)^(3/4))/15 + (8*2^(1/4)*Ell
ipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \frac{2}{15}x\left(2+3x^2\right)^{3/4} - \frac{4}{15}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{8x}{15\sqrt[4]{2+3x^2}} + \frac{2}{15}x\left(2+3x^2\right)^{3/4} + \frac{8}{15}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx$$
$$= -\frac{8x}{15\sqrt[4]{2+3x^2}} + \frac{2}{15}x\left(2+3x^2\right)^{3/4} + \frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

Mathematica [C] time = 0.0091322, size = 41, normalized size = 0.65

$$\frac{2}{15}x\left(\left(3x^2+2\right)^{3/4} - 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)
/2]))/15
```

Maple [C] time = 0.02, size = 31, normalized size = 0.5

$$\frac{2x}{15}\left(3x^2+2\right)^{\frac{3}{4}} - \frac{2\,2^{3/4}x}{15}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2+2)^(3/4)-2/15*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(3x^2+2\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] time = 0.597907, size = 27, normalized size = 0.43

$$\frac{2^{\frac{3}{4}} x^{3} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{3x^{2} e^{i\pi}}{2}} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(3 x^{2} + 2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

3.866 $\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] (2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/S
qrt[3]
```

Rubi [A] time = 0.0069099, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {227, 196}

$$\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)^(-1/4),x]
```

```
[Out] (2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/S
qrt[3]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx &= \frac{2x}{\sqrt[4]{2+3x^2}}-2\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx\\ &= \frac{2x}{\sqrt[4]{2+3x^2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0033747, size = 24, normalized size = 0.56

$$\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)^(-1/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/2^(1/4)
```

Maple [C] time = 0.01, size = 18, normalized size = 0.4

$$\frac{2^{\frac{3}{4}}x}{2}{}_2\mathrm{F}_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/4),x)
```

```
[Out] 1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(3x^2+2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(-1/4), x)
```

Sympy [C] time = 0.58462, size = 26, normalized size = 0.6

$$\frac{2^{\frac{3}{4}}x{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)^(-1/4), x)
```

3.867 $\int \frac{1}{x^2 \sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{3x}{2\sqrt[4]{3x^2+2}}-\frac{\left(3x^2+2\right)^{3/4}}{2x}-\frac{\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

```
[Out] (3*x)/(2*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[
ArcTan[Sqrt[3/2]*x]/2, 2])/2^(3/4)
```

Rubi [A] time = 0.0127797, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 227, 196}

$$\frac{3x}{2\sqrt[4]{3x^2+2}}-\frac{\left(3x^2+2\right)^{3/4}}{2x}-\frac{\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (3*x)/(2*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[
ArcTan[Sqrt[3/2]*x]/2, 2])/2^(3/4)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx = -\frac{\left(2+3x^2\right)^{3/4}}{2x} + \frac{3}{4}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= \frac{3x}{2\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{2x} - \frac{3}{2}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx$$
$$= \frac{3x}{2\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

Mathematica [C] time = 0.0054375, size = 27, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2]/(2^(1/4)*x))
```

Maple [C] time = 0.021, size = 33, normalized size = 0.5

$$-\frac{1}{2x}\left(3x^2+2\right)^{\frac{3}{4}} + \frac{3\,2^{3/4}x}{8}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2+2)^(1/4),x)
```

```
[Out] -1/2*(3*x^2+2)^(3/4)/x+3/8*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3x^2+2\right)^{\frac{3}{4}}}{3x^4+2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^4 + 2*x^2), x)
```

Sympy [C] time = 0.664222, size = 29, normalized size = 0.46

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

3.868 $\int \frac{1}{x^4 \sqrt[4]{2+3x^2}} \, dx$

Optimal. Leaf size=83

$$-\frac{9x}{8\sqrt[4]{3x^2+2}}+\frac{3\left(3x^2+2\right)^{3/4}}{8x}-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}+\frac{3\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

```
[Out] (-9*x)/(8*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(6*x^3) + (3*(2 + 3*x^2)^(
3/4))/(8*x) + (3*Sqrt[3]*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rubi [A] time = 0.0192217, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 227, 196}

$$-\frac{9x}{8\sqrt[4]{3x^2+2}}+\frac{3\left(3x^2+2\right)^{3/4}}{8x}-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}+\frac{3\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-9*x)/(8*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(6*x^3) + (3*(2 + 3*x^2)^(
3/4))/(8*x) + (3*Sqrt[3]*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx &= -\frac{\left(2+3x^2\right)^{3/4}}{6x^3} - \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx \\ &= -\frac{\left(2+3x^2\right)^{3/4}}{6x^3} + \frac{3\left(2+3x^2\right)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx \\ &= -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{6x^3} + \frac{3\left(2+3x^2\right)^{3/4}}{8x} + \frac{9}{8}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx \\ &= -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{6x^3} + \frac{3\left(2+3x^2\right)^{3/4}}{8x} + \frac{3\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.006043, size = 29, normalized size = 0.35

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3\sqrt[4]{2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2]/(3*2^(1/4)*x^3)
```

Maple [C] time = 0.021, size = 45, normalized size = 0.5

$$\frac{27\,x^4+6\,x^2-8}{24\,x^3}\frac{1}{\sqrt[4]{3\,x^2+2}} - \frac{9\,2^{3/4}x}{32}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2+2)^(1/4),x)
```

```
[Out] 1/24*(27*x^4+6*x^2-8)/x^3/(3*x^2+2)^(1/4)-9/32*2^(3/4)*x*hypergeom([1/4,1/2
],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x^2+2)^{\frac{3}{4}}}{3x^6+2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^6 + 2*x^4), x)
```

Sympy [C] time = 0.814624, size = 32, normalized size = 0.39

$$-\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+2)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

3.869 $\int \frac{1}{x^6 \sqrt[4]{2+3x^2}} \, dx$

Optimal. Leaf size=101

$$\frac{189x}{160\sqrt[4]{3x^2+2}} - \frac{63\left(3x^2+2\right)^{3/4}}{160x} + \frac{7\left(3x^2+2\right)^{3/4}}{40x^3} - \frac{\left(3x^2+2\right)^{3/4}}{10x^5} - \frac{63\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

```
[Out] (189*x)/(160*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(10*x^5) + (7*(2 + 3*x^
2)^(3/4))/(40*x^3) - (63*(2 + 3*x^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE
[ArcTan[Sqrt[3/2]*x]/2, 2])/(80*2^(3/4))
```

Rubi [A] time = 0.0295966, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 227, 196}

$$\frac{189x}{160\sqrt[4]{3x^2+2}} - \frac{63\left(3x^2+2\right)^{3/4}}{160x} + \frac{7\left(3x^2+2\right)^{3/4}}{40x^3} - \frac{\left(3x^2+2\right)^{3/4}}{10x^5} - \frac{63\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (189*x)/(160*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(10*x^5) + (7*(2 + 3*x^
2)^(3/4))/(40*x^3) - (63*(2 + 3*x^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE
[ArcTan[Sqrt[3/2]*x]/2, 2])/(80*2^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx &= -\frac{\left(2+3x^2\right)^{3/4}}{10x^5}-\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx \\
&= -\frac{\left(2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(2+3x^2\right)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx \\
&= -\frac{\left(2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(2+3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2+3x^2\right)^{3/4}}{160x}+\frac{189}{320}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx \\
&= \frac{189x}{160\sqrt[4]{2+3x^2}}-\frac{\left(2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(2+3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2+3x^2\right)^{3/4}}{160x}-\frac{189}{160}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx \\
&= \frac{189x}{160\sqrt[4]{2+3x^2}}-\frac{\left(2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(2+3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2+3x^2\right)^{3/4}}{160x}-\frac{63\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0050861, size = 29, normalized size = 0.29

$$
-\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5\sqrt[4]{2}x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] -Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2]/(5*2^(1/4)*x^5)
```

Maple [C] time = 0.022, size = 50, normalized size = 0.5

$$
-\frac{189\,x^6+42\,x^4-8\,x^2+32}{160\,x^5}\frac{1}{\sqrt[4]{3\,x^2+2}}+\frac{189\,2^{3/4}x}{640}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2+2)^(1/4),x)
```

```
[Out] -1/160*(189*x^6+42*x^4-8*x^2+32)/x^5/(3*x^2+2)^(1/4)+189/640*2^(3/4)*x*hype
rgeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^6}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x^2+2)^{\frac{3}{4}}}{3x^8+2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^8 + 2*x^6), x)
```

Sympy [C] time = 1.02844, size = 32, normalized size = 0.32

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(3x^2+2)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

3.870 $\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{39}\left(2-3x^2\right)^{3/4}x^5-\frac{40\left(2-3x^2\right)^{3/4}x^3}{1053}-\frac{32\left(2-3x^2\right)^{3/4}x}{1053}+\frac{128\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

```
[Out] (-32*x*(2 - 3*x^2)^(3/4))/1053 - (40*x^3*(2 - 3*x^2)^(3/4))/1053 - (2*x^5*(
2 - 3*x^2)^(3/4))/39 + (128*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(1
053*Sqrt[3])
```

Rubi [A] time = 0.0224705, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 228}

$$-\frac{2}{39}\left(2-3x^2\right)^{3/4}x^5-\frac{40\left(2-3x^2\right)^{3/4}x^3}{1053}-\frac{32\left(2-3x^2\right)^{3/4}x}{1053}+\frac{128\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-32*x*(2 - 3*x^2)^(3/4))/1053 - (40*x^3*(2 - 3*x^2)^(3/4))/1053 - (2*x^5*(
2 - 3*x^2)^(3/4))/39 + (128*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(1
053*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx &= -\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{20}{39}\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{40x^3\left(2-3x^2\right)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{80}{351}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{32x\left(2-3x^2\right)^{3/4}}{1053}-\frac{40x^3\left(2-3x^2\right)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{64\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx}{1053}\\
&= -\frac{32x\left(2-3x^2\right)^{3/4}}{1053}-\frac{40x^3\left(2-3x^2\right)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{128\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0244062, size = 54, normalized size = 0.65

$$-\frac{2x\left(\left(2-3x^2\right)^{3/4}\left(27x^4+20x^2+16\right)-16\ 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4)*(16 + 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/1053
```

Maple [C] time = 0.031, size = 50, normalized size = 0.6

$$\frac{2x\left(27x^4+20x^2+16\right)\left(3x^2-2\right)}{1053}\frac{1}{\sqrt[4]{-3x^2+2}}+\frac{32\,2^{3/4}x}{1053}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4+20*x^2+16)*(3*x^2-2)/(-3*x^2+2)^(1/4)+32/1053*2^(3/4)*x*hy
pergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}x^6}{3x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^6/(3*x^2 - 2), x)
```

Sympy [C] time = 0.803028, size = 29, normalized size = 0.35

$$\frac{2^{\frac{3}{4}}x^7{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix} \middle| \frac{3x^2e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

3.871 $\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2}{27}\left(2-3x^2\right)^{3/4}x^3-\frac{8}{135}\left(2-3x^2\right)^{3/4}x+\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

```
[Out] (-8*x*(2 - 3*x^2)^(3/4))/135 - (2*x^3*(2 - 3*x^2)^(3/4))/27 + (32*2^(1/4)*E
llipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(135*Sqrt[3])
```

Rubi [A] time = 0.0144044, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 228}

$$-\frac{2}{27}\left(2-3x^2\right)^{3/4}x^3-\frac{8}{135}\left(2-3x^2\right)^{3/4}x+\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-8*x*(2 - 3*x^2)^(3/4))/135 - (2*x^3*(2 - 3*x^2)^(3/4))/27 + (32*2^(1/4)*E
llipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(135*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx &= -\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{4}{9}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0192284, size = 49, normalized size = 0.75

$$-\frac{2}{135}x\left(\left(2-3x^2\right)^{3/4}\left(5x^2+4\right)-4\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4)*(4 + 5*x^2) - 4*2^(3/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, (3*x^2)/2]))/135
```

Maple [C] time = 0.026, size = 45, normalized size = 0.7

$$\frac{2\,x\left(5\,x^{2}+4\right)\left(3\,x^{2}-2\right)}{135}\frac{1}{\sqrt[4]{-3\,x^{2}+2}}+\frac{8\,2^{3/4}x}{135}{}_{2}\mathrm{F}_{1}(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3\,x^{2}}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2+4)*(3*x^2-2)/(-3*x^2+2)^(1/4)+8/135*2^(3/4)*x*hypergeom([1/4
,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(-3\,x^{2}+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3\,x^{2}+2\right)^{\frac{3}{4}}x^{4}}{3\,x^{2}-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^4/(3*x^2 - 2), x)
```

Sympy [C] time = 0.680588, size = 29, normalized size = 0.45

$$\frac{2^{\frac{3}{4}}x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^{2}e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(1/4), x)
```

3.872 $\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

```
[Out] (-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2
])/(15*Sqrt[3])
```

Rubi [A] time = 0.0083717, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 228}

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2
])/(15*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx &= -\frac{2}{15}x\left(2-3x^2\right)^{3/4}+\frac{4}{15}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\\ &= -\frac{2}{15}x\left(2-3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0080976, size = 41, normalized size = 0.87

$$-\frac{2}{15}x\left(\left(2-3x^2\right)^{3/4}-2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)
/2]))/15
```

Maple [C] time = 0.026, size = 38, normalized size = 0.8

$$\frac{2x\left(3x^2-2\right)}{15}\frac{1}{\sqrt[4]{-3x^2+2}}+\frac{2\,2^{3/4}x}{15}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2-2)/(-3*x^2+2)^(1/4)+2/15*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],
3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}x^2}{3x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^2/(3*x^2 - 2), x)
```

Sympy [C] time = 0.610562, size = 29, normalized size = 0.62

$$\frac{2^{\frac{3}{4}}x^3{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

3.873 $\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx$

Optimal. Leaf size=28

$$\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] (2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rubi [A] time = 0.0030895, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {228}

$$\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)^(-1/4),x]
```

```
[Out] (2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx = \frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.0030215, size = 24, normalized size = 0.86

$$\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)^(-1/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/2^(1/4)
```

Maple [C] time = 0.018, size = 18, normalized size = 0.6

$$\frac{2^{\frac{3}{4}}x}{2}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4),x)
```

```
[Out] 1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 + 2)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^2 - 2), x)
```

Sympy [C] time = 0.589562, size = 27, normalized size = 0.96

$$\frac{2^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 + 2)^(-1/4), x)
```

3.874 $\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx$

Optimal. Leaf size=47

$$-\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2^
(3/4)
```

Rubi [A] time = 0.0082851, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 228}

$$-\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2^
(3/4)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx &= -\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{3}{4}\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx\\ &= -\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.0043975, size = 27, normalized size = 0.57

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2]/(2^(1/4)*x))
```

Maple [C] time = 0.027, size = 40, normalized size = 0.9

$$\frac{3x^{2}-2}{2x}\frac{1}{\sqrt[4]{-3x^{2}+2}}-\frac{3\,2^{3/4}x}{8}{}_{2}\mathrm{F}_{1}(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^{2}}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(1/4),x)
```

```
[Out] 1/2*(3*x^2-2)/x/(-3*x^2+2)^(1/4)-3/8*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/
2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^{2}+2\right)^{\frac{1}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3x^{2}+2\right)^{\frac{3}{4}}}{3x^{4}-2x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] time = 0.677284, size = 31, normalized size = 0.66

$$-\frac{2^{\frac{3}{4}}{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^{2}e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

3.875 $\int \frac{1}{x^4 \sqrt[4]{2-3x^2}} \, dx$

Optimal. Leaf size=67

$$-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(6*x^3) - (3*(2 - 3*x^2)^(3/4))/(8*x) - (3*Sqrt[3]*Ellip
ticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rubi [A] time = 0.0144736, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 228}

$$-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(6*x^3) - (3*(2 - 3*x^2)^(3/4))/(8*x) - (3*Sqrt[3]*Ellip
ticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[4]{2-3x^2}} \, dx &= -\frac{\left(2-3x^2\right)^{3/4}}{6x^3}+\frac{3}{4}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx\\
&= -\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{9}{16}\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx\\
&= -\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0047798, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3\sqrt[4]{2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2]/(3*2^(1/4)*x^3)
```

Maple [C] time = 0.027, size = 45, normalized size = 0.7

$$\frac{27x^4-6x^2-8}{24x^3}\frac{1}{\sqrt[4]{-3x^2+2}}-\frac{9\,2^{3/4}x}{32}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(1/4),x)
```

```
[Out] 1/24*(27*x^4-6*x^2-8)/x^3/(-3*x^2+2)^(1/4)-9/32*2^(3/4)*x*hypergeom([1/4,1/
2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}}{3x^6-2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] time = 0.813079, size = 34, normalized size = 0.51

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

3.876 $\int \frac{1}{x^6 \sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(10*x^5) - (7*(2 - 3*x^2)^(3/4))/(40*x^3) - (63*(2 - 3*x
^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(80*2
^(3/4))
```

Rubi [A] time = 0.0212376, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 228}

$$-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(10*x^5) - (7*(2 - 3*x^2)^(3/4))/(40*x^3) - (63*(2 - 3*x
^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(80*2
^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[4]{2-3x^2}}\,dx &= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}+\frac{21}{20}\int \frac{1}{x^4 \sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{189}{320}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{63\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0044946, size = 29, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5\sqrt[4]{2}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2]/(5*2^(1/4)*x^5)
```

Maple [C] time = 0.028, size = 50, normalized size = 0.6

$$\frac{189\,x^6-42\,x^4-8\,x^2-32}{160\,x^5}\frac{1}{\sqrt[4]{-3\,x^2+2}}-\frac{189\,2^{3/4}x}{640}{}_2\mathrm{F}_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3\,x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2+2)^(1/4),x)
```

```
[Out] 1/160*(189*x^6-42*x^4-8*x^2-32)/x^5/(-3*x^2+2)^(1/4)-189/640*2^(3/4)*x*hype
rgeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^8-2\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] time = 1.00831, size = 34, normalized size = 0.4

$$-\frac{2^{\frac{3}{4}} {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

3.877 $\int \frac{x^6}{\left(2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{320\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2079\sqrt{3}}+\frac{2}{33}\sqrt[4]{3x^2+2}x^5-\frac{40}{693}\sqrt[4]{3x^2+2}x^3+\frac{160\sqrt[4]{3x^2+2}x}{2079}$$

```
[Out] (160*x*(2 + 3*x^2)^(1/4))/2079 - (40*x^3*(2 + 3*x^2)^(1/4))/693 + (2*x^5*(2
 + 3*x^2)^(1/4))/33 - (320*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(20
79*Sqrt[3])
```

Rubi [A] time = 0.0249696, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 231}

$$\frac{2}{33}\sqrt[4]{3x^2+2}x^5-\frac{40}{693}\sqrt[4]{3x^2+2}x^3+\frac{160\sqrt[4]{3x^2+2}x}{2079}-\frac{320\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2079\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (160*x*(2 + 3*x^2)^(1/4))/2079 - (40*x^3*(2 + 3*x^2)^(1/4))/693 + (2*x^5*(2
 + 3*x^2)^(1/4))/33 - (320*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(20
79*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(2+3x^2\right)^{3/4}}\,dx &= \frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{20}{33}\int \frac{x^4}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= -\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}+\frac{80}{231}\int \frac{x^2}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= \frac{160x\sqrt[4]{2+3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{320\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx}{2079}\\ &= \frac{160x\sqrt[4]{2+3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{320\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2079\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0193365, size = 54, normalized size = 0.65

$$\frac{2x\left(\sqrt[4]{3x^2+2}\left(63x^4-60x^2+80\right)-80\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{2079}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(1/4)*(80 - 60*x^2 + 63*x^4) - 80*2^(1/4)*Hypergeometric2
F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/2079
```

Maple [C] time = 0.031, size = 43, normalized size = 0.5

$$\frac{2\,x\left(63\,x^4-60\,x^2+80\right)}{2079}\sqrt[4]{3\,x^2+2}-\frac{160\,\sqrt[4]{2}x}{2079}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\,\frac{3}{2};\,-\frac{3\,x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2)^(3/4),x)
```

```
[Out] 2/2079*x*(63*x^4-60*x^2+80)*(3*x^2+2)^(1/4)-160/2079*2^(1/4)*x*hypergeom([1
/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\left(3\,x^2+2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] time = 0.773891, size = 27, normalized size = 0.33

$$\frac{\sqrt[4]{2}x^7{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

3.878 $\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=65

$$\frac{16\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{63\sqrt{3}}+\frac{2}{21}\sqrt[4]{3x^2+2}x^3-\frac{8}{63}\sqrt[4]{3x^2+2}x$$

```
[Out] (-8*x*(2 + 3*x^2)^(1/4))/63 + (2*x^3*(2 + 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rubi [A] time = 0.0148995, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 231}

$$\frac{2}{21}\sqrt[4]{3x^2+2}x^3-\frac{8}{63}\sqrt[4]{3x^2+2}x+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (-8*x*(2 + 3*x^2)^(1/4))/63 + (2*x^3*(2 + 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(2+3x^2\right)^{3/4}}\,dx &= \frac{2}{21}x^3\sqrt[4]{2+3x^2}-\frac{4}{7}\int \frac{x^2}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= -\frac{8}{63}x\sqrt[4]{2+3x^2}+\frac{2}{21}x^3\sqrt[4]{2+3x^2}+\frac{16}{63}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= -\frac{8}{63}x\sqrt[4]{2+3x^2}+\frac{2}{21}x^3\sqrt[4]{2+3x^2}+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0170208, size = 49, normalized size = 0.75

$$\frac{2}{63}x\left(4\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+\sqrt[4]{3x^2+2}\,(3x^2-4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((-4 + 3*x^2)*(2 + 3*x^2)^(1/4) + 4*2^(1/4)*Hypergeometric2F1[1/2, 3/4
, 3/2, (-3*x^2)/2]))/63
```

Maple [C] time = 0.02, size = 38, normalized size = 0.6

$$\frac{2\,x\left(3\,x^2-4\right)}{63}\sqrt[4]{3\,x^2+2}+\frac{8\,\sqrt[4]{2}x}{63}\,{}_2\mathrm{F}_1(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3\,x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2+2)^(3/4),x)
```

```
[Out] 2/63*x*(3*x^2-4)*(3*x^2+2)^(1/4)+8/63*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-
3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^4}{\left(3\,x^2+2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] time = 0.651126, size = 27, normalized size = 0.42

$$\frac{\sqrt[4]{2}x^5{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

3.879 $\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=47

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\,2^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

```
[Out] (2*x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])
/(9*Sqrt[3])
```

Rubi [A] time = 0.0086441, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 231}

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])
/(9*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(2+3x^2\right)^{3/4}}\,dx &= \frac{2}{9}x\sqrt[4]{2+3x^2}-\frac{4}{9}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= \frac{2}{9}x\sqrt[4]{2+3x^2}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0080869, size = 41, normalized size = 0.87

$$\frac{2}{9}x\left(\sqrt[4]{3x^2+2}-\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(1/4) - 2^(1/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)
/2]))/9
```

Maple [C] time = 0.02, size = 31, normalized size = 0.7

$$\frac{2x}{9}\sqrt[4]{3x^2+2}-\frac{2\sqrt[4]{2}x}{9}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(3/4),x)
```

```
[Out] 2/9*x*(3*x^2+2)^(1/4)-2/9*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\left(3x^2+2\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] time = 0.609564, size = 27, normalized size = 0.57

$$\frac{\sqrt[4]{2}x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

3.880 $\int \frac{1}{(2+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=27

$$\frac{2^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

```
[Out] (2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rubi [A] time = 0.0029293, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {231}

$$\frac{2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(2+3x^2)^{3/4}} \, dx = \frac{2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.003973, size = 24, normalized size = 0.89

$$\frac{x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)^(-3/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/2^(3/4)
```

Maple [C] time = 0.013, size = 18, normalized size = 0.7

$$\frac{\sqrt[4]{2}x}{2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(3/4),x)
```

```
[Out] 1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2+2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(-3/4), x)
```

Sympy [C] time = 0.588789, size = 26, normalized size = 0.96

$$\frac{\sqrt[4]{2}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)^(-3/4), x)
```

3.881 $\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3x^2+2}}{2x}$$

```
[Out] -(2 + 3*x^2)^(1/4)/(2*x) - (Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rubi [A] time = 0.0083085, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 231}

$$-\frac{\sqrt[4]{3x^2+2}}{2x}-\frac{\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(2 + 3*x^2)^(1/4)/(2*x) - (Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2+3x^2}}{2x}-\frac{3}{4}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{2+3x^2}}{2x}-\frac{\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.0047161, size = 27, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2]/(2^(3/4)*x))
```

Maple [C] time = 0.021, size = 33, normalized size = 0.7

$$-\frac{1}{2x}\sqrt[4]{3x^2+2}-\frac{3\sqrt[4]{2}x}{8}{}_2\mathrm{F}_1(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2+2)^(3/4),x)
```

```
[Out] -1/2*(3*x^2+2)^(1/4)/x-3/8*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(3x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^2+2\right)^{\frac{1}{4}}}{3x^4+2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^4 + 2*x^2), x)
```

Sympy [C] time = 0.768287, size = 29, normalized size = 0.59

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

3.882 $\int \frac{1}{x^4(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=67

$$\frac{5\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{8\sqrt[4]{2}}+\frac{5\sqrt[4]{3x^2+2}}{8x}-\frac{\sqrt[4]{3x^2+2}}{6x^3}$$

```
[Out] -(2 + 3*x^2)^(1/4)/(6*x^3) + (5*(2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rubi [A] time = 0.0148175, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 231}

$$\frac{5\sqrt[4]{3x^2+2}}{8x}-\frac{\sqrt[4]{3x^2+2}}{6x^3}+\frac{5\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(2 + 3*x^2)^(1/4)/(6*x^3) + (5*(2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(2+3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2+3x^2}}{6x^3}-\frac{5}{4}\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{2+3x^2}}{6x^3}+\frac{5\sqrt[4]{2+3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{2+3x^2}}{6x^3}+\frac{5\sqrt[4]{2+3x^2}}{8x}+\frac{5\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.0048023, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3\ 2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2]/(3*2^(3/4)*x^3)
```

Maple [C] time = 0.021, size = 45, normalized size = 0.7

$$\frac{45x^4+18x^2-8}{24x^3}\left(3x^2+2\right)^{-\frac{3}{4}}+\frac{15\sqrt[4]{2}x}{32}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2+2)^(3/4),x)
```

```
[Out] 1/24*(45*x^4+18*x^2-8)/x^3/(3*x^2+2)^(3/4)+15/32*2^(1/4)*x*hypergeom([1/2,3
/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^2+2\right)^{\frac{1}{4}}}{3x^6+2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^6 + 2*x^4), x)
```

Sympy [C] time = 0.963091, size = 32, normalized size = 0.48

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

3.883 $\int \frac{1}{x^6\left(2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}-\frac{27\sqrt[4]{3x^2+2}}{32x}+\frac{9\sqrt[4]{3x^2+2}}{40x^3}-\frac{\sqrt[4]{3x^2+2}}{10x^5}$$

```
[Out] -(2 + 3*x^2)^(1/4)/(10*x^5) + (9*(2 + 3*x^2)^(1/4))/(40*x^3) - (27*(2 + 3*x
^2)^(1/4))/(32*x) - (27*Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rubi [A] time = 0.0227747, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 231}

$$-\frac{27\sqrt[4]{3x^2+2}}{32x}+\frac{9\sqrt[4]{3x^2+2}}{40x^3}-\frac{\sqrt[4]{3x^2+2}}{10x^5}-\frac{27\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(2 + 3*x^2)^(1/4)/(10*x^5) + (9*(2 + 3*x^2)^(1/4))/(40*x^3) - (27*(2 + 3*x
^2)^(1/4))/(32*x) - (27*Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6\left(2+3x^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{2+3x^2}}{10x^5} - \frac{27}{20}\int \frac{1}{x^4\left(2+3x^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2+3x^2}}{10x^5} + \frac{9\sqrt[4]{2+3x^2}}{40x^3} + \frac{27}{16}\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2+3x^2}}{10x^5} + \frac{9\sqrt[4]{2+3x^2}}{40x^3} - \frac{27\sqrt[4]{2+3x^2}}{32x} - \frac{81}{64}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2+3x^2}}{10x^5} + \frac{9\sqrt[4]{2+3x^2}}{40x^3} - \frac{27\sqrt[4]{2+3x^2}}{32x} - \frac{27\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Mathematica [C] time = 0.0051366, size = 29, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5\ 2^{3/4}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2]/(5*2^(3/4)*x^5)
```

Maple [C] time = 0.022, size = 50, normalized size = 0.6

$$-\frac{405\,x^6+162\,x^4-24\,x^2+32}{160\,x^5}\left(3\,x^2+2\right)^{-\frac{3}{4}} - \frac{81\,\sqrt[4]{2}x}{128}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\,\frac{3}{2};\,-\frac{3\,x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2+2)^(3/4),x)
```

```
[Out] -1/160*(405*x^6+162*x^4-24*x^2+32)/x^5/(3*x^2+2)^(3/4)-81/128*2^(1/4)*x*hyp
ergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x^2+2)^{\frac{1}{4}}}{3x^8+2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^8 + 2*x^6), x)
```

Sympy [C] time = 1.18746, size = 32, normalized size = 0.38

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+2)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

3.884 $\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{320\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2079\sqrt{3}}-\frac{2}{33}\sqrt[4]{2-3x^2}x^5-\frac{40}{693}\sqrt[4]{2-3x^2}x^3-\frac{160\sqrt[4]{2-3x^2}x}{2079}$$

```
[Out] (-160*x*(2 - 3*x^2)^(1/4))/2079 - (40*x^3*(2 - 3*x^2)^(1/4))/693 - (2*x^5*(
2 - 3*x^2)^(1/4))/33 + (320*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
079*Sqrt[3])
```

Rubi [A] time = 0.0226623, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 232}

$$-\frac{2}{33}\sqrt[4]{2-3x^2}x^5-\frac{40}{693}\sqrt[4]{2-3x^2}x^3-\frac{160\sqrt[4]{2-3x^2}x}{2079}+\frac{320\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2079\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-160*x*(2 - 3*x^2)^(1/4))/2079 - (40*x^3*(2 - 3*x^2)^(1/4))/693 - (2*x^5*(
2 - 3*x^2)^(1/4))/33 + (320*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
079*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{20}{33}\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{40}{693}x^3\sqrt[4]{2-3x^2} - \frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{80}{231}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{160x\sqrt[4]{2-3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2-3x^2} - \frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{320\int \frac{1}{(2-3x^2)^{3/4}}\,dx}{2079}$$
$$= -\frac{160x\sqrt[4]{2-3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2-3x^2} - \frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{320\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2079\sqrt{3}}$$

Mathematica [A] time = 0.0353716, size = 59, normalized size = 0.71

$$\frac{320\ 2^{3/4}\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right) - 6x\sqrt[4]{2-3x^2}\left(63x^4+60x^2+80\right)}{6237}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-6*x*(2 - 3*x^2)^(1/4)*(80 + 60*x^2 + 63*x^4) + 320*2^(3/4)*Sqrt[3]*Ellipt
icF[ArcSin[Sqrt[3/2]*x]/2, 2])/6237
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^6\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(x^6/(-3*x^2+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^6}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^6/(3*x^2 - 2), x)
```

Sympy [C] time = 0.80729, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}x^7{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

3.885 $\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=65

$$\frac{16\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{63\sqrt{3}}-\frac{2}{21}\sqrt[4]{2-3x^2}x^3-\frac{8}{63}\sqrt[4]{2-3x^2}x$$

```
[Out] (-8*x*(2 - 3*x^2)^(1/4))/63 - (2*x^3*(2 - 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rubi [A] time = 0.0155594, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 232}

$$-\frac{2}{21}\sqrt[4]{2-3x^2}x^3-\frac{8}{63}\sqrt[4]{2-3x^2}x+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-8*x*(2 - 3*x^2)^(1/4))/63 - (2*x^3*(2 - 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{4}{7}\int \frac{x^2}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{8}{63}x\sqrt[4]{2-3x^2}-\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{16}{63}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{8}{63}x\sqrt[4]{2-3x^2}-\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.024523, size = 54, normalized size = 0.83

$$-\frac{2}{189}\left(3x\sqrt[4]{2-3x^2}\left(3x^2+4\right)-8\,2^{3/4}\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*(3*x*(2 - 3*x^2)^(1/4)*(4 + 3*x^2) - 8*2^(3/4)*Sqrt[3]*EllipticF[ArcSin
[Sqrt[3/2]*x]/2, 2]))/189
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int x^4\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(x^4/(-3*x^2+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{1}{4}}x^4}{3x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^4/(3*x^2 - 2), x)
```

Sympy [C] time = 0.682182, size = 29, normalized size = 0.45

$$\frac{\sqrt[4]{2}x^5{}_2F_1\left(\begin{matrix} \frac{3}{4},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

3.886 $\int \frac{x^2}{(2-3x^2)^{3/4}} \, dx$

Optimal. Leaf size=47

$$\frac{4\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

```
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Rubi [A] time = 0.0085179, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 232}

$$\frac{4\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(2-3x^2)^{3/4}} \, dx &= -\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4}{9}\int \frac{1}{(2-3x^2)^{3/4}} \, dx\\ &= -\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0145681, size = 47, normalized size = 1.

$$\frac{4\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int x^{2} \left(-3 x^{2}+2\right)^{-\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(-3 x^{2}+2\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3 x^{2}+2\right)^{\frac{1}{4}} x^{2}}{3 x^{2}-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^2/(3*x^2 - 2), x)
```

Sympy [C] time = 0.635517, size = 29, normalized size = 0.62

$$\frac{\sqrt[4]{2} x^{3} {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{3 x^{2} e^{2 i \pi}}{2}} \right)}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

3.887 $\int \frac{1}{(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=27

$$\frac{2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

```
[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rubi [A] time = 0.0033811, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {232}

$$\frac{2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx=\frac{2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.002695, size = 27, normalized size = 1.

$$\frac{2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Maple [C] time = 0.02, size = 18, normalized size = 0.7

$$\frac{\sqrt[4]{2}x}{2}{}_2\mathrm{F}_1(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(3/4),x)
```

```
[Out] 1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 + 2)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{1}{4}}}{3x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^2 - 2), x)
```

Sympy [C] time = 0.610807, size = 27, normalized size = 1.

$$\frac{\sqrt[4]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 + 2)^(-3/4), x)
```

3.888 $\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{2x}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rubi [A] time = 0.0089599, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 232}

$$\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2-3x^2}}{2x}+\frac{3}{4}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{2-3x^2}}{2x}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.0049404, size = 27, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2]/(2^(3/4)*x))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(-3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(1/x^2/(-3*x^2+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{1}{4}}}{3x^4-2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] time = 0.768468, size = 31, normalized size = 0.63

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

3.889 $\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=67

$$\frac{5\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{8\sqrt[4]{2}}-\frac{5\sqrt[4]{2-3x^2}}{8x}-\frac{\sqrt[4]{2-3x^2}}{6x^3}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(6*x^3) - (5*(2 - 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rubi [A] time = 0.0150862, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 232}

$$-\frac{5\sqrt[4]{2-3x^2}}{8x}-\frac{\sqrt[4]{2-3x^2}}{6x^3}+\frac{5\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(6*x^3) - (5*(2 - 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2-3x^2}}{6x^3}+\frac{5}{4}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{5\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.0044461, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3\ 2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2]/(3*2^(3/4)*x^3)
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(1/x^4/(-3*x^2+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{1}{4}}}{3x^6-2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] time = 0.968133, size = 34, normalized size = 0.51

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

3.890 $\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{27\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}-\frac{27\sqrt[4]{2-3x^2}}{32x}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{\sqrt[4]{2-3x^2}}{10x^5}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(10*x^5) - (9*(2 - 3*x^2)^(1/4))/(40*x^3) - (27*(2 - 3*x
^2)^(1/4))/(32*x) + (27*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rubi [A] time = 0.0218717, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 232}

$$-\frac{27\sqrt[4]{2-3x^2}}{32x}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{\sqrt[4]{2-3x^2}}{10x^5}+\frac{27\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(10*x^5) - (9*(2 - 3*x^2)^(1/4))/(40*x^3) - (27*(2 - 3*x
^2)^(1/4))/(32*x) + (27*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2-3x^2}}{10x^5} + \frac{27}{20}\int \frac{1}{x^4(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2-3x^2}}{10x^5} - \frac{9\sqrt[4]{2-3x^2}}{40x^3} + \frac{27}{16}\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2-3x^2}}{10x^5} - \frac{9\sqrt[4]{2-3x^2}}{40x^3} - \frac{27\sqrt[4]{2-3x^2}}{32x} + \frac{81}{64}\int \frac{1}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2-3x^2}}{10x^5} - \frac{9\sqrt[4]{2-3x^2}}{40x^3} - \frac{27\sqrt[4]{2-3x^2}}{32x} + \frac{27\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Mathematica [C] time = 0.0047718, size = 29, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5\,2^{3/4}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2]/(5*2^(3/4)*x^5)
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(1/x^6/(-3*x^2+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-3\,x^{2}+2\right)^{\frac{1}{4}}}{3\,x^{8}-2\,x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] time = 1.15853, size = 34, normalized size = 0.4

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^{2}e^{2i\pi}}{2}\right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^{2}+2\right)^{\frac{3}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

3.891 $\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=258

$$\frac{64\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{1053\sqrt{3}\,x}+\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5+\frac{40\left(3x^2-2\right)^{3/4}x^3}{1053}+\frac{32\,(3\cdots}{}$$

```
[Out] (32*x*(-2 + 3*x^2)^(3/4))/1053 + (40*x^3*(-2 + 3*x^2)^(3/4))/1053 + (2*x^5*
(-2 + 3*x^2)^(3/4))/39 + (128*x*(-2 + 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[-
2 + 3*x^2])) - (128*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[
2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2]
)/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(
Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(1053*Sqrt[3]*x)
```

Rubi [A] time = 0.14362, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5+\frac{40\left(3x^2-2\right)^{3/4}x^3}{1053}+\frac{32\left(3x^2-2\right)^{3/4}x}{1053}+\frac{128\sqrt[4]{3x^2-2}\,x}{1053\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{64\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\cdots\right.}{1053\sqrt{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (32*x*(-2 + 3*x^2)^(3/4))/1053 + (40*x^3*(-2 + 3*x^2)^(3/4))/1053 + (2*x^5*
(-2 + 3*x^2)^(3/4))/39 + (128*x*(-2 + 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[-
2 + 3*x^2])) - (128*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[
2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2]
)/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(
Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(1053*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{20}{39}\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{64\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx}{1053}$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{\left(64\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx\right.}{1053x}$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{\left(128\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x\right.}{1053\sqrt{3}x}$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{128x\sqrt[4]{-2+3x^2}}{1053\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} - \frac{128\sqrt[4]{}}{}$$

Mathematica [C] time = 0.0169025, size = 68, normalized size = 0.26

$$\frac{2x\left(16\ 2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)+81x^6+6x^4+8x^2-32\right)}{1053\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-32 + 8*x^2 + 6*x^4 + 81*x^6 + 16*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeom
etric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/(1053*(-2 + 3*x^2)^(1/4))
```

Maple [C] time = 0.038, size = 65, normalized size = 0.3

$$\frac{2x\left(27x^4+20x^2+16\right)}{1053}\left(3x^2-2\right)^{\frac{3}{4}}+\frac{32\,2^{3/4}x}{1053}\sqrt[4]{-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\frac{1}{\sqrt[4]{\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4+20*x^2+16)*(3*x^2-2)^(3/4)+32/1053*2^(3/4)/signum(-1+3/2*x
^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^6}{\left(3x^2-2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] time = 0.809982, size = 29, normalized size = 0.11

$$\frac{2^{\frac{3}{4}}x^7e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(1/4), x)
```

3.892 $\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=240

$$\frac{16\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{135\sqrt{3}x}+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3+\frac{8}{135}\left(3x^2-2\right)^{3/4}x+\frac{3}{135\left(\right.}$$

```
[Out] (8*x*(-2 + 3*x^2)^(3/4))/135 + (2*x^3*(-2 + 3*x^2)^(3/4))/27 + (32*x*(-2 +
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 + 3*x^2])) - (32*2^(1/4)*Sqrt[x^2/(Sq
rt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTa
n[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) + (16*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rubi [A] time = 0.111758, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3+\frac{8}{135}\left(3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{3x^2-2}x}{135\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{16\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2}}{\sqrt[4]{2}}\right.\right.}{135\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 + 3*x^2)^(1/4),x]
```


```
[Out] (8*x*(-2 + 3*x^2)^(3/4))/135 + (2*x^3*(-2 + 3*x^2)^(3/4))/27 + (32*x*(-2 +
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 + 3*x^2])) - (32*2^(1/4)*Sqrt[x^2/(Sq
rt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTa
n[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) + (16*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{135x}$$

$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{\left(32\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{135\sqrt{3}x}-\frac{\left(32\sqrt{\right.}\cdots}{\cdots}$$

$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2+3x^2}}{135\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}-\frac{32\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.0149713, size = 63, normalized size = 0.26

$$\frac{2x\left(4\,2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)+15x^4+2x^2-8\right)}{135\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-8 + 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1
/4, 1/2, 3/2, (3*x^2)/2]))/(135*(-2 + 3*x^2)^(1/4))
```

Maple [C] time = 0.037, size = 60, normalized size = 0.3

$$\frac{2\,x\left(5\,x^2+4\right)}{135}\left(3\,x^2-2\right)^{\frac{3}{4}}+\frac{8\,2^{3/4}x}{135}\sqrt[4]{-\text{signum}\left(-1+\frac{3\,x^2}{2}\right)}\,{}_2\text{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3\,x^2}{2}\right)\frac{1}{\sqrt[4]{\text{signum}\left(-1+\frac{3\,x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2+4)*(3*x^2-2)^(3/4)+8/135*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-
signum(-1+3/2*x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(3x^2-2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] time = 0.695069, size = 29, normalized size = 0.12

$$\frac{2^{\frac{3}{4}} x^5 e^{-\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="giac")

[Out] integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

3.893 $\int \frac{x^2}{\sqrt[4]{-2+3x^2}} \, dx$

Optimal. Leaf size=222

$$\frac{4\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-2}x}{15\left(\sqrt{3x^2-2}+\sqrt{2}\right)}-\frac{8\sqrt[4]{2}\ldots}{}$$

```
[Out] (2*x*(-2 + 3*x^2)^(3/4))/15 + (8*x*(-2 + 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt[
-2 + 3*x^2])) - (8*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt
[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2
])/(15*Sqrt[3]*x)
```

Rubi [A] time = 0.090511, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$\frac{2}{15}\left(3x^2-2\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-2}x}{15\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{4\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}\ldots\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(3/4))/15 + (8*x*(-2 + 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt[
-2 + 3*x^2])) - (8*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt
[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2
])/(15*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{4}{15}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{15x}$$

$$= \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{\left(8\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{15\sqrt{3}x} - \frac{\left(8\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx, x,\right.}{15\sqrt{3}x}$$

$$= \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{8x\sqrt[4]{-2+3x^2}}{15\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} - \frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan^{-1}\right.}{15\sqrt{3}x}$$

Mathematica [C] time = 0.0113824, size = 57, normalized size = 0.26

$$\frac{2x\left(2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)+3x^2-2\right)}{15\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2 + 2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/
2, (3*x^2)/2]))/(15*(-2 + 3*x^2)^(1/4))
```

Maple [C] time = 0.035, size = 53, normalized size = 0.2

$$\frac{2x}{15}\left(3x^2-2\right)^{\frac{3}{4}} + \frac{2\,2^{3/4}x}{15}\sqrt[4]{-\text{signum}\left(-1+\frac{3x^2}{2}\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\frac{1}{\sqrt[4]{\text{signum}\left(-1+\frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2-2)^(3/4)+2/15*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/
2*x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(3\,x^2-2\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] time = 0.620308, size = 29, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}x^3e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

3.894 $\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=199

$$\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt{3}x}+\frac{2\sqrt[4]{3x^2-2}\,x}{\sqrt{3x^2-2}+\sqrt{2}}-\frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-\right.}{}$$

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) + (2^(1/4)*Sqrt[x^2/(S
qrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcT
an[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rubi [A] time = 0.0741029, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {230, 305, 220, 1196}

$$\frac{2\sqrt[4]{3x^2-2}\,x}{\sqrt{3x^2-2}+\sqrt{2}}+\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x}-\frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right.}{\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^2)^(-1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) + (2^(1/4)*Sqrt[x^2/(S
qrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcT
an[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \frac{\left(\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{x}$$

$$= \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{\sqrt{3}x} - \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{\sqrt{3}x}$$

$$= \frac{2x\sqrt[4]{-2+3x^2}}{\sqrt{2}+\sqrt{-2+3x^2}} - \frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x} + \frac{\sqrt[4]{2}\sqrt{\frac{x}{\left(\sqrt{2}+\sqrt{-\right.}}}}{}$$

Mathematica [C] time = 0.0057571, size = 43, normalized size = 0.22

$$\frac{x\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^2)^(-1/4),x]
```

```
[Out] (x*(1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/(-2 +
 3*x^2)^(1/4)
```

Maple [C] time = 0.028, size = 40, normalized size = 0.2

$$\frac{2^{\frac{3}{4}}x}{2}\sqrt[4]{-\text{signum}\left(-1+\frac{3x^2}{2}\right)}\,{}_2\text{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\frac{1}{\sqrt[4]{\text{signum}\left(-1+\frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(1/4),x)
```

```
[Out] 1/2*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hypergeo
m([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="maxima")

[Out] integrate((3*x^2 - 2)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="fricas")

[Out] integral((3*x^2 - 2)^(-1/4), x)
```

Sympy [C] time = 0.600331, size = 27, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}xe^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(1/4),x)

[Out] 2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2/2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="giac")

[Out] integrate((3*x^2 - 2)^(-1/4), x)
```

3.895 $\int \frac{1}{x^2 \sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=221

$$-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2\,2^{3/4}x}-\frac{3\sqrt[4]{3x^2-2}\,x}{2\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{\left(3x^2-2\right)^{3/4}}{2x}+\frac{\sqrt{3}\sqrt{\frac{\cdots}{\left(\sqrt{3x^2\cdots}\right.}}}{\cdots}$$

```
[Out] (-2 + 3*x^2)^(3/4)/(2*x) - (3*x*(-2 + 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sq
rt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3
/4)*x) - (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[
-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2*2^(3/
4)*x)
```

Rubi [A] time = 0.0896784, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{3\sqrt[4]{3x^2-2}\,x}{2\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2\,2^{3/4}x}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(2*x) - (3*x*(-2 + 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sq
rt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3
/4)*x) - (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[
-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2*2^(3/
4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{3}{4}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{2x}\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{3}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{2x}+\frac{\left(\sqrt{3}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx,\right.}{2x}\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{3x\sqrt[4]{-2+3x^2}}{2\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan^{-1}\left(\right.\right.}{2^{3/4}x}
\end{aligned}
$$

Mathematica [C] time = 0.0059243, size = 46, normalized size = 0.21

$$
-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{x\sqrt[4]{3x^2-2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -(((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2])/(x*(
-2 + 3*x^2)^(1/4)))
```

Maple [C] time = 0.037, size = 55, normalized size = 0.3

$$
\frac{1}{2x}\left(3x^2-2\right)^{\frac{3}{4}}-\frac{3\,2^{3/4}x}{8}\sqrt[4]{-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\frac{1}{\sqrt[4]{\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)^(1/4),x)
```

```
[Out] 1/2*(3*x^2-2)^(3/4)/x-3/8*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*
x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{3}{4}}}{3\,x^4-2\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] time = 0.685476, size = 31, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2/2)/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

3.896 $\int \frac{1}{x^4 \sqrt[4]{-2+3x^2}} \, dx$

Optimal. Leaf size=242

$$-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{8\ 2^{3/4}x}-\frac{9\sqrt[4]{3x^2-2}\,x}{8\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{3\left(3x^2-2\right)^{3/4}}{8x}+\frac{(3x^2}{}$$

```
[Out] (-2 + 3*x^2)^(3/4)/(6*x^3) + (3*(-2 + 3*x^2)^(3/4))/(8*x) - (9*x*(-2 + 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 + 3*x^2])) + (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2] +
 Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 +
 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) - (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2]
+ Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2
+ 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rubi [A] time = 0.109254, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{9\sqrt[4]{3x^2-2}\,x}{8\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{3\left(3x^2-2\right)^{3/4}}{8x}+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{8\ 2^{3/4}x}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(6*x^3) + (3*(-2 + 3*x^2)^(3/4))/(8*x) - (9*x*(-2 + 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 + 3*x^2])) + (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2] +
 Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 +
 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) - (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2]
+ Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2
+ 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{9}{16}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{\left(3\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{8x}$$

$$= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{\left(3\sqrt{3}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{8x}+\frac{\left(3\sqrt{3}\sqrt{x^2}\right)\text{Su}}{}$$

$$= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{9x\sqrt[4]{-2+3x^2}}{8\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right.}{4\,2^{3/4}x}$$

Mathematica [C] time = 0.0072733, size = 48, normalized size = 0.2

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2])/(3*x
^3*(-2 + 3*x^2)^(1/4))
```

Maple [C] time = 0.037, size = 67, normalized size = 0.3

$$\frac{27x^4-6x^2-8}{24x^3}\frac{1}{\sqrt[4]{3x^2-2}}-\frac{9\,2^{3/4}x}{32}\sqrt[4]{-\text{signum}\left(-1+\frac{3x^2}{2}\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\frac{1}{\sqrt[4]{\text{signum}\left(-1+\frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)^(1/4),x)
```

```
[Out] 1/24*(27*x^4-6*x^2-8)/x^3/(3*x^2-2)^(1/4)-9/32*2^(3/4)/signum(-1+3/2*x^2)^(
1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^2-2\right)^{\frac{3}{4}}}{3x^6-2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] time = 0.82427, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

3.897 $\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{160\ 2^{3/4}x}-\frac{189\sqrt[4]{3x^2-2}x}{160\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{63\left(3x^2-2\right)^{3/4}}{160x}+$$

```
[Out] (-2 + 3*x^2)^(3/4)/(10*x^5) + (7*(-2 + 3*x^2)^(3/4))/(40*x^3) + (63*(-2 + 3
*x^2)^(3/4))/(160*x) - (189*x*(-2 + 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] +
 Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8
0*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(160*2^(3/4)*x)
```

Rubi [A] time = 0.126316, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{189\sqrt[4]{3x^2-2}x}{160\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{63\left(3x^2-2\right)^{3/4}}{160x}+\frac{7\left(3x^2-2\right)^{3/4}}{40x^3}+\frac{\left(3x^2-2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(\right.}{160\ 2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```


```
[Out] (-2 + 3*x^2)^(3/4)/(10*x^5) + (7*(-2 + 3*x^2)^(3/4))/(40*x^3) + (63*(-2 + 3
*x^2)^(3/4))/(160*x) - (189*x*(-2 + 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] +
 Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8
0*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(160*2^(3/4)*x)
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{189}{320}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{\left(63\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+\right.}{160x}\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{\left(63\sqrt{3}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3\right.}{160x}\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{189x\sqrt[4]{-2+3x^2}}{160\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.0065384, size = 48, normalized size = 0.18

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2])/(5*x
^5*(-2 + 3*x^2)^(1/4))
```

Maple [C] time = 0.037, size = 72, normalized size = 0.3

$$\frac{189x^{6}-42x^{4}-8x^{2}-32}{160x^{5}}\frac{1}{\sqrt[4]{3x^{2}-2}}-\frac{189\,2^{3/4}x}{640}\sqrt[4]{-\mathrm{signum}\left(-1+\frac{3x^{2}}{2}\right)}{}_{2}\mathrm{F}_{1}\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^{2}}{2}\right)\frac{1}{\sqrt[4]{\mathrm{signum}\left(-1+\frac{3x^{2}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2-2)^(1/4),x)
```

```
[Out] 1/160*(189*x^6-42*x^4-8*x^2-32)/x^5/(3*x^2-2)^(1/4)-189/640*2^(3/4)/signum(
-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3
/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(3x^{2}-2\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^{2}-2\right)^{\frac{3}{4}}}{3x^{8}-2x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] time = 1.04363, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^{2}}{2}\right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

3.898 $\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=260

$$\frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{1053\sqrt{3}x}-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5+\frac{40\left(-3x^2-2\right)^{3/4}}{1053}$$

```
[Out] (-32*x*(-2 - 3*x^2)^(3/4))/1053 + (40*x^3*(-2 - 3*x^2)^(3/4))/1053 - (2*x^5
*(-2 - 3*x^2)^(3/4))/39 - (128*x*(-2 - 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[
-2 - 3*x^2])) - (128*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(S
qrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
1/2])/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2]
)^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(1053*Sqrt[3]*x)
```

Rubi [A] time = 0.133789, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5+\frac{40\left(-3x^2-2\right)^{3/4}x^3}{1053}-\frac{32\left(-3x^2-2\right)^{3/4}x}{1053}-\frac{128\sqrt[4]{-3x^2-2}x}{1053\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (-32*x*(-2 - 3*x^2)^(3/4))/1053 + (40*x^3*(-2 - 3*x^2)^(3/4))/1053 - (2*x^5
*(-2 - 3*x^2)^(3/4))/39 - (128*x*(-2 - 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[
-2 - 3*x^2])) - (128*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(S
qrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
1/2])/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2]
)^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(1053*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{20}{39}\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{64\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx}{1053}$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{\left(64\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,\right.}{1053x}$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{\left(128\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x,\right.}{1053\sqrt{3}x}$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{128x\sqrt[4]{-2-3x^2}}{1053\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} - \frac{128\sqrt[4]{2}}{}$$

Mathematica [C] time = 0.0185613, size = 68, normalized size = 0.26

$$\frac{2x\left(-16\ 2^{3/4}\sqrt[4]{3x^2+2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+81x^6-6x^4+8x^2+32\right)}{1053\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(32 + 8*x^2 - 6*x^4 + 81*x^6 - 16*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeome
tric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/(1053*(-2 - 3*x^2)^(1/4))
```

Maple [C] time = 0.023, size = 53, normalized size = 0.2

$$\frac{2x\left(27x^{4}-20x^{2}+16\right)\left(3x^{2}+2\right)}{1053}\frac{1}{\sqrt[4]{-3x^{2}-2}}+\frac{32\left(-1\right)^{3/4}x2^{3/4}}{1053}{}_{2}\mathrm{F}_{1}\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2-2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4-20*x^2+16)*(3*x^2+2)/(-3*x^2-2)^(1/4)+32/1053*(-1)^(3/4)*2
^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(-3x^{2}-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3159\,x\mathrm{integral}\left(\frac{256\left(-3x^{2}-2\right)^{\frac{3}{4}}}{3159\left(3x^{4}+2x^{2}\right)},x\right)-2\left(81x^{6}-60x^{4}+48x^{2}-64\right)\left(-3x^{2}-2\right)^{\frac{3}{4}}}{3159x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3159*(3159*x*integral(256/3159*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2
*(81*x^6 - 60*x^4 + 48*x^2 - 64)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] time = 0.808737, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}x^{7}e^{-\frac{i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{3x^{2}e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)
/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

3.899 $\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{16\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{135\sqrt{3}x}-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3+\frac{8}{135}\left(-3x^2-2\right)$$

```
[Out] (8*x*(-2 - 3*x^2)^(3/4))/135 - (2*x^3*(-2 - 3*x^2)^(3/4))/27 + (32*x*(-2 -
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 - 3*x^2])) + (32*2^(1/4)*Sqrt[-(x^2/(
Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*Ar
cTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) - (16*2^(1/4)*Sqrt[
-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipti
cF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rubi [A] time = 0.110469, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3+\frac{8}{135}\left(-3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{-3x^2-2}x}{135\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{16\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(\right.}{135\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (8*x*(-2 - 3*x^2)^(3/4))/135 - (2*x^3*(-2 - 3*x^2)^(3/4))/27 + (32*x*(-2 -
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 - 3*x^2])) + (32*2^(1/4)*Sqrt[-(x^2/(
Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*Ar
cTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) - (16*2^(1/4)*Sqrt[
-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipti
cF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx &= -\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx\\ &= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx\\ &= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{135x}\\ &= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{\left(32\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{135\sqrt{3}x}+\frac{\left(32\sqrt{-x\right.}}{}\\ &= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2-3x^2}}{135\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{32\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\right.}{1}\end{aligned}$$

Mathematica [C] time = 0.0141214, size = 63, normalized size = 0.26

$$\frac{2x\left(4\ 2^{3/4}\sqrt[4]{3x^2+2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+15x^4-2x^2-8\right)}{135\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-8 - 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1
/4, 1/2, 3/2, (-3*x^2)/2]))/(135*(-2 - 3*x^2)^(1/4))
```

Maple [C] time = 0.02, size = 48, normalized size = 0.2

$$\frac{2\,x\left(5\,x^2-4\right)\left(3\,x^2+2\right)}{135}\frac{1}{\sqrt[4]{-3\,x^2-2}}-\frac{8\,(-1)^{3/4}\,x2^{3/4}}{135}{}_2\mathrm{F}_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2-2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2-4)*(3*x^2+2)/(-3*x^2-2)^(1/4)-8/135*(-1)^(3/4)*2^(3/4)*x*hyp
ergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{405\,x\mathrm{integral}\left(-\frac{64\,(-3\,x^2-2)^{\frac{3}{4}}}{405\,(3\,x^4+2\,x^2)},x\right)-2\,(15\,x^4-12\,x^2+16)(-3\,x^2-2)^{\frac{3}{4}}}{405\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/405*(405*x*integral(-64/405*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(1
5*x^4 - 12*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] time = 0.693486, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)
/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

3.900 $\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=224

$$\frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x-\frac{8\sqrt[4]{-3x^2-2}x}{15\left(\sqrt{-3x^2-2}+\sqrt{}\right.}$$

```
[Out] (-2*x*(-2 - 3*x^2)^(3/4))/15 - (8*x*(-2 - 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt
[-2 - 3*x^2])) - (8*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sq
rt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1
/2])/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)
]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(15*Sqrt[3]*x)
```

Rubi [A] time = 0.0939805, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x-\frac{8\sqrt[4]{-3x^2-2}x}{15\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}-\ldots 8$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*(-2 - 3*x^2)^(3/4))/15 - (8*x*(-2 - 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt
[-2 - 3*x^2])) - (8*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sq
rt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1
/2])/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)
]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(15*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}+\frac{\left(4\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{15x}$$

$$= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}+\frac{\left(8\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{15\sqrt{3}x}-\frac{\left(8\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx,x\right.}{15\sqrt{3}x}$$

$$= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{8x\sqrt[4]{-2-3x^2}}{15\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\tan^{-1}\left(\right.\right.}{15\sqrt{3}x}$$

Mathematica [C] time = 0.0113575, size = 58, normalized size = 0.26

$$\frac{2x\left(-2^{3/4}\sqrt[4]{3x^2+2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+3x^2+2\right)}{15\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(2 + 3*x^2 - 2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2
, (-3*x^2)/2]))/(15*(-2 - 3*x^2)^(1/4))
```

Maple [C] time = 0.019, size = 41, normalized size = 0.2

$$\frac{2\,x\left(3\,x^2+2\right)}{15}\frac{1}{\sqrt[4]{-3\,x^2-2}}+\frac{2\,(-1)^{3/4}\,x2^{3/4}}{15}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2+2)/(-3*x^2-2)^(1/4)+2/15*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,
1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{45\,x\mathrm{integral}\left(\frac{16\left(-3\,x^2-2\right)^{\frac{3}{4}}}{45\left(3\,x^4+2\,x^2\right)},x\right)-2\left(3\,x^2-4\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{45\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/45*(45*x*integral(16/45*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(3*x^2
 - 4)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] time = 0.604949, size = 34, normalized size = 0.15

$$\frac{2^{\frac{3}{4}}x^3e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)
/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")

[Out] integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

3.901 $\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt{3}x}+\frac{2\sqrt[4]{-3x^2-2}x}{\sqrt{-3x^2-2}+\sqrt{2}}+\frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{\right.}}}}{}$$

```
[Out] (2*x*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) + (2*2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) - (2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rubi [A] time = 0.0784781, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {230, 305, 220, 1196}

$$\frac{2\sqrt[4]{-3x^2-2}x}{\sqrt{-3x^2-2}+\sqrt{2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x}+\frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^{\ldots}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x^2)^(-1/4),x]
```

```
[Out] (2*x*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) + (2*2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) - (2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{\left(\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{x}$$

$$= -\frac{\left(2\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{\sqrt{3}x} + \frac{\left(2\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{\sqrt{3}x}$$

$$= \frac{2x\sqrt[4]{-2-3x^2}}{\sqrt{2}+\sqrt{-2-3x^2}} + \frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x} - \frac{\sqrt[4]{2}\sqrt{-\frac{\ \ }{\left(\sqrt{2}+\right.}}}{}$$

Mathematica [C] time = 0.0060325, size = 43, normalized size = 0.21

$$\frac{x\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x^2)^(-1/4),x]
```

```
[Out] (x*(1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/(-2
- 3*x^2)^(1/4)
```

Maple [C] time = 0.01, size = 21, normalized size = 0.1

$$-\frac{(-1)^{\frac{3}{4}}\,x2^{\frac{3}{4}}}{2}{}_2\mathrm{F}_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(1/4),x)
```

```
[Out] -1/2*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 - 2)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,x\mathrm{integral}\left(-\frac{4\left(-3\,x^2-2\right)^{\frac{3}{4}}}{3\left(3\,x^4+2\,x^2\right)},x\right)-2\left(-3\,x^2-2\right)^{\frac{3}{4}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*integral(-4/3*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(-3*x^2 -
 2)^(3/4))/x
```

Sympy [C] time = 0.598373, size = 32, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}xe^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)
/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-1/4), x)
```

3.902 $\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2\,2^{3/4}x}+\frac{3\sqrt[4]{-3x^2-2}\,x}{2\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{\left(-3x^2-2\right)^{3/4}}{2x}+$$

```
[Out] (-2 - 3*x^2)^(3/4)/(2*x) + (3*x*(-2 - 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
^(3/4)*x) - (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
*2^(3/4)*x)
```

Rubi [A] time = 0.092718, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$\frac{3\sqrt[4]{-3x^2-2}\,x}{2\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2\,2^{3/4}x}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(2*x) + (3*x*(-2 - 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
^(3/4)*x) - (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
*2^(3/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = \frac{\left(-2-3x^2\right)^{3/4}}{2x}+\frac{3}{4}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{2x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{2x}+\frac{\left(\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,d\right.}{2x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x}+\frac{3x\sqrt[4]{-2-3x^2}}{2\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\tan^{-1}\left(\sqrt[4]{-}\right.\right.}{2^{3/4}x}$$

Mathematica [C] time = 0.0074077, size = 46, normalized size = 0.21

$$-\frac{\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{x\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2])/(x*
(-2 - 3*x^2)^(1/4)))
```

Maple [C] time = 0.019, size = 43, normalized size = 0.2

$$-\frac{3\,x^2+2}{2\,x}\frac{1}{\sqrt[4]{-3\,x^2-2}}-\frac{3\,(-1)^{3/4}\,x2^{3/4}}{8}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3\,x^2}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2-2)^(1/4),x)
```

[Out] -1/2*(3*x^2+2)/x/(-3*x^2-2)^(1/4)-3/8*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x\mathrm{integral}\left(-\frac{3\left(-3\,x^2-2\right)^{\frac{3}{4}}}{4\left(3\,x^2+2\right)},x\right)+\left(-3\,x^2-2\right)^{\frac{3}{4}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")

[Out] 1/2*(2*x*integral(-3/4*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(3/4))/x

---

Sympy [C] time = 0.669019, size = 36, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(-3*x**2-2)**(1/4),x)

[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

3.903 $\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}} \, dx$

Optimal. Leaf size=244

$$\frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{8\ 2^{3/4}x}-\frac{9\sqrt[4]{-3x^2-2}\,x}{8\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{3\left(-3x^2-2\right)^{3/4}}{8x}+\ (\ldots$$

```
[Out] (-2 - 3*x^2)^(3/4)/(6*x^3) - (3*(-2 - 3*x^2)^(3/4))/(8*x) - (9*x*(-2 - 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 - 3*x^2])) - (3*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2]
 + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-
2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) + (3*Sqrt[3]*Sqrt[-(x^2/(Sqr
t[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTa
n[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rubi [A] time = 0.110542, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{9\sqrt[4]{-3x^2-2}\,x}{8\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{3\left(-3x^2-2\right)^{3/4}}{8x}+\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}+\frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right.\right.}{8\ 2^{3/4}x}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(6*x^3) - (3*(-2 - 3*x^2)^(3/4))/(8*x) - (9*x*(-2 - 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 - 3*x^2])) - (3*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2]
 + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-
2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) + (3*Sqrt[3]*Sqrt[-(x^2/(Sqr
t[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTa
n[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx = \frac{\left(-2-3x^2\right)^{3/4}}{6x^3} - \frac{3}{4}\int \frac{1}{x^2 \sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-2-3x^2\right)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-2-3x^2\right)^{3/4}}{8x} + \frac{\left(3\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{8x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-2-3x^2\right)^{3/4}}{8x} + \frac{\left(3\sqrt{3}\sqrt{-x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{8x} - \frac{\left(3\sqrt{3}\sqrt{-x\right.}}{}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-2-3x^2\right)^{3/4}}{8x} - \frac{9x\sqrt[4]{-2-3x^2}}{8\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} - \frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3\right.}}{4\,2^{3/4}x}$$

Mathematica [C] time = 0.0073808, size = 48, normalized size = 0.2

$$-\frac{\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2])/(3*
x^3*(-2 - 3*x^2)^(1/4))
```

Maple [C] time = 0.021, size = 48, normalized size = 0.2

$$\frac{27\,x^4+6\,x^2-8}{24\,x^3}\frac{1}{\sqrt[4]{-3\,x^2-2}}+\frac{9\,(-1)^{3/4}\,x2^{3/4}}{32}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\,\frac{3}{2};\,-\frac{3\,x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2-2)^(1/4),x)
```

```
[Out] 1/24*(27*x^4+6*x^2-8)/x^3/(-3*x^2-2)^(1/4)+9/32*(-1)^(3/4)*2^(3/4)*x*hyperg
eom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,x^3\mathrm{integral}\left(\frac{9\left(-3\,x^2-2\right)^{\frac{3}{4}}}{16\left(3\,x^2+2\right)},x\right)-\left(9\,x^2-4\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(9/16*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) - (9*x^2 - 4)
*(-3*x^2 - 2)^(3/4))/x^3
```

Sympy [C] time = 0.811443, size = 39, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2
)/(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")

[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

3.904 $\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}} \, dx$

Optimal. Leaf size=262

$$-\frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{160\ 2^{3/4}x}+\frac{189\sqrt[4]{-3x^2-2}\,x}{160\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{63\left(-3x^2-2\right)}{160x}$$

```
[Out] (-2 - 3*x^2)^(3/4)/(10*x^5) - (7*(-2 - 3*x^2)^(3/4))/(40*x^3) + (63*(-2 - 3
*x^2)^(3/4))/(160*x) + (189*x*(-2 - 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2
] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(80*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(
Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(160*2^(3/4)*x)
```

Rubi [A] time = 0.128881, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$\frac{189\sqrt[4]{-3x^2-2}\,x}{160\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{63\left(-3x^2-2\right)^{3/4}}{160x}-\frac{7\left(-3x^2-2\right)^{3/4}}{40x^3}+\frac{\left(-3x^2-2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right.}{160\ 2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(10*x^5) - (7*(-2 - 3*x^2)^(3/4))/(40*x^3) + (63*(-2 - 3
*x^2)^(3/4))/(160*x) + (189*x*(-2 - 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2
] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(80*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(
Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(160*2^(3/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx = \frac{\left(-2-3x^2\right)^{3/4}}{10x^5} - \frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5} - \frac{7\left(-2-3x^2\right)^{3/4}}{40x^3} + \frac{63}{80}\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5} - \frac{7\left(-2-3x^2\right)^{3/4}}{40x^3} + \frac{63\left(-2-3x^2\right)^{3/4}}{160x} + \frac{189}{320}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5} - \frac{7\left(-2-3x^2\right)^{3/4}}{40x^3} + \frac{63\left(-2-3x^2\right)^{3/4}}{160x} - \frac{\left(63\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-\right.}{160x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5} - \frac{7\left(-2-3x^2\right)^{3/4}}{40x^3} + \frac{63\left(-2-3x^2\right)^{3/4}}{160x} - \frac{\left(63\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-\right.}{160x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5} - \frac{7\left(-2-3x^2\right)^{3/4}}{40x^3} + \frac{63\left(-2-3x^2\right)^{3/4}}{160x} + \frac{189x\sqrt[4]{-2-3x^2}}{160\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} + \frac{63\sqrt{3}\sqrt{-\frac{\cdots}{\left(\sqrt{2}+\cdots\right.}}}{\cdots}$$

Mathematica [C] time = 0.0076757, size = 48, normalized size = 0.18

$$-\frac{\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2])/(5*
x^5*(-2 - 3*x^2)^(1/4))
```

Maple [C] time = 0.022, size = 53, normalized size = 0.2

$$-\frac{189 x^{6}+42 x^{4}-8 x^{2}+32}{160 x^{5}}\frac{1}{\sqrt[4]{-3 x^{2}-2}}-\frac{189 \left(-1\right)^{3/4} x 2^{3/4}}{640}{}_{2}\mathrm{F}_{1}\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3 x^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2-2)^(1/4),x)
```

```
[Out] -1/160*(189*x^6+42*x^4-8*x^2+32)/x^5/(-3*x^2-2)^(1/4)-189/640*(-1)^(3/4)*2^
(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3 x^{2}-2\right)^{\frac{1}{4}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{160 x^{5} \mathrm{integral}\left(-\frac{189 \left(-3 x^{2}-2\right)^{\frac{3}{4}}}{320 \left(3 x^{2}+2\right)}, x\right)+\left(63 x^{4}-28 x^{2}+16\right)\left(-3 x^{2}-2\right)^{\frac{3}{4}}}{160 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/160*(160*x^5*integral(-189/320*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (63*x
^4 - 28*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x^5
```

Sympy [C] time = 1.02355, size = 39, normalized size = 0.15

$$\frac{2^{\frac{3}{4}} e^{\frac{3 i \pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3 x^{2} e^{i \pi}}{2}\right)}{10 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2
)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

3.905 $\int \frac{x^6}{\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=138

$$\frac{160\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2079\sqrt{3}x}+\frac{2}{33}\sqrt[4]{3x^2-2}x^5+\frac{40}{693}\sqrt[4]{3x^2-2}x^3+\frac{160\sqrt[4]{3x}}{207}$$

```
[Out] (160*x*(-2 + 3*x^2)^(1/4))/2079 + (40*x^3*(-2 + 3*x^2)^(1/4))/693 + (2*x^5*
(-2 + 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])
^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1
/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rubi [A] time = 0.0568948, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 234, 220}

$$\frac{2}{33}\sqrt[4]{3x^2-2}x^5+\frac{40}{693}\sqrt[4]{3x^2-2}x^3+\frac{160\sqrt[4]{3x^2-2}x}{2079}+\frac{160\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2079\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (160*x*(-2 + 3*x^2)^(1/4))/2079 + (40*x^3*(-2 + 3*x^2)^(1/4))/693 + (2*x^5*
(-2 + 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])
^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1
/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\left(-2+3x^2\right)^{3/4}}\,dx = \frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{20}{33}\int \frac{x^4}{\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{80}{231}\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{320\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx}{2079}$$

$$= \frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{\left(320\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x\right.}{2079x}$$

$$= \frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{160\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2\right.}}{2079\sqrt{}}$$

Mathematica [C] time = 0.0186276, size = 68, normalized size = 0.49

$$\frac{2x\left(80\sqrt[4]{2}\left(2-3x^2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)+189x^6+54x^4+120x^2-160\right)}{2079\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-160 + 120*x^2 + 54*x^4 + 189*x^6 + 80*2^(1/4)*(2 - 3*x^2)^(3/4)*Hype
rgeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/(2079*(-2 + 3*x^2)^(3/4))
```

Maple [C] time = 0.042, size = 65, normalized size = 0.5

$$\frac{2\,x\left(63\,x^4+60\,x^2+80\right)}{2079}\sqrt[4]{3\,x^2-2}+\frac{160\,\sqrt[4]{2}x}{2079}\left(-\text{signum}\left(-1+\frac{3\,x^2}{2}\right)\right)^{\frac{3}{4}}{}_2\text{F}_1(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3\,x^2}{2})\left(\text{signum}\left(-1+\frac{3\,x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)^(3/4),x)
```

```
[Out] 2/2079*x*(63*x^4+60*x^2+80)*(3*x^2-2)^(1/4)+160/2079*2^(1/4)/signum(-1+3/2*
x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\left(3\,x^2-2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] time = 0.787967, size = 31, normalized size = 0.22

$$\frac{\sqrt[4]{2}x^7e^{-\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

3.906 $\int \frac{x^4}{(-2+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=120

$$\frac{8\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{63\sqrt{3}x}+\frac{2}{21}\sqrt[4]{3x^2-2}x^3+\frac{8}{63}\sqrt[4]{3x^2-2}x$$

```
[Out] (8*x*(-2 + 3*x^2)^(1/4))/63 + (2*x^3*(-2 + 3*x^2)^(1/4))/21 + (8*2^(3/4)*Sq
rt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Ellipti
cF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rubi [A] time = 0.0456249, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 234, 220}

$$\frac{2}{21}\sqrt[4]{3x^2-2}x^3+\frac{8}{63}\sqrt[4]{3x^2-2}x+\frac{8\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{63\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (8*x*(-2 + 3*x^2)^(1/4))/63 + (2*x^3*(-2 + 3*x^2)^(1/4))/21 + (8*2^(3/4)*Sq
rt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Ellipti
cF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\left(-2+3x^2\right)^{3/4}}\,dx = \frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{4}{7}\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$=\frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{16}{63}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$=\frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{63x}$$
$$=\frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{8\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right.\right.}{63\sqrt{3}x}$$

Mathematica [C] time = 0.0147773, size = 63, normalized size = 0.52

$$\frac{2x\left(4\sqrt[4]{2}\left(2-3x^2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)+9x^4+6x^2-8\right)}{63\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-8 + 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, (3*x^2)/2]))/(63*(-2 + 3*x^2)^(3/4))
```

Maple [C] time = 0.037, size = 60, normalized size = 0.5

$$\frac{2x\left(3x^2+4\right)}{63}\sqrt[4]{3x^2-2}+\frac{8\sqrt[4]{2}x}{63}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\left(\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)^(3/4),x)
```

```
[Out] 2/63*x*(3*x^2+4)*(3*x^2-2)^(1/4)+8/63*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-si
gnum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(3x^2-2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] time = 0.674112, size = 31, normalized size = 0.26

$$\frac{\sqrt[4]{2}x^5e^{-\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2-2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

3.907 $\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=102

$$\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}+\frac{2}{9}\sqrt[4]{3x^2-2}\,x$$

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2
])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rubi [A] time = 0.0367001, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 234, 220}

$$\frac{2}{9}\sqrt[4]{3x^2-2}\,x+\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2
])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx = \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{4}{9}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$= \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{9x}$$
$$= \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}x}$$

Mathematica [C] time = 0.0124234, size = 57, normalized size = 0.56

$$\frac{2x\left(\sqrt[4]{2}\left(2-3x^2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)+3x^2-2\right)}{9\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2 + 2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/
2, (3*x^2)/2]))/(9*(-2 + 3*x^2)^(3/4))
```

Maple [C] time = 0.037, size = 53, normalized size = 0.5

$$\frac{2x}{9}\sqrt[4]{3x^2-2}+\frac{2\sqrt[4]{2}x}{9}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}{}_2\text{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\left(\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)^(3/4),x)
```

```
[Out] 2/9*x*(3*x^2-2)^(1/4)+2/9*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*
x^2))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(3\,x^2-2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] time = 0.650613, size = 31, normalized size = 0.3

$$\frac{\sqrt[4]{2}x^3e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

3.908 $\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

```
[Out] (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Elli
pticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x)
```

Rubi [A] time = 0.0259494, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {234, 220}

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^2)^(-3/4),x]
```

```
[Out] (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Elli
pticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x)
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx &= \frac{\left(\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{x}\\ &= \frac{\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}\end{aligned}$$

Mathematica [C] time = 0.0060872, size = 43, normalized size = 0.52

$$\frac{x\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)}{\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^2)^(-3/4),x]
```

```
[Out] (x*(1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2])/(-2 +
 3*x^2)^(3/4)
```

Maple [C] time = 0.028, size = 40, normalized size = 0.5

$$\frac{\sqrt[4]{2}x}{2}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\left(\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/2*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeo
m([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(3x^2-2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(-3/4), x)
```

Sympy [C] time = 0.610121, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}xe^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2/2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

3.909 $\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}+\frac{\sqrt[4]{3x^2-2}}{2x}$$

```
[Out] (-2 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2
]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(4*2^(1/4)*x)
```

Rubi [A] time = 0.0353238, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 234, 220}

$$\frac{\sqrt[4]{3x^2-2}}{2x}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2
]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(4*2^(1/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{3}{4}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{\left(\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{2x}$$
$$= \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

Mathematica [C] time = 0.0071818, size = 46, normalized size = 0.44

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{x\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2])/(x*(
-2 + 3*x^2)^(3/4)))
```

Maple [C] time = 0.038, size = 55, normalized size = 0.5

$$\frac{1}{2x}\sqrt[4]{3x^2-2}+\frac{3\sqrt[4]{2}x}{8}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}{}_2\text{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\left(\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/2*(3*x^2-2)^(1/4)/x+3/8*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*
x^2))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^4-2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] time = 0.768383, size = 29, normalized size = 0.28

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2/2)/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

3.910 $\int \frac{1}{x^4 (-2+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=122

$$\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{16\sqrt[4]{2}x}+\frac{5\sqrt[4]{3x^2-2}}{8x}+\frac{\sqrt[4]{3x^2-2}}{6x^3}$$

```
[Out] (-2 + 3*x^2)^(1/4)/(6*x^3) + (5*(-2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Sqrt
[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF
[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rubi [A] time = 0.0463736, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 234, 220}

$$\frac{5\sqrt[4]{3x^2-2}}{8x}+\frac{\sqrt[4]{3x^2-2}}{6x^3}+\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(6*x^3) + (5*(-2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Sqrt
[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF
[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\left(-2+3x^2\right)^{3/4}}\,dx=\frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5}{4}\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx$$
$$=\frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$=\frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{\left(5\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{8x}$$
$$=\frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}x}$$

Mathematica [C] time = 0.0078814, size = 48, normalized size = 0.39

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3x^3\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2])/(3*x
^3*(-2 + 3*x^2)^(3/4))
```

Maple [C] time = 0.038, size = 67, normalized size = 0.6

$$\frac{45x^4-18x^2-8}{24x^3}\left(3x^2-2\right)^{-\frac{3}{4}}+\frac{15\sqrt[4]{2}x}{32}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}{}_2\text{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\left(\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/24*(45*x^4-18*x^2-8)/x^3/(3*x^2-2)^(3/4)+15/32*2^(1/4)/signum(-1+3/2*x^2)
^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^6-2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")

[Out] integral((3*x^2 - 2)^(1/4)/(3*x^6 - 2*x^4), x)

Sympy [C] time = 0.955271, size = 32, normalized size = 0.26

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(3*x**2-2)**(3/4),x)

[Out] 2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2/2)/(6*x**3)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="giac")

[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)

3.911 $\int \frac{1}{x^6(-2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=140

$$\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{64\sqrt[4]{2}x}+\frac{27\sqrt[4]{3x^2-2}}{32x}+\frac{9\sqrt[4]{3x^2-2}}{40x^3}+\frac{\sqrt[4]{3x^2-2}}{10x^5}$$

```
[Out] (-2 + 3*x^2)^(1/4)/(10*x^5) + (9*(-2 + 3*x^2)^(1/4))/(40*x^3) + (27*(-2 + 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*
(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)]
, 1/2])/(64*2^(1/4)*x)
```

Rubi [A] time = 0.056775, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 234, 220}

$$\frac{27\sqrt[4]{3x^2-2}}{32x}+\frac{9\sqrt[4]{3x^2-2}}{40x^3}+\frac{\sqrt[4]{3x^2-2}}{10x^5}+\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{64\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(10*x^5) + (9*(-2 + 3*x^2)^(1/4))/(40*x^3) + (27*(-2 + 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*
(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)]
, 1/2])/(64*2^(1/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{27}{20}\int \frac{1}{x^4\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27}{16}\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{81}{64}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{\left(27\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x}\right.}{32x}$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\right.}{64\sqrt[4]{2}x}$$

Mathematica [C] time = 0.0073827, size = 48, normalized size = 0.34

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5x^5\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2])/(5*x
^5*(-2 + 3*x^2)^(3/4))
```

Maple [C] time = 0.039, size = 72, normalized size = 0.5

$$\frac{405x^6-162x^4-24x^2-32}{160x^5}\left(3x^2-2\right)^{-\frac{3}{4}}+\frac{81\sqrt[4]{2}x}{128}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\left(\text{signum}\left(-1+\frac{3x^2}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/160*(405*x^6-162*x^4-24*x^2-32)/x^5/(3*x^2-2)^(3/4)+81/128*2^(1/4)/signum
(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2,3/4],[3/2],
3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^8-2\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] time = 1.20733, size = 32, normalized size = 0.23

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2},\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle|\frac{3x^2}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

3.912 $\int \frac{x^6}{\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=139

$$\frac{160\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2079\sqrt{3}x}-\frac{2}{33}\sqrt[4]{-3x^2-2}x^5+\frac{40}{693}\sqrt[4]{-3x^2-2}x^3$$

```
[Out] (-160*x*(-2 - 3*x^2)^(1/4))/2079 + (40*x^3*(-2 - 3*x^2)^(1/4))/693 - (2*x^5
*(-2 - 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^
2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/
2^(1/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rubi [A] time = 0.0575328, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 234, 220}

$$-\frac{2}{33}\sqrt[4]{-3x^2-2}x^5+\frac{40}{693}\sqrt[4]{-3x^2-2}x^3-\frac{160\sqrt[4]{-3x^2-2}x}{2079}+\frac{160\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\ldots\right)\right)}{2079\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (-160*x*(-2 - 3*x^2)^(1/4))/2079 + (40*x^3*(-2 - 3*x^2)^(1/4))/693 - (2*x^5
*(-2 - 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^
2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/
2^(1/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^6}{\left(-2-3x^2\right)^{3/4}}\,dx = -\frac{2}{33}x^5\sqrt[4]{-2-3x^2} - \frac{20}{33}\int \frac{x^4}{\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} + \frac{80}{231}\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= -\frac{160x\sqrt[4]{-2-3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} - \frac{320\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx}{2079}$$
$$= -\frac{160x\sqrt[4]{-2-3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} + \frac{\left(320\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x,\right.}{2079x}$$
$$= -\frac{160x\sqrt[4]{-2-3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} + \frac{160\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-\right.}{2079\sqrt{3}}$$

Mathematica [C] time = 0.0195166, size = 68, normalized size = 0.49

$$\frac{2x\left(-80\sqrt[4]{2}\left(3x^2+2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+189x^6-54x^4+120x^2+160\right)}{2079\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(160 + 120*x^2 - 54*x^4 + 189*x^6 - 80*2^(1/4)*(2 + 3*x^2)^(3/4)*Hyper
geometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/(2079*(-2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^6\left(-3x^2-2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(x^6/(-3*x^2-2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{2079}\left(63\,x^{5}-60\,x^{3}+80\,x\right)\left(-3\,x^{2}-2\right)^{\frac{1}{4}}+\mathrm{integral}\left(\frac{320\left(-3\,x^{2}-2\right)^{\frac{1}{4}}}{2079\left(3\,x^{2}+2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/2079*(63*x^5 - 60*x^3 + 80*x)*(-3*x^2 - 2)^(1/4) + integral(320/2079*(-3
*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] time = 0.802413, size = 36, normalized size = 0.26

$$\frac{\sqrt[4]{2}x^{7}e^{-\frac{3i\pi}{4}}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^{2}e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*p
i)/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{6}}{\left(-3\,x^{2}-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

3.913 $\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=121

$$-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{63\sqrt{3}x}-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3+\frac{8}{63}\sqrt[4]{-3x^2-2}x$$

```
[Out] (8*x*(-2 - 3*x^2)^(1/4))/63 - (2*x^3*(-2 - 3*x^2)^(1/4))/21 - (8*2^(3/4)*Sq
rt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Elli
pticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rubi [A] time = 0.0461336, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 234, 220}

$$-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3+\frac{8}{63}\sqrt[4]{-3x^2-2}x-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{63\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (8*x*(-2 - 3*x^2)^(1/4))/63 - (2*x^3*(-2 - 3*x^2)^(1/4))/21 - (8*2^(3/4)*Sq
rt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Elli
pticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\left(-2-3x^2\right)^{3/4}}dx = -\frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{4}{7}\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}dx$$
$$= \frac{8}{63}x\sqrt[4]{-2-3x^2} - \frac{2}{21}x^3\sqrt[4]{-2-3x^2} + \frac{16}{63}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}dx$$
$$= \frac{8}{63}x\sqrt[4]{-2-3x^2} - \frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{\left(16\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}dx, x, \sqrt[4]{-2-3x^2}\right)}{63x}$$
$$= \frac{8}{63}x\sqrt[4]{-2-3x^2} - \frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{8\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2}}{}\right.\right.}{63\sqrt{3}x}$$

Mathematica [C] time = 0.0153001, size = 63, normalized size = 0.52

$$\frac{2x\left(4\sqrt[4]{2}\left(3x^2+2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+9x^4-6x^2-8\right)}{63\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-8 - 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, (-3*x^2)/2]))/(63*(-2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int x^4\left(-3x^2-2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(x^4/(-3*x^2-2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-3x^2-2\right)^{\frac{3}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{63}\left(3x^3-4x\right)\left(-3x^2-2\right)^{\frac{1}{4}}+\mathrm{integral}\left(-\frac{16\left(-3x^2-2\right)^{\frac{1}{4}}}{63\left(3x^2+2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/63*(3*x^3 - 4*x)*(-3*x^2 - 2)^(1/4) + integral(-16/63*(-3*x^2 - 2)^(1/4)
/(3*x^2 + 2), x)
```

Sympy [C] time = 0.685824, size = 36, normalized size = 0.3

$$\frac{\sqrt[4]{2}x^5e^{-\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*p
i)/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

3.914 $\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=103

$$\frac{2\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}$$

```
[Out] (-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*
x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4
)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rubi [A] time = 0.0355607, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {321, 234, 220}

$$\frac{2\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*
x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4
)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx = -\frac{2}{9}x\sqrt[4]{-2-3x^2} - \frac{4}{9}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$$

$$= -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{9x}$$

$$= -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{2\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}x}$$

Mathematica [C] time = 0.0117778, size = 58, normalized size = 0.56

$$\frac{2x\left(-\sqrt[4]{2}\left(3x^2+2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+3x^2+2\right)}{9\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(2 + 3*x^2 - 2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2
, (-3*x^2)/2]))/(9*(-2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int x^2\left(-3x^2-2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2-2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{9}\left(-3x^2-2\right)^{\frac{1}{4}}x+\mathrm{integral}\left(\frac{4\left(-3x^2-2\right)^{\frac{1}{4}}}{9\left(3x^2+2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/9*(-3*x^2 - 2)^(1/4)*x + integral(4/9*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] time = 0.619544, size = 36, normalized size = 0.35

$$\frac{\sqrt[4]{2}x^3e^{-\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*p
i)/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

3.915 $\int \frac{1}{\left(-2-3x^2\right)^{3/4}} \, dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

```
[Out] -((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])
*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x))
```

Rubi [A] time = 0.0270031, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {234, 220}

$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x^2)^(-3/4),x]
```

```
[Out] -((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])
*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-2-3x^2\right)^{3/4}} \, dx &= -\frac{\left(\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}} \, dx, x, \sqrt[4]{-2-3x^2}\right)}{x}\\
&= -\frac{\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}
\end{aligned}$$

Mathematica [C] time = 0.0067792, size = 43, normalized size = 0.51

$$\frac{x\left(\frac{3x^2}{2}+1\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)}{\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x^2)^(-3/4),x]
```

```
[Out] (x*(1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/(-2
- 3*x^2)^(3/4)
```

Maple [C] time = 0.011, size = 21, normalized size = 0.3

$$-\frac{\sqrt[4]{-1}\,\sqrt[4]{2}x}{2}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(3/4),x)
```

```
[Out] -1/2*(-1)^(1/4)*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-3x^2-2\right)^{\frac{1}{4}}}{3x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] time = 0.598736, size = 34, normalized size = 0.4

$$\frac{\sqrt[4]{2}xe^{-\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/
2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```

3.916 $\int \frac{1}{x^2(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}+\frac{\sqrt[4]{-3x^2-2}}{2x}$$

```
[Out] (-2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])
^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(
1/4)], 1/2])/(4*2^(1/4)*x)
```

Rubi [A] time = 0.0353193, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 234, 220}

$$\frac{\sqrt[4]{-3x^2-2}}{2x}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])
^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(
1/4)], 1/2])/(4*2^(1/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{2x} - \frac{3}{4}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\left(\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{2x}$$

$$= \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

Mathematica [C] time = 0.0067861, size = 46, normalized size = 0.44

$$-\frac{\left(\frac{3x^2}{2}+1\right)^{3/4}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{x\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2])/(x*
(-2 - 3*x^2)^(3/4)))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(-3x^2-2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(1/x^2/(-3*x^2-2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x\mathrm{integral}\left(\frac{3\left(-3\,x^2-2\right)^{\frac{1}{4}}}{4\left(3\,x^2+2\right)},x\right)+\left(-3\,x^2-2\right)^{\frac{1}{4}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*integral(3/4*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(1/
4))/x
```

Sympy [C] time = 0.754902, size = 34, normalized size = 0.32

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(
2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

3.917 $\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=123

$$-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{16\sqrt[4]{2}x}-\frac{5\sqrt[4]{-3x^2-2}}{8x}+\frac{\sqrt[4]{-3x^2-2}}{6x^3}$$

```
[Out] (-2 - 3*x^2)^(1/4)/(6*x^3) - (5*(-2 - 3*x^2)^(1/4))/(8*x) - (5*Sqrt[3]*Sqrt
[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipt
icF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rubi [A] time = 0.0485651, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 234, 220}

$$-\frac{5\sqrt[4]{-3x^2-2}}{8x}+\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(6*x^3) - (5*(-2 - 3*x^2)^(1/4))/(8*x) - (5*Sqrt[3]*Sqrt
[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipt
icF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(-2-3x^2\right)^{3/4}}\,dx &= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5}{4}\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx \\
&= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx \\
&= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}-\frac{\left(5\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{8x} \\
&= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)\right.}{16\sqrt[4]{2}x}
\end{aligned}
$$

Mathematica [C] time = 0.0079847, size = 48, normalized size = 0.39

$$
-\frac{\left(\frac{3x^2}{2}+1\right)^{3/4}{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3x^3\left(-3x^2-2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] -((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2])/(3*
x^3*(-2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$
\int \frac{1}{x^4}\left(-3x^2-2\right)^{-\frac{3}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(1/x^4/(-3*x^2-2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}}x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,x^{3}\mathrm{integral}\left(-\frac{15\,(-3\,x^{2}-2)^{\frac{1}{4}}}{16\,(3\,x^{2}+2)},x\right)-\left(15\,x^{2}-4\right)\left(-3\,x^{2}-2\right)^{\frac{1}{4}}}{24\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(-15/16*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) - (15*x^2 -
 4)*(-3*x^2 - 2)^(1/4))/x^3
```

Sympy [C] time = 0.955024, size = 37, normalized size = 0.3

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^{2}e^{i\pi}}{2}\right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/
(6*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{3}{4}}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

3.918 $\int \frac{1}{x^6\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=141

$$\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{64\sqrt[4]{2}x}+\frac{27\sqrt[4]{-3x^2-2}}{32x}-\frac{9\sqrt[4]{-3x^2-2}}{40x^3}+\frac{\sqrt[4]{-3x^2-2}}{10x^5}$$

```
[Out] (-2 - 3*x^2)^(1/4)/(10*x^5) - (9*(-2 - 3*x^2)^(1/4))/(40*x^3) + (27*(-2 - 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2
)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/
4)], 1/2])/(64*2^(1/4)*x)
```

Rubi [A] time = 0.057696, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 234, 220}

$$\frac{27\sqrt[4]{-3x^2-2}}{32x}-\frac{9\sqrt[4]{-3x^2-2}}{40x^3}+\frac{\sqrt[4]{-3x^2-2}}{10x^5}+\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{64\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(10*x^5) - (9*(-2 - 3*x^2)^(1/4))/(40*x^3) + (27*(-2 - 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2
)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/
4)], 1/2])/(64*2^(1/4)*x)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^6\left(-2-3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{27}{20}\int \frac{1}{x^4\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27}{16}\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} - \frac{81}{64}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} + \frac{\left(27\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{32x}$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} + \frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\,\text{ta}\right.}{64\sqrt[4]{2}x}$$

Mathematica [C] time = 0.0079166, size = 48, normalized size = 0.34

$$-\frac{\left(\frac{3x^2}{2}+1\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5x^5\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] -((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2])/(5*
x^5*(-2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(-3x^2-2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(1/x^6/(-3*x^2-2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{160\,x^5\mathrm{integral}\left(\frac{81\left(-3\,x^2-2\right)^{\frac{1}{4}}}{64\left(3\,x^2+2\right)},x\right)+\left(135\,x^4-36\,x^2+16\right)\left(-3\,x^2-2\right)^{\frac{1}{4}}}{160\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/160*(160*x^5*integral(81/64*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (135*x^4
 - 36*x^2 + 16)*(-3*x^2 - 2)^(1/4))/x^5
```

Sympy [C] time = 1.14833, size = 37, normalized size = 0.26

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/
(10*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

3.919 $\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=152

$$-\frac{a^{5/2}c^2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}$$

```
[Out] -(a^2*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4))/(12*b^2) + (a*c*(c*x)^(5/2)*(a + b*x
^2)^(1/4))/(30*b) + ((c*x)^(9/2)*(a + b*x^2)^(1/4))/(5*c) - (a^(5/2)*c^2*(1
 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(12*b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.107769, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 237, 335, 275, 231}

$$-\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2}-\frac{a^{5/2}c^2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] -(a^2*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4))/(12*b^2) + (a*c*(c*x)^(5/2)*(a + b*x
^2)^(1/4))/(30*b) + ((c*x)^(9/2)*(a + b*x^2)^(1/4))/(5*c) - (a^(5/2)*c^2*(1
 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(12*b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{1}{10}a\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{\left(a^2c^2\right)\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{12b}$$

$$= -\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{\left(a^3c^4\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{24b^2}$$

$$= -\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{\left(a^3c^3\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x\right.}{12b^2}$$

$$= -\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{\left(a^3c^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subs}}{12b^2\left(a+\right.}$$

$$= -\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{\left(a^3c^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subs}}{12b^2\left(a+b\right.}$$

$$= -\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{\left(a^3c^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subs}}{24b^2\left(a+b\right.}$$

$$= -\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{a^{5/2}c^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}c\right.}{12b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0653525, size = 102, normalized size = 0.67

$$\frac{c^3\sqrt{cx}\sqrt[4]{a+bx^2}\left(\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2+abx^2+6b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{30b^2\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-5*a^2 + a*b*x^2 +
 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(30*b
^2*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}c^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c^3*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

3.920 $\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}$$

```
[Out] (a*c*Sqrt[c*x]*(a + b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a + b*x^2)^(1/4))/(
3*c) + (a^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0845446, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 237, 335, 275, 231}

$$\frac{a^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```


```
[Out] (a*c*Sqrt[c*x]*(a + b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a + b*x^2)^(1/4))/(
3*c) + (a^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a + b*x^2)^(3/4))
```


Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx &= \frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{1}{6}a\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{12b}\\
&= \frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{6b}\\
&= \frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{6b\left(a+bx^2\right)^{3/4}}\\
&= \frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{6b\left(a+bx^2\right)^{3/4}}\\
&= \frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{12b\left(a+bx^2\right)^{3/4}}\\
&= \frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{a^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0423786, size = 85, normalized size = 0.72

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}\left((a+bx^2)\sqrt[4]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hyper
geometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

Sympy [C] time = 5.19315, size = 46, normalized size = 0.39

$$\frac{\sqrt[4]{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

3.921 $\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] (Sqrt[c*x]*(a + b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a + b*x^2)^(3/
4))
```

Rubi [A] time = 0.0728459, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 329, 237, 335, 275, 231}

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*(a + b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a + b*x^2)^(3/
4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx &= \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}+\frac{1}{2}a\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx \\
&= \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}+\frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c} \\
&= \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}+\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{c\left(a+bx^2\right)^{3/4}} \\
&= \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{c\left(a+bx^2\right)^{3/4}} \\
&= \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{2c\left(a+bx^2\right)^{3/4}} \\
&= \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0115177, size = 54, normalized size = 0.61

$$
\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)])/(Sq
rt[c*x]*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt[4]{b x^{2}+a}\,\frac{1}{\sqrt{c x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{\frac{1}{4}}}{\sqrt{c x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b x^{2}+a\right)^{\frac{1}{4}}\sqrt{c x}}{c x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

Sympy [C] time = 1.42959, size = 46, normalized size = 0.52

$$\frac{\sqrt[4]{a}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{b x^{2} e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(1/2),x)
```

```
[Out] a**(1/4)*sqrt(x)*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{\sqrt{cx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

3.922 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) - (2*b^(3/2)*(1 + a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 + b*x^2)^(3/4))
```

Rubi [A] time = 0.0742942, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 329, 237, 335, 275, 231}

$$-\frac{2b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) - (2*b^(3/2)*(1 + a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 + b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}+\frac{b\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{3c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}+\frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3c^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3c^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{\left(b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3c^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{2b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}c^4\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0119783, size = 56, normalized size = 0.6

$$
-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x^2)/a)])/(
3*(c*x)^(5/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt[4]{b x^{2}+a}\,(c x)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{\frac{1}{4}}}{(c x)^{\frac{5}{2}}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^{2}+a\right)^{\frac{1}{4}} \sqrt{c x}}{c^{3} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] time = 11.206, size = 32, normalized size = 0.34

$$-\frac{\sqrt[4]{b}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a e^{i\pi}}{b x^{2}}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

```
[Out] -b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(5/2)*
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

3.923 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}} dx$

Optimal. Leaf size=123

$$\frac{4b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) - (2*b*(a + b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Cot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.088923, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {277, 325, 329, 237, 335, 275, 231}

$$\frac{4b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```


```
[Out] (-2*(a + b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) - (2*b*(a + b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Cot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a + b*x^2)^(3/4))
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}+\frac{b\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{7c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{21ac^4}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{21ac^5}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{21ac^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{21ac^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(2b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{21ac^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{4b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0132993, size = 56, normalized size = 0.46

$$-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x^2)/a)])/
(7*(c*x)^(9/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2+a}\,(cx)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{c^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

3.924 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{8b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) - (2*b*(a + b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a + b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) - (8*
b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.110039, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {277, 325, 329, 237, 335, 275, 231}

$$\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{8b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) - (2*b*(a + b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a + b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) - (8*
b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a + b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} + \frac{b\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx}{11c^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} - \frac{\left(6b^2\right)\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{77ac^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(4b^3\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{77a^2c^6}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(8b^3\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{77a^2c^7}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(8b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \ldots\right.}{77a^2c^7\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(8b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \ldots\right.}{77a^2c^7\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(4b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \ldots\right.}{77a^2c^7\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{8b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.012955, size = 56, normalized size = 0.36

$$-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x^2)/a)])
/(11*(c*x)^(13/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2+a}\,(cx)^{-\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^7x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

3.925 $\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=147

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

```
[Out] (a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a + b*x^2)^(1/4)
)/(4*c) + (3*a^2*c^(5/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1
/4))])/(32*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(32*b^(7/4))
```

Rubi [A] time = 0.0921969, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 331, 298, 205, 208}

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a + b*x^2)^(1/4)
)/(4*c) + (3*a^2*c^(5/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1
/4))])/(32*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(32*b^(7/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{1}{8}a\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{32b}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{16b}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{16b}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c^3\right)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{32b^{3/2}}+\frac{\left(3a^2c^3\right)\text{Sub}}{}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right.}{32b^{7/4}}$$

Mathematica [C] time = 0.0442451, size = 85, normalized size = 0.58

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{4b\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hyp
ergeometric2F1[-1/4, 3/4, 7/4, -((b*x^2)/a)]))/(4*b*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}} \sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 43.4649, size = 46, normalized size = 0.31

$$\frac{\sqrt[4]{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

Giac [B] time = 3.05243, size = 570, normalized size = 3.88

$$-\frac{1}{128}a^2c^6\left(\frac{6\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}+\frac{2\left(bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^2c^4}+\frac{6\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}-\frac{2\left(bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^2c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/128*a^2*c^6*(6*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(
2)*(-b)^(1/4)*sqrt(abs(c)) + 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(
c*x))/((-b)^(1/4)*sqrt(abs(c))))/(b^2*c^4) + 6*sqrt(2)*(-b)^(1/4)*sqrt(abs(
c))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4)*sqrt(abs(c)) - 2*(b*c^2*x^2 + a
*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/((-b)^(1/4)*sqrt(abs(c))))/(b^2*c^4) +
3*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*log(sqrt(2)*(b*c^2*x^2 + a*c^2)^(1/4)*(-b
)^(1/4)*abs(c)/sqrt(c*x) + sqrt(-b)*abs(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)
/(c*x))/(b^2*c^4) - 3*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*log(-sqrt(2)*(b*c^2*x
^2 + a*c^2)^(1/4)*(-b)^(1/4)*abs(c)/sqrt(c*x) + sqrt(-b)*abs(c) + sqrt(b*c^
2*x^2 + a*c^2)*abs(c)/(c*x))/(b^2*c^4) - 8*(3*(b*c^2*x^2 + a*c^2)^(1/4)*b*c
^2*sqrt(abs(c))/sqrt(c*x) + (b*c^2*x^2 + a*c^2)^(1/4)*(b*c^2 + a*c^2/x^2)*s
qrt(abs(c))/sqrt(c*x))*x^4/(a^2*b*c^6))
```

3.926 $\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=116

$$-\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

```
[Out] ((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4)) + (a*Sqrt[c]*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4))
```

Rubi [A] time = 0.0698382, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 329, 331, 298, 205, 208}

$$-\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4)) + (a*Sqrt[c]*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx &= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{1}{4}a\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{2c}\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2c}\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4\sqrt{b}}-\frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4\sqrt{b}}\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}-\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.010098, size = 56, normalized size = 0.48

$$\frac{2x\sqrt{cx}\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{cx}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 2.50043, size = 46, normalized size = 0.4

$$\frac{\sqrt[4]{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

Giac [B] time = 1.64458, size = 498, normalized size = 4.29

$$\frac{1}{16}ac^2\left(\frac{8\left(bc^2x^2+ac^2\right)^{\frac{1}{4}}x^2\sqrt{|c|}}{\sqrt{cx}ac^2}+\frac{2\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}+\frac{2\left(bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)}{bc^2}+\frac{2\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\ldots}{\ldots}\right)}{bc^2}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/16*a*c^2*(8*(b*c^2*x^2 + a*c^2)^(1/4)*x^2*sqrt(abs(c))/(sqrt(c*x)*a*c^2)
+ 2*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4)*
sqrt(abs(c)) + 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/((-b)^(1
/4)*sqrt(abs(c))))/(b*c^2) + 2*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*arctan(-1/2*
sqrt(2)*(sqrt(2)*(-b)^(1/4)*sqrt(abs(c)) - 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt
(abs(c))/sqrt(c*x))/((-b)^(1/4)*sqrt(abs(c))))/(b*c^2) + sqrt(2)*(-b)^(1/4)
*sqrt(abs(c))*log(sqrt(2)*(b*c^2*x^2 + a*c^2)^(1/4)*(-b)^(1/4)*abs(c)/sqrt(
c*x) + sqrt(-b)*abs(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)/(c*x))/(b*c^2) - sq
rt(2)*(-b)^(1/4)*sqrt(abs(c))*log(-sqrt(2)*(b*c^2*x^2 + a*c^2)^(1/4)*(-b)^(
1/4)*abs(c)/sqrt(c*x) + sqrt(-b)*abs(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)/(c
*x))/(b*c^2))
```

3.927 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) - (b^(1/4)*ArcTan[(b^(1/4)*Sqrt[c*x])/
(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2) + (b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2)
```

Rubi [A] time = 0.0692396, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 329, 331, 298, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) - (b^(1/4)*ArcTan[(b^(1/4)*Sqrt[c*x])/
(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2) + (b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2)
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{b\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}-\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0117696, size = 54, normalized size = 0.5

$$
-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -((b*x^2)/a)])/(
(c*x)^(3/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$
\int \sqrt[4]{bx^2+a}\,(cx)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 3.92682, size = 49, normalized size = 0.46

$$\frac{\sqrt[4]{a}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [B] time = 1.85829, size = 452, normalized size = 4.22

$$2\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}+\frac{2\left(bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)+2\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}-\frac{2\left(bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)+\sqrt{2}(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/
4)*sqrt(abs(c)) + 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/((-b)
^(1/4)*sqrt(abs(c)))) + 2*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*arctan(-1/2*sqrt(
2)*(sqrt(2)*(-b)^(1/4)*sqrt(abs(c)) - 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(
c))/sqrt(c*x))/((-b)^(1/4)*sqrt(abs(c)))) + sqrt(2)*(-b)^(1/4)*sqrt(abs(c))
*log(sqrt(2)*(b*c^2*x^2 + a*c^2)^(1/4)*(-b)^(1/4)*abs(c)/sqrt(c*x) + sqrt(-
b)*abs(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)/(c*x)) - sqrt(2)*(-b)^(1/4)*sqrt
(abs(c))*log(-sqrt(2)*(b*c^2*x^2 + a*c^2)^(1/4)*(-b)^(1/4)*abs(c)/sqrt(c*x)
 + sqrt(-b)*abs(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)/(c*x)) - 8*(b*c^2*x^2 +
 a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/c^2
```

3.928 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}} dx$

Optimal. Leaf size=28

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rubi [A] time = 0.0060003, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}} dx = -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A] time = 0.008432, size = 26, normalized size = 0.93

$$-\frac{2x\left(a+bx^2\right)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$-\frac{2\,x}{5\,a}\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(7/2),x)
```

```
[Out] -2/5*x*(b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Fricas [A] time = 1.96398, size = 62, normalized size = 2.21

$$-\frac{2\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b*x^2 + a)^(5/4)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [B] time = 50.3123, size = 78, normalized size = 2.79

$$\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

```
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4))
+ b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4))
```

Giac [A] time = 2.01632, size = 58, normalized size = 2.07

$$-\frac{2\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2+\frac{ac^2}{x^2}\right)}{5\sqrt{cx}ac^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")

[Out] -2/5*(b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 + a*c^2/x^2)/(sqrt(c*x)*a*c^6)
```

3.929 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rubi [A] time = 0.0145266, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0148063, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{5/4}\left(4bx^2-5a\right)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[c*x]*(a + b*x^2)^(5/4)*(-5*a + 4*b*x^2))/(45*a^2*c^6*x^5)
```

Maple [A] time = 0.005, size = 31, normalized size = 0.5

$$-\frac{2x\left(-4bx^2+5a\right)}{45a^2}\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(5/4)*(-4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Fricas [A] time = 1.69684, size = 105, normalized size = 1.84

$$\frac{2\left(4b^2x^4-abx^2-5a^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 - a*b*x^2 - 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.25524, size = 144, normalized size = 2.53

$$\frac{2\left(\frac{9\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2+\frac{ac^2}{x^2}\right)bc^2}{\sqrt{cx}}-\frac{5\left(b^2c^8x^4+2\,abc^8x^2+a^2c^8\right)\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}}{\sqrt{cx}c^4x^4}\right)}{45\,a^2c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] 2/45*(9*(b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 + a*c^2/x^2)*b*c^2/sqrt(c*x) - 5*(
b^2*c^8*x^4 + 2*a*b*c^8*x^2 + a^2*c^8)*(b*c^4*x^2 + a*c^4)^(1/4)/(sqrt(c*x)
*c^4*x^4))/(a^2*c^10)
```

3.930 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rubi [A] time = 0.0244733, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}-\frac{8\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{15/2}}\,dx}{5a}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}
\end{aligned}$$

Mathematica [A] time = 0.0169091, size = 52, normalized size = 0.61

$$-\frac{2\sqrt{cx}\left(a+bx^2\right)^{5/4}\left(45a^2-40abx^2+32b^2x^4\right)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(5/4)*(45*a^2 - 40*a*b*x^2 + 32*b^2*x^4))/(585*a^
3*c^8*x^7)
```

Maple [A] time = 0.004, size = 42, normalized size = 0.5

$$-\frac{2x\left(32b^2x^4-40abx^2+45a^2\right)}{585a^3}\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -2/585*x*(b*x^2+a)^(5/4)*(32*b^2*x^4-40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Fricas [A] time = 1.77874, size = 135, normalized size = 1.59

$$-\frac{2\left(32b^3x^6-8ab^2x^4+5a^2bx^2+45a^3\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{585a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*b^3*x^6 - 8*a*b^2*x^4 + 5*a^2*b*x^2 + 45*a^3)*(b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.14117, size = 244, normalized size = 2.87

$$-\frac{2\left(\frac{117\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2+\frac{ac^2}{x^2}\right)b^2c^4}{\sqrt{cx}}-\frac{130\left(b^2c^8x^4+2\,abc^8x^2+a^2c^8\right)\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}b}{\sqrt{cx}c^2x^4}+\frac{45\left(b^3c^{12}x^6+3\,ab^2c^{12}x^4+3\,a^2bc^{12}x^2+a^3c^{12}\right)\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}}{\sqrt{cx}c^6x^6}\right)}{585\,a^3c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

```
[Out] -2/585*(117*(b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 + a*c^2/x^2)*b^2*c^4/sqrt(c*x)
 - 130*(b^2*c^8*x^4 + 2*a*b*c^8*x^2 + a^2*c^8)*(b*c^4*x^2 + a*c^4)^(1/4)*b/
(sqrt(c*x)*c^2*x^4) + 45*(b^3*c^12*x^6 + 3*a*b^2*c^12*x^4 + 3*a^2*b*c^12*x^
2 + a^3*c^12)*(b*c^4*x^2 + a*c^4)^(1/4)/(sqrt(c*x)*c^6*x^6))/(a^3*c^14)
```

3.931 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=113

$$\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a + b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a + b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a + b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rubi [A] time = 0.0384764, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a + b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a + b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a + b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}-\frac{12\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{19/2}}\,dx}{5a}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}-\frac{128\int \frac{\left(a+bx^2\right)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}
\end{aligned}$$

Mathematica [A] time = 0.0160607, size = 63, normalized size = 0.56

$$\frac{2 \left(a+b x^2\right)^{5/4} \left(180 a^2 b x^2-195 a^3-160 a b^2 x^4+128 b^3 x^6\right)}{3315 a^4 c^9 x^8 \sqrt{c x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (2*(a + b*x^2)^(5/4)*(-195*a^3 + 180*a^2*b*x^2 - 160*a*b^2*x^4 + 128*b^3*x^
6))/(3315*a^4*c^9*x^8*Sqrt[c*x])
```

Maple [A] time = 0.005, size = 53, normalized size = 0.5

$$-\frac{2\,x\left(-128\,b^3x^6+160\,ab^2x^4-180\,a^2bx^2+195\,a^3\right)}{3315\,a^4}\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -2/3315*x*(b*x^2+a)^(5/4)*(-128*b^3*x^6+160*a*b^2*x^4-180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Fricas [A] time = 1.80267, size = 165, normalized size = 1.46

$$\frac{2\left(128\,b^4x^8-32\,ab^3x^6+20\,a^2b^2x^4-15\,a^3bx^2-195\,a^4\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 - 32*a*b^3*x^6 + 20*a^2*b^2*x^4 - 15*a^3*b*x^2 - 195*a^
4)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.84309, size = 359, normalized size = 3.18

$$\frac{2\left(\frac{663\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2+\frac{ac^2}{x^2}\right)b^3c^6}{\sqrt{cx}}-\frac{1105\left(b^2c^8x^4+2\,abc^8x^2+a^2c^8\right)\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}b^2}{\sqrt{cx}x^4}+\frac{765\left(b^3c^{12}x^6+3\,ab^2c^{12}x^4+3\,a^2bc^{12}x^2+a^3c^{12}\right)\left(bc^4x^2+ac^4\right)^{\frac{1}{4}}b}{\sqrt{cx}c^4x^6}-1\cdots\right.}{3315\,a^4c^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] 2/3315*(663*(b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 + a*c^2/x^2)*b^3*c^6/sqrt(c*x)
 - 1105*(b^2*c^8*x^4 + 2*a*b*c^8*x^2 + a^2*c^8)*(b*c^4*x^2 + a*c^4)^(1/4)*b
^2/(sqrt(c*x)*x^4) + 765*(b^3*c^12*x^6 + 3*a*b^2*c^12*x^4 + 3*a^2*b*c^12*x^
2 + a^3*c^12)*(b*c^4*x^2 + a*c^4)^(1/4)*b/(sqrt(c*x)*c^4*x^6) - 195*(b^4*c^
16*x^8 + 4*a*b^3*c^16*x^6 + 6*a^2*b^2*c^16*x^4 + 4*a^3*b*c^16*x^2 + a^4*c^1
6)*(b*c^4*x^2 + a*c^4)^(1/4)/(sqrt(c*x)*c^8*x^8))/(a^4*c^18)
```

3.932 $\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=122

$$-\frac{a^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}$$

```
[Out] -(a*c*Sqrt[c*x]*(a - b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a - b*x^2)^(1/4))/
(3*c) - (a^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b
]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0920301, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {279, 321, 329, 237, 335, 275, 232}

$$-\frac{a^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c*Sqrt[c*x]*(a - b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a - b*x^2)^(1/4))/
(3*c) - (a^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b
]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{1}{6}a\int \frac{(cx)^{3/2}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{\left(a^2c^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{12b}$$

$$= -\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{\left(a^2c\right)\operatorname{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{6b}$$

$$= -\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{c}\right)}{6b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{6b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{12b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{a^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0534662, size = 88, normalized size = 0.72

$$\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}\left(a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^2}{a}\right)+\sqrt[4]{1-\frac{bx^2}{a}}\left(bx^2-a\right)\right)}{3b\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hype
rgeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a]))/(3*b*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}}\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

Sympy [C] time = 5.09224, size = 48, normalized size = 0.39

$$\frac{\sqrt[4]{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*exp
_polar(2*I*pi)/a)/(2*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2 + a\right)^{\frac{1}{4}} (cx)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

3.933 $\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

```
[Out] (Sqrt[c*x]*(a - b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a - b*x^2)^(3/
4))
```

Rubi [A] time = 0.0791617, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {279, 329, 237, 335, 275, 232}

$$\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*(a - b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a - b*x^2)^(3/
4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} + \frac{1}{2}a\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} + \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} + \frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} - \frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} - \frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{2c\left(a-bx^2\right)^{3/4}}$$

$$= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0135242, size = 55, normalized size = 0.6

$$\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a])/(Sqrt[
c*x]*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt[4]{-b x^{2}+a}\,\frac{1}{\sqrt{c x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^{2}+a\right)^{\frac{1}{4}}}{\sqrt{c x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-b x^{2}+a\right)^{\frac{1}{4}}\sqrt{c x}}{c x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

Sympy [C] time = 1.44166, size = 39, normalized size = 0.42

$$-\frac{i\sqrt[4]{b}xe^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{a}{bx^{2}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(1/2),x)
```

```
[Out] -I*b**(1/4)*x*exp(3*I*pi/4)*hyper((-1/2, -1/4), (1/2,), a/(b*x**2))/sqrt(c)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

3.934 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}} \, dx$

Optimal. Leaf size=97

$$\frac{2b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) + (2*b^(3/2)*(1 - a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 - b*x^2)^(3/4))
```

Rubi [A] time = 0.0785771, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {277, 329, 237, 335, 275, 232}

$$\frac{2b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) + (2*b^(3/2)*(1 - a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 - b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} - \frac{b\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{3c^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{3c^3}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} - \frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{3c^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{3c^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{\left(b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}c^4\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0153646, size = 57, normalized size = 0.59

$$
-\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, (b*x^2)/a])/(3*(
c*x)^(5/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \sqrt[4]{-b x^{2}+a}\, (c x)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^{2}+a\right)^{\frac{1}{4}}}{(c x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-b x^{2}+a\right)^{\frac{1}{4}} \sqrt{c x}}{c^{3} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] time = 11.0769, size = 36, normalized size = 0.37

$$-\frac{i \sqrt[4]{b} e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{a}{b x^{2}}} \right)}{c^{\frac{5}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

```
[Out] -I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 1/2), (3/2,), a/(b*x**2))/(c**(5/2)*x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

3.935 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{4b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) + (2*b*(a - b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Csc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0958838, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {277, 325, 329, 237, 335, 275, 232}

$$\frac{4b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```



```
[Out] (-2*(a - b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) + (2*b*(a - b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Csc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a - b*x^2)^(3/4))
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}-\frac{b\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{7c^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{21ac^4}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{21ac^5}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{21ac^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{21ac^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(2b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{21ac^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{4b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0146951, size = 57, normalized size = 0.45

$$-\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, (b*x^2)/a])/(7*
(c*x)^(9/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2+a}\,(cx)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

3.936 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx$

Optimal. Leaf size=159

$$\frac{8b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) + (2*b*(a - b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a - b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) + (8*
b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.111031, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {277, 325, 329, 237, 335, 275, 232}

$$\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{8b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) + (2*b*(a - b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a - b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) + (8*
b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a - b*x^2)^(3/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} - \frac{b\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx}{11c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} - \frac{\left(6b^2\right)\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{77ac^4}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(4b^3\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{77a^2c^6}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(8b^3\right)\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{77a^2c^7}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(8b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x,\right.}{77a^2c^7\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(8b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x,\right.}{77a^2c^7\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(4b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x,\right.}{77a^2c^7\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{8b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0145603, size = 57, normalized size = 0.36

$$-\frac{2x\sqrt[4]{a-bx^2}\,_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (b*x^2)/a])/(1
1*(c*x)^(13/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2+a}\,(cx)^{-\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

3.937 $\int (cx)^{5/2} \sqrt[4]{a - bx^2}\, dx$

Optimal. Leaf size=343

$$\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}} + \frac{3a^2c^{5/2}\text{ta}}{}$$

```
[Out] -(a*c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a - b*x^2)^(1/4
))/(4*c) - (3*a^2*c^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) + (3*a^2*c^(5/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) +
(3*a^2*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4)) - (3*a^2*c^(5/
2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqr
t[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4))
```

Rubi [A] time = 0.367634, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {279, 321, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}} + \frac{3a^2c^{5/2}\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a - b*x^2)^(1/4
))/(4*c) - (3*a^2*c^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) + (3*a^2*c^(5/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) +
(3*a^2*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4)) - (3*a^2*c^(5/
2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqr
t[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{1}{8}a\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{32b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} - \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{32b^{3/2}} + \frac{\left(3a^2c\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{64\sqrt{2}b^{7/4}} + \frac{\left(3a^2c\right.}{}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{3a^2c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2}\log\left(\sqrt{c}\right.}{64}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} - \frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}} + \frac{3a^2c^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt{c}\sqrt[4]{a}}\right.}{32\sqrt{2}b^{7/4}}$$

Mathematica [C] time = 0.0468912, size = 88, normalized size = 0.26

$$\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}\left(a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)+\sqrt[4]{1-\frac{bx^2}{a}}\left(bx^2-a\right)\right)}{4b\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hy
pergeometric2F1[-1/4, 3/4, 7/4, (b*x^2)/a]))/(4*b*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}}\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 44.795, size = 48, normalized size = 0.14

$$\frac{\sqrt[4]{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(2*I*pi)/a)/(2*gamma(11/4))
```

Giac [A] time = 2.10758, size = 533, normalized size = 1.55

$$-\frac{1}{128}a^2c^6\left(\frac{6\sqrt{2}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}+\frac{2\left(-bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{7}{4}}c^4}+\frac{6\sqrt{2}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}-\frac{2\left(-bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{7}{4}}c^4}+\frac{3\sqrt{2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/128*a^2*c^6*(6*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
sqrt(abs(c)) + 2*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/(b^(1/4
)*sqrt(abs(c))))/(b^(7/4)*c^4) + 6*sqrt(2)*sqrt(abs(c))*arctan(-1/2*sqrt(2)
*(sqrt(2)*b^(1/4)*sqrt(abs(c)) - 2*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/
sqrt(c*x))/(b^(1/4)*sqrt(abs(c))))/(b^(7/4)*c^4) + 3*sqrt(2)*sqrt(abs(c))*l
og(sqrt(2)*(-b*c^2*x^2 + a*c^2)^(1/4)*b^(1/4)*abs(c)/sqrt(c*x) + sqrt(b)*ab
s(c) + sqrt(-b*c^2*x^2 + a*c^2)*abs(c)/(c*x))/(b^(7/4)*c^4) - 3*sqrt(2)*sqr
t(abs(c))*log(-sqrt(2)*(-b*c^2*x^2 + a*c^2)^(1/4)*b^(1/4)*abs(c)/sqrt(c*x)
+ sqrt(b)*abs(c) + sqrt(-b*c^2*x^2 + a*c^2)*abs(c)/(c*x))/(b^(7/4)*c^4) - 8
*(3*(-b*c^2*x^2 + a*c^2)^(1/4)*b*c^2*sqrt(abs(c))/sqrt(c*x) + (-b*c^2*x^2 +
 a*c^2)^(1/4)*(b*c^2 - a*c^2/x^2)*sqrt(abs(c))/sqrt(c*x))*x^4/(a^2*b*c^6))
```

3.938 $\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=307

$$\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{3/4}}-\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{3/4}}-\frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}}+\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{c}}\right.}{4\sqrt{2}b^{3}}$$

```
[Out] ((c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(3/4)) + (a*Sqrt[
c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*
Sqrt[2]*b^(3/4)) + (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*
x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
- (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b
^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
```

Rubi [A] time = 0.268361, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {279, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{3/4}}-\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{3/4}}-\frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}}+\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{c}}\right.}{4\sqrt{2}b^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(3/4)) + (a*Sqrt[
c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*
Sqrt[2]*b^(3/4)) + (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*
x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
- (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b
^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{1}{4}a\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\,\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{bc}} + \frac{a\,\text{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{bc}}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{\left(a\sqrt{c}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{3/4}} + \frac{\left(a\sqrt{c}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\right.}{8\sqrt{2}b^{3/4}}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{3/4}} - \frac{a\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{3/4}} + \cdots$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}} + \frac{a\sqrt{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}} + \frac{a\sqrt{c}\log\left(\sqrt{c}+\cdots\right.}{8\sqrt{2}b\cdots}$$

Mathematica [C] time = 0.0110217, size = 57, normalized size = 0.19

$$\frac{2x\sqrt{cx}\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)}{3\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (b*x^2)/
a])/(3*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt{cx}\sqrt[4]{-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 2.65611, size = 48, normalized size = 0.16

$$\frac{\sqrt[4]{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(2*I*pi)/a)/(2*gamma(7/4))
```

Giac [A] time = 2.24555, size = 459, normalized size = 1.5

$$\frac{1}{16}ac^2\left(\frac{8\left(-bc^2x^2+ac^2\right)^{\frac{1}{4}}x^2\sqrt{|c|}}{\sqrt{cx}ac^2}-\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}+\frac{2\left(-bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{3}{4}}c^2}-\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}-\frac{2\left(-bc^2\right.}{}\right.}{2b^{\frac{1}{4}}\sqrt{|c|}}\right.}{b^{\frac{3}{4}}c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/16*a*c^2*(8*(-b*c^2*x^2 + a*c^2)^(1/4)*x^2*sqrt(abs(c))/(sqrt(c*x)*a*c^2)
 - 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*sqrt(abs(c))
+ 2*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/(b^(1/4)*sqrt(abs(c)
)))/(b^(3/4)*c^2) - 2*sqrt(2)*sqrt(abs(c))*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*sqrt(abs(c)) - 2*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/(b
^(1/4)*sqrt(abs(c))))/(b^(3/4)*c^2) - sqrt(2)*sqrt(abs(c))*log(sqrt(2)*(-b*
c^2*x^2 + a*c^2)^(1/4)*b^(1/4)*abs(c)/sqrt(c*x) + sqrt(b)*abs(c) + sqrt(-b*
c^2*x^2 + a*c^2)*abs(c)/(c*x))/(b^(3/4)*c^2) + sqrt(2)*sqrt(abs(c))*log(-sq
rt(2)*(-b*c^2*x^2 + a*c^2)^(1/4)*b^(1/4)*abs(c)/sqrt(c*x) + sqrt(b)*abs(c)
+ sqrt(-b*c^2*x^2 + a*c^2)*abs(c)/(c*x))/(b^(3/4)*c^2))
```

3.939 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}c^{3/2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right.}{\sqrt{2}c^{3/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(c*Sqrt[c*x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c^(3/2)) - (b^(1/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c
^(3/2)) - (b^(1/4)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqr
t[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2)) + (b^(1/4)*
Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c
*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2))
```

Rubi [A] time = 0.274709, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {277, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}c^{3/2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right.}{\sqrt{2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(c*Sqrt[c*x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c^(3/2)) - (b^(1/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c
^(3/2)) - (b^(1/4)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqr
t[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2)) + (b^(1/4)*
Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c
*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{b\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx}{c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}c^{3/2}} -$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}c^{3/2}} + \frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \ldots\right.}{}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right.}{2\sqrt{2}c^{3/2}}$$

Mathematica [C] time = 0.0128392, size = 55, normalized size = 0.19

$$-\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (b*x^2)/a])/((c*
x)^(3/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2+a}\,(cx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 4.25267, size = 51, normalized size = 0.17

$$\frac{\sqrt[4]{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(2*I*pi)/a
)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [A] time = 2.79996, size = 429, normalized size = 1.45

$$2\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}+\frac{2\left(-bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)+2\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}-\frac{2\left(-bc^2x^2+ac^2\right)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)+\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*b^(1/4)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*sqr
t(abs(c)) + 2*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/(b^(1/4)*s
```

```
qrt(abs(c)))) + 2*sqrt(2)*b^(1/4)*sqrt(abs(c))*arctan(-1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*sqrt(abs(c)) - 2*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x)
)/(b^(1/4)*sqrt(abs(c)))) + sqrt(2)*b^(1/4)*sqrt(abs(c))*log(sqrt(2)*(-b*c^
2*x^2 + a*c^2)^(1/4)*b^(1/4)*abs(c)/sqrt(c*x) + sqrt(b)*abs(c) + sqrt(-b*c^
2*x^2 + a*c^2)*abs(c)/(c*x)) - sqrt(2)*b^(1/4)*sqrt(abs(c))*log(-sqrt(2)*(-
b*c^2*x^2 + a*c^2)^(1/4)*b^(1/4)*abs(c)/sqrt(c*x) + sqrt(b)*abs(c) + sqrt(-
b*c^2*x^2 + a*c^2)*abs(c)/(c*x)) - 8*(-b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c)
)/sqrt(c*x))/c^2
```

3.940 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rubi [A] time = 0.0061529, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {264}

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx=-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A] time = 0.0081107, size = 27, normalized size = 0.93

$$-\frac{2x\left(a-bx^2\right)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-\frac{2\,x}{5\,a}\left(-bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(7/2),x)
```

```
[Out] -2/5*x*(-b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Fricas [A] time = 2.22486, size = 78, normalized size = 2.69

$$\frac{2\left(bx^2-a\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^2 - a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [B] time = 52.9355, size = 182, normalized size = 6.28

$$\begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for } \frac{|a|}{|b||x^2|}>1 \\ \frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^2}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*ga
mma(-1/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gam
ma(-1/4)), Abs(a)/(Abs(b)*Abs(x**2)) > 1), (b**(1/4)*(-a/(b*x**2) + 1)**(1/
4)*exp(I*pi/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4)) - b**(5/4)*(-a/(b*
x**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4)), True)
)
```

Giac [A] time = 2.43532, size = 61, normalized size = 2.1

$$\frac{2\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2-\frac{ac^2}{x^2}\right)}{5\sqrt{cx}ac^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] 2/5*(-b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 - a*c^2/x^2)/(sqrt(c*x)*a*c^6)
```

3.941 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rubi [A] time = 0.0162985, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0167363, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{5/4}\left(5a+4bx^2\right)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(5/4)*(5*a + 4*b*x^2))/(45*a^2*c^6*x^5)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.5

$$-\frac{2\,x\left(4\,b x^{2}+5\,a\right)}{45\,a^{2}}\left(-b x^{2}+a\right)^{\frac{5}{4}}(cx)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -2/45*x*(-b*x^2+a)^(5/4)*(4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-b x^{2}+a\right)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Fricas [A] time = 2.06019, size = 107, normalized size = 1.81

$$\frac{2\left(4\,b^{2}x^{4}+abx^{2}-5\,a^{2}\right)\left(-bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^{2}c^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 + a*b*x^2 - 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^
5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.62605, size = 149, normalized size = 2.53

$$\frac{2\left(\frac{9\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2-\frac{ac^2}{x^2}\right)bc^2}{\sqrt{cx}}-\frac{5\left(b^2c^8x^4-2\,abc^8x^2+a^2c^8\right)\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}}{\sqrt{cx}c^4x^4}\right)}{45\,a^2c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] 2/45*(9*(-b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 - a*c^2/x^2)*b*c^2/sqrt(c*x) - 5*
(b^2*c^8*x^4 - 2*a*b*c^8*x^2 + a^2*c^8)*(-b*c^4*x^2 + a*c^4)^(1/4)/(sqrt(c*
x)*c^4*x^4))/(a^2*c^10)
```

3.942 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rubi [A] time = 0.0278238, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}-\frac{8\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{15/2}}\,dx}{5a}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}\end{aligned}$$

Mathematica [A] time = 0.0183541, size = 53, normalized size = 0.6

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{5/4}\left(45a^2+40abx^2+32b^2x^4\right)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(5/4)*(45*a^2 + 40*a*b*x^2 + 32*b^2*x^4))/(585*a^
3*c^8*x^7)
```

Maple [A] time = 0.003, size = 43, normalized size = 0.5

$$-\frac{2x\left(32b^2x^4+40abx^2+45a^2\right)}{585a^3}\left(-bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -2/585*x*(-b*x^2+a)^(5/4)*(32*b^2*x^4+40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Fricas [A] time = 1.56927, size = 135, normalized size = 1.53

$$\frac{2\left(32b^3x^6+8ab^2x^4+5a^2bx^2-45a^3\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{585a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*b^3*x^6 + 8*a*b^2*x^4 + 5*a^2*b*x^2 - 45*a^3)*(-b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.68627, size = 251, normalized size = 2.85

$$\frac{2\left(\frac{117\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2-\frac{ac^2}{x^2}\right)b^2c^4}{\sqrt{cx}}-\frac{130\left(b^2c^8x^4-2\,abc^8x^2+a^2c^8\right)\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}b}{\sqrt{cx}c^2x^4}+\frac{45\left(b^3c^{12}x^6-3\,ab^2c^{12}x^4+3\,a^2bc^{12}x^2-a^3c^{12}\right)\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}}{\sqrt{cx}c^6x^6}\right)}{585\,a^3c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

```
[Out] 2/585*(117*(-b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 - a*c^2/x^2)*b^2*c^4/sqrt(c*x)
 - 130*(b^2*c^8*x^4 - 2*a*b*c^8*x^2 + a^2*c^8)*(-b*c^4*x^2 + a*c^4)^(1/4)*b
/(sqrt(c*x)*c^2*x^4) + 45*(b^3*c^12*x^6 - 3*a*b^2*c^12*x^4 + 3*a^2*b*c^12*x
^2 - a^3*c^12)*(-b*c^4*x^2 + a*c^4)^(1/4)/(sqrt(c*x)*c^6*x^6))/(a^3*c^14)
```

3.943 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=117

$$\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a - b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a - b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a - b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rubi [A] time = 0.0402291, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a - b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a - b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a - b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}-\frac{12\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{19/2}}\,dx}{5a}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}-\frac{128\int \frac{\left(a-bx^2\right)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}\end{aligned}$$

Mathematica [A] time = 0.0165824, size = 64, normalized size = 0.55

$$-\frac{2\left(a-bx^2\right)^{5/4}\left(180a^2bx^2+195a^3+160ab^2x^4+128b^3x^6\right)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4)*(195*a^3 + 180*a^2*b*x^2 + 160*a*b^2*x^4 + 128*b^3*x^
6))/(3315*a^4*c^9*x^8*Sqrt[c*x])
```

Maple [A] time = 0.005, size = 54, normalized size = 0.5

$$-\frac{2\,x\left(128\,b^3x^6+160\,ab^2x^4+180\,a^2bx^2+195\,a^3\right)}{3315\,a^4}\left(-bx^2+a\right)^{\frac{5}{4}}(cx)^{-\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -2/3315*x*(-b*x^2+a)^(5/4)*(128*b^3*x^6+160*a*b^2*x^4+180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Fricas [A] time = 1.569, size = 166, normalized size = 1.42

$$\frac{2\left(128\,b^4x^8+32\,ab^3x^6+20\,a^2b^2x^4+15\,a^3bx^2-195\,a^4\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 + 32*a*b^3*x^6 + 20*a^2*b^2*x^4 + 15*a^3*b*x^2 - 195*a^
4)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.12245, size = 367, normalized size = 3.14

$$\frac{2\left(\frac{663\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}\left(bc^2-\frac{ac^2}{x^2}\right)b^3c^6}{\sqrt{cx}}-\frac{1105\left(b^2c^8x^4-2\,abc^8x^2+a^2c^8\right)\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}b^2}{\sqrt{cx}x^4}+\frac{765\left(b^3c^{12}x^6-3\,ab^2c^{12}x^4+3\,a^2bc^{12}x^2-a^3c^{12}\right)\left(-bc^4x^2+ac^4\right)^{\frac{1}{4}}b}{\sqrt{cx}c^4x^6}-\right.}{3315\,a^4c^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] 2/3315*(663*(-b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 - a*c^2/x^2)*b^3*c^6/sqrt(c*x
) - 1105*(b^2*c^8*x^4 - 2*a*b*c^8*x^2 + a^2*c^8)*(-b*c^4*x^2 + a*c^4)^(1/4)
*b^2/(sqrt(c*x)*x^4) + 765*(b^3*c^12*x^6 - 3*a*b^2*c^12*x^4 + 3*a^2*b*c^12*
x^2 - a^3*c^12)*(-b*c^4*x^2 + a*c^4)^(1/4)*b/(sqrt(c*x)*c^4*x^6) - 195*(b^4
*c^16*x^8 - 4*a*b^3*c^16*x^6 + 6*a^2*b^2*c^16*x^4 - 4*a^3*b*c^16*x^2 + a^4*
c^16)*(-b*c^4*x^2 + a*c^4)^(1/4)/(sqrt(c*x)*c^8*x^8))/(a^4*c^18)
```

3.944 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4)) - (a*c^(3/2)*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4))
```

Rubi [A] time = 0.0648402, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 240, 212, 208, 205}

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4)) - (a*c^(3/2)*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx}{4b} \\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b} \\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b} \\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{\left(ac^2\right)\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b} - \frac{\left(ac^2\right)\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b} \\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} - \frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.0362232, size = 97, normalized size = 0.83

$$\frac{(cx)^{3/2}\left(2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^{3/4} - a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) - a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(a + b*x^2)^(3/4) - a*ArcTan[(b^(1/4)*Sqrt[
x])/(a + b*x^2)^(1/4)] - a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(
4*b^(5/4)*x^(3/2))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}} \frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

Fricas [B] time = 1.79081, size = 689, normalized size = 5.89

$$\frac{4\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}c+4\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}ab^4c-\left(b^5x^2+ab^4\right)\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}a^2c^3x+\sqrt{\frac{a^4c^6}{b^5}}\left(b^3x^2+ab^2\right)}{bx^2+a}}}{a^4bc^6x^2+a^5c^6}\right)-\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\,\mathrm{l}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(a^4*c^6/b^5)^(1/4)*b*arctan(-((a^
4*c^6/b^5)^(3/4)*(b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 + a*b^4)*(a
^4*c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*a^2*c^3*x + sqrt(a^4*c^6/b^5)*(b^3*
x^2 + a*b^2))/(b*x^2 + a)))/(a^4*b*c^6*x^2 + a^5*c^6)) - (a^4*c^6/b^5)^(1/4
)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a
*b))/(b*x^2 + a)) + (a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*
a*c - (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a*b))/(b*x^2 + a)))/b
```

Sympy [C] time = 3.63552, size = 44, normalized size = 0.38

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(1/4)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

3.945 $\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

```
[Out] ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c]) +
 ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c])
```

Rubi [A] time = 0.0502655, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 240, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c]) +
 ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right) + \text{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0113856, size = 65, normalized size = 0.78

$$\frac{\sqrt{x}\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{\sqrt[4]{b}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)]))/(b^(1/4)*Sqrt[c*x])
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Fricas [B] time = 1.71301, size = 576, normalized size = 6.94

$$-2\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}bc\left(\frac{1}{bc^2}\right)^{\frac{3}{4}}-\left(b^2cx^2+abc\right)\sqrt{\frac{\sqrt{bx^2+a}cx+\left(bc^2x^2+ac^2\right)\sqrt{\frac{1}{bc^2}}}{bx^2+a}}\left(\frac{1}{bc^2}\right)^{\frac{3}{4}}}{bx^2+a}\right)+\frac{1}{2}\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(1/(b*c^2))^(1/4)*arctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(1/(b*c^2))^(
3/4) - (b^2*c*x^2 + a*b*c)*sqrt((sqrt(b*x^2 + a)*c*x + (b*c^2*x^2 + a*c^2)*
sqrt(1/(b*c^2)))/(b*x^2 + a))*(1/(b*c^2))^(3/4))/(b*x^2 + a)) + 1/2*(1/(b*c
^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 + a*c)*(1/(b*c^2))^(
1/4))/(b*x^2 + a)) - 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
 - (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a))
```

Sympy [C] time = 1.7304, size = 44, normalized size = 0.53

$$\frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(1/4)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

3.946 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rubi [A] time = 0.005981, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]

[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx=-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A] time = 0.0057442, size = 26, normalized size = 0.93

$$-\frac{2x\left(a+bx^2\right)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]

[Out] (-2*x*(a + b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$-\frac{2x}{3a}\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/3*x*(b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Fricas [A] time = 1.54449, size = 62, normalized size = 2.21

$$-\frac{2\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/3*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [A] time = 11.2281, size = 36, normalized size = 1.29

$$\frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

3.947 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rubi [A] time = 0.0158406, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\ &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}+\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0176363, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/4}\left(4bx^2-3a\right)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (2*Sqrt[c*x]*(a + b*x^2)^(3/4)*(-3*a + 4*b*x^2))/(21*a^2*c^5*x^4)
```

Maple [A] time = 0.003, size = 31, normalized size = 0.5

$$-\frac{2x\left(-4bx^2+3a\right)}{21a^2}\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/21*x*(b*x^2+a)^(3/4)*(-4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Fricas [A] time = 1.57777, size = 86, normalized size = 1.51

$$\frac{2\left(4bx^2-3a\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/21*(4*b*x^2 - 3*a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

3.948 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rubi [A] time = 0.0264572, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}-\frac{8\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{3a}\\
&= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}\\
&= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.0237282, size = 52, normalized size = 0.61

$$-\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/4}\left(21a^2-24abx^2+32b^2x^4\right)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(3/4)*(21*a^2 - 24*a*b*x^2 + 32*b^2*x^4))/(231*a^
3*c^7*x^6)
```

Maple [A] time = 0.004, size = 42, normalized size = 0.5

$$-\frac{2x\left(32b^2x^4-24abx^2+21a^2\right)}{231a^3}\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/231*x*(b*x^2+a)^(3/4)*(32*b^2*x^4-24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Fricas [A] time = 1.44771, size = 115, normalized size = 1.35

$$-\frac{2\left(32b^2x^4-24abx^2+21a^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{231a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*b^2*x^4 - 24*a*b*x^2 + 21*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*
c^7*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

3.949 $\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=156

$$\frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}}+\frac{7a^{5/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2}+\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}$$

```
[Out] (7*a^2*c^4*x*Sqrt[c*x])/(20*b^2*(a + b*x^2)^(1/4)) - (7*a*c^3*(c*x)^(3/2)*(
a + b*x^2)^(3/4))/(30*b^2) + (c*(c*x)^(7/2)*(a + b*x^2)^(3/4))/(5*b) + (7*a
^(5/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(20*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0671365, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {321, 314, 284, 335, 196}

$$\frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}}+\frac{7a^{5/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2}+\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (7*a^2*c^4*x*Sqrt[c*x])/(20*b^2*(a + b*x^2)^(1/4)) - (7*a*c^3*(c*x)^(3/2)*(
a + b*x^2)^(3/4))/(30*b^2) + (c*(c*x)^(7/2)*(a + b*x^2)^(3/4))/(5*b) + (7*a
^(5/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(20*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 314

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(x*Sqrt[
c*x])/(a + b*x^2)^(1/4), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7ac^2\right)\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx}{10b}$$

$$= -\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(7a^2c^4\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx}{20b^2}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7a^3c^4\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{40b^2}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7a^3c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,d}{40b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(7a^3c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{}{\left(1+\frac{a}{}\right.}\right.}{40b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{7a^{5/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{20b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0312011, size = 87, normalized size = 0.56

$$\frac{c^3(cx)^{3/2}\left(7a^2\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-7a^2-abx^2+6b^2x^4\right)}{30b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c^3*(c*x)^(3/2)*(-7*a^2 - a*b*x^2 + 6*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(1/4
)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(30*b^2*(a + b*x^2)^(1/4
))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (cx)^{\frac{9}{2}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^4x^4}{(bx^2+a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^4*x^4/(b*x^2 + a)^(1/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

3.950 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}-\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}$$

```
[Out] -(a*c^2*x*Sqrt[c*x])/(2*b*(a + b*x^2)^(1/4)) + (c*(c*x)^(3/2)*(a + b*x^2)^(
3/4))/(3*b) - (a^(3/2)*c^2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0465621, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {321, 314, 284, 335, 196}

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}-\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] -(a*c^2*x*Sqrt[c*x])/(2*b*(a + b*x^2)^(1/4)) + (c*(c*x)^(3/2)*(a + b*x^2)^(
3/4))/(3*b) - (a^(3/2)*c^2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 314

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(x*Sqrt[
c*x])/(a + b*x^2)^(1/4), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{\left(ac^2\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx}{2b}$$

$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} + \frac{\left(a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b}$$

$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} + \frac{\left(a^2c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{\left(a^2c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4b^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{a^{3/2}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0238553, size = 69, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(-a\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
3/4, 7/4, -((b*x^2)/a)]))/(3*b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 30.3746, size = 44, normalized size = 0.35

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

3.951 $\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

```
[Out] (x*Sqrt[c*x])/(a + b*x^2)^(1/4) + (Sqrt[a]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*
EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0324452, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {314, 284, 335, 196}

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

```
[Out] (x*Sqrt[c*x])/(a + b*x^2)^(1/4) + (Sqrt[a]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*
EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 314

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(x*Sqrt[
c*x])/(a + b*x^2)^(1/4), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} - \frac{1}{2}a\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} - \frac{\left(a\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\left(a\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0098727, size = 56, normalized size = 0.67

$$\frac{2x\sqrt{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*
x^2)/a)])/(3*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \sqrt{cx}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 1.05564, size = 44, normalized size = 0.53

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(1/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

3.952 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (2*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[
c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^
(1/4))
```

Rubi [A] time = 0.0345417, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {316, 284, 335, 196}

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] -2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (2*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[
c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^
(1/4))
```

Rule 316

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{b\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{c^2}$$

$$= -\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{c^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{c^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{2\sqrt{b}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.011586, size = 54, normalized size = 0.6

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x^2)/a)]
)/((c*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{2}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^{2}x^{4}+ac^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] time = 3.99342, size = 31, normalized size = 0.34

$$-\frac{{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{\sqrt[4]{b}c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(1/4)*c**(3/2)*x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

3.953 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=126

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}+\frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

```
[Out] (4*b)/(5*a*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(5*a*c*
(c*x)^(5/2)) - (4*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0490704, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 316, 284, 335, 196}

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}+\frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```


```
[Out] (4*b)/(5*a*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(5*a*c*
(c*x)^(5/2)) - (4*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a + b*x^2)^(1/4))
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 316

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx}{5ac^2}\\
&= \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ac^4}\\
&= \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5ac^4\sqrt[4]{a+bx^2}}\\
&= \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{5ac^4\sqrt[4]{a+bx^2}}\\
&= \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0122788, size = 56, normalized size = 0.44

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x^2)/a)
])/(5*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{7}{2}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^4x^6+ac^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 + a*c^4*x^4), x)
```

Sympy [C] time = 96.643, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{3\sqrt[4]{b}c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*b**(1/4)*c**(7/2)
*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

3.954 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=157

$$-\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

```
[Out] (-8*b^2)/(15*a^2*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(
9*a*c*(c*x)^(9/2)) + (4*b*(a + b*x^2)^(3/4))/(15*a^2*c^3*(c*x)^(5/2)) + (8*
b^(5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(15*a^(5/2)*c^6*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0678214, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 316, 284, 335, 196}

$$-\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-8*b^2)/(15*a^2*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(
9*a*c*(c*x)^(9/2)) + (4*b*(a + b*x^2)^(3/4))/(15*a^2*c^3*(c*x)^(5/2)) + (8*
b^(5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(15*a^(5/2)*c^6*(a + b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 316

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} - \frac{(2b)\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx}{3ac^2}$$

$$= -\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx}{15a^2c^4}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{\left(4b^3\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{15a^2c^6}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}}{15a^2c^6\sqrt[4]{a+bx^2}}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{}{(1+}\right.}{15a^2c^6\sqrt[4]{a+bx^2}}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.013287, size = 56, normalized size = 0.36

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x^2)/a)
])/(9*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{11}{2}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^6x^8+ac^6x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 + a*c^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

3.955 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=308

$$-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{5/4}}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}}+\frac{ac^{3/2}\tan^{-1}}{4\sqrt{\ }}$$

```
[Out] -(c*Sqrt[c*x]*(a - b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(5/4)) + (a*c^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4
*Sqrt[2]*b^(5/4)) - (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b
*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
 + (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*
b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
```

Rubi [A] time = 0.265121, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {321, 329, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{5/4}}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}}+\frac{ac^{3/2}\tan^{-1}}{4\sqrt{\ }}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -(c*Sqrt[c*x]*(a - b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(5/4)) + (a*c^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4
*Sqrt[2]*b^(5/4)) - (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b
*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
 + (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*
b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx}{4b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{(ac)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{(ac)\operatorname{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{a\operatorname{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b} + \frac{a\operatorname{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} - \frac{\left(ac^{3/2}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{5/4}} - \frac{\left(ac^{3/2}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\right.}{8\sqrt{2}b^{5/4}}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{5/4}} + \frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{5/4}} + \frac{(a}{}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}} + \frac{ac^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}} - \frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}}{\sqrt{a}}\right.}{8\sqrt{2}b}$$

Mathematica [A] time = 0.144358, size = 241, normalized size = 0.78

$$-\frac{(cx)^{3/2}\left(8\sqrt[4]{b}\sqrt{x}\left(a-bx^2\right)^{3/4}+\sqrt{2}a\log\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)-\sqrt{2}a\log\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)+2\sqrt{2}a\tan^{-1}\left(1-\right.\right.}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -((c*x)^(3/2)*(8*b^(1/4)*Sqrt[x]*(a - b*x^2)^(3/4) + 2*Sqrt[2]*a*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - 2*Sqrt[2]*a*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + Sqrt[2]*a*Log[1 + (Sqrt[b]*x)/Sq
rt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - Sqrt[2]*a*Lo
g[1 + (Sqrt[b]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(
1/4)]))/(16*b^(5/4)*x^(3/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}} \frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

Fricas [A] time = 1.8139, size = 709, normalized size = 2.3

$$-\frac{4\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}c+4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}ab^4c-\left(b^5x^2-ab^4\right)\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{-\frac{\sqrt{-bx^2+a}a^2c^3x-\sqrt{-\frac{a^4c^6}{b^5}}\left(b^3x^2-ab^2\right)}{bx^2-a}}}{a^4bc^6x^2-a^5c^6}\right)+\left(-\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(-a^4*c^6/b^5)^(1/4)*b*arctan(-(
(-a^4*c^6/b^5)^(3/4)*(-b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 - a*b^
4)*(-a^4*c^6/b^5)^(3/4)*sqrt(-(sqrt(-b*x^2 + a)*a^2*c^3*x - sqrt(-a^4*c^6/b
^5)*(b^3*x^2 - a*b^2))/(b*x^2 - a)))/(a^4*b*c^6*x^2 - a^5*c^6)) + (-a^4*c^6
/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (-a^4*c^6/b^5)^(1/4)*
(b^2*x^2 - a*b))/(b*x^2 - a)) - (-a^4*c^6/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3
/4)*sqrt(c*x)*a*c - (-a^4*c^6/b^5)^(1/4)*(b^2*x^2 - a*b))/(b*x^2 - a)))/b
```

Sympy [C] time = 3.8288, size = 46, normalized size = 0.15

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(1/4)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

3.956 $\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=272

$$-\frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt
[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a
- b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]
*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqr
t[2]*b^(1/4)*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c])
```

Rubi [A] time = 0.227996, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {329, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt
[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a
- b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]
*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqr
t[2]*b^(1/4)*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2\ldots}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}\ldots}\ldots\right)}{2\sqrt{2}\sqrt[4]{\ldots}}$$

$$= -\frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\ldots}{\sqrt{c}\sqrt[4]{a-\ldots}}\ldots\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} - \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c\ldots}}{\sqrt{a-b\ldots}}\ldots\right)}{2\sqrt{2}\sqrt[4]{\ldots}}$$

Mathematica [A] time = 0.0417946, size = 197, normalized size = 0.72

$$\frac{\sqrt{x}\left(-\log\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)+\log\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - Log[1 + (Sqrt[b]*x)
/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + Log[1 + (
Sqrt[b]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)]))
/(2*Sqrt[2]*b^(1/4)*Sqrt[c*x])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Fricas [A] time = 1.68594, size = 594, normalized size = 2.18

$$-2\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}bc\left(-\frac{1}{bc^2}\right)^{\frac{3}{4}}-\left(b^2cx^2-abc\right)\sqrt{-\frac{\sqrt{-bx^2+a}cx-\left(bc^2x^2-ac^2\right)\sqrt{-\frac{1}{bc^2}}}{bx^2-a}}\left(-\frac{1}{bc^2}\right)^{\frac{3}{4}}}{bx^2-a}\right)-\frac{1}{2}\left(-\frac{1}{bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^2))^(1/4)*arctan(-((-b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(-1/(b*c^2)
)^(3/4) - (b^2*c*x^2 - a*b*c)*sqrt(-(sqrt(-b*x^2 + a)*c*x - (b*c^2*x^2 - a*
c^2)*sqrt(-1/(b*c^2)))/(b*x^2 - a))*(-1/(b*c^2))^(3/4))/(b*x^2 - a)) - 1/2*
(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 - a*c)*(-1/
(b*c^2))^(1/4))/(b*x^2 - a)) + 1/2*(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/
4)*sqrt(c*x) - (b*c*x^2 - a*c)*(-1/(b*c^2))^(1/4))/(b*x^2 - a))
```

Sympy [C] time = 1.81993, size = 46, normalized size = 0.17

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(2*I*pi)/a)/(2
*a**(1/4)*sqrt(c)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

3.957 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rubi [A] time = 0.0060532, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {264}

$$-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]

[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A] time = 0.0063869, size = 27, normalized size = 0.93

$$-\frac{2x\left(a-bx^2\right)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]

[Out] (-2*x*(a - b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-\frac{2\,x}{3\,a}\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] -2/3*x*(-b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Fricas [A] time = 1.55234, size = 63, normalized size = 2.17

$$-\frac{2\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/3*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [A] time = 11.2316, size = 92, normalized size = 3.17

$$\begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \frac{|a|}{|b||x^2|}>1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Piecewise((b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma
(1/4)), Abs(a)/(Abs(b)*Abs(x**2)) > 1), (-b**(3/4)*(-a/(b*x**2) + 1)**(3/4)
*exp(-I*pi/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

3.958 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a - b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rubi [A] time = 0.0153705, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a - b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\ &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}+\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0182048, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{3/4}\left(3a+4bx^2\right)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(3/4)*(3*a + 4*b*x^2))/(21*a^2*c^5*x^4)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.5

$$-\frac{2x\left(4bx^2+3a\right)}{21a^2}\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] -2/21*x*(-b*x^2+a)^(3/4)*(4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Fricas [A] time = 1.5763, size = 89, normalized size = 1.51

$$-\frac{2\left(4bx^2+3a\right)\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/21*(4*b*x^2 + 3*a)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

3.959 $\int \frac{1}{(cx)^{13/2} \sqrt[4]{a-bx^2}} \, dx$

Optimal. Leaf size=88

$$-\frac{64 \left(a-b x^2\right)^{11/4}}{231 a^3 c (c x)^{11/2}}+\frac{16 \left(a-b x^2\right)^{7/4}}{21 a^2 c (c x)^{11/2}}-\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{11/2}}$$

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rubi [A] time = 0.0269606, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$-\frac{64 \left(a-b x^2\right)^{11/4}}{231 a^3 c (c x)^{11/2}}+\frac{16 \left(a-b x^2\right)^{7/4}}{21 a^2 c (c x)^{11/2}}-\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{13/2} \sqrt[4]{a-bx^2}} \, dx &= -\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{11/2}}-\frac{8 \int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{13/2}} \, dx}{3 a}\\
&= -\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{11/2}}+\frac{16 \left(a-b x^2\right)^{7/4}}{21 a^2 c (c x)^{11/2}}+\frac{32 \int \frac{\left(a-bx^2\right)^{7/4}}{(cx)^{13/2}} \, dx}{21 a^2}\\
&= -\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{11/2}}+\frac{16 \left(a-b x^2\right)^{7/4}}{21 a^2 c (c x)^{11/2}}-\frac{64 \left(a-b x^2\right)^{11/4}}{231 a^3 c (c x)^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.0248092, size = 53, normalized size = 0.6

$$-\frac{2 \sqrt{c x} \left(a-b x^2\right)^{3/4} \left(21 a^2+24 a b x^2+32 b^2 x^4\right)}{231 a^3 c^7 x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(3/4)*(21*a^2 + 24*a*b*x^2 + 32*b^2*x^4))/(231*a^
3*c^7*x^6)
```

Maple [A] time = 0.005, size = 43, normalized size = 0.5

$$-\frac{2x\left(32b^2x^4+24abx^2+21a^2\right)}{231a^3}\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] -2/231*x*(-b*x^2+a)^(3/4)*(32*b^2*x^4+24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Fricas [A] time = 1.54446, size = 116, normalized size = 1.32

$$-\frac{2\left(32b^2x^4+24abx^2+21a^2\right)\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{231a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*b^2*x^4 + 24*a*b*x^2 + 21*a^2)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3
*c^7*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

3.960 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=128

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}-\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$

```
[Out] -(a*c^3*(a - b*x^2)^(3/4))/(2*b^2*Sqrt[c*x]) - (c*(c*x)^(3/2)*(a - b*x^2)^(
3/4))/(3*b) + (a^(3/2)*c^2*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0494462, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {321, 315, 317, 335, 228}

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}-\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c^3*(a - b*x^2)^(3/4))/(2*b^2*Sqrt[c*x]) - (c*(c*x)^(3/2)*(a - b*x^2)^(
3/4))/(3*b) + (a^(3/2)*c^2*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 315

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(c*(a +
b*x^2)^(3/4))/(b*Sqrt[c*x]), x] + Dist[(a*c^2)/(2*b), Int[1/((c*x)^(3/2)*(a
 + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{\left(ac^2\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx}{2b}$$

$$= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}-\frac{\left(a^2c^4\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{4b^2}$$

$$= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}-\frac{\left(a^2c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}\,dx}{4b^2\sqrt[4]{a-bx^2}}$$

$$= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{\left(a^2c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx,x,\frac{1}{x}\right)}{4b^2\sqrt[4]{a-bx^2}}$$

$$= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{a^{3/2}c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0291706, size = 71, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(a\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)-a+bx^2\right)}{3b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, (b*x^2)/a]))/(3*b*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}c^{2}x^{2}}{bx^{2}-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b*x^2 - a), x)
```

Sympy [C] time = 29.7991, size = 46, normalized size = 0.36

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(1/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cx)^{\frac{5}{2}}}{\left(-bx^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

3.961 $\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}-\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}}$$

```
[Out] -((c*(a - b*x^2)^(3/4))/(b*Sqrt[c*x])) + (Sqrt[a]*(1 - a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(1
/4))
```

Rubi [A] time = 0.0357489, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {315, 317, 335, 228}

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}-\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

```
[Out] -((c*(a - b*x^2)^(3/4))/(b*Sqrt[c*x])) + (Sqrt[a]*(1 - a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(1
/4))
```

Rule 315

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(c*(a +
b*x^2)^(3/4))/(b*Sqrt[c*x]), x] + Dist[(a*c^2)/(2*b), Int[1/((c*x)^(3/2)*(a
 + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} - \frac{\left(ac^2\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{2b}$$

$$= -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} - \frac{\left(a\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}\,dx}{2b\sqrt[4]{a-bx^2}}$$

$$= -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} + \frac{\left(a\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a-bx^2}}$$

$$= -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} + \frac{\sqrt{a}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0121084, size = 57, normalized size = 0.63

$$\frac{2x\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)}{3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (b*x^
2)/a])/(3*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \sqrt{cx}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*x^2 - a), x)
```

Sympy [C] time = 1.01386, size = 46, normalized size = 0.51

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*
pi)/a)/(2*a**(1/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

3.962 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a-bx^2}}$$

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.0235431, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {317, 335, 228}

$$-\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(1/4))
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\left(\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}\,dx}{c^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{c^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{2\sqrt{b}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0123118, size = 55, normalized size = 0.81

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x^2)/a])/(
(c*x)^(3/2)*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{2}} \frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^2x^4-ac^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 - a*c^2*x^2), x)
```

Sympy [C] time = 3.79929, size = 32, normalized size = 0.47

$$\frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt[4]{b}c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x**2))/(b**(1/4)*c**(3/2)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

3.963 $\int \frac{1}{(cx)^{7/2} \sqrt[4]{a-bx^2}} \, dx$

Optimal. Leaf size=100

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

```
[Out] (-2*(a - b*x^2)^(3/4))/(5*a*c*(c*x)^(5/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(1/
4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a
 - b*x^2)^(1/4))
```

Rubi [A] time = 0.0358012, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 317, 335, 228}

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(5*a*c*(c*x)^(5/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(1/
4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a
 - b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{5ac^2}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}\,dx}{5ac^4\sqrt[4]{a-bx^2}}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{\left(2b\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{5ac^4\sqrt[4]{a-bx^2}}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.0128299, size = 57, normalized size = 0.57

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (b*x^2)/a])/
(5*(c*x)^(7/2)*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{7}{2}}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^4x^6-ac^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 - a*c^4*x^4), x)
```

Sympy [C] time = 98.5107, size = 39, normalized size = 0.39

$$-\frac{ie^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{3\sqrt[4]{b}c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] -I*exp(-3*I*pi/4)*hyper((1/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(1/4)*c**(7/2
)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

3.964 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=130

$$-\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

```
[Out] (-2*(a - b*x^2)^(3/4))/(9*a*c*(c*x)^(9/2)) - (4*b*(a - b*x^2)^(3/4))/(15*a^
2*c^3*(c*x)^(5/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[A
rcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(5/2)*c^6*(a - b*x^2)^(1/4))
```

Rubi [A] time = 0.051073, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {325, 317, 335, 228}

$$-\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(9*a*c*(c*x)^(9/2)) - (4*b*(a - b*x^2)^(3/4))/(15*a^
2*c^3*(c*x)^(5/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[A
rcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(5/2)*c^6*(a - b*x^2)^(1/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}+\frac{(2b)\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx}{3ac^2}\\
&= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}+\frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{15a^2c^4}\\
&= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}+\frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}\,dx}{15a^2c^6\sqrt[4]{a-bx^2}}\\
&= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx,x,\frac{1}{x}\right)}{15a^2c^6\sqrt[4]{a-bx^2}}\\
&= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{8b^{5/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.013212, size = 57, normalized size = 0.44

$$
-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, (b*x^2)/a])/
(9*(c*x)^(11/2)*(a - b*x^2)^(1/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$
\int (cx)^{-\frac{11}{2}}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^6x^8-ac^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 - a*c^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

3.965 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}$$

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3
/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4)
)
```

Rubi [A] time = 0.0696213, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 237, 335, 275, 231}

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}+\frac{\sqrt{a}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3
/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4)
)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx &= \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} - \frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{2b} \\
&= \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} - \frac{(ac)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{b} \\
&= \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} - \frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{b\left(a+bx^2\right)^{3/4}} \\
&= \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} + \frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{b\left(a+bx^2\right)^{3/4}} \\
&= \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} + \frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{2b\left(a+bx^2\right)^{3/4}} \\
&= \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} + \frac{\sqrt{a}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0258063, size = 66, normalized size = 0.77

$$
\frac{c\sqrt{cx}\left(-a\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{b\left(a+bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/
4, 5/4, -((b*x^2)/a)]))/(b*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}} \left(bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}cx}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c*x/(b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 3.69933, size = 44, normalized size = 0.51

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/4)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

3.966 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{b}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}c^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0629385, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 237, 335, 275, 231}

$$-\frac{2\sqrt{b}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^(3/4))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}\\
&= \frac{\left(2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\left(a+bx^2\right)^{3/4}}\\
&= -\frac{\left(2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\left(a+bx^2\right)^{3/4}}\\
&= -\frac{\left(\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{c\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0108494, size = 54, normalized size = 0.82

$$\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)])/
(Sqrt[c*x]*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bcx^3+acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] time = 2.56298, size = 31, normalized size = 0.47

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

3.967 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=97

$$\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0734007, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {325, 329, 237, 335, 275, 231}

$$\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a + b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}-\frac{(2b)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{3ac^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}-\frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3ac^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}-\frac{\left(4b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}+\frac{\left(4b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}+\frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}+\frac{4b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0121644, size = 56, normalized size = 0.58

$$-\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)]
)/(3*(c*x)^(5/2)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{2}} \left(b x^2+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^2+a\right)^{\frac{3}{4}} (cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^2+a\right)^{\frac{1}{4}} \sqrt{cx}}{b c^3 x^5 + a c^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] time = 38.2484, size = 48, normalized size = 0.49

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right)}{2 a^{\frac{3}{4}} c^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

3.968 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{8b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) + (4*b*(a + b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0876074, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {325, 329, 237, 335, 275, 231}

$$-\frac{8b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) + (4*b*(a + b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a + b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} - \frac{(6b)\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{7ac^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(4b^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{7a^2c^4}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(8b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{7a^2c^5}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(8b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{7a^2c^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(8b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{7a^2c^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{7a^2c^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0132673, size = 56, normalized size = 0.44

$$
-\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\left(a+bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a)
])/(7*(c*x)^(9/2)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{9}{2}} \left(bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^5x^7+ac^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 + a*c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

3.969 $\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=157

$$\frac{80b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) + (20*b*(a + b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a + b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 + (80*b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.107442, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {325, 329, 237, 335, 275, 231}

$$-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{80b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) + (20*b*(a + b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a + b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 + (80*b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a + b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}-\frac{(10b)\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx}{11ac^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}+\frac{\left(60b^2\right)\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{77a^2c^4}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{\left(40b^3\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{77a^3c^6}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{\left(80b^3\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{77a^3c^7}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{\left(80b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \cdots\right.}{77a^3c^7\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{\left(80b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \cdots\right.}{77a^3c^7\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{\left(40b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \cdots\right.}{77a^3c^7\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{80b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\cdots\right.\right.}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0135153, size = 56, normalized size = 0.36

$$
-\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a+bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x^2)/a
)])/(11*(c*x)^(13/2)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{13}{2}} \left(bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^7x^9+ac^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 + a*c^7*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

3.970 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=117

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) + (3*a*c^(5/2)*ArcTan[(b^(1/4)*Sqrt
[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) - (3*a*c^(5/2)*ArcTanh[(b^
(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rubi [A] time = 0.0668808, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 331, 298, 205, 208}

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) + (3*a*c^(5/2)*ArcTan[(b^(1/4)*Sqrt
[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) - (3*a*c^(5/2)*ArcTanh[(b^
(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{4b}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{\left(3ac^3\right)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} + \frac{\left(3ac^3\right)\text{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} + \frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

Mathematica [A] time = 0.037342, size = 97, normalized size = 0.83

$$\frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}+3a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] ((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) + 3*a*ArcTan[(b^(1/4)*Sqr
t[x])/(a + b*x^2)^(1/4)] - 3*a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]
))/(4*b^(7/4)*x^(5/2))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 33.5981, size = 44, normalized size = 0.38

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

3.971 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

```
[Out] -((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3/4)
) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3
/4)
```

Rubi [A] time = 0.0557165, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 331, 298, 205, 208}

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3/4)
) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3
/4)
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}\\ &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\ &= \frac{c\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}} - \frac{c\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}}\\ &= -\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0131137, size = 67, normalized size = 0.8

$$\frac{\sqrt{cx}\left(\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/4),x]

[Out] (Sqrt[c*x]*(-ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)
*Sqrt[x])/(a + b*x^2)^(1/4)]))/(b^(3/4)*Sqrt[x])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt{cx}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)

[Out] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [C] time = 1.84899, size = 44, normalized size = 0.52

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

3.972 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rubi [A] time = 0.0059991, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Mathematica [A] time = 0.0053811, size = 24, normalized size = 0.92

$$-\frac{2x\sqrt[4]{a+bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$-2\,\frac{x\sqrt[4]{bx^2+a}}{a\,(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2*x*(b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Fricas [A] time = 1.38803, size = 57, normalized size = 2.19

$$-\frac{2\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

Sympy [A] time = 6.23434, size = 36, normalized size = 1.38

$$\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

3.973 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=55

$$\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rubi [A] time = 0.0143003, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx}{a}\\ &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0169781, size = 34, normalized size = 0.62

$$-\frac{2x\left(a-4bx^2\right)\sqrt[4]{a+bx^2}}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - 4*b*x^2)*(a + b*x^2)^(1/4))/(5*a^2*(c*x)^(7/2))
```

Maple [A] time = 0.004, size = 29, normalized size = 0.5

$$-\frac{2x\left(-4bx^{2}+a\right)}{5a^{2}}\sqrt[4]{bx^{2}+a}\,(cx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(b*x^2+a)^(1/4)*(-4*b*x^2+a)/a^2/(c*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Fricas [A] time = 1.38041, size = 82, normalized size = 1.49

$$\frac{2\left(4bx^{2}-a\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{5a^{2}c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(4*b*x^2 - a)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

3.974 $\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rubi [A] time = 0.024621, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}-\frac{8\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx}{a}\\ &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}\\ &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0233931, size = 52, normalized size = 0.63

$$-\frac{2\sqrt{cx}\sqrt[4]{a+bx^2}\left(5a^2-8abx^2+32b^2x^4\right)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(1/4)*(5*a^2 - 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c
^6*x^5)
```

Maple [A] time = 0.006, size = 42, normalized size = 0.5

$$-\frac{2x\left(32b^2x^4-8abx^2+5a^2\right)}{45a^3}\sqrt[4]{bx^2+a}\,(cx)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(1/4)*(32*b^2*x^4-8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Fricas [A] time = 1.29528, size = 111, normalized size = 1.34

$$-\frac{2\left(32b^2x^4-8abx^2+5a^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 - 8*a*b*x^2 + 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^6
*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

3.975 $\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

```
[Out] -((c*Sqrt[c*x]*(a - b*x^2)^(1/4))/b) - (Sqrt[a]*(1 - a/(b*x^2))^(3/4)*(c*x)
^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(3
/4))
```

Rubi [A] time = 0.0764107, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {321, 329, 237, 335, 275, 232}

$$-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}-\frac{\sqrt{a}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -((c*Sqrt[c*x]*(a - b*x^2)^(1/4))/b) - (Sqrt[a]*(1 - a/(b*x^2))^(3/4)*(c*x)
^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(3
/4))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}+\frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{2b}$$

$$= -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}+\frac{(ac)\operatorname{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}+\frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}-\frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}-\frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{2b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}-\frac{\sqrt{a}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0258591, size = 68, normalized size = 0.75

$$\frac{c\sqrt{cx}\left(a\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^2}{a}\right)-a+bx^2\right)}{b\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3
/4, 5/4, (b*x^2)/a]))/(b*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}} \left(-bx^{2}+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}cx}{bx^{2}-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x/(b*x^2 - a), x)
```

Sympy [C] time = 3.95103, size = 46, normalized size = 0.51

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(3/4)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

3.976 $\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}c^2\left(a-bx^2\right)^{3/4}}$$

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0635725, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {329, 237, 335, 275, 232}

$$-\frac{2\sqrt{b}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(3/4))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\left(2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\left(\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0127629, size = 55, normalized size = 0.81

$$\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^2}{a}\right)}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

```
[Out] (2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a])/(Sq
rt[c*x]*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}}\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bcx^3-acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 - a*c*x), x)
```

Sympy [C] time = 2.63855, size = 32, normalized size = 0.47

$$\frac{ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), a/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

3.977 $\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0761773, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {325, 329, 237, 335, 275, 232}

$$-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a - b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{(2b)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{3ac^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3ac^3}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{\left(4b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{\left(4b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{4b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0122904, size = 57, normalized size = 0.57

$$-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (b*x^2)/a])/(
3*(c*x)^(5/2)*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{2}} \left(-bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}} \sqrt{cx}}{bc^3x^5 - ac^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 - a*c^3*x^3), x)
```

Sympy [C] time = 40.1752, size = 39, normalized size = 0.39

$$-\frac{ie^{\frac{3i\pi}{4}}\, {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{a}{bx^2}\right)}{3b^{\frac{3}{4}}c^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] -I*exp(3*I*pi/4)*hyper((3/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(3/4)*c**(5/2)
*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

3.978 $\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}} dx$

Optimal. Leaf size=130

$$-\frac{8b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) - (4*b*(a - b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.0943923, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {325, 329, 237, 335, 275, 232}

$$-\frac{8b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) - (4*b*(a - b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a - b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}+\frac{(6b)\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{7ac^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}+\frac{\left(4b^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{7a^2c^4}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}+\frac{\left(8b^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{7a^2c^5}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}+\frac{\left(8b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{7a^2c^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{\left(8b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{7a^2c^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{7a^2c^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{8b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0126375, size = 57, normalized size = 0.44

$$
-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{bx^2}{a}\right)}{7(cx)^{9/2}\left(a-bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (b*x^2)/a])/
(7*(c*x)^(9/2)*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{9}{2}} \left(-bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}} \sqrt{cx}}{bc^5x^7 - ac^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 - a*c^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

3.979 $\int \frac{1}{(cx)^{13/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=162

$$-\frac{80b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) - (20*b*(a - b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a - b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 - (80*b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a - b*x^2)^(3/4))
```

Rubi [A] time = 0.111149, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {325, 329, 237, 335, 275, 232}

$$-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{80b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) - (20*b*(a - b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a - b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 - (80*b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a - b*x^2)^(3/4))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}+\frac{(10b)\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx}{11ac^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}+\frac{\left(60b^2\right)\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{77a^2c^4}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{\left(40b^3\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{77a^3c^6}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{\left(80b^3\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{\left(80b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)}\right.}{77a^3c^7\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{\left(80b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right.}\right.}{77a^3c^7\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{\left(40b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right.}\right.}{77a^3c^7\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{80b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right|}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0125299, size = 57, normalized size = 0.35

$$-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, (b*x^2)/a])
/(11*(c*x)^(13/2)*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{13}{2}} \left(-bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}} \sqrt{cx}}{bc^7x^9 - ac^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 - a*c^7*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

3.980 $\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=308

$$\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{7/4}}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}}+\frac{3ac^{5/2}\tan}{}$$

```
[Out] -(c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*b) - (3*a*c^(5/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(7/4)) + (3*
a*c^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4)
)])/(4*Sqrt[2]*b^(7/4)) + (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*
b^(7/4)) - (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(7/4))
```

Rubi [A] time = 0.261505, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {321, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}b^{7/4}}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}}+\frac{3ac^{5/2}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```


```
[Out] -(c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*b) - (3*a*c^(5/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(7/4)) + (3*
a*c^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4)
)])/(4*Sqrt[2]*b^(7/4)) + (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*
b^(7/4)) - (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(7/4))
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{4b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{(3ac)\,\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b^{3/2}} + \frac{(3ac)\,\text{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b^{3/2}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{\left(3ac^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{7/4}} + \frac{\left(3ac^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-\cdots}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt[4]{\cdots}}\cdots}\cdots\right)}{8\sqrt{2}b^{7/4}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{7/4}} - \frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{7/4}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}} + \frac{3ac^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}} + \frac{3ac^{5/2}\log\left(\sqrt{c}\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.0562795, size = 112, normalized size = 0.36

$$-\frac{(cx)^{5/2}\left(2bx^{3/2}\sqrt[4]{a-bx^2}-3a\sqrt[4]{-b}\tan^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+3a\sqrt[4]{-b}\tanh^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)\right)}{4b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -((c*x)^(5/2)*(2*b*x^(3/2)*(a - b*x^2)^(1/4) - 3*a*(-b)^(1/4)*ArcTan[((-b)^
(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + 3*a*(-b)^(1/4)*ArcTanh[((-b)^(1/4)*Sqrt
[x])/(a - b*x^2)^(1/4)]))/(4*b^2*x^(5/2))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}}\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 36.5276, size = 46, normalized size = 0.15

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

3.981 $\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=272

$$\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}$$

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4
))])/(Sqrt[2]*b^(3/4))) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*b^(3/4)) + (Sqrt[c]*Log[Sqrt[c] + (Sq
rt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^
(1/4)])/(2*Sqrt[2]*b^(3/4)) - (Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*
b^(3/4))
```

Rubi [A] time = 0.231778, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4
))])/(Sqrt[2]*b^(3/4))) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*b^(3/4)) + (Sqrt[c]*Log[Sqrt[c] + (Sq
rt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^
(1/4)])/(2*Sqrt[2]*b^(3/4)) - (Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*
b^(3/4))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}c} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}c}$$

$$= \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}b^{3/4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}b^{3/4}} + \frac{c\,\text{Subst}\left(\right.}{}$$

$$= \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}b^{3/4}} - \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}b^{3/4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x,\right.}{\sqrt{2}b^{3/4}}$$

$$= -\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}} + \frac{\sqrt{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}} + \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}b^{3/4}} - \frac{\sqrt{c}\,}{}$$

Mathematica [A] time = 0.0231313, size = 75, normalized size = 0.28

$$\frac{\sqrt{cx}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)\right)}{(-b)^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c*x]*(-ArcTan[((-b)^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + ArcTanh[((-b)
^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)]))/((-b)^(3/4)*Sqrt[x])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \sqrt{cx}\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 1.9962, size = 46, normalized size = 0.17

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*
pi)/a)/(2*a**(3/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

3.982 $\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

[Out] (-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])

Rubi [A] time = 0.0067443, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {264}

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]

[Out] (-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Mathematica [A] time = 0.005298, size = 25, normalized size = 0.93

$$-\frac{2x\sqrt[4]{a-bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]

[Out] (-2*x*(a - b*x^2)^(1/4))/(a*(c*x)^(3/2))

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-2\,\frac{x\sqrt[4]{-bx^2+a}}{a\,(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2*x*(-b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Fricas [A] time = 2.12474, size = 58, normalized size = 2.15

$$-\frac{2\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

Sympy [A] time = 7.03884, size = 94, normalized size = 3.48

$$\begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{for } \frac{|a|}{|b||x^2|}>1 \\ -\frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^2}+1}e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma
(3/4)), Abs(a)/(Abs(b)*Abs(x**2)) > 1), (-b**(1/4)*(-a/(b*x**2) + 1)**(1/4)
*exp(-3*I*pi/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

3.983 $\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a - b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rubi [A] time = 0.0140473, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a - b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx&=-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx}{a}\\&=-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0177282, size = 35, normalized size = 0.61

$$-\frac{2x\sqrt[4]{a-bx^2}\left(a+4bx^2\right)}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*(a + 4*b*x^2))/(5*a^2*(c*x)^(7/2))
```

Maple [A] time = 0.004, size = 30, normalized size = 0.5

$$-\frac{2\,x\left(4\,bx^{2}+a\right)}{5\,a^{2}}\sqrt[4]{-bx^{2}+a}\,(cx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(-b*x^2+a)^(1/4)*(4*b*x^2+a)/a^2/(c*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Fricas [A] time = 2.07214, size = 85, normalized size = 1.49

$$-\frac{2\left(4\,bx^{2}+a\right)\left(-bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,a^{2}c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/5*(4*b*x^2 + a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

3.984 $\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a - b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a - b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rubi [A] time = 0.025203, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a - b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a - b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}-\frac{8\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx}{a}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0236336, size = 53, normalized size = 0.62

$$-\frac{2\sqrt{cx}\sqrt[4]{a-bx^2}\left(5a^2+8abx^2+32b^2x^4\right)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(1/4)*(5*a^2 + 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c
^6*x^5)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.5

$$-\frac{2x\left(32b^2x^4+8abx^2+5a^2\right)}{45a^3}\sqrt[4]{-bx^2+a}\,(cx)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(-b*x^2+a)^(1/4)*(32*b^2*x^4+8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Fricas [A] time = 2.1201, size = 112, normalized size = 1.3

$$-\frac{2\left(32b^2x^4+8abx^2+5a^2\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 + 8*a*b*x^2 + 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^
6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

3.985 $\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=146

$$\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

```
[Out] (5*a*c^3*Sqrt[c*x])/(2*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(5/2))/(2*b*(a + b
*x^2)^(1/4)) - (5*a*c^(7/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)
^(1/4))])/(4*b^(9/4)) - (5*a*c^(7/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rubi [A] time = 0.0763353, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {285, 288, 329, 240, 212, 208, 205}

$$\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (5*a*c^3*Sqrt[c*x])/(2*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(5/2))/(2*b*(a + b
*x^2)^(1/4)) - (5*a*c^(7/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)
^(1/4))])/(4*b^(9/4)) - (5*a*c^(7/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^2\right)\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^4\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx}{4b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^3\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^4\right)\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^2} - \frac{\left(5ac^4\right)\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,d\right.}{4b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}$$

Mathematica [C] time = 0.0365638, size = 63, normalized size = 0.43

$$\frac{c(cx)^{5/2}\left(1-\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)\right)}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(5/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4,
-((b*x^2)/a)]))/(2*b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}} \left(bx^2 + a\right)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2 + a\right)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [B] time = 2.29662, size = 841, normalized size = 5.76

$$4\left(bc^3x^2 + 5\,ac^3\right)\left(bx^2 + a\right)^{\frac{3}{4}}\sqrt{cx} + 20\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2 + ab^2\right)\arctan\left(-\frac{\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}ab^7c^3-\left(b^8x^2+ab^7\right)\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}a^2c^7x+\cdots}{b\cdots}}}{a^4bc^{14}x^2+a^5c^{14}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3*x^2 + 5*a*c^3)*(b*x^2 + a)^(3/4)*sqrt(c*x) + 20*(a^4*c^14/b^9
)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((a^4*c^14/b^9)^(3/4)*(b*x^2 + a)^(3/4)*s
qrt(c*x)*a*b^7*c^3 - (b^8*x^2 + a*b^7)*(a^4*c^14/b^9)^(3/4)*sqrt((sqrt(b*x^
2 + a)*a^2*c^7*x + sqrt(a^4*c^14/b^9)*(b^5*x^2 + a*b^4))/(b*x^2 + a)))/(a^4
*b*c^14*x^2 + a^5*c^14)) - 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*(
(b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 + (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))
/(b*x^2 + a)) + 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*((b*x^2 + a)
^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))/(b*x^2 + a
)))/(b^3*x^2 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

3.986 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=107

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4) + (c^(3/2)*ArcTanh[(b^(1/4)*Sqrt[c*
x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4)
```

Rubi [A] time = 0.0594885, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {288, 329, 240, 212, 208, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*c*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4) + (c^(3/2)*ArcTanh[(b^(1/4)*Sqrt[c*
x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4)
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^2\operatorname{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}+\frac{c^2\operatorname{Subst}\left(\int \frac{1}{c+\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0119004, size = 59, normalized size = 0.55

$$\frac{2x(cx)^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)}{5a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*(c*x)^(3/2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -((
b*x^2)/a)])/(5*a*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [B] time = 1.72883, size = 701, normalized size = 6.55

$$-\frac{4\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}c+4\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}b^4c\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}-\left(b^5x^2+ab^4\right)\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}c^3x+\left(b^3x^2+ab^2\right)\sqrt{\frac{c^6}{b^5}}}{bx^2+a}}}{bc^6x^2+ac^6}\right)-\left(b^2x^2+ab\right.}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*a
rctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b^4*c*(c^6/b^5)^(3/4) - (b^5*x^2 + a*b^
4)*(c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*c^3*x + (b^3*x^2 + a*b^2)*sqrt(c^6
/b^5))/(b*x^2 + a)))/(b*c^6*x^2 + a*c^6)) - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)
*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x
^2 + a)) + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
*c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)))/(b^2*x^2 + a*b)
```

Sympy [C] time = 13.3458, size = 44, normalized size = 0.41

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/4)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

3.987 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

[Out] (2*Sqrt[c*x])/(a*c*(a + b*x^2)^(1/4))

Rubi [A] time = 0.0055901, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]

[Out] (2*Sqrt[c*x])/(a*c*(a + b*x^2)^(1/4))

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Mathematica [A] time = 0.0060322, size = 24, normalized size = 0.92

$$\frac{2x}{a\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]

[Out] (2*x)/(a*Sqrt[c*x]*(a + b*x^2)^(1/4))

Maple [A] time = 0.005, size = 21, normalized size = 0.8

$$2\,\frac{x}{\sqrt[4]{bx^2+a}a\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] 2*x/(b*x^2+a)^(1/4)/a/(c*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

Fricas [A] time = 1.50464, size = 69, normalized size = 2.65

$$\frac{2\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{abcx^2+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*b*c*x^2 + a^2*c)
```

Sympy [A] time = 7.04361, size = 34, normalized size = 1.31

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt{c}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(1/4)/(2*a*b**(1/4)*sqrt(c)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

3.988 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=55

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

```
[Out] 2/(a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*c*(c*x
)^(3/2))
```

Rubi [A] time = 0.0148584, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*c*(c*x
)^(3/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx&=\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}+\frac{4\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\&=\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0096554, size = 34, normalized size = 0.62

$$-\frac{2x\left(a+4bx^2\right)}{3a^2(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(a + 4*b*x^2))/(3*a^2*(c*x)^(5/2)*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.004, size = 29, normalized size = 0.5

$$-\frac{2x(4bx^2+a)}{3a^2}(cx)^{-\frac{5}{2}}\frac{1}{\sqrt[4]{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/3*x*(4*b*x^2+a)/(b*x^2+a)^(1/4)/a^2/(c*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

Fricas [A] time = 1.53265, size = 105, normalized size = 1.91

$$-\frac{2(4bx^2+a)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{3(a^2bc^3x^4+a^3c^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/3*(4*b*x^2 + a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*b*c^3*x^4 + a^3*c^3*x^2
)
```

Sympy [A] time = 128.074, size = 78, normalized size = 1.42

$$\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}c^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^2c^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-3/4)/(8*a*b**(1/4)*c**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
 + b**(3/4)*gamma(-3/4)/(2*a**2*c**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

3.989 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/(a*c*(c*x)^(7/2)*(a + b*x^2)^(1/4)) - (16*(a + b*x^2)^(3/4))/(3*a^2*c*(c*
x)^(7/2)) + (64*(a + b*x^2)^(7/4))/(21*a^3*c*(c*x)^(7/2))
```

Rubi [A] time = 0.0235764, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(7/2)*(a + b*x^2)^(1/4)) - (16*(a + b*x^2)^(3/4))/(3*a^2*c*(c*
x)^(7/2)) + (64*(a + b*x^2)^(7/4))/(21*a^3*c*(c*x)^(7/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}+\frac{8\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\
&= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}-\frac{32\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a^2}\\
&= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0109534, size = 47, normalized size = 0.57

$$-\frac{2x\left(3a^2-8abx^2-32b^2x^4\right)}{21a^3(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(3*a^2 - 8*a*b*x^2 - 32*b^2*x^4))/(21*a^3*(c*x)^(9/2)*(a + b*x^2)^(1/
4))
```

Maple [A] time = 0.004, size = 42, normalized size = 0.5

$$-\frac{2x\left(-32b^{2}x^{4}-8abx^{2}+3a^{2}\right)}{21a^{3}}\frac{1}{\sqrt[4]{bx^{2}+a}}(cx)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/21*x*(-32*b^2*x^4-8*a*b*x^2+3*a^2)/(b*x^2+a)^(1/4)/a^3/(c*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

Fricas [A] time = 1.59913, size = 131, normalized size = 1.58

$$\frac{2\left(32b^{2}x^{4}+8abx^{2}-3a^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\left(a^{3}bc^{5}x^{6}+a^{4}c^{5}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(32*b^2*x^4 + 8*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*b*c^
5*x^6 + a^4*c^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

3.990 $\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=109

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/(a*c*(c*x)^(11/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(a^2*c*(c*x)
^(11/2)) + (64*(a + b*x^2)^(7/4))/(7*a^3*c*(c*x)^(11/2)) - (256*(a + b*x^2)
^(11/4))/(77*a^4*c*(c*x)^(11/2))
```

Rubi [A] time = 0.0374668, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(11/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(a^2*c*(c*x)
^(11/2)) + (64*(a + b*x^2)^(7/4))/(7*a^3*c*(c*x)^(11/2)) - (256*(a + b*x^2)
^(11/4))/(77*a^4*c*(c*x)^(11/2))
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}+\frac{12\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\ &= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}-\frac{32\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{a^2}\\ &= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}+\frac{128\int \frac{\left(a+bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{7a^3}\\ &= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}\end{aligned}$$

Mathematica [A] time = 0.0122965, size = 58, normalized size = 0.53

$$-\frac{2x\left(-12a^2bx^2+7a^3+32ab^2x^4+128b^3x^6\right)}{77a^4(cx)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(7*a^3 - 12*a^2*b*x^2 + 32*a*b^2*x^4 + 128*b^3*x^6))/(77*a^4*(c*x)^(1
3/2)*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.004, size = 53, normalized size = 0.5

$$-\frac{2x\left(128b^3x^6+32ab^2x^4-12a^2bx^2+7a^3\right)}{77a^4}\frac{1}{\sqrt[4]{bx^2+a}}(cx)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/77*x*(128*b^3*x^6+32*a*b^2*x^4-12*a^2*b*x^2+7*a^3)/(b*x^2+a)^(1/4)/a^4/(
c*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

Fricas [A] time = 1.61597, size = 158, normalized size = 1.45

$$-\frac{2\left(128b^3x^6+32ab^2x^4-12a^2bx^2+7a^3\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{77\left(a^4bc^7x^8+a^5c^7x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/77*(128*b^3*x^6 + 32*a*b^2*x^4 - 12*a^2*b*x^2 + 7*a^3)*(b*x^2 + a)^(3/4)
*sqrt(c*x)/(a^4*b*c^7*x^8 + a^5*c^7*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

3.991 $\int \frac{(cx)^{13/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=155

$$\frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}+\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}-\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

```
[Out] (77*a^2*c^5*(c*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) - (11*a*c^3*(c*x)^(7/2)
)/(30*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(11/2))/(5*b*(a + b*x^2)^(1/4)) + (
77*a^(5/2)*c^6*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(20*b^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0678403, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {285, 284, 335, 196}

$$\frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}+\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}-\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (77*a^2*c^5*(c*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) - (11*a*c^3*(c*x)^(7/2)
)/(30*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(11/2))/(5*b*(a + b*x^2)^(1/4)) + (
77*a^(5/2)*c^6*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(20*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{13/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(11ac^2\right)\int \frac{(cx)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{10b}$$

$$= -\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{\left(77a^2c^4\right)\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{60b^2}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(77a^3c^6\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{40b^3}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(77a^3c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{40b^4\sqrt[4]{a+bx^2}}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{\left(77a^3c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{77a^{5/2}c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0353649, size = 87, normalized size = 0.56

$$\frac{c^5(cx)^{3/2}\left(-77a^2\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+77a^2-22abx^2+12b^2x^4\right)}{60b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^5*(c*x)^(3/2)*(77*a^2 - 22*a*b*x^2 + 12*b^2*x^4 - 77*a^2*(1 + (b*x^2)/a)
^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)
^(1/4))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int (cx)^{\frac{13}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}c^6x^6}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^6*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

3.992 $\int \frac{(cx)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{7a^{3/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

```
[Out] (-7*a*c^3*(c*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(7/2))/(3*b*(a
+ b*x^2)^(1/4)) - (7*a^(3/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.048874, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {285, 284, 335, 196}

$$-\frac{7a^{3/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-7*a*c^3*(c*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(7/2))/(3*b*(a
+ b*x^2)^(1/4)) - (7*a^(3/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{\left(7ac^2\right)\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{6b}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} + \frac{\left(7a^2c^4\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b^2}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} + \frac{\left(7a^2c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{\left(7a^2c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{7a^{3/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0312727, size = 74, normalized size = 0.6

$$\frac{c^3(cx)^{3/2}\left(7a\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-7a+2bx^2\right)}{6b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^3*(c*x)^(3/2)*(-7*a + 2*b*x^2 + 7*a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (cx)^{\frac{9}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}c^4x^4}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^4*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

3.993 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{a}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

```
[Out] (c*(c*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/4
)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^
2)^(1/4))
```

Rubi [A] time = 0.0330505, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {285, 284, 335, 196}

$$\frac{3\sqrt{a}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/4
)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^
2)^(1/4))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} - \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{2b}$$

$$= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} - \frac{\left(3ac^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{2b^2\sqrt[4]{a+bx^2}}$$

$$= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} + \frac{\left(3ac^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b^2\sqrt[4]{a+bx^2}}$$

$$= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} + \frac{3\sqrt{a}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0283402, size = 60, normalized size = 0.67

$$\frac{c(cx)^{3/2}\left(1-\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4,
-((b*x^2)/a)]))/(b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}c^2x^2}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

Sympy [C] time = 37.5172, size = 44, normalized size = 0.49

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

3.994 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0202479, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {284, 335, 196}

$$-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx=\frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{b\sqrt[4]{a+bx^2}}$$
$$=-\frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{b\sqrt[4]{a+bx^2}}$$
$$=-\frac{2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0105382, size = 59, normalized size = 0.94

$$\frac{2x\sqrt{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*
x^2)/a)])/(3*a*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \sqrt{cx}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 4.16699, size = 44, normalized size = 0.7

$$\frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(5/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

3.995 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=93

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/(a*c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*c^2*(a + b*x^2
)^(1/4))
```

Rubi [A] time = 0.0346462, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {286, 284, 335, 196}

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(a*c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*c^2*(a + b*x^2
)^(1/4))
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{(2b)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{ac^2}$$

$$= -\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{\left(2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{ac^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{\left(2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{ac^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{4\sqrt{b}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0116247, size = 57, normalized size = 0.61

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x^2)/a)]
)/(a*(c*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2c^2x^6+2\,abc^2x^4+a^2c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^2
*x^2), x)
```

Sympy [C] time = 27.3635, size = 48, normalized size = 0.52

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

3.996 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{24b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}+\frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/(5*a*c*(c*x)^(5/2)*(a + b*x^2)^(1/4)) + (12*b)/(5*a^2*c^3*Sqrt[c*x]*(a +
 b*x^2)^(1/4)) - (24*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcC
ot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*c^4*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0510784, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {286, 284, 335, 196}

$$-\frac{24b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}+\frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(5*a*c*(c*x)^(5/2)*(a + b*x^2)^(1/4)) + (12*b)/(5*a^2*c^3*Sqrt[c*x]*(a +
 b*x^2)^(1/4)) - (24*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcC
ot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*c^4*(a + b*x^2)^(1/4))
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{(6b)\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{5ac^2}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{\left(12b^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{5a^2c^4}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5a^2c^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{5a^2c^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{24b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0123385, size = 59, normalized size = 0.47

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a)
])/(5*a*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{7}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2c^4x^8+2\,abc^4x^6+a^2c^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^4*x^8 + 2*a*b*c^4*x^6 + a^2*c^4
*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

3.997 $\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=157

$$-\frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{16b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/(9*a*c*(c*x)^(9/2)*(a + b*x^2)^(1/4)) + (4*b)/(9*a^2*c^3*(c*x)^(5/2)*(a
+ b*x^2)^(1/4)) - (8*b^2)/(3*a^3*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (16*b^(
5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(3*a^(7/2)*c^6*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0674036, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {286, 284, 335, 196}

$$-\frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{16b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(9*a*c*(c*x)^(9/2)*(a + b*x^2)^(1/4)) + (4*b)/(9*a^2*c^3*(c*x)^(5/2)*(a
+ b*x^2)^(1/4)) - (8*b^2)/(3*a^3*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (16*b^(
5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(3*a^(7/2)*c^6*(a + b*x^2)^(1/4))
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}-\frac{(10b)\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx}{9ac^2}\\
&= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}+\frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{3a^2c^4}\\
&= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{\left(8b^3\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,d\cdots}{3a^3c^6}\\
&= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{\left(8b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\cdots\right.}{3a^3c^6\sqrt[4]{a\cdots}}\\
&= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{\left(8b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\cdots\right.}{3a\cdots}\\
&= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{16b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{c\cdots}}{3a^{7/2}c\cdots}
\end{aligned}$$

Mathematica [C] time = 0.0125433, size = 59, normalized size = 0.38

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9a(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x^2)/a)
])/(9*a*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{11}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2c^6x^{10}+2\,abc^6x^8+a^2c^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^6*x^10 + 2*a*b*c^6*x^8 + a^2*c^
6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

3.998 $\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{9/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -((b
*x^2)/a)])/(9*c*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.017512, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{9/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -((b
*x^2)/a)])/(9*c*(a + b*x^2)^(1/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{(cx)^{5/4}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{9/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0113514, size = 56, normalized size = 0.97

$$\frac{4x(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -(
(b*x^2)/a)])/(9*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{4}} \frac{1}{\sqrt[4]{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2 + a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}}\, cx}{\left(bx^2 + a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/4)*c*x/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 53.3204, size = 44, normalized size = 0.76

$$\frac{c^{\frac{5}{4}} x^{\frac{9}{4}} \Gamma\left(\frac{9}{8}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{9}{8} \\ \frac{17}{8} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(5/4)*x**(9/4)*gamma(9/8)*hyper((1/4, 9/8), (17/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(17/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

3.999 $\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{7/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -((b
*x^2)/a)])/(7*c*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0168453, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{7/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -((b
*x^2)/a)])/(7*c*(a + b*x^2)^(1/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{(cx)^{3/4}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{7/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.009834, size = 56, normalized size = 0.97

$$\frac{4x(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -(
(b*x^2)/a)])/(7*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{4}} \frac{1}{\sqrt[4]{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(bx^2 + a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx)^{\frac{3}{4}}}{\left(bx^2 + a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 8.52489, size = 44, normalized size = 0.76

$$\frac{c^{\frac{3}{4}} x^{\frac{7}{4}} \Gamma\left(\frac{7}{8}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{8} \\ \frac{15}{8} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/4)*x**(7/4)*gamma(7/8)*hyper((1/4, 7/8), (15/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(15/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

3.1000 $\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0166135, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(1/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{\sqrt[4]{cx}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01133, size = 56, normalized size = 0.97

$$\frac{4x\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -(
(b*x^2)/a)])/(5*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx}\,\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] time = 1.39535, size = 44, normalized size = 0.76

$$\frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{8}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(1/4)*x**(5/4)*gamma(5/8)*hyper((1/4, 5/8), (13/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(13/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

3.1001 $\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b
*x^2)/a)])/(3*c*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.018118, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b
*x^2)/a)])/(3*c*(a + b*x^2)^(1/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0112992, size = 56, normalized size = 0.97

$$\frac{4x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b*x^2)/a)])
/(3*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{cx}}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{4}}}{bcx^3+acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] time = 1.48144, size = 44, normalized size = 0.76

$$\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{8}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] x**(3/4)*gamma(3/8)*hyper((1/4, 3/8), (11/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(1/4)*c**(1/4)*gamma(11/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

3.1002 $\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{4\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*
x^2)/a)])/(c*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0168982, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*
x^2)/a)])/(c*(a + b*x^2)^(1/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4\sqrt[4]{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0103394, size = 54, normalized size = 0.96

$$\frac{4x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*x^2)/a)])/
((c*x)^(3/4)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{4}} \frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{1}{4}}}{bcx^3+acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(1/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] time = 3.50702, size = 44, normalized size = 0.79

$$\frac{\sqrt[4]{x}\Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix}\frac{1}{8}, \frac{1}{4} \\ \frac{9}{8}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] x**(1/4)*gamma(1/8)*hyper((1/8, 1/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(1/4)*c**(3/4)*gamma(9/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

3.1003 $\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a+bx^2}}$$

```
[Out] (-4*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)])/
(c*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0170952, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-4*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)])/
(c*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= -\frac{4\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0109496, size = 54, normalized size = 0.96

$$-\frac{4x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)]
)/((c*x)^(5/4)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{4}} \frac{1}{\sqrt[4]{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{1}{4}} (cx)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2 + a\right)^{\frac{3}{4}} (cx)^{\frac{3}{4}}}{bc^2x^4 + ac^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] time = 10.0957, size = 48, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{8}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{8}, \frac{1}{4} \\ \frac{7}{8} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a} c^{\frac{5}{4}} \sqrt[4]{x}\, \Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] gamma(-1/8)*hyper((-1/8, 1/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

3.1004 $\int \frac{(cx)^{5/4}}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{9/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -((b
*x^2)/a)])/(9*a*c*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0185229, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{9/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -((b
*x^2)/a)])/(9*a*c*(a + b*x^2)^(3/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{5/4}}{\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{(cx)^{5/4}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\ &= \frac{4(cx)^{9/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.012388, size = 59, normalized size = 0.97

$$\frac{4x(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -(
(b*x^2)/a)])/(9*a*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{4}}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}cx}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 124.957, size = 44, normalized size = 0.72

$$\frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{7}{4}\\ \frac{17}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(5/4)*x**(9/4)*gamma(9/8)*hyper((9/8, 7/4), (17/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(17/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

3.1005 $\int \frac{(cx)^{3/4}}{\left(a+bx^2\right)^{7/4}} \, dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{7/4}\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -((b
*x^2)/a)])/(7*a*c*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.018248, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{7/4}\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -((b
*x^2)/a)])/(7*a*c*(a + b*x^2)^(3/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/4}}{\left(a+bx^2\right)^{7/4}} \, dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4} \int \frac{(cx)^{3/4}}{\left(1+\frac{bx^2}{a}\right)^{7/4}} \, dx}{a\left(a+bx^2\right)^{3/4}}\\
&= \frac{4(cx)^{7/4}\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0114778, size = 59, normalized size = 0.97

$$\frac{4x(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -(
(b*x^2)/a)])/(7*a*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{4}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 61.7267, size = 44, normalized size = 0.72

$$\frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right){}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{7}{4}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(3/4)*x**(7/4)*gamma(7/8)*hyper((7/8, 7/4), (15/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(15/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

3.1006 $\int \frac{\sqrt[4]{cx}}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -((b
*x^2)/a)])/(5*a*c*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0185868, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -((b
*x^2)/a)])/(5*a*c*(a + b*x^2)^(3/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{cx}}{\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{\sqrt[4]{cx}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\ &= \frac{4(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0099126, size = 59, normalized size = 0.97

$$\frac{4x\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -(
(b*x^2)/a)])/(5*a*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 26.7134, size = 44, normalized size = 0.72

$$\frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{7}{4}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(1/4)*x**(5/4)*gamma(5/8)*hyper((5/8, 7/4), (13/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(13/8))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1007 $\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b
*x^2)/a)])/(3*a*c*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0183507, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b
*x^2)/a)])/(3*a*c*(a + b*x^2)^(3/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\ &= \frac{4(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0104656, size = 59, normalized size = 0.97

$$\frac{4x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3a\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b*x^2)/a)])
/(3*a*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{cx}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}{b^2cx^5+2\,abcx^3+a^2cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
 x)
```

Sympy [C] time = 28.7939, size = 44, normalized size = 0.72

$$\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{7}{4}\\\frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt[4]{c}\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] x**(3/4)*gamma(3/8)*hyper((3/8, 7/4), (11/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(7/4)*c**(1/4)*gamma(11/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

3.1008 $\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=59

$$\frac{4\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*
x^2)/a)])/(a*c*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0181921, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*
x^2)/a)])/(a*c*(a + b*x^2)^(3/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\ &= \frac{4\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.010729, size = 57, normalized size = 0.97

$$\frac{4x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{a(cx)^{3/4}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*x^2)/a)])/
(a*(c*x)^(3/4)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{4}}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}{b^2cx^5+2\,abcx^3+a^2cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
 x)
```

Sympy [C] time = 65.3303, size = 44, normalized size = 0.75

$$\frac{\sqrt[4]{x}\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{7}{4}\\\frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] x**(1/4)*gamma(1/8)*hyper((1/8, 7/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(7/4)*c**(3/4)*gamma(9/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

3.1009 $\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=59

$$-\frac{4\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-4*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)])/
(a*c*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0190109, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{4\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-4*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)])/
(a*c*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\
&= -\frac{4\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0108342, size = 57, normalized size = 0.97

$$-\frac{4x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{a(cx)^{5/4}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)]
)/(a*(c*x)^(5/4)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{4}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}{b^2c^2x^6+2\,abc^2x^4+a^2c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c
^2*x^2), x)
```

Sympy [C] time = 133.22, size = 48, normalized size = 0.81

$$\frac{\Gamma\left(-\frac{1}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{7}{4}\\ \frac{7}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] gamma(-1/8)*hyper((-1/8, 7/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

3.1010 $\int x^6 \sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=345

$$-\frac{81\ 3^{3/4}\sqrt{2-\sqrt{3}}a^4\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{2816b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9a^2x^3\sqrt[6]{\cdots}}{70\cdots}$$

```
[Out] (81*a^3*x*(a + b*x^2)^(1/6))/(2816*b^3) - (9*a^2*x^3*(a + b*x^2)^(1/6))/(70
4*b^2) + (3*a*x^5*(a + b*x^2)^(1/6))/(352*b) + (3*x^7*(a + b*x^2)^(1/6))/22
 - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2816*b
^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
- (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.425017, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 241, 236, 219}

$$-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{81\ 3^{3/4}\sqrt{2-\sqrt{3}}a^4\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}\cdots}{-\sqrt[3]{\frac{a}{bx^2+a}}\cdots}\right)\cdots\right)}{2816b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^(1/6),x]
```

```
[Out] (81*a^3*x*(a + b*x^2)^(1/6))/(2816*b^3) - (9*a^2*x^3*(a + b*x^2)^(1/6))/(70
4*b^2) + (3*a*x^5*(a + b*x^2)^(1/6))/(352*b) + (3*x^7*(a + b*x^2)^(1/6))/22
 - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2816*b
^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
- (a/(a + b*x^2))^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int x^6\sqrt[6]{a+bx^2}\,dx=\frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{1}{22}a\int\frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$=\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{\left(15a^2\right)\int\frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx}{352b}$$

$$=-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{\left(27a^3\right)\int\frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{704b^2}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{\left(81a^4\right)\int\frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{2816b^3}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{\left(81a^4\right)\text{Subst}\left(\int\frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,\right.}{2816b^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+b}}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{\left(243a^4\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Su}}{5632b^4}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{81\ 3^{3/4}\sqrt{2-\sqrt{3}}a^4\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{}}{}$$

Mathematica [C] time = 0.0591379, size = 105, normalized size = 0.3

$$\frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{\frac{bx^2}{a}+1}\left(-3a^2bx^2+27a^3+2ab^2x^4+32b^3x^6\right)-27a^3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{704b^3\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(27*a^3 - 3*a^2*b*x^2 + 2*a*b
^2*x^4 + 32*b^3*x^6) - 27*a^3*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a
)]))/(704*b^3*(1 + (b*x^2)/a)^(1/6))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^6\sqrt[6]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^6*(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{6}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{6}}x^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)*x^6, x)
```

Sympy [A] time = 1.45636, size = 29, normalized size = 0.08

$$\frac{\sqrt[6]{a}x^7{}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x**7*hyper((-1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{6}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^6, x)
```

3.1011 $\int x^4 \sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=321

$$\frac{27\, 3^{3/4}\sqrt{2-\sqrt{3}}a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{640b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27a^2x\sqrt[6]{a}}{640}$$

```
[Out] (-27*a^2*x*(a + b*x^2)^(1/6))/(640*b^2) + (3*a*x^3*(a + b*x^2)^(1/6))/(160*
b) + (3*x^5*(a + b*x^2)^(1/6))/16 + (27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a +
b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) +
(a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))], -7 + 4*Sqrt[3]])/(640*b^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(
a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.291074, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 241, 236, 219}

$$-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{27\, 3^{3/4}\sqrt{2-\sqrt{3}}a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{}\right)}{640b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/6),x]
```

```
[Out] (-27*a^2*x*(a + b*x^2)^(1/6))/(640*b^2) + (3*a*x^3*(a + b*x^2)^(1/6))/(160*
b) + (3*x^5*(a + b*x^2)^(1/6))/16 + (27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a +
b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) +
(a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))], -7 + 4*Sqrt[3]])/(640*b^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(
a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int x^4\sqrt[6]{a+bx^2}\,dx = \frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{1}{16}a\int\frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$= \frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}-\frac{\left(9a^2\right)\int\frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{160b}$$

$$= -\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{\left(27a^3\right)\int\frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{640b^2}$$

$$= -\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{\left(27a^3\right)\text{Subst}\left(\int\frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{640b^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}-\frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^3}}\,dx,\right.}{1280b^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\ldots}\right.}{640b^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.0460341, size = 93, normalized size = 0.29

$$\frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{\frac{bx^2}{a}+1}\left(-9a^2+abx^2+10b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{160b^2\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(-9*a^2 + a*b*x^2 + 10*b^2*x^
4) + 9*a^2*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)]))/(160*b^2*(1 +
(b*x^2)/a)^(1/6))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^{4}\sqrt[6]{b x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^4*(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{\frac{1}{6}} x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^{2}+a\right)^{\frac{1}{6}} x^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)*x^4, x)
```

Sympy [A] time = 1.23883, size = 29, normalized size = 0.09

$$\frac{\sqrt[6]{a} x^{5} {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x**5*hyper((-1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{6}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^4, x)
```

3.1012 $\int x^2 \sqrt[6]{a + bx^2}\, dx$

Optimal. Leaf size=297

$$-\frac{3\, 3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3}{10}x^3\sqrt[6]{a$$

```
[Out] (3*a*x*(a + b*x^2)^(1/6))/(40*b) + (3*x^3*(a + b*x^2)^(1/6))/10 - (3*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^2*x*(a/(a + b*
x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2
))^(1/3))^2)])
```

Rubi [A] time = 0.247237, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 241, 236, 219}

$$-\frac{3\, 3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/6))/(40*b) + (3*x^3*(a + b*x^2)^(1/6))/10 - (3*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^2*x*(a/(a + b*
x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2
))^(1/3))^2)])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sqrt[6]{a+bx^2}\,dx &= \frac{3}{10}x^3\sqrt[6]{a+bx^2}+\frac{1}{10}a\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx\\
&= \frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{\left(3a^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{40b}\\
&= \frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{40b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= \frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}+\frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{80b^2x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= \frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right.}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right.}}}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.0479826, size = 62, normalized size = 0.21

$$
\frac{3x\sqrt[6]{a+bx^2}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{\frac{bx^2}{a}+1}}+a+bx^2\right)}{10b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*(a + b*x^2 - (a*Hypergeometric2F1[-1/6, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/6)))/(10*b)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^2 \sqrt[6]{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^2*(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{6}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{6}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)*x^2, x)
```

Sympy [A] time = 0.960266, size = 29, normalized size = 0.1

$$\frac{\sqrt[6]{a} x^3 {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x**3*hyper((-1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{1}{6}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^2, x)
```

3.1013 $\int \sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=273

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3}{4}x\sqrt[6]{a+bx^2}$$

```
[Out] (3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^(1/6)*
(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2
))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.208586, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {195, 241, 236, 219}

$$\frac{3}{4}x\sqrt[6]{a+bx^2}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^(1/6)*
(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2
))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{1}{4}a\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$= \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= \frac{3}{4}x\sqrt[6]{a+bx^2} - \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{a}{a+b}}}{1-\sqrt{3}-\sqrt[3]{\frac{a}{a+b}}}\right.\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.0047909, size = 46, normalized size = 0.17

$$\frac{x\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6),x]
```

```
[Out] (x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/6)
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6),x)
```

```
[Out] int((b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6), x)
```

Sympy [A] time = 0.917706, size = 26, normalized size = 0.1

$$\sqrt[6]{a}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{6},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x*hyper((-1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6), x)
```

3.1014 $\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

```
[Out] -((a + b*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a +
b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/
(3^(1/4)*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.209954, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 241, 236, 219}

$$\frac{\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a +
b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/
(3^(1/4)*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{1}{3}b\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$= -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{x} - \frac{\left(\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}{1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}\right.\right.}{\sqrt[4]{3}x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.0089048, size = 49, normalized size = 0.18

$$-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/6)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[6]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^2,x)

[Out] int((b*x^2+a)^(1/6)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/6)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx^2+a)^{\frac{1}{6}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/6)/x^2, x)
```

Sympy [A] time = 0.95833, size = 29, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**2,x)

[Out] -a**(1/6)*hyper((-1/2, -1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(1/6)/x^2, x)
```

3.1015 $\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=297

$$-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\ldots$$

```
[Out] -(a + b*x^2)^(1/6)/(3*x^3) - (b*(a + b*x^2)^(1/6))/(9*a*x) - (2*Sqrt[2 - Sq
rt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*
x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(1/3
)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2)])
```

Rubi [A] time = 0.248183, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 241, 236, 219}

$$-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\ldots\right)}{9\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^4,x]
```



```
[Out] -(a + b*x^2)^(1/6)/(3*x^3) - (b*(a + b*x^2)^(1/6))/(9*a*x) - (2*Sqrt[2 - Sq
rt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*
x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(1/3
)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2)])
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[6]{a+bx^2}}{3x^3} + \frac{1}{9}b\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{\left(2b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{27a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{27a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} + \frac{\left(b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{9ax\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{2\sqrt{2-\sqrt{3}}\,b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{9\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.0087802, size = 51, normalized size = 0.17

$$-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^4,x]
```

```
[Out] -((a + b*x^2)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, -((b*x^2)/a)])/(3*x
^3*(1 + (b*x^2)/a)^(1/6))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt[6]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^4,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^4, x)
```

Sympy [A] time = 1.25758, size = 34, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2}, -\frac{1}{6} \\ -\frac{1}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**4,x)
```

```
[Out] -a**(1/6)*hyper((-3/2, -1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{6}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^4, x)
```

3.1016 $\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx$

Optimal. Leaf size=323

$$\frac{16\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{135\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}$$

```
[Out] -(a + b*x^2)^(1/6)/(5*x^5) - (b*(a + b*x^2)^(1/6))/(45*a*x^3) + (8*b^2*(a +
 b*x^2)^(1/6))/(135*a^2*x) + (16*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2)^(1/6)*(1
 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))
^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4
*Sqrt[3]])/(135*3^(1/4)*a^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.282843, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 241, 236, 219}

$$\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{16\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{b\sqrt[6]{}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^6,x]
```

```
[Out] -(a + b*x^2)^(1/6)/(5*x^5) - (b*(a + b*x^2)^(1/6))/(45*a*x^3) + (8*b^2*(a +
 b*x^2)^(1/6))/(135*a^2*x) + (16*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2)^(1/6)*(1
 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))
^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4
*Sqrt[3]])/(135*3^(1/4)*a^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{5x^5}+\frac{1}{15}b\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}-\frac{\left(8b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{135a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{\left(16b^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{405a^2}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{\left(16b^3\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{405a^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}-\frac{\left(8b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{135a^2x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{16\sqrt{2-\sqrt{3}}b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\cdots}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}{135\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.0089334, size = 51, normalized size = 0.16

$$-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^6,x]
```

```
[Out] -((a + b*x^2)^(1/6)*Hypergeometric2F1[-5/2, -1/6, -3/2, -((b*x^2)/a)])/(5*x
^5*(1 + (b*x^2)/a)^(1/6))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\sqrt[6]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^6,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^6, x)
```

Sympy [A] time = 1.54019, size = 34, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{6} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**6,x)
```

```
[Out] -a**(1/6)*hyper((-5/2, -1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^6, x)
```

3.1017 $\int \frac{\sqrt[6]{a+bx^2}}{x^8} dx$

Optimal. Leaf size=347

$$-\frac{32\sqrt{2-\sqrt{3}}b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{405\sqrt[4]{3}a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{16b^3\sqrt[6]{a+b}}{405a^3x}$$

```
[Out] -(a + b*x^2)^(1/6)/(7*x^7) - (b*(a + b*x^2)^(1/6))/(105*a*x^5) + (2*b^2*(a
+ b*x^2)^(1/6))/(135*a^2*x^3) - (16*b^3*(a + b*x^2)^(1/6))/(405*a^3*x) - (3
2*Sqrt[2 - Sqrt[3]]*b^3*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(405*3^(1/4)*a^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.317764, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 241, 236, 219}

$$-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{32\sqrt{2-\sqrt{3}}b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\right.}{405\sqrt[4]{3}a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^8,x]
```


```
[Out] -(a + b*x^2)^(1/6)/(7*x^7) - (b*(a + b*x^2)^(1/6))/(105*a*x^5) + (2*b^2*(a
+ b*x^2)^(1/6))/(135*a^2*x^3) - (16*b^3*(a + b*x^2)^(1/6))/(405*a^3*x) - (3
2*Sqrt[2 - Sqrt[3]]*b^3*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(405*3^(1/4)*a^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```


Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[6]{a+bx^2}}{7x^7} + \frac{1}{21}b\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} - \frac{\left(2b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx}{45a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} + \frac{\left(16b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{405a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{\left(32b^4\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{1215a^3}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{\left(32b^4\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{1215a^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} + \frac{\left(16b^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}} \cdots\right.}{405a^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\cdots}\right.}{405\sqrt[4]{3}a^3x\sqrt[3]{\cdots}}$$

Mathematica [C] time = 0.0091563, size = 51, normalized size = 0.15

$$-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{7}{2},-\frac{1}{6};-\frac{5}{2};-\frac{bx^2}{a}\right)}{7x^7\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^8,x]
```

```
[Out] -((a + b*x^2)^(1/6)*Hypergeometric2F1[-7/2, -1/6, -5/2, -((b*x^2)/a)])/(7*x
^7*(1 + (b*x^2)/a)^(1/6))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^8}\sqrt[6]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^8,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^8,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{6}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{1}{6}}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^8, x)
```

Sympy [A] time = 2.03402, size = 34, normalized size = 0.1

$$-\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix}-\frac{7}{2},-\frac{1}{6}\\-\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**8,x)
```

```
[Out] -a**(1/6)*hyper((-7/2, -1/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^8, x)
```

3.1018 $\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=659

$$\frac{81\ 3^{3/4}a^4\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{448\sqrt{2}b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3}-\frac{24}{896b^3}$$

```
[Out] (-243*a^3*x)/(896*b^3*(a + b*x^2)^(1/6)) + (81*a^2*x*(a + b*x^2)^(5/6))/(44
8*b^3) - (9*a*x^3*(a + b*x^2)^(5/6))/(56*b^2) + (3*x^5*(a + b*x^2)^(5/6))/(
20*b) - (243*a^4*x)/(896*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - S
qrt[3] - (a/(a + b*x^2))^(1/3))) - (243*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 -
(a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2
/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sq
rt[3]])/(1792*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (81*3^(3/4
)*a^4*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(448*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rubi [A] time = 0.69329, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {321, 238, 198, 235, 304, 219, 1879}

$$\frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3}-\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}}-\frac{243a^4x}{896b^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{81\ 3^{3/4}a^4\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)}{448\sqrt{2}b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(1/6),x]
```



```
[Out] (-243*a^3*x)/(896*b^3*(a + b*x^2)^(1/6)) + (81*a^2*x*(a + b*x^2)^(5/6))/(44
8*b^3) - (9*a*x^3*(a + b*x^2)^(5/6))/(56*b^2) + (3*x^5*(a + b*x^2)^(5/6))/(
20*b) - (243*a^4*x)/(896*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - S
qrt[3] - (a/(a + b*x^2))^(1/3))) - (243*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 -
(a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2
/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sq
rt[3]])/(1792*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (81*3^(3/4
)*a^4*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(448*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
```

```
^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} - \frac{(3a)\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx}{4b}$$

$$= -\frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(27a^2\right)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{56b^2}$$

$$= \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} - \frac{\left(81a^3\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{448b^3}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(81a^4\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}}{896b^3}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(81a^4\right)\text{Subst}\left(\int \ldots\right.}{896b^3\left(\frac{a}{a+bx^2}\right)\ldots}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} - \frac{\left(243a^4\sqrt{-\frac{bx^2}{a+bx^2}}\right)\ldots}{1792b^4x\ldots}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(243a^4\sqrt{-\frac{bx^2}{a+bx^2}}\right)\ldots}{1792b^4x\ldots}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{243a^4\sqrt{\ldots}}{896b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\ldots}$$

Mathematica [C] time = 0.0313801, size = 90, normalized size = 0.14

$$\frac{3\left(-135a^3x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+15a^2bx^3+135a^3x-8ab^2x^5+112b^3x^7\right)}{2240b^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*(135*a^3*x + 15*a^2*b*x^3 - 8*a*b^2*x^5 + 112*b^3*x^7 - 135*a^3*x*(1 + (
b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(2240*b^3*
(a + b*x^2)^(1/6))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x^6\frac{1}{\sqrt[6]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^6/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(1/6), x)
```

Sympy [A] time = 1.16688, size = 27, normalized size = 0.04

$$\frac{x^7 {}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/6),x)
```

```
[Out] x**7*hyper((1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/6), x)
```

3.1019 $\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=635

$$-\frac{27\,3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{112\sqrt{2}b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81a\cdots}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)\cdots}$$

```
[Out] (81*a^2*x)/(224*b^2*(a + b*x^2)^(1/6)) - (27*a*x*(a + b*x^2)^(5/6))/(112*b^
2) + (3*x^3*(a + b*x^2)^(5/6))/(14*b) + (81*a^3*x)/(224*b^2*(a/(a + b*x^2))
^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (81*3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^
2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]
*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(448*b^3*x*(a/(a + b*x^2))^(2/3)*(a +
 b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))^2)]) - (27*3^(3/4)*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a
/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^3*x*(a/(a +
b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.608733, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {321, 238, 198, 235, 304, 219, 1879}

$$\frac{81a^3x}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}}-\frac{27\,3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\cdots}{112\sqrt{2}b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/6),x]
```


```
[Out] (81*a^2*x)/(224*b^2*(a + b*x^2)^(1/6)) - (27*a*x*(a + b*x^2)^(5/6))/(112*b^
2) + (3*x^3*(a + b*x^2)^(5/6))/(14*b) + (81*a^3*x)/(224*b^2*(a/(a + b*x^2))
^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (81*3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^
2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]
*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(448*b^3*x*(a/(a + b*x^2))^(2/3)*(a +
 b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))^2)]) - (27*3^(3/4)*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a
/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^3*x*(a/(a +
b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{(9a)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{14b}$$

$$= -\frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} + \frac{\left(27a^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{112b^2}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{\left(27a^3\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{224b^2}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{\left(27a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} + \frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\cdots}\right.}{448b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\cdots}\right.}{448b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}\cdots}{224b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}$$

Mathematica [C] time = 0.0205943, size = 79, normalized size = 0.12

$$\frac{3\left(9a^2x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-abx^3+8b^2x^5\right)}{112b^2\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*(-9*a^2*x - a*b*x^3 + 8*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(112*b^2*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^4\frac{1}{\sqrt[6]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^4/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(1/6), x)
```

Sympy [A] time = 0.946965, size = 27, normalized size = 0.04

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/6),x)
```

```
[Out] x**5*hyper((1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/6), x)
```

3.1020 $\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=611

$$\frac{3\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{8\sqrt{2}b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{32b^2x\left(\right.}$$

```
[Out] (-9*a*x)/(16*b*(a + b*x^2)^(1/6)) + (3*x*(a + b*x^2)^(5/6))/(8*b) - (9*a^2*
x)/(16*b*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))
*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/
3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(32*b^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (3*3^(3/4)*a^2*(1 - (a/(a + b*x^2)
)^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqr
t[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.546757, antiderivative size = 611, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {321, 238, 198, 235, 304, 219, 1879}

$$\frac{3\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{8\sqrt{2}b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\cdots}}{32b^2x\left(\frac{a}{a+bx^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/6),x]
```


```
[Out] (-9*a*x)/(16*b*(a + b*x^2)^(1/6)) + (3*x*(a + b*x^2)^(5/6))/(8*b) - (9*a^2*
x)/(16*b*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))
*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/
3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(32*b^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (3*3^(3/4)*a^2*(1 - (a/(a + b*x^2)
)^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqr
t[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{(3a)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{8b}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{\left(3a^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{16b}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} - \frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{\right.}\right.}\right.}{}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{16b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}a}}{}$$

Mathematica [C] time = 0.0169111, size = 62, normalized size = 0.1

$$\frac{3x\left(-a\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{8b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2,
-((b*x^2)/a)]))/(8*b*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[6]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^2/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(1/6), x)
```

Sympy [A] time = 0.820236, size = 27, normalized size = 0.04

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/6),x)
```

```
[Out] x**3*hyper((1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/6), x)
```

3.1021 $\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=577

$$-\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3x}{2\sqrt[6]{a+bx^2}}+\frac{}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+}$$

```
[Out] (3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*x)/(2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^
(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]
*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (3^(3/4
)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b
*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1
+ Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))],
-7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[
-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.467236, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {238, 198, 235, 304, 219, 1879}

$$\frac{3x}{2\sqrt[6]{a+bx^2}}+\frac{3ax}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+}}}{-\sqrt[3]{\frac{a}{bx^2+}}}\right.\right.}{\sqrt{2}bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+b}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/6),x]
```

```
[Out] (3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*x)/(2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^
(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]
*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (3^(3/4
)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b
*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1
+ Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))],
-7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[
-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{1}{2}a\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} + \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int\right.}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+\right.}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+}}$$

Mathematica [C] time = 0.0058211, size = 46, normalized size = 0.08

$$\frac{x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/6),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/6)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-1/6), x)
```

Sympy [A] time = 0.805116, size = 24, normalized size = 0.04

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/6),x)
```

```
[Out] x*hyper((1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/6), x)
```

3.1022 $\int \frac{1}{x^2 \sqrt[6]{a+bx^2}} dx$

Optimal. Leaf size=586

$$-\frac{\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{bx}{a\sqrt[6]{a+bx^2}}+\frac{}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx\right.}$$

```
[Out] (b*x)/(a*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(a*x) + (b*x)/((a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^
(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1
/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)^2)]) - (Sqrt[2]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/
3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)
^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2)])
```

Rubi [A] time = 0.532992, antiderivative size = 586, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {325, 238, 198, 235, 304, 219, 1879}

$$\frac{bx}{a\sqrt[6]{a+bx^2}}+\frac{bx}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{\left(a+bx^2\right)^{5/6}}{ax}-\frac{\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\text{si}\right.}{\sqrt[4]{3}x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/6)),x]
```


```
[Out] (b*x)/(a*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(a*x) + (b*x)/((a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^
(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1
/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)^2)]) - (Sqrt[2]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/
3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)
^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2)])
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{5/6}}{ax}+\frac{(2b)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{3a} \\
&= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{ax}-\frac{1}{3}b\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx \\
&= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{ax}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}} \\
&= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{ax}+\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} \\
&= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{ax}-\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\,\mathrm{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{-\frac{bx^2}{a+bx^2}}\cdots\right.}{x\left(\frac{a}{a+b\cdots}\right.} \\
&= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{ax}-\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\left(1\cdots\right.}{2x\left(\cdots\right.}
\end{aligned}
$$

Mathematica [C] time = 0.0090326, size = 49, normalized size = 0.08

$$
-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[6]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/6)))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\frac{1}{\sqrt[6]{bx^2+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^4 + a*x^2), x)
```

Sympy [A] time = 0.992973, size = 27, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[6]{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/6),x)
```

```
[Out] -hyper((-1/2, 1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/6)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

3.1023 $\int \frac{1}{x^4 \sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=633

$$\frac{4\sqrt{2}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}$$

```
[Out] (-4*b^2*x)/(9*a^2*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(3*a*x^3) + (4*b*(
a + b*x^2)^(5/6))/(9*a^2*x) - (4*b^2*x)/(9*a*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (2*Sqrt[2 + Sqrt[3]]*b*(
1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2)
)^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqr
t[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
4*Sqrt[3]])/(3*3^(3/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((
1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (4*
Sqrt[2]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/
(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/
3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rubi [A] time = 0.59463, antiderivative size = 633, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {325, 238, 198, 235, 304, 219, 1879}

$$-\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}-\frac{4b^2x}{9a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4\sqrt{2}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{9\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/6)),x]
```

```
[Out] (-4*b^2*x)/(9*a^2*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(3*a*x^3) + (4*b*(
a + b*x^2)^(5/6))/(9*a^2*x) - (4*b^2*x)/(9*a*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (2*Sqrt[2 + Sqrt[3]]*b*(
1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2)
)^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqr
t[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
4*Sqrt[3]])/(3*3^(3/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((
1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (4*
Sqrt[2]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/
(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/
3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}-\frac{(4b)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{9a}$$

$$= -\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}-\frac{\left(8b^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{27a^2}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}+\frac{\left(4b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{27a}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}+\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{27a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}-\frac{\left(2b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}+\frac{\left(2b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}+\frac{4b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}-$$

Mathematica [C] time = 0.0087501, size = 51, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/6)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, -((b*x^2)/a)])/(
3*x^3*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt[6]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{bx^6+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^6 + a*x^4), x)
```

Sympy [A] time = 1.26845, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[6]{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/6),x)
```

```
[Out] -hyper((-3/2, 1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

3.1024 $\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=661

$$-\frac{8\sqrt{2}b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{27\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}}+\frac{\cdots}{27a^2\left(\frac{a}{a+bx^2}\cdots\right.}$$

```
[Out] (8*b^3*x)/(27*a^3*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(5*a*x^5) + (2*b*(
a + b*x^2)^(5/6))/(9*a^2*x^3) - (8*b^2*(a + b*x^2)^(5/6))/(27*a^3*x) + (8*b
^3*x)/(27*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a
+ b*x^2))^(1/3))) + (4*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sq
rt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^2*x*
(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (8*Sqrt[2]*b^2*(1 - (a/(a + b*
x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2
7*3^(1/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.67218, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {325, 238, 198, 235, 304, 219, 1879}

$$\frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}}+\frac{8b^3x}{27a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x}-\frac{8\sqrt{2}b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2}\cdots}{\left(-\sqrt[3]{\cdots}\right.}}}{27\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(1/6)),x]
```


```
[Out] (8*b^3*x)/(27*a^3*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(5*a*x^5) + (2*b*(
a + b*x^2)^(5/6))/(9*a^2*x^3) - (8*b^2*(a + b*x^2)^(5/6))/(27*a^3*x) + (8*b
^3*x)/(27*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a
+ b*x^2))^(1/3))) + (4*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sq
rt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^2*x*
(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (8*Sqrt[2]*b^2*(1 - (a/(a + b*
x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2
7*3^(1/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{5/6}}{5ax^5} - \frac{(2b)\int \frac{1}{x^4 \sqrt[6]{a+bx^2}}\,dx}{3a} \\
&= -\frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} + \frac{\left(8b^2\right)\int \frac{1}{x^2 \sqrt[6]{a+bx^2}}\,dx}{27a^2} \\
&= -\frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} + \frac{\left(16b^3\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{81a^3} \\
&= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{\left(8b^3\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{81a^2} \\
&= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{\left(8b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x \cdots\right.}{81a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx \cdots\right.} \\
&= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} + \frac{\left(4b^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{-\cdots}}\right.}{27a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\cdots} \\
&= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{\left(4b^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\cdots}{\sqrt{-\cdots}}\right.}{27a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\cdots} \\
&= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{8b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1\cdots}}{27a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(\cdots\right.}
\end{aligned}
$$

Mathematica [C] time = 0.0089599, size = 51, normalized size = 0.08

$$
-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(1/6)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 1/6, -3/2, -((b*x^2)/a)])/(
5*x^5*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$
\int \frac{1}{x^6}\frac{1}{\sqrt[6]{bx^2+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{bx^8+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^8 + a*x^6), x)
```

Sympy [A] time = 1.61469, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{6} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[6]{a}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(1/6),x)
```

```
[Out] -hyper((-5/2, 1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

3.1025 $\int \frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=324

$$-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{128b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81a^2x\sqrt[6]{\ldots}}{12\ldots}$$

```
[Out] (81*a^2*x*(a + b*x^2)^(1/6))/(128*b^3) - (9*a*x^3*(a + b*x^2)^(1/6))/(32*b^
2) + (3*x^5*(a + b*x^2)^(1/6))/(16*b) - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(
a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3
) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipti
cF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))], -7 + 4*Sqrt[3]])/(128*b^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.292085, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 241, 236, 219}

$$\frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\ldots\right)}{128b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/6),x]
```

```
[Out] (81*a^2*x*(a + b*x^2)^(1/6))/(128*b^3) - (9*a*x^3*(a + b*x^2)^(1/6))/(32*b^
2) + (3*x^5*(a + b*x^2)^(1/6))/(16*b) - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(
a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3
) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipti
cF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))], -7 + 4*Sqrt[3]])/(128*b^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
```

```
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx = \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{(15a)\int \frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx}{16b}$$

$$= -\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} + \frac{\left(27a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{32b^2}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{\left(81a^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{128b^3}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{\left(81a^3\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128b^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} + \frac{\left(243a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,\right.}{256b^4x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx}}\right.}{128b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{}}$$

Mathematica [C] time = 0.0313999, size = 89, normalized size = 0.27

$$\frac{3x\left(-27a^3\left(\frac{bx^2}{a}+1\right)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)+15a^2bx^2+27a^3-4ab^2x^4+8b^3x^6\right)}{128b^3\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(27*a^3 + 15*a^2*b*x^2 - 4*a*b^2*x^4 + 8*b^3*x^6 - 27*a^3*(1 + (b*x^2)
/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(128*b^3*(a + b*
```

```
x^2)^(5/6))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^{6} \left(b x^{2}+a \right)^{-\frac{5}{6}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/6),x)
```

```
[Out] int(x^6/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(bx^{2}+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{6}}{\left(bx^{2}+a\right)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(5/6), x)
```

Sympy [A] time = 0.976871, size = 27, normalized size = 0.08

$$\frac{x^{7}{}_{2}F_{1}\left(\begin{matrix} \frac{5}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{7a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/6),x)
```

```
[Out] x**7*hyper((5/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2 + a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/6), x)
```

3.1026 $\int \frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=300

$$\frac{27\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27ax\sqrt[6]{a}}{40b}$$

```
[Out] (-27*a*x*(a + b*x^2)^(1/6))/(40*b^2) + (3*x^3*(a + b*x^2)^(1/6))/(10*b) + (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/
3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
 (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.251414, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 241, 236, 219}

$$\frac{27\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/6),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(1/6))/(40*b^2) + (3*x^3*(a + b*x^2)^(1/6))/(10*b) + (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/
3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
 (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
```

```
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx &= \frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{(9a)\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{10b}\\
&= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b}+\frac{\left(27a^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{40b^2}\\
&= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b}+\frac{\left(27a^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{40b^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{80b^3x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b}+\frac{27\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+(\cdots}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.0215768, size = 79, normalized size = 0.26

$$
\frac{3\left(9a^2x\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-5abx^3+4b^2x^5\right)}{40b^2\left(a+bx^2\right)^{5/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*(-9*a^2*x - 5*a*b*x^3 + 4*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(5/6)*Hyperg
eometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(40*b^2*(a + b*x^2)^(5/6))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^{4} \left(b x^{2}+a \right)^{-\frac{5}{6}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/6),x)
```

```
[Out] int(x^4/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(bx^{2}+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{4}}{\left(bx^{2}+a\right)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(5/6), x)
```

Sympy [A] time = 0.925291, size = 27, normalized size = 0.09

$$\frac{x^{5}{}_{2}F_{1}\left(\begin{matrix} \frac{5}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{5a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/6),x)
```

```
[Out] x**5*hyper((5/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/6), x)
```

3.1027 $\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=276

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^
(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.212147, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 241, 236, 219}

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^
(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx &= \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{(3a)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{4b} \\
&= \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}} \\
&= \frac{3x\sqrt[6]{a+bx^2}}{4b} + \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}} \\
&= \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0177022, size = 62, normalized size = 0.22

$$
\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{4b\left(a+bx^2\right)^{5/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2,
-((b*x^2)/a)]))/(4*b*(a + b*x^2)^(5/6))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$
\int x^2\left(bx^2+a\right)^{-\frac{5}{6}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/6),x)
```

```
[Out] int(x^2/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(5/6), x)
```

Sympy [A] time = 0.899105, size = 27, normalized size = 0.1

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/6),x)
```

```
[Out] x**3*hyper((5/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/6), x)
```

3.1028 $\int \frac{1}{(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=252

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3^(3/4)*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sq
rt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a + b*x^
2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)])
```

Rubi [A] time = 0.183953, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {241, 236, 219}

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/6),x]
```

```
[Out] (3^(3/4)*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sq
rt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a + b*x^
2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)])
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}{1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}\right)|-7+4\sqrt{}\right.}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.0062051, size = 46, normalized size = 0.18

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{5/6} {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/6),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(5/6)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-5/6), x)
```

Sympy [A] time = 0.854266, size = 24, normalized size = 0.1

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/6),x)
```

```
[Out] x*hyper((1/2, 5/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-5/6), x)
```

3.1029 $\int \frac{1}{x^2(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=273

$$-\frac{2\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}$$

```
[Out] -((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a
/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3
))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt
[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.211623, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 241, 236, 219}

$$-\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{2\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/6)),x]
```


```
[Out] -((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a
/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3
))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt
[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{(2b)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{3a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{ax}+\frac{\left(\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{ax\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0090108, size = 49, normalized size = 0.18

$$
-\frac{\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{5/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(5/6)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\left(bx^2+a\right)^{-\frac{5}{6}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/(b*x^4 + a*x^2), x)
```

Sympy [A] time = 1.21817, size = 27, normalized size = 0.1

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-1/2, 5/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/6)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

3.1030 $\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=300

$$\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}-$$

```
[Out] -(a + b*x^2)^(1/6)/(3*a*x^3) + (8*b*(a + b*x^2)^(1/6))/(9*a^2*x) + (16*Sqrt
[2 - Sqrt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/
(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*(a/(a + b*x
^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))^2)])
```

Rubi [A] time = 0.242957, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 241, 236, 219}

$$\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/6)),x]
```

```
[Out] -(a + b*x^2)^(1/6)/(3*a*x^3) + (8*b*(a + b*x^2)^(1/6))/(9*a^2*x) + (16*Sqrt
[2 - Sqrt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/
(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*(a/(a + b*x
^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))^2)])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
```

```
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx = -\frac{\sqrt[6]{a+bx^2}}{3ax^3} - \frac{(8b)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{9a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{\left(16b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{27a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{\left(16b^2\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{27a^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} - \frac{\left(8b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{9a^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{16\sqrt{2-\sqrt{3}}\,b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F}{9\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1}}$$

Mathematica [C] time = 0.0089693, size = 51, normalized size = 0.17

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/6}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/6)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, -((b*x^2)/a)])/(
3*x^3*(a + b*x^2)^(5/6))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{-\frac{5}{6}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/(b*x^6 + a*x^4), x)
```

Sympy [A] time = 1.49648, size = 32, normalized size = 0.11

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{6}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-3/2, 5/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

3.1031 $\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=326

$$-\frac{224\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{135\sqrt[4]{3}a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{112b^2\sqrt[6]{a+}}{135a^3}$$

```
[Out] -(a + b*x^2)^(1/6)/(5*a*x^5) + (14*b*(a + b*x^2)^(1/6))/(45*a^2*x^3) - (112
*b^2*(a + b*x^2)^(1/6))/(135*a^3*x) - (224*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2
)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
 + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)], -7 + 4*Sqrt[3]])/(135*3^(1/4)*a^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (
a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.278822, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 241, 236, 219}

$$-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{224\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(5/6)),x]
```


```
[Out] -(a + b*x^2)^(1/6)/(5*a*x^5) + (14*b*(a + b*x^2)^(1/6))/(45*a^2*x^3) - (112
*b^2*(a + b*x^2)^(1/6))/(135*a^3*x) - (224*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2
)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
 + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)], -7 + 4*Sqrt[3]])/(135*3^(1/4)*a^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (
a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
```

```
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx = -\frac{\sqrt[6]{a+bx^2}}{5ax^5}-\frac{(14b)\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx}{15a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}+\frac{\left(112b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{135a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{\left(224b^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{405a^3}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{\left(224b^3\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{405a^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}+\frac{\left(112b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x \cdots\right.}{135a^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{224\sqrt{2-\sqrt{3}}b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\cdots}{135\sqrt[4]{3}a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\cdots}}$$

Mathematica [C] time = 0.0096064, size = 51, normalized size = 0.16

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/6}\,_2F_1\left(-\frac{5}{2},\frac{5}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(5/6)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-5/2, 5/6, -3/2, -((b*x^2)/a)])/(
5*x^5*(a + b*x^2)^(5/6))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(bx^2+a\right)^{-\frac{5}{6}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{bx^8+ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/(b*x^8 + a*x^6), x)
```

Sympy [A] time = 1.9733, size = 32, normalized size = 0.1

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2},\frac{5}{6} \\ -\frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-5/2, 5/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

3.1032 $\int \frac{x^6}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=654

$$-\frac{405\ 3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{112\sqrt{2}b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}+\frac{}{224b^3}$$

```
[Out] (1215*a^2*x)/(224*b^3*(a + b*x^2)^(1/6)) - (3*x^5)/(b*(a + b*x^2)^(1/6)) -
(405*a*x*(a + b*x^2)^(5/6))/(112*b^3) + (45*x^3*(a + b*x^2)^(5/6))/(14*b^2)
 + (1215*a^3*x)/(224*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))) + (1215*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/
(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[
3]])/(448*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (405*3^(3/4)*a
^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*
Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)
])
```

Rubi [A] time = 0.668826, antiderivative size = 654, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 321, 238, 198, 235, 304, 219, 1879}

$$\frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}+\frac{1215a^3x}{224b^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{405\ 3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F}{112\sqrt{2}b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(7/6),x]
```

```
[Out] (1215*a^2*x)/(224*b^3*(a + b*x^2)^(1/6)) - (3*x^5)/(b*(a + b*x^2)^(1/6)) -
(405*a*x*(a + b*x^2)^(5/6))/(112*b^3) + (45*x^3*(a + b*x^2)^(5/6))/(14*b^2)
 + (1215*a^3*x)/(224*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))) + (1215*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/
(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[
3]])/(448*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (405*3^(3/4)*a
^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*
```

```
Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)
])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
```

```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{7/6}}\,dx = -\frac{3x^5}{b\sqrt[6]{a+bx^2}}+\frac{15\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= -\frac{3x^5}{b\sqrt[6]{a+bx^2}}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}-\frac{(135a)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{14b^2}$$

$$= -\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}+\frac{\left(405a^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{112b^3}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}-\frac{\left(405a^3\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{224b^3}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}-\frac{\left(405a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-}\cdots}\right.}{224b^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(\cdots\right.}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}+\frac{\left(1215a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Sub}\cdots}{448b^4x\left(\frac{\cdots}{a+\cdots}\right.}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}-\frac{\left(1215a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Sub}\cdots}{448b^4x\left(\frac{\cdots}{a+\cdots}\right.}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3}+\frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2}-\frac{1215a^3\sqrt{-\frac{bx\cdots}{a+b\cdots}}}{224b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+\cdots}}$$

Mathematica [C] time = 0.0232628, size = 79, normalized size = 0.12

$$\frac{-405a^2x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)+405a^2x-90abx^3+48b^2x^5}{224b^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(7/6),x]
```

```
[Out] (405*a^2*x - 90*a*b*x^3 + 48*b^2*x^5 - 405*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hype
rgeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(224*b^3*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x^6\left(bx^2+a\right)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(7/6),x)
```

```
[Out] int(x^6/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}x^6}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 1.1415, size = 27, normalized size = 0.04

$$\frac{x^7{}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(7/6),x)
```

```
[Out] x**7*hyper((7/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="giac")

[Out] integrate(x^6/(b*x^2 + a)^(7/6), x)
```

3.1033 $\int \frac{x^4}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=630

$$\frac{27\ 3^{3/4} a^2 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{8\sqrt{2}b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{81a^2x}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\right.}$$

```
[Out] (-81*a*x)/(16*b^2*(a + b*x^2)^(1/6)) - (3*x^3)/(b*(a + b*x^2)^(1/6)) + (27*
x*(a + b*x^2)^(5/6))/(8*b^2) - (81*a^2*x)/(16*b^2*(a/(a + b*x^2))^(2/3)*(a
+ b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (81*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(32*b^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)]) + (27*3^(3/4)*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2
))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqrt[2]*b^3*x*(a/(a + b*x^2))^(2/3)
*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
+ b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.591613, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 321, 238, 198, 235, 304, 219, 1879}

$$-\frac{81a^2x}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{27\ 3^{3/4} a^2 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right.\right.}{8\sqrt{2}b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(7/6),x]
```

```
[Out] (-81*a*x)/(16*b^2*(a + b*x^2)^(1/6)) - (3*x^3)/(b*(a + b*x^2)^(1/6)) + (27*
x*(a + b*x^2)^(5/6))/(8*b^2) - (81*a^2*x)/(16*b^2*(a/(a + b*x^2))^(2/3)*(a
+ b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (81*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(32*b^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)]) + (27*3^(3/4)*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2
))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqrt[2]*b^3*x*(a/(a + b*x^2))^(2/3)
*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
+ b*x^2))^(1/3))^2)])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{7/6}}\,dx = -\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{9\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= -\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x\left(a+bx^2\right)^{5/6}}{8b^2}-\frac{(27a)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{8b^2}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x\left(a+bx^2\right)^{5/6}}{8b^2}+\frac{\left(27a^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{16b^2}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x\left(a+bx^2\right)^{5/6}}{8b^2}+\frac{\left(27a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x\left(a+bx^2\right)^{5/6}}{8b^2}-\frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x\left(a+bx^2\right)^{5/6}}{8b^2}+\frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x\left(a+bx^2\right)^{5/6}}{8b^2}+\frac{81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{16b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}-$$

Mathematica [C] time = 0.0201148, size = 65, normalized size = 0.1

$$\frac{3x\left(9a\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)-9a+2bx^2\right)}{16b^2\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x*(-9*a + 2*b*x^2 + 9*a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6
, 3/2, -((b*x^2)/a)]))/(16*b^2*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int x^4\left(bx^2+a\right)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(7/6),x)
```

```
[Out] int(x^4/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{5}{6}}x^4}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 1.11077, size = 27, normalized size = 0.04

$$\frac{x^5\,{}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(7/6),x)
```

```
[Out] x**5*hyper((7/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(7/6), x)
```

3.1034 $\int \frac{x^2}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=583

$$-\frac{3\,3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\cdots}{4b^2x\left(\frac{a}{a+b\cdots}\right.\cdots}$$

```
[Out] (3*x)/(2*b*(a + b*x^2)^(1/6)) + (9*a*x)/(2*b*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
+ b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[
(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-
((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (
3*3^(3/4)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (
a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(
1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3
))^2)])
```

Rubi [A] time = 0.522428, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {288, 238, 198, 235, 304, 219, 1879}

$$-\frac{3\,3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\cdots\right.}{\left(-\sqrt[3]{\cdots}\right.}}\cdots}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x)/(2*b*(a + b*x^2)^(1/6)) + (9*a*x)/(2*b*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
+ b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[
(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-
((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (
3*3^(3/4)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (
a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(
1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3
))^2)])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{7/6}}\,dx = -\frac{3x}{b\sqrt[6]{a+bx^2}} + \frac{3\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{(3a)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{2b}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} + \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\right.}{2b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{2b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a}}$$

Mathematica [C] time = 0.0137239, size = 58, normalized size = 0.1

$$\frac{3x - 3x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{2b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x - 3*x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)
/a)])/(2*b*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^2\left(bx^2+a\right)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(7/6),x)
```

```
[Out] int(x^2/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}x^2}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 1.09921, size = 27, normalized size = 0.05

$$\frac{x^3{}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(7/6),x)
```

```
[Out] x**3*hyper((7/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(7/6), x)
```

3.1035 $\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=555

$$\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}\right.}$$

```
[Out] (-3*x)/((a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt
[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
 b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*(a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2]) + (Sqrt[2]*3^(3/4)*(1 - (a/(a + b*x^2))^(1/3
))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
(a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(
1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a +
 b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2])
```

Rubi [A] time = 0.394179, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {198, 235, 304, 219, 1879}

$$-\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+\right.}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-7/6),x]
```


```
[Out] (-3*x)/((a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt
[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
 b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*(a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2]) + (Sqrt[2]*3^(3/4)*(1 - (a/(a + b*x^2))^(1/3
))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
(a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(
1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a +
 b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2])
```


Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= -\frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} - \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \cdots\right.}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\cdots\right.}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\cdots}{\left(1-\sqrt{\cdots}\right.}}}$$

Mathematica [C] time = 0.007328, size = 49, normalized size = 0.09

$$\frac{x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{a\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-7/6),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(a
*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 1.03937, size = 24, normalized size = 0.04

$$\frac{x_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/6),x)
```

```
[Out] x*hyper((1/2, 7/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-7/6), x)
```

3.1036 $\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=614

$$-\frac{4\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}+\frac{\ }{a}$$

```
[Out] 3/(a*x*(a + b*x^2)^(1/6)) + (4*b*x)/(a^2*(a + b*x^2)^(1/6)) - (4*(a + b*x^2
)^(5/6))/(a^2*x) + (4*b*x)/(a*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a
 + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]
])/(a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))
^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (4*Sqrt[2]*(1 - (a/(a
+ b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.573968, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 325, 238, 198, 235, 304, 219, 1879}

$$\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}+\frac{4bx}{a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{3}{ax\sqrt[6]{a+bx^2}}-\frac{4\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right.}{\left(-\sqrt[3]{\right.}}}}{\sqrt[4]{3}ax\left(\frac{a}{a+b\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x*(a + b*x^2)^(1/6)) + (4*b*x)/(a^2*(a + b*x^2)^(1/6)) - (4*(a + b*x^2
)^(5/6))/(a^2*x) + (4*b*x)/(a*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a
 + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]
])/(a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))
^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (4*Sqrt[2]*(1 - (a/(a
+ b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$=\frac{3}{ax\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}+\frac{(8b)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{3a^2}$$

$$=\frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}-\frac{(4b)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{3a}$$

$$=\frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}-\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{3a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$=\frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}+\frac{\left(2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$=\frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}-\frac{\left(2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{(2}{}$$

$$=\frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}-\frac{4\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+\frac{2\sqrt[4]{3}}{}$$

Mathematica [C] time = 0.0087606, size = 52, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{7}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(7/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/6)))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(bx^2+a\right)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [A] time = 1.55032, size = 27, normalized size = 0.04

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(7/6),x)
```

```
[Out] -hyper((-1/2, 7/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(7/6)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

3.1037 $\int \frac{1}{x^4\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=652

$$\frac{40\sqrt{2}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}}-\frac{}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a\right.}$$

```
[Out] 3/(a*x^3*(a + b*x^2)^(1/6)) - (40*b^2*x)/(9*a^3*(a + b*x^2)^(1/6)) - (10*(a
 + b*x^2)^(5/6))/(3*a^2*x^3) + (40*b*(a + b*x^2)^(5/6))/(9*a^3*x) - (40*b^2
*x)/(9*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))) - (20*Sqrt[2 + Sqrt[3]]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(
1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (40*Sqrt[2]*b*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1
/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.653809, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 325, 238, 198, 235, 304, 219, 1879}

$$-\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}}-\frac{40b^2x}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x}-\frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3}+\frac{40\sqrt{2}b\left(1-\sqrt[3]{\frac{a}{a+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(7/6)),x]
```


```
[Out] 3/(a*x^3*(a + b*x^2)^(1/6)) - (40*b^2*x)/(9*a^3*(a + b*x^2)^(1/6)) - (10*(a
 + b*x^2)^(5/6))/(3*a^2*x^3) + (40*b*(a + b*x^2)^(5/6))/(9*a^3*x) - (40*b^2
*x)/(9*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))) - (20*Sqrt[2 + Sqrt[3]]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(
1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (40*Sqrt[2]*b*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1
/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax^3\sqrt[6]{a+bx^2}} + \frac{10\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} - \frac{(40b)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{9a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} - \frac{\left(80b^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{27a^3}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{\left(40b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{27a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{\left(40b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, \right.}{27a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx\right.}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} - \frac{\left(20b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{-}}\right.}{9a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{}}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{\left(20b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+}{\sqrt{-}}\right.}{9a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{}}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{40b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1}}{9a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1\right.}$$

Mathematica [C] time = 0.0088248, size = 54, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{7}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(7/6)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, -((b*x^2)/a)])/(
3*a*x^3*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(bx^2+a\right)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [A] time = 2.00891, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{7}{6}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(7/6),x)
```

```
[Out] -hyper((-3/2, 7/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6)*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

3.1038 $\int \frac{1}{x^6\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=680

$$-\frac{128\sqrt{2}b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{27\sqrt[4]{3}a^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}+\frac{\cdots}{27a^3\left(\frac{a}{a+bx^2}\cdots\right.}$$

```
[Out] 3/(a*x^5*(a + b*x^2)^(1/6)) + (128*b^3*x)/(27*a^4*(a + b*x^2)^(1/6)) - (16*
(a + b*x^2)^(5/6))/(5*a^2*x^5) + (32*b*(a + b*x^2)^(5/6))/(9*a^3*x^3) - (12
8*b^2*(a + b*x^2)^(5/6))/(27*a^4*x) + (128*b^3*x)/(27*a^3*(a/(a + b*x^2))^(
2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (64*Sqrt[2
+ Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)]) - (128*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a
 + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^3*x*(a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2)])
```

Rubi [A] time = 0.741427, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 325, 238, 198, 235, 304, 219, 1879}

$$\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}-\frac{128\sqrt{2}b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\cdots\right.}{\left(-\sqrt[3]{\cdots}\right.}}}{27\sqrt[4]{3}a^3x\left(\frac{a}{a+bx^2}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x^5*(a + b*x^2)^(1/6)) + (128*b^3*x)/(27*a^4*(a + b*x^2)^(1/6)) - (16*
(a + b*x^2)^(5/6))/(5*a^2*x^5) + (32*b*(a + b*x^2)^(5/6))/(9*a^3*x^3) - (12
8*b^2*(a + b*x^2)^(5/6))/(27*a^4*x) + (128*b^3*x)/(27*a^3*(a/(a + b*x^2))^(
2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (64*Sqrt[2
+ Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)]) - (128*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a
 + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^3*x*(a/(a + b*x^
```

```
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
```

```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax^5\sqrt[6]{a+bx^2}}+\frac{16\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}-\frac{(32b)\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{3a^2}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}+\frac{\left(128b^2\right)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{27a^3}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}+\frac{\left(256b^3\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{81a^4}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}-\frac{(128b}{}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}-\frac{(128b}{}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}+\frac{(64b^2}{}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}-\frac{(64b^2}{}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5}+\frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}-\frac{}{27a^3x}$$

Mathematica [C] time = 0.0095485, size = 54, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5ax^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(7/6)),x]
```

```
[Out] -((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 7/6, -3/2, -((b*x^2)/a)])/(
5*a*x^5*(a + b*x^2)^(1/6))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^{6}} \left(b x^{2}+a\right)^{-\frac{7}{6}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(b x^{2} + a\right)}^{\frac{7}{6}} x^{6}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(b x^{2} + a\right)}^{\frac{5}{6}}}{b^{2} x^{10} + 2 \, a b x^{8} + a^{2} x^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [A] time = 2.77424, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{7}{6} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5 a^{\frac{7}{6}} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(7/6),x)
```

```
[Out] -hyper((-5/2, 7/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6)*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

3.1039 $\int x^7 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=100

$$-\frac{a^3\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}+\frac{3a^2\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}-\frac{3a\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}$$

```
[Out] -(a^3*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/(2
*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(4
+ p)/(2*b^4*(4 + p))
```

Rubi [A] time = 0.0635418, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^3\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}+\frac{3a^2\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}-\frac{3a\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^p,x]
```

```
[Out] -(a^3*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/(2
*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(4
+ p)/(2*b^4*(4 + p))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+bx^2\right)^p\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^p\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3a^2\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}-\frac{3a\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}
\end{aligned}$$

Mathematica [A] time = 0.0500538, size = 95, normalized size = 0.95

$$\frac{1}{2}\left(-\frac{a^3\left(a+bx^2\right)^{p+1}}{b^4(p+1)}+\frac{3a^2\left(a+bx^2\right)^{p+2}}{b^4(p+2)}-\frac{3a\left(a+bx^2\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{b^4(p+4)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^p,x]
```

```
[Out] (-((a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^2)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (a + b*x^2)^(4 + p
)/(b^4*(4 + p)))/2
```

Maple [A] time = 0.005, size = 132, normalized size = 1.3

$$-\frac{\left(bx^2+a\right)^{1+p}\left(-b^3p^3x^6-6\,b^3p^2x^6-11\,b^3px^6+3\,ab^2p^2x^4-6\,b^3x^6+9\,ab^2px^4+6\,ab^2x^4-6\,a^2bpx^2-6\,a^2bx^2+6\,a^3\right)}{2\,b^4\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(1+p)*(-b^3*p^3*x^6-6*b^3*p^2*x^6-11*b^3*p*x^6+3*a*b^2*p^2*x
^4-6*b^3*x^6+9*a*b^2*p*x^4+6*a*b^2*x^4-6*a^2*b*p*x^2-6*a^2*b*x^2+6*a^3)/b^4
/(p^4+10*p^3+35*p^2+50*p+24)
```

Maxima [A] time = 1.28818, size = 143, normalized size = 1.43

$$\frac{\left(\left(p^3+6\,p^2+11\,p+6\right)b^4x^8+\left(p^3+3\,p^2+2\,p\right)ab^3x^6-3\left(p^2+p\right)a^2b^2x^4+6\,a^3bpx^2-6\,a^4\right)\left(bx^2+a\right)^p}{2\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^8 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^6 - 3*(
p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 6*a^4)*(b*x^2 + a)^p/((p^4 + 10*p^3
+ 35*p^2 + 50*p + 24)*b^4)
```

Fricas [A] time = 1.61686, size = 302, normalized size = 3.02

$$\frac{\left(\left(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4\right)x^8+6\,a^3bpx^2+\left(ab^3p^3+3\,ab^3p^2+2\,ab^3p\right)x^6-3\left(a^2b^2p^2+a^2b^2p\right)x^4-6\,a^4\right)\left(bx^2+a\right)^p}{2\left(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^8 + 6*a^3*b*p*x^2 + (a*b^3*
p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^6 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^4 - 6*a^
4)*(b*x^2 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

Sympy [A] time = 11.4469, size = 2025, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(1
2*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*l
og(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x
**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6
*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a*
*3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x*
*2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b*
*6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2
+ 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 1
8*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**
2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt(a)*sq
rt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x
**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(-I*sqrt(a
)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(I
*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 3*a**
3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x
**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4
) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**
2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4
+ 8*a*b**5*x**2 + 4*b**6*x**4) + 6*a*b**2*x**4/(4*a**2*b**4 + 8*a*b**5*x**2
 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4),
Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2)
+ 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*
a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b*
*4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 +
4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*x**6/(4*a*b**4
 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**4)
- a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/
(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(2*b**4*p**4 +
20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*b*p*x**2*(a +
b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**
4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70
*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a + b*x**2)**p/(2*
b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + a*b**3*p*
*3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b*
*4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4
*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x**6*(a + b*x**2)
**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + b*
*4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 1
00*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b
**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x**8*(a + b*x**
2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) +
6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 10
0*b**4*p + 48*b**4), True))
```

Giac [B] time = 2.4378, size = 554, normalized size = 5.54

$$\frac{\left(bx^2+a\right)^4\left(bx^2+a\right)^p p^3 - 3\left(bx^2+a\right)^3\left(bx^2+a\right)^p ap^3 + 3\left(bx^2+a\right)^2\left(bx^2+a\right)^p a^2p^3 - \left(bx^2+a\right)\left(bx^2+a\right)^p a^3p^3 + 6\left(bx^2+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^4*(b*x^2 + a)^p*p^3 - 3*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^3
+ 3*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^3 - (b*x^2 + a)*(b*x^2 + a)^p*a^3*p^3
 + 6*(b*x^2 + a)^4*(b*x^2 + a)^p*p^2 - 21*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^2
 + 24*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^2 - 9*(b*x^2 + a)*(b*x^2 + a)^p*a^3
*p^2 + 11*(b*x^2 + a)^4*(b*x^2 + a)^p*p - 42*(b*x^2 + a)^3*(b*x^2 + a)^p*a*
p + 57*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p - 26*(b*x^2 + a)*(b*x^2 + a)^p*a^3
*p + 6*(b*x^2 + a)^4*(b*x^2 + a)^p - 24*(b*x^2 + a)^3*(b*x^2 + a)^p*a + 36*
(b*x^2 + a)^2*(b*x^2 + a)^p*a^2 - 24*(b*x^2 + a)*(b*x^2 + a)^p*a^3)/((b^3*p
^4 + 10*b^3*p^3 + 35*b^3*p^2 + 50*b^3*p + 24*b^3)*b)
```

3.1040 $\int x^5 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=72

$$\frac{a^2 \left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{a \left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}$$

```
[Out] (a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2
 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))
```

Rubi [A] time = 0.0431496, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{a \left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^p,x]
```


```
[Out] (a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2
 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^p \, dx &= \frac{1}{2} \text{Subst}\left(\int x^2(a+bx)^p \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2 \left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{a \left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}
\end{aligned}$$

Mathematica [A] time = 0.0272372, size = 64, normalized size = 0.89

$$\frac{\left(a+bx^2\right)^{p+1} \left(2a^2-2ab(p+1)x^2+b^2 \left(p^2+3p+2\right) x^4\right)}{2b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x^4))
/(2*b^3*(1 + p)*(2 + p)*(3 + p))
```

Maple [A] time = 0.004, size = 80, normalized size = 1.1

$$\frac{\left(bx^{2}+a\right)^{1+p}\left(b^{2}p^{2}x^{4}+3\,b^{2}px^{4}+2\,b^{2}x^{4}-2\,abpx^{2}-2\,abx^{2}+2\,a^{2}\right)}{2\,b^{3}\left(p^{3}+6\,p^{2}+11\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^p,x)
```

```
[Out] 1/2*(b*x^2+a)^(1+p)*(b^2*p^2*x^4+3*b^2*p*x^4+2*b^2*x^4-2*a*b*p*x^2-2*a*b*x^
2+2*a^2)/b^3/(p^3+6*p^2+11*p+6)
```

Maxima [A] time = 2.04432, size = 99, normalized size = 1.38

$$\frac{\left(\left(p^{2}+3\,p+2\right)b^{3}x^{6}+\left(p^{2}+p\right)ab^{2}x^{4}-2\,a^{2}bpx^{2}+2\,a^{3}\right)\left(bx^{2}+a\right)^{p}}{2\left(p^{3}+6\,p^{2}+11\,p+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Fricas [A] time = 1.59954, size = 197, normalized size = 2.74

$$\frac{\left(\left(b^{3}p^{2}+3\,b^{3}p+2\,b^{3}\right)x^{6}-2\,a^{2}bpx^{2}+\left(ab^{2}p^{2}+ab^{2}p\right)x^{4}+2\,a^{3}\right)\left(bx^{2}+a\right)^{p}}{2\left(b^{3}p^{3}+6\,b^{3}p^{2}+11\,b^{3}p+6\,b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^6 - 2*a^2*b*p*x^2 + (a*b^2*p^2 + a*b^2*p
)*x^4 + 2*a^3)*(b*x^2 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [A] time = 5.32673, size = 979, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8*a*
b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2
*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) +
 x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a
)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*
log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) -
2*b**2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a*
*2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sq
rt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x
**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) -
2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**
4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)
/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) +
x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12
*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**
3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x*
*6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b
**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
, True))
```

Giac [B] time = 1.47293, size = 312, normalized size = 4.33

$$\frac{\left(bx^2+a\right)^3\left(bx^2+a\right)^p p^2 - 2\left(bx^2+a\right)^2\left(bx^2+a\right)^p a p^2 + \left(bx^2+a\right)\left(bx^2+a\right)^p a^2 p^2 + 3\left(bx^2+a\right)^3\left(bx^2+a\right)^p p - 8\left(bx^2+a\right.}{2\left(b^2p^3+6\,b^2p\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^3*(b*x^2 + a)^p*p^2 - 2*(b*x^2 + a)^2*(b*x^2 + a)^p*a*p^2
+ (b*x^2 + a)*(b*x^2 + a)^p*a^2*p^2 + 3*(b*x^2 + a)^3*(b*x^2 + a)^p*p - 8*(
b*x^2 + a)^2*(b*x^2 + a)^p*a*p + 5*(b*x^2 + a)*(b*x^2 + a)^p*a^2*p + 2*(b*x
^2 + a)^3*(b*x^2 + a)^p - 6*(b*x^2 + a)^2*(b*x^2 + a)^p*a + 6*(b*x^2 + a)*(
b*x^2 + a)^p*a^2)/((b^2*p^3 + 6*b^2*p^2 + 11*b^2*p + 6*b^2)*b)
```

3.1041 $\int x^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}$$

```
[Out] -(a*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2 +
p))
```

Rubi [A] time = 0.0289054, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2 +
p))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^p \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.0183847, size = 40, normalized size = 0.83

$$\frac{\left(a+bx^2\right)^{p+1}\left(b(p+1)x^2-a\right)}{2b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^p,x]

[Out] ((a + b*x^2)^(1 + p)*(-a + b*(1 + p)*x^2))/(2*b^2*(1 + p)*(2 + p))
```

Maple [A] time = 0.003, size = 42, normalized size = 0.9

$$-\frac{\left(bx^{2}+a\right)^{1+p}\left(-x^{2}pb-bx^{2}+a\right)}{2\,b^{2}\left(p^{2}+3\,p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p,x)

[Out] -1/2*(b*x^2+a)^(1+p)*(-b*p*x^2-b*x^2+a)/b^2/(p^2+3*p+2)
```

Maxima [A] time = 2.28799, size = 63, normalized size = 1.31

$$\frac{\left(b^{2}\left(p+1\right)x^{4}+abpx^{2}-a^{2}\right)\left(bx^{2}+a\right)^{p}}{2\left(p^{2}+3\,p+2\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="maxima")

[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p/((p^2 + 3*p + 2)*b^2)
```

Fricas [A] time = 1.5574, size = 115, normalized size = 2.4

$$\frac{\left(abpx^{2}+\left(b^{2}p+b^{2}\right)x^{4}-a^{2}\right)\left(bx^{2}+a\right)^{p}}{2\left(b^{2}p^{2}+3\,b^{2}p+2\,b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="fricas")

[Out] 1/2*(a*b*p*x^2 + (b^2*p + b^2)*x^4 - a^2)*(b*x^2 + a)^p/(b^2*p^2 + 3*b^2*p
+ 2*b^2)
```

Sympy [A] time = 2.05159, size = 364, normalized size = 7.58

$$\begin{cases} \frac{a^{p}x^{4}}{4} & \text{for}\: b=0 \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}} & \text{for}\: p=-2 \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{2}}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{2}}+\frac{x^{2}}{2b} & \text{for}\: p=-1 \\ -\frac{a^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b*
*2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
 + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3
*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sq
rt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*
p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b*
*2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2
), True))
```

Giac [B] time = 1.54358, size = 127, normalized size = 2.65

$$\frac{\left(bx^2+a\right)^2\left(bx^2+a\right)^p p-\left(bx^2+a\right)\left(bx^2+a\right)^p ap+\left(bx^2+a\right)^2\left(bx^2+a\right)^p-2\left(bx^2+a\right)\left(bx^2+a\right)^p a}{2\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^2*(b*x^2 + a)^p*p - (b*x^2 + a)*(b*x^2 + a)^p*a*p + (b*x^2
 + a)^2*(b*x^2 + a)^p - 2*(b*x^2 + a)*(b*x^2 + a)^p*a)/((p^2 + 3*p + 2)*b^2
)
```

3.1042 $\int x\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] (a + b*x^2)^(1 + p)/(2*b*(1 + p))
```

Rubi [A] time = 0.0047815, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^p,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^p\,dx=\frac{\left(a+bx^2\right)^{1+p}}{2b(1+p)}$$

Mathematica [A] time = 0.0028317, size = 22, normalized size = 0.96

$$\frac{\left(a+bx^2\right)^{p+1}}{2bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^p,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b + 2*b*p)
```

Maple [A] time = 0.001, size = 22, normalized size = 1.

$$\frac{\left(bx^2+a\right)^{1+p}}{2\,b\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p,x)
```

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54206, size = 55, normalized size = 2.39

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^p}{2\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^p/(b*p + b)
```

Sympy [A] time = 0.709601, size = 97, normalized size = 4.22

$$\begin{cases} \frac{x^2}{2a} & \text{for}\ b=0 \wedge p=-1 \\ \frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b}+\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} & \text{for}\ p=-1 \\ \frac{a\left(a+bx^2\right)^p}{2bp+2b}+\frac{bx^2\left(a+bx^2\right)^p}{2bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/2, Eq(b, 0)), (log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b), Eq(p
, -1)), (a*(a + b*x**2)**p/(2*b*p + 2*b) + b*x**2*(a + b*x**2)**p/(2*b*p +
2*b), True))
```

Giac [A] time = 1.77907, size = 28, normalized size = 1.22

$$\frac{\left(bx^2+a\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

3.1043 $\int \frac{(a+bx^2)^p}{x} dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[Out] -((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2
*a*(1 + p))
```

Rubi [A] time = 0.0222451, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 65}

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x,x]
```

```
[Out] -((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2
*a*(1 + p))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p}{x} dx, x, x^2\right)\\ &= -\frac{(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0110894, size = 41, normalized size = 1.

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x,x]
```

```
[Out] -((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2
*a*(1 + p))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x,x)
```

```
[Out] int((b*x^2+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x, x)
```

Sympy [C] time = 3.20257, size = 39, normalized size = 0.95

$$-\frac{b^p x^{2p} \Gamma\left(-p\right) {}_{2}F_{1}\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x,x)
```

```
[Out] -b**p*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**
2))/(2*gamma(1 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x, x)
```

3.1044 $\int \frac{(a+bx^2)^p}{x^3} dx$

Optimal. Leaf size=42

$$\frac{b(a+bx^2)^{p+1} {}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

```
[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Rubi [A] time = 0.0225637, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 65}

$$\frac{b(a+bx^2)^{p+1} {}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p}{x^2} dx, x, x^2\right)\\ &= \frac{b(a+bx^2)^{1+p} {}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0071793, size = 42, normalized size = 1.

$$\frac{b(a+bx^2)^{p+1} {}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^3,x)
```

```
[Out] int((b*x^2+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^3, x)
```

Sympy [C] time = 7.0958, size = 42, normalized size = 1.

$$-\frac{b^p x^{2p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma(2-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**3,x)
```

```
[Out] -b**p*x**(2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/
(b*x**2))/(2*x**2*gamma(2 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^3, x)
```

3.1045 $\int x^6 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=40

$$\frac{x^7 \left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{9}{2};\frac{9}{2};-\frac{bx^2}{a}\right)}{7a}$$

```
[Out] (x^7*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 9/2 + p, 9/2, -((b*x^2)/a)])/
(7*a)
```

Rubi [A] time = 0.0129535, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$\frac{1}{7}x^7 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^p,x]
```

```
[Out] (x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (
b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^6 \left(1+\frac{bx^2}{a}\right)^p \, dx\\ &= \frac{1}{7}x^7 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0080813, size = 49, normalized size = 1.22

$$\frac{1}{7}x^7 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^p,x]
```

```
[Out] (x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (
b*x^2)/a)^p)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^{6} \left(b x^{2}+a \right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^p,x)
```

```
[Out] int(x^6*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{2} + a\right)^{p} x^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^{2} + a\right)^{p} x^{6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^6, x)
```

Sympy [C] time = 15.3736, size = 26, normalized size = 0.65

$$\frac{a^{p} x^{7} {{}_{2}F_{1}\left(\begin{matrix} \frac{7}{2}, -p \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^6, x)
```

3.1046 $\int x^4 (a+bx^2)^p \, dx$

Optimal. Leaf size=40

$$\frac{x^5 (a+bx^2)^{p+1} {}_2F_1\left(1,p+\frac{7}{2};\frac{7}{2};-\frac{bx^2}{a}\right)}{5a}$$

```
[Out] (x^5*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 7/2 + p, 7/2, -((b*x^2)/a)])/
(5*a)
```

Rubi [A] time = 0.0129529, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$\frac{1}{5} x^5 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^p,x]
```

```
[Out] (x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (
b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4 (a+bx^2)^p \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^4 \left(1+\frac{bx^2}{a}\right)^p \, dx \\
&= \frac{1}{5} x^5 (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.0055965, size = 49, normalized size = 1.22

$$\frac{1}{5} x^5 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^p,x]
```

```
[Out] (x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (
b*x^2)/a)^p)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^{4} \left(b x^{2}+a \right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^4 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx^2 + a)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^4, x)
```

Sympy [C] time = 8.94206, size = 26, normalized size = 0.65

$$\frac{a^{p} x^{5} {}_{2}F_{1}\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^4, x)
```

3.1047 $\int x^2 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=40

$$\frac{x^3 \left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{5}{2};\frac{5}{2};-\frac{bx^2}{a}\right)}{3a}$$

```
[Out] (x^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 5/2 + p, 5/2, -((b*x^2)/a)])/
(3*a)
```

Rubi [A] time = 0.0138384, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^p,x]
```

```
[Out] (x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (
b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^2 \left(1+\frac{bx^2}{a}\right)^p \, dx \\
&= \frac{1}{3}x^3 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.0056372, size = 49, normalized size = 1.22

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^p,x]
```

```
[Out] (x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (
b*x^2)/a)^p)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x^2 \left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2, x)
```

Sympy [C] time = 4.75786, size = 26, normalized size = 0.65

$$\frac{a^p x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2, x)
```

3.1048 $\int \left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=35

$$\frac{x\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{3}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{a}$$

```
[Out] (x*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 3/2 + p, 3/2, -((b*x^2)/a)])/a
```

Rubi [A] time = 0.0094728, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0044344, size = 44, normalized size = 1.26

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p,x)
```

```
[Out] int((b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p, x)
```

Sympy [C] time = 2.66019, size = 22, normalized size = 0.63

$$a^p x_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p,x)
```

```
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

3.1049 $\int \frac{(a+bx^2)^p}{x^2} \, dx$

Optimal. Leaf size=38

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1,p+\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax}$$

```
[Out] -(((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1/2 + p, 1/2, -((b*x^2)/a)])/(a
*x))
```

Rubi [A] time = 0.0133995, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b
*x^2)/a)^p))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^p}{x^2} \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2} \, dx \\
&= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0060888, size = 47, normalized size = 1.24

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b
*x^2)/a)^p))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2,x)
```

```
[Out] int((b*x^2+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^2, x)
```

Sympy [C] time = 4.60555, size = 26, normalized size = 0.68

$$-\frac{a^p {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^2, x)
```

3.1050 $\int x^{7/2}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{9/2}\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+\frac{13}{4};\frac{13}{4};-\frac{bx^2}{a}\right)}{9a}$$

```
[Out] (2*x^(9/2)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 13/4 + p, 13/4, -((b*x^
2)/a)])/(9*a)
```

Rubi [A] time = 0.0134661, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(9
*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^p\,dx&=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{7/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\&=\frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0069319, size = 51, normalized size = 1.21

$$\frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(9
*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x^{\frac{7}{2}} \left(b x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(7/2)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^p x^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b x^2 + a\right)^p x^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(7/2), x)
```

3.1051 $\int x^{5/2}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{7/2}\left(a+bx^2\right)^{p+1}\,_2F_1\left(1,p+\frac{11}{4};\frac{11}{4};-\frac{bx^2}{a}\right)}{7a}$$

```
[Out] (2*x^(7/2)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 11/4 + p, 11/4, -((b*x^
2)/a)])/(7*a)
```

Rubi [A] time = 0.0137521, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{7}x^{7/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{5/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{7}x^{7/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0064023, size = 51, normalized size = 1.21

$$\frac{2}{7}x^{7/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}} \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(5/2)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(5/2), x)
```

3.1052 $\int x^{3/2} \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=42

$$\frac{2x^{5/2}\left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{9}{4};\frac{9}{4};-\frac{bx^2}{a}\right)}{5a}$$

[Out] (2*x^(5/2)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 9/4 + p, 9/4, -((b*x^2)/a)])/(5*a)

Rubi [A] time = 0.0133974, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(a + b*x^2)^p,x]

[Out] (2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{3/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0070871, size = 51, normalized size = 1.21

$$\frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(3/2)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(3/2), x)
```

3.1053 $\int \sqrt{x}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{3/2}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{7}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a}$$

```
[Out] (2*x^(3/2)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 7/4 + p, 7/4, -((b*x^2)
/a)])/(3*a)
```

Rubi [A] time = 0.012401, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \sqrt{x}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0073355, size = 51, normalized size = 1.21

$$\frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \sqrt{x}\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(1/2)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^p\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*sqrt(x), x)
```

3.1054 $\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}\left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{5}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{a}$$

```
[Out] (2*Sqrt[x]*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 5/4 + p, 5/4, -((b*x^2)
/a)])/a
```

Rubi [A] time = 0.0125484, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$2\sqrt{x}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{\sqrt{x}}\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\sqrt{x}}\,dx\\ &= 2\sqrt{x}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0063282, size = 49, normalized size = 1.22

$$2\sqrt{x}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \frac{1}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(1/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/sqrt(x), x)
```

Sympy [C] time = 79.3878, size = 37, normalized size = 0.92

$$\frac{a^p\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(1/2),x)
```

```
[Out] a**p*sqrt(x)*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**2*exp_polar(I*pi)/a)/
(2*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/sqrt(x), x)
```

3.1055 $\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{3}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

```
[Out] (-2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 3/4 + p, 3/4, -((b*x^2)/a)])/(
a*Sqrt[x])
```

Rubi [A] time = 0.0131761, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]*
(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x^{3/2}}\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{3/2}}\,dx\\ &= -\frac{2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0071059, size = 49, normalized size = 1.22

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]*
(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{p} x^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(3/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{p}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^{2}+a\right)^{p}}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^p}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(3/2), x)
```

3.1056 $\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+\frac{1}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax^{3/2}}$$

```
[Out] (-2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1/4 + p, 1/4, -((b*x^2)/a)])/(
3*a*x^(3/2))
```

Rubi [A] time = 0.0135999, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2
)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{5/2}}\,dx\\ &= -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0069577, size = 51, normalized size = 1.21

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2
)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(b x^{2}+a \right)^{p} x^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(5/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(5/2),x)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^p}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2 + a\right)^p}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(5/2), x)
```

3.1057 $\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p-\frac{1}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

```
[Out] (-2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, -1/4 + p, -1/4, -((b*x^2)/a)])
/(5*a*x^(5/2))
```

Rubi [A] time = 0.0130226, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5/
2)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x^{7/2}}\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{7/2}}\,dx\\ &= -\frac{2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0070434, size = 51, normalized size = 1.21

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5/
2)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(7/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(7/2), x)
```

3.1058 $\int x^m \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=53

$$\frac{x^{m+1}\left(a+bx^2\right)^{p+1} {}_2F_1\left(1,\frac{1}{2}(m+2p+3);\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, (3 + m + 2*p)/2, (3 + m
)/2, -((b*x^2)/a)])/(a*(1 + m))
```

Rubi [A] time = 0.0167581, antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$\frac{x^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((b*x
^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.010189, size = 63, normalized size = 1.19

$$\frac{x^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^m \left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^p,x)
```

```
[Out] int(x^m*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^m, x)
```

Sympy [C] time = 33.1953, size = 51, normalized size = 0.96

$$\frac{a^p x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**p,x)
```

```
[Out] a**p*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^m, x)
```

3.1059 $\int (cx)^m \left(a+bx^2\right)^p dx$

Optimal. Leaf size=66

$$\frac{(cx)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{c(m+1)}$$

```
[Out] ((c*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(b*x^2)/a)])/(c*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0189753, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{(cx)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^2)^p,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(b*x^2)/a)])/(c*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (cx)^m\left(a+bx^2\right)^p dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int (cx)^m\left(1+\frac{bx^2}{a}\right)^p dx\\
&= \frac{(cx)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{c(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0094413, size = 64, normalized size = 0.97

$$\frac{x(cx)^m\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^2)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int (cx)^m \left(bx^2 + a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2+a)^p,x)
```

```
[Out] int((c*x)^m*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p (cx)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(c*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(c*x)^m, x)
```

Sympy [C] time = 33.2968, size = 54, normalized size = 0.82

$$\frac{a^p c^m x x^m \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2+a)**p,x)
```

```
[Out] a**p*c**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(c*x)^m, x)
```

3.1060 $\int x^{-8-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-7}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(-\frac{5}{2},1;\frac{1}{2}(-2p-5);-\frac{bx^2}{a}\right)}{a(2p+7)}$$

```
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[-5/2, 1, (-5 - 2*p)/2
, -((b*x^2)/a)])/(a*(7 + 2*p)))
```

Rubi [A] time = 0.0208351, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-7}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-7),-p;\frac{1}{2}(-2p-5);-\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-8 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-7 - 2*p)/2, -p, (-5 - 2*p
)/2, -((b*x^2)/a)])/((7 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-8-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-8-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-7-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-7-2p),-p;\frac{1}{2}(-5-2p);-\frac{bx^2}{a}\right)}{7+2p}\end{aligned}$$

Mathematica [A] time = 0.0153562, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-7}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-\frac{7}{2},-p;-p-\frac{5}{2};-\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-8 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-7/2 - p, -p, -5/2 - p, -((
b*x^2)/a)])/((7 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x^{-8-2p} \left(b x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-8-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-8-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^p x^{-2p-8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^2 + a\right)^p x^{-2p-8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-8-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p-8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 8), x)
```

3.1061 $\int x^{-7-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=105

$$-\frac{b^2x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

```
[Out] -((b^2*(a + b*x^2)^(1 + p))/(a^3*(1 + p)*(2 + p)*(3 + p)*x^(2*(1 + p)))) +
(b*(a + b*x^2)^(1 + p))/(a^2*(2 + p)*(3 + p)*x^(2*(2 + p))) - (a + b*x^2)^(
1 + p)/(2*a*(3 + p)*x^(2*(3 + p)))
```

Rubi [A] time = 0.0571453, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$-\frac{b^2x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((b^2*(a + b*x^2)^(1 + p))/(a^3*(1 + p)*(2 + p)*(3 + p)*x^(2*(1 + p)))) +
(b*(a + b*x^2)^(1 + p))/(a^2*(2 + p)*(3 + p)*x^(2*(2 + p))) - (a + b*x^2)^(
1 + p)/(2*a*(3 + p)*x^(2*(3 + p)))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{-7-2p}\left(a+bx^2\right)^p\,dx &= -\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}-\frac{(2b)\int x^{-5-2p}\left(a+bx^2\right)^p\,dx}{a(3+p)}\\ &= \frac{bx^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}+\frac{\left(2b^2\right)\int x^{-3-2p}\left(a+bx^2\right)^p\,dx}{a^2(2+p)(3+p)}\\ &= -\frac{b^2x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{a^3(1+p)(2+p)(3+p)}+\frac{bx^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}\end{aligned}$$

Mathematica [C] time = 0.0139091, size = 62, normalized size = 0.59

$$-\frac{x^{-2(p+3)}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-p-3,-p;-p-2;-\frac{bx^2}{a}\right)}{2(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((a + b*x^2)^p*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^2)/a)])/(2*(3
+ p)*x^(2*(3 + p))*(1 + (b*x^2)/a)^p)
```

Maple [A] time = 0.004, size = 81, normalized size = 0.8

$$-\frac{\left(bx^{2}+a\right)^{1+p}x^{-6-2p}\left(2b^{2}x^{4}-2abpx^{2}+a^{2}p^{2}-2abx^{2}+3a^{2}p+2a^{2}\right)}{\left(6+2p\right)\left(2+p\right)\left(1+p\right)a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-7-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(1+p)*x^(-6-2*p)*(2*b^2*x^4-2*a*b*p*x^2+a^2*p^2-2*a*b*x^2+3*
a^2*p+2*a^2)/(3+p)/(2+p)/(1+p)/a^3
```

Maxima [A] time = 1.65909, size = 113, normalized size = 1.08

$$-\frac{\left(2b^{3}x^{6}-2ab^{2}px^{4}+\left(p^{2}+p\right)a^{2}bx^{2}+\left(p^{2}+3p+2\right)a^{3}\right)e^{\left(p\log\left(bx^{2}+a\right)-2p\log\left(x\right)\right)}}{2\left(p^{3}+6p^{2}+11p+6\right)a^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*x^6 - 2*a*b^2*p*x^4 + (p^2 + p)*a^2*b*x^2 + (p^2 + 3*p + 2)*a^3
)*e^(p*log(b*x^2 + a) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*a^3*x^6)
```

Fricas [A] time = 1.61979, size = 219, normalized size = 2.09

$$-\frac{\left(2b^{3}x^{7}-2ab^{2}px^{5}+\left(a^{2}bp^{2}+a^{2}bp\right)x^{3}+\left(a^{3}p^{2}+3a^{3}p+2a^{3}\right)x\right)\left(bx^{2}+a\right)^{p}x^{-2p-7}}{2\left(a^{3}p^{3}+6a^{3}p^{2}+11a^{3}p+6a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*x^7 - 2*a*b^2*p*x^5 + (a^2*b*p^2 + a^2*b*p)*x^3 + (a^3*p^2 + 3*
a^3*p + 2*a^3)*x)*(b*x^2 + a)^p*x^(-2*p - 7)/(a^3*p^3 + 6*a^3*p^2 + 11*a^3*
p + 6*a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-7-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p-7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 7), x)
```

3.1062 $\int x^{-6-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(-\frac{3}{2},1;\frac{1}{2}(-2p-3);-\frac{bx^2}{a}\right)}{a(2p+5)}$$

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[-3/2, 1, (-3 - 2*p)/2
, -((b*x^2)/a)])/(a*(5 + 2*p)))
```

Rubi [A] time = 0.0203274, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-5),-p;\frac{1}{2}(-2p-3);-\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-5 - 2*p)/2, -p, (-3 - 2*p
)/2, -((b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-6-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-6-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-5-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-5-2p),-p;\frac{1}{2}(-3-2p);-\frac{bx^2}{a}\right)}{5+2p}\end{aligned}$$

Mathematica [A] time = 0.0153312, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-\frac{5}{2},-p;-p-\frac{3}{2};-\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-5/2 - p, -p, -3/2 - p, -((
b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{-6-2p} \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p-6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^{-2p-6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-6-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-2p-6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

3.1063 $\int x^{-5-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

```
[Out] (b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)
^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))
```

Rubi [A] time = 0.0202722, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)
^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{-5-2p}\left(a+bx^2\right)^p\,dx &= -\frac{x^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{2a(2+p)}-\frac{b\int x^{-3-2p}\left(a+bx^2\right)^p\,dx}{a(2+p)}\\ &= \frac{bx^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{2a(2+p)}\end{aligned}$$

Mathematica [C] time = 0.013967, size = 62, normalized size = 0.93

$$-\frac{x^{-2(p+2)}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-2,-p;-p-1;-\frac{bx^2}{a}\right)}{2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((a + b*x^2)^p*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^2)/a)])/(2*(2
+ p)*x^(2*(2 + p))*(1 + (b*x^2)/a)^p)
```

Maple [A] time = 0.003, size = 45, normalized size = 0.7

$$-\frac{\left(bx^{2}+a\right)^{1+p} x^{-4-2p}\left(-bx^{2}+ap+a\right)}{\left(4+2p\right)\left(1+p\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-5-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(1+p)*x^(-4-2*p)*(-b*x^2+a*p+a)/(2+p)/(1+p)/a^2
```

Maxima [A] time = 1.49775, size = 80, normalized size = 1.19

$$\frac{\left(b^{2}x^{4}-abpx^{2}-a^{2}\left(p+1\right)\right)e^{\left(p\log\left(bx^{2}+a\right)-2p\log(x)\right)}}{2\left(p^{2}+3p+2\right)a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^4 - a*b*p*x^2 - a^2*(p + 1))*e^(p*log(b*x^2 + a) - 2*p*log(x))/(
(p^2 + 3*p + 2)*a^2*x^4)
```

Fricas [A] time = 1.61193, size = 135, normalized size = 2.01

$$\frac{\left(b^{2}x^{5}-abpx^{3}-\left(a^{2}p+a^{2}\right)x\right)\left(bx^{2}+a\right)^{p} x^{-2p-5}}{2\left(a^{2}p^{2}+3a^{2}p+2a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^5 - a*b*p*x^3 - (a^2*p + a^2)*x)*(b*x^2 + a)^p*x^(-2*p - 5)/(a^2
*p^2 + 3*a^2*p + 2*a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-5-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-2p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 5), x)
```

3.1064 $\int x^{-4-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-3}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2}(-2p-1);-\frac{bx^2}{a}\right)}{a(2p+3)}$$

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[-1/2, 1, (-1 - 2*p)/2
, -((b*x^2)/a)])/(a*(3 + 2*p)))
```

Rubi [A] time = 0.0210447, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-3}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-3),-p;\frac{1}{2}(-2p-1);-\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -p, (-1 - 2*p
)/2, -((b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-4-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-4-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-3-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-3-2p),-p;\frac{1}{2}(-1-2p);-\frac{bx^2}{a}\right)}{3+2p}\end{aligned}$$

Mathematica [A] time = 0.0143668, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-3}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-\frac{3}{2},-p;-p-\frac{1}{2};-\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-3/2 - p, -p, -1/2 - p, -((
b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int x^{-4-2p} \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p-4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^{-2p-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2\,p-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

3.1065 $\int x^{-3-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=30

$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*(1 + p)*x^(2*(1 + p)))
```

Rubi [A] time = 0.0060034, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {264}

$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*(1 + p)*x^(2*(1 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a(1+p)}$$

Mathematica [A] time = 0.0116172, size = 29, normalized size = 0.97

$$\frac{x^{-2p-2}\left(a+bx^2\right)^{p+1}}{a(-2p-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(-2 - 2*p)*(a + b*x^2)^(1 + p))/(a*(-2 - 2*p))
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$-\frac{x^{-2-2p}\left(bx^2+a\right)^{1+p}}{2a\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*x^(-2-2*p)*(b*x^2+a)^(1+p)/a/(1+p)
```

Maxima [A] time = 1.83406, size = 50, normalized size = 1.67

$$-\frac{\left(bx^2+a\right)e^{\left(p\log\left(bx^2+a\right)-2p\log\left(x\right)\right)}}{2a\left(p+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)*e^(p*log(b*x^2 + a) - 2*p*log(x))/(a*(p + 1)*x^2)
```

Fricas [A] time = 1.61178, size = 77, normalized size = 2.57

$$-\frac{\left(bx^3+ax\right)\left(bx^2+a\right)^p x^{-2p-3}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3 + a*x)*(b*x^2 + a)^p*x^(-2*p - 3)/(a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 3), x)
```

3.1066 $\int x^{-2-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(\frac{1}{2},1;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{a(2p+1)}$$

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1/2, 1, (1 - 2*p)/2,
-((b*x^2)/a)])/(a*(1 + 2*p)))
```

Rubi [A] time = 0.0211372, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-1),-p;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-1 - 2*p)/2, -p, (1 - 2*p)
/2, -((b*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-2-2p}\left(a+bx^2\right)^p\,dx&=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-2-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\&=-\frac{x^{-1-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-1-2p),-p;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.0159522, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-\frac{1}{2},-p;\frac{1}{2}-p;-\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((b
*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{-2-2p} \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-2-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2 + a\right)^p x^{-2p-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

3.1067 $\int x^{-1-2p} \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=43

$$-\frac{x^{-2p} \left(a+bx^2\right)^{p+1} {}_2F_1\left(1,1;1-p;-\frac{bx^2}{a}\right)}{2ap}$$

```
[Out] -((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1, 1 - p, -((b*x^2)/a)])/(2*a*p*
x^(2*p))
```

Rubi [A] time = 0.0157644, antiderivative size = 56, normalized size of antiderivative = 1.3, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(2*p*x^(2*p
)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-2p} \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^{-1-2p} \left(1+\frac{bx^2}{a}\right)^p \, dx \\ &= -\frac{x^{-2p} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}\end{aligned}$$

Mathematica [A] time = 0.0071147, size = 56, normalized size = 1.3

$$-\frac{x^{-2p} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(2*p*x^(2*p
)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{-1-2p} \left(b x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^p x^{-2p-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b x^2 + a\right)^p x^{-2p-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

3.1068 $\int x^{-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x^{1-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,\frac{3}{2};\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{a(1-2p)}$$

```
[Out] (x^(1 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 3/2, (3 - 2*p)/2, -((
b*x^2)/a)])/(a*(1 - 2*p))
```

Rubi [A] time = 0.0191409, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{1-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),-p;\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(2*p),x]
```

```
[Out] (x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(1 - 2*p)/2, -p, (3 - 2*p)/2,
-((b*x^2)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-2p}\left(a+bx^2\right)^p\,dx&=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\&=\frac{x^{1-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),-p;\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{1-2p}\end{aligned}$$

Mathematica [A] time = 0.0161678, size = 65, normalized size = 1.25

$$\frac{x^{1-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}-p,-p;\frac{3}{2}-p;-\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(2*p),x]
```

```
[Out] (x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1/2 - p, -p, 3/2 - p, -((b*x^2
)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{2p}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(x^(2*p)),x)
```

```
[Out] int((b*x^2+a)^p/(x^(2*p)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{2p}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(2*p), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^{2p}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(2*p), x)
```

Sympy [C] time = 24.1166, size = 24, normalized size = 0.46

$$b^p x \, {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\right| \frac{ae^{i\pi}}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(x**(2*p)),x)
```

```
[Out] b**p*x*hyper((-1/2, -p), (1/2,), a*exp_polar(I*pi)/(b*x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(2*p), x)
```

3.1069 $\int x^{1-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{x^{2-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,2;2-p;-\frac{bx^2}{a}\right)}{2a(1-p)}$$

```
[Out] (x^(2 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 2, 2 - p, -((b*x^2)/a
)])/(2*a*(1 - p))
```

Rubi [A] time = 0.0208196, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{2-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a)
])/(2*(1 - p)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{1-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{1-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{2-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2(1-p)}\end{aligned}$$

Mathematica [A] time = 0.01706, size = 61, normalized size = 1.24

$$\frac{x^{2-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a)
])/((2 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{1-2p} \left(b x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(1-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^p x^{-2p+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b x^2 + a\right)^p x^{-2p+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p+1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

3.1070 $\int x^{2-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x^{3-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,\frac{5}{2};\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{a(3-2p)}$$

```
[Out] (x^(3 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 5/2, (5 - 2*p)/2, -((
b*x^2)/a)])/(a*(3 - 2*p))
```

Rubi [A] time = 0.0196595, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{3-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(3-2p),-p;\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(3 - 2*p)/2, -p, (5 - 2*p)/2,
-((b*x^2)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{2-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{3-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(3-2p),-p;\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{3-2p}\end{aligned}$$

Mathematica [A] time = 0.0169598, size = 65, normalized size = 1.25

$$\frac{x^{3-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{2}-p,-p;\frac{5}{2}-p;-\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[3/2 - p, -p, 5/2 - p, -((b*x^2
)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x^{2-2p} \left(b x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(2-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2 + a\right)^p x^{-2p+2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^2 + a\right)^p x^{-2p+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2p+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

3.1071 $\int x^{3-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{x^{4-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,3;3-p;-\frac{bx^2}{a}\right)}{2a(2-p)}$$

```
[Out] (x^(4 - 2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 3, 3 - p, -((b*x^2)/a
)])/(2*a*(2 - p))
```

Rubi [A] time = 0.0206146, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{4-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{2(2-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a)
])/(2*(2 - p)*(1 + (b*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{3-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{4-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{2(2-p)}\end{aligned}$$

Mathematica [A] time = 0.0152446, size = 61, normalized size = 1.24

$$\frac{x^{4-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{4-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a)
])/((4 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{3-2p}\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(3-2*p)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p x^{-2p+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^p x^{-2p+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p x^{-2\,p+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```